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Erratum to

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p.9. Rewrite the second part of the **Corollary** as follows :

... that x_M is nilpotent for every $x \in \mathfrak{p}$ and is injective for every $x \notin \mathfrak{p}$.

p.92, line 3. Replace M if by M is.