

# References

1. Adams, J.F.: On the cobar construction. Colloque de Topologie Algebrique. Louvain (1958).
2. Adams, J.F.: On the non-existence of elements of Hopf invariant one. *Ann. of Math.* **72** (1960) 20-104.
3. Adams, J.F.: Vector fields on spheres. *Ann. of Math.* **75** (1962) 603-32.
4. Adams, J.F.: Vector fields on spheres. *Topology* **1**(1962) 63-65.
5. Adams, J.F., and Hilton, P.J.: On the chain algebra of a loop space. *Comm. Math. Helv.* **30** (1956), 305-30.
6. Alexander, J.W.: Deformations of an  $n$ -cell. *Proc. Nat. Acad. Sci. U.S.A.* **9** (1923) 406-407.
7. Artin, E.: Theory of Braids. *Ann. of Math.* **48** (1947) 101-26.
8. Bahri, A., and Rabinowitz, P.H.: Periodic solutions of Hamiltonian systems of 3-body type. *Ann. Inst. H. Poincaré, Anal. Nonlin.* **8** (1991) 561-49.
9. Birman, J.S.: Braids, links, and mapping class groups. *Ann. of Math. Studies* **82**. Princeton University Press (1984).
10. Bödigheimer, F.-C., Cohen, F.R., and Milgram, R.J.: Truncated symmetric products and configuration spaces. *Math. Z.* **214** (1994) 179-216.
11. Bott, R.: Configuration spaces and imbedding invariants. *Turk. J. Math.* **20** (1996) 1-17.
12. Bott, R., and Samelson, H.: On the Pontryagin product in spaces of paths. *Comment. Math. Helv.* **27** (1953) 320-37.
13. Bott, R., and Taubes, C.: On the self-linking of knots. *J. Math. Phys.* **35** (1994) 5247-87.
14. Bourbaki, N.: Groupes et algèbres de Lie, chap. 2-3. Hermann, Paris (1972).
15. Brown, R.F., and White, J.H.: Homology and Morse theory of third configuration spaces. *Indiana U. Math. J.* **30** (1981) 501-12.
16. Chow, W-L.: On the algebraic braid group. *Ann. of Math.* **49** (1948), 654-58.
17. Cohen, F.R.: The homology of  $C_{n+1}$ -spaces,  $n \geq 0$ . The homology of iterated loop spaces, *LNM* **533** 207-351. Springer-Verlag, New York (1976).
18. Cohen, F.R., and Gitler, S.: On loop spaces of configuration spaces. Preliminary report.
19. Cohen, F.R., and Taylor, L.R.: Configuration spaces: applications to Gelfand-Fuks cohomology. *Bull. Amer. Math. Soc.* **84** (1978) 134-36.
20. Cohen, F.R., and Taylor, L.R.: Computations of Gelfand-Fuks cohomology, the cohomology of function spaces, and the cohomology of configuration spaces. Geometric applications of homotopy theory I, *LNM* **657** 106-43. Springer-Verlag, New York (1978).
21. Cohen, R.L.: A model for the free loop space of a suspension. *LNM* **1286** 193-207. Springer-Verlag, New York (1987).
22. Crabb, M.C., and James, I.M.: Fiberwise configuration spaces. *Bol. de la Soc. Mat. Mex.* **37** (1992) 83-97.

