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## List of Symbols

$M_*$ , 1  
 $(M_*)^*$ , 1  
 $\sigma(M, M_*)$ , 2  
 $\mathcal{N}^0$ , 2  
 $\sum_{\alpha \in \mathbb{I}} \oplus \mathcal{A}_\alpha$ , 2  
 $\text{Sp}(a)$ , 3  
 $C(K)$ , 4  
 $C_0(\Omega)$ , 4  
 $\bar{K}_1$ , 5  
 $\Lambda$ , 6  
 $f|_{\mathcal{A}}$ , 7  
 $\mathcal{A}^s$ , 8  
 $a^+$ , 8  
 $a^-$ , 8  
 $|a|$ , 8  
 $f^*$ , 9  
 $T$ , 15  
 $\bar{\mathcal{A}}$ , 18  
 $\langle x, f \rangle$ , 19  
 $L_a f$ , 19  
 $R_a f$ , 19  
 $\tau(M, M_*)$ , 19  
 $\alpha_\varphi$ , 20  
 $s(M, M_*)$ , 20  
 $s$ , 20  
 $\alpha_\varphi^*$ , 20  
 $s^*(M, M_*)$ , 20  
 $s^*$ , 20  
 $\mathbb{C}$ , 23  
 $M^p$ , 24  
 $l(a)$ , 25  
 $r(a)$ , 25  
 $s(h)$ , 25  
 $c(p)$ , 25  
 $\sum_{\alpha \in \mathbb{I}} e_\alpha$ , 30  
 $s(\varphi)$ , 31  
 $f^+$ , 31

$f^-$ , 31  
 $|g|$ , 32  
 $B(\mathcal{H})$ , 33  
 $H$ , 35  
 $T(\mathcal{H})$ , 36  
 $\varphi_\alpha$ , 37  
 $\text{Tr}(h)$ , 36  
 $\dim(\mathcal{H})$ , 38  
 $\{\pi, \mathcal{H}\}$ , 40  
 $\{\pi_\varphi, \mathcal{H}_\varphi\}$ , 40  
 $\{\pi^\nu, \mathcal{H}\}$ , 40  
 $x_\varphi$ , 40  
 $\mathcal{L}$ , 41  
 $\{U, K\}$ , 41  
 $\mathcal{A}^{**}$ , 42  
 $L^1(\Omega, \mu)$ , 45  
 $L^\infty(\Omega, \mu)$ , 45  
 $C(\mathcal{H})$ , 46  
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 $\mathcal{L}'$ , 48  
 $\mathcal{L}''$ , 48  
 $1_{\mathcal{X}_1}$ , 48  
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 $\mathcal{A} \otimes \mathcal{B}$ , 66  
 $\mathcal{L}_{\mathcal{A}}$ , 66  
 $\alpha_0^*$ , 66  
 $\mathcal{M} \bar{\otimes} \mathcal{N}$ , 67  
 $\text{Card}(\mathbb{I})$ , 69  
 $\odot \mathcal{A}_\alpha$ , 75  
 $\otimes_{\alpha \in \mathbb{I}} A_\alpha$ , 75  
 $\otimes_{\alpha \in \mathbb{I}} \varphi_\alpha$ , 75  
 $p \sim q$ , 79  
 $p \preceq q$ , 79  
 $p < q$ , 79  
 $M^u$ , 82  
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 $\varphi_\varphi$ , 103  
 $P_{\{\mathcal{N}'\}^c}$ , 104  
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 $J$ , 111  
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$\int a(t) d\mu(t)$ , 134	$(\mathcal{N})_1$ , 183	$RQ_i^2[(h,k)(r,s)]$ , 189
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$\lim_{\alpha} \{\mathcal{A}_{\alpha}; \Phi_{\beta,\alpha} | (\beta,\alpha) \in \mathbb{I} \times \mathbb{I} \text{ and } \beta \geq \alpha\}$ , 71

$\lim_{\alpha} \{\varphi_{\alpha}; \Phi_{\beta,\alpha} | (\beta,\alpha) \in \mathbb{I} \times \mathbb{I}, \beta \geq \alpha\}$ , 74

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