

Bibliography

Our original intention was to provide a complete bibliography with full documentation, but we were soon discouraged by the complexity of such a task. Instead we have decided to give the following short list of books and articles¹ in order to enable the reader to trace individual results to their source and to lead him to other fields of interest in number theory and the theory of quadratic forms.

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¹ Additional references to specific points of interest are given in footnotes in the text.

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