23. Crowell, R.H., and Fox, R.H.: Introduction to knot theory. GTM 57. Springer-Verlag, New York (1963).
24. Dold, A.: Partitions of unity in the theory of fibrations. *Ann. of Math.*, 2d ser. **78** (1963) 223-55.
25. Dold, A.: Lectures on Algebraic Topology. Springer-Verlag, New York (1972).
26. Dold, A., and Lashof, R.: Principal quasi-fibrations and fibre homotopy equivalence of bundles. *Ill. J. Math.* **3** (1959) 285-305.
27. Eilenberg, S. and Mac Lane, S.: On the groups  $H(\Pi, n)$ , I. *Ann. of Math.* **58** (1953) 55-106.
28. Eilenberg, S., and Moore, J.C.: Homology and fibrations, I. Colagebras, cotensor product and its derived functors. *Comm. Math. Helv.* **40** (1966) 199-236.
29. Fadell, E.R.: Homotopy groups of configuration spaces and the string problem of Dirac. *Duke Math. J.* **29** (1962) 119-26.
30. Fadell, E.: Generalized normal bundles for locally-flat imbeddings. *Trans. Amer. Soc.* **114** (1965) 488-513.
31. Fadell, E.: Lectures on cohomological index theories of  $G$ -spaces with applications to critical point theory. *Raccolta di Sem. Univ. della Calabria* (1985).
32. Fadell, E.R., and Hurewicz, W.: On the structure of higher differential operators in spectral sequences. *Ann. of Math.*, 2d ser. **68** (1958) 314-47.
33. Fadell, E., and Husseini, S.: Relative cohomological index theories. *Advances in Math.* **64** (1987) 1-31.
34. Fadell, E., and Husseini, S.: An ideal-valued cohomological index theory with applications to Borsuk-Ulam and Bourgin-Yang theorems. *Ergod. Th. and Dynam. Sys.* **8**, Charles Conley Memorial Issue (1988) 73-85.
35. Fadell, E., and Husseini, S.: A note on the category of the free loop space. *Proc. Amer. Math. Soc.* **107** (1985) 527-36.
36. Fadell, E., and Husseini, S.: Category of loop spaces of open subsets in Euclidean space. *J. Nonlin. Anal.* **17** (1991) 1153-61.
37. Fadell, E., and Husseini, S.: Infinite cup length in free loop spaces with an application to a problem of N-body type. *Ann. Inst. H. Poincaré, Anal. Nonlin.* **9** (1992) 305-19.
38. Fadell, E. and Husseini, S.: Relative category, products and coproducts. *Rendiconti del Sem. Mat. e Fis. di Milano* **64** (1994) 99-115.
39. Fadell, E., and Husseini, S.: On the growth properties of the homology of free loop spaces of configuration spaces. *Proc. of the First World Congress of Nonlin. Anal.*, pp. 2127-34. Walter de Gruyter, Berlin and New York (1996).
40. Fadell, E., and Husseini, S.: The space of loops on configuration spaces and the Majer-Terracini index. *Topological Methods in Nonlinear Analysis, J. of the Schauder Center* **11** (1998) 249-71.
41. Fadell, E., and Husseini, S.: When are configuration bundles fiber homotopically trivial? *Bull. des Sc. Math. Quebec*, **22** (1998) 149-59.
42. Fadell, E., Husseini, S., and Rabinowitz, P.: Borsuk-Ulam Theorems for arbitrary  $S^1$  actions and applications. *Trans. Amer. Math. Soc.* **274** (1982) 345-60.
43. Fadell, E., and Neuwirth, L.: Configuration spaces. *Math. Scand.* **10** (1962) 111-18.
44. Fadell, E., and Van Buskirk, J.: The Braid groups of  $E^2$  and  $S^2$ . *Duke Math. J.* **29** (1962) 243-57.
45. Falk, M., and Randell, R. : Lower central series of a fiber-type arrangement. *Inventiones Math.* **82** (1985) 77-88.
46. Feichtner, E.M., and Ziegler, G.M.: The integral cohomology of ordered configuration spaces of spheres. Preprint (1996).
47. Fournier, G., and Willem, M.: Multiple solutions of the forced pendulum equation. *Ann. Inst. H. Poincaré, Anal. Nonlin.* **6** suppl. (1989) 259-81.

