

# Subject Index

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# Glossary of Formulas

## Law of Sines:

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

where  $a$ ,  $b$  and  $c$  are the sides of a triangle and  $\alpha$ ,  $\beta$  and  $\gamma$  are the angles opposite to these sides.

## Law of Cosines:

$$a^2 + b^2 - 2ab \cos \gamma = c^2$$

with Pythagoras's Theorem  $a^2 + b^2 = c^2$  as special case when  $\gamma = \pi / 2$

## Cosine and Sine for Sum and Difference:

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

## Special cases:

$$\cos(2\alpha) = \cos^2 \alpha - \sin^2 \alpha$$

$$\sin(2\alpha) = 2 \sin \alpha \cos \alpha$$

## Trigonometric Identity:

$$\cos^2 \alpha + \sin^2 \alpha = 1$$

**Gradient of Scalar Field  $\lambda(x, y)$ :**

$$\nabla \lambda = \left( \frac{\partial \lambda}{\partial x}, \frac{\partial \lambda}{\partial y} \right)$$

**Divergence of Vector Field**

$\phi = (\phi_1(x, y), \phi_2(x, y))$ :

$$\nabla \cdot \phi = \frac{\partial \phi_1}{\partial x} + \frac{\partial \phi_2}{\partial y}$$

**Dot product of vectors**

$\phi = (\phi_1(x, y), \phi_2(x, y))$  and  $\psi = (\psi_1(x, y), \psi_2(x, y))$ :

$$\phi \cdot \psi = \phi_1 \psi_1 + \phi_2 \psi_2$$

**Tangent and Normal**

to parameterized curve  $(x(t), y(t))$ :

$$\mathbf{t} = (x'(t), y'(t))$$

$$\mathbf{n} = (y'(t), -x'(t))$$

The dot product of normal and tangent is zero:

$$\mathbf{t} \cdot \mathbf{n} = 0$$

**Laplacian:**

$$\nabla^2 \lambda = \nabla \cdot \nabla \lambda = \frac{\partial^2 \lambda}{\partial x^2} + \frac{\partial^2 \lambda}{\partial y^2}$$

**Gauss's Integral Theorem:**

$$\iint_R \nabla \cdot \phi \, dx \, dy = \oint_C \phi \cdot \mathbf{n} \, ds$$

**Euler's Equation:**

$$F_y - \frac{d}{dx} F_{y'} = 0$$

minimizes the integral

$$\int_a^b F(x, y, y') \, dx$$

with respect to the function  $y(x)$ ,  $a < x < b$

**Special Case**

when  $x$  is not included:

$$F - y' F_{y'} = c$$

**Euler's Equation in 2D**

$$F_u - \frac{\partial}{\partial x} F_p - \frac{\partial}{\partial y} F_q = 0$$

minimizes the integral

$$\iint_R F(x, y, u, p, q) \, dx \, dy$$

with respect to  $u(x, y)$  on  $R$  where

$$p = \frac{\partial u}{\partial x} \quad \text{and} \quad q = \frac{\partial u}{\partial y}$$



**Jacobian Determinant**for change of coordinates  $x = f(u, v), y = g(u, v)$ :

$$dxdy = |J|dudv$$

with

$$J = \begin{vmatrix} f_u & f_v \\ g_u & g_v \end{vmatrix}$$

**Special case for polar coordinates** $x = r \cos \theta, y = r \sin \theta$ 

$$dxdy = r drd\theta$$

**Beckmann's Equations:**

$$k \frac{\phi}{|\phi|} = \nabla \lambda$$

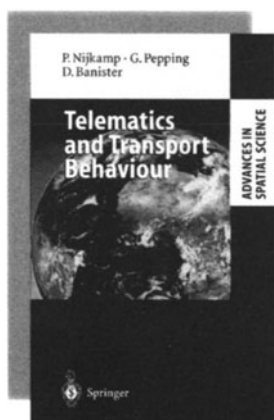
minimizes

$$\iint_R k |\phi| dx dy$$

subject to the constraint

$$\nabla \cdot \phi + z = 0$$

# Spatial Science



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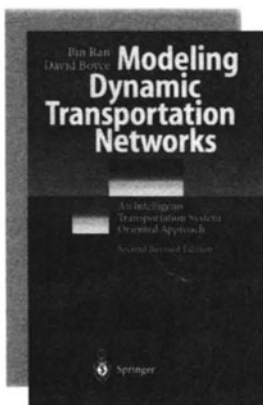
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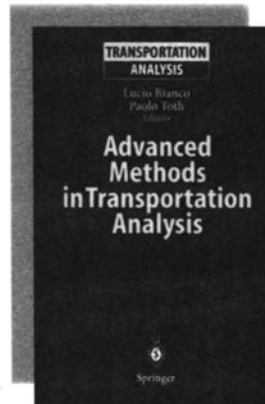
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