48. Fulton, W., and MacPherson, R.: A compactification of configuration spaces. *Ann. of Math.* **139** (1994) 187-229.
49. Gillette, R., and Van Buskirk, J.: The word problem and its consequences for the braid group and mapping class groups of the 2-sphere. *Trans. Amer. Math. Soc.* **131** (1968) 277-96.
50. Gottlieb, D.H.: A certain subgroup of the fundamental group. *Amer. J. Math.* **87** (1965) 840-56.
51. Haefliger, A.: Plongements différentiables de variétés dans variétés. *Comm. Math. Helv.* **36** (1961) 47-82.
52. Haefliger, A.: Plongements différentiables dans le domaine stable. *Comm. Math. Helv.* **37** (1962) 155-76.
53. Hall, M. Jr.: *The Theory of Groups*. The Macmillan Company, New York (1959).
54. Hansen, V.L.: *Braids and coverings: selected topics*. Lond. Math. Soc. Student Texts **18** (1989). Cambridge Univ. Press.
55. Hilton, P.J.: On the homotopy groups of the union of spheres. *J. London Math. Soc.* **30** (1955) 154-72.
56. Hopf, H.: Über die Abbildungen von Sphären auf Sphären niedrigen Dimension. *Fund. Math.* **5** (1935) 427-40.
57. Husseini, S.Y.: Constructions of the reduced product type. *Topology* **2** (1963) 213-37.
58. Husseini, S.Y.: Constructions of the reduced product type, II. *Topology* **3** (1965) 59-79.
59. Husseini, S.Y.: When is a complex fibered by a subcomplex? *Trans. Amer. Math. Soc.* **124** (1966) 249-91.
60. Husseini, S.Y.: *Topology of the classical groups*. Gordon and Breach, N.Y. (1969)
61. Husseini, S.Y.: Generalized Lefschetz Numbers. *Trans. Amer. Math. Soc.* **272** (1982) 247-74.
62. Jambu, M., and Papadima, S.: A generalization of fiber-type arrangements and a new deformation method. *Topology* **37** (1998) 1135-64.
63. James, I.M.: Reduced product spaces. *Ann. of Math.* **62** (1955) 170-97.
64. James, I.M.: *The topology of Stiefel manifolds*. London Math. Soc. Lecture Notes **24** (1976). Cambridge University Press.
65. James, I.M., and Whitehead, J.H.C.: On the homotopy theory of sphere-bundles over spheres, I, II. *Proc. London Math. Soc.*, **4**(1954) 196-218, **5** (1955) 148-66.
66. Kallel, S.: *Brace products, homology operations and the structure of free loop spaces*. (To appear).
67. Kohno, T.: Monodromy representations of braid groups and Yang-Baxter equations. *Annales Inst. Fourier*, **37** (1987) 139-60.
68. Kohno, T.: Linear representations of braid groups and classical Yang-Baxter equations. *Contemp. Math.* **78** (1988) 339-63.
69. Kohno, T.: Vassiliev Invariants and the de Rham Complex on the space of knots. *Symplectic Geometry and Quantization*, *Contemp. Math.* **179** (1994) 123-38.
70. Loday, J.-L., and Quillen, D.: Cyclic homology and the Lie algebra homology of matrices. *Comm. Math. Helv.* **59** (1984) 565-91.
71. Lyndon, R.C., and Schupp, P.E.: *Combinatorial group theory*. Springer-Verlag, New York (1977).
72. Mac Lane, S.: *Homology*. Springer-Verlag, New York (1963).
73. Magnus, W., Karrass, A., and Solitar, D.: *Combinatorial group theory*. Wiley, New York (1966).

74. Majer, P.: Two variational methods on manifolds with boundary. *Topology* **34** (1994), 1-12.
75. Majer, P., and Terracini, S.: Periodic solutions to some problems of  $n$ -body type. *Arch. Rational Anal.* **124** (1993) 381-404.
76. Majer, P., and Terracini, S.: Periodic solutions to some  $N$ -body type problems: the fixed energy case. *Duke Math. J.* **63** (1993) 683-97.
77. Majer, P., and Terracini, S.: Multiple periodic solutions to some  $n$ -body type problems via a collision index. *Variational Methods in Nonlinear Analyses*, Arice, 1992, pp. 245-62. Gordon and Breach, Basel (1995).
78. Majer, P., and Terracini, S.: On the existence of infinitely many periodic solutions to some problems of  $n$ -body type. *Comm. Pure Appl. Math.* **48** (1995) 449-70.
79. Massey, W.S.: The homotopy type of certain configuration spaces. *Bol. de la Soc. Mat. Mex.* **37** (1992) 355-65.
80. Mathews, H.V.: Cellular twisted products. Ph.D. Thesis, University of Wisconsin-Madison (1993).
81. Mawhin, J., and Willem, M.: *Critical Point Theory and Hamiltonian Systems*. Applied Mathematical Sciences **74** (1989). Springer-Verlag, New York.
82. Milnor, J.W.: Construction of universal bundles I, II. *Ann. of Math.* **63** (1956) 272-84, 430-36.
83. Milnor, J.W., and Moore, J.C.: On the structure of Hopf algebras. *Ann. of Math.* **81** (1965) 211-64.
84. Moore, J.C.: Le Théorème de Freudenthal, la suite exacte de James at l'invariant de Hopf généralisé. Exposé 22, Séminaire H. Cartan, E.N.S. (1954/55).
85. Moulton, V. L.: Vector braids. *J. Pure App. Algebra* **131** (1998) 245-96.
86. Orlik, P., and Terao, H.: *Arrangements of hyperplanes*. Grundlehren **300** (1992), Springer-Verlag, New York.
87. Palais, R. S.: Local triviality of the restriction map for embeddings. *Comm. Math. Helv.* **34** (1960) 305-12.
88. Palais, R. S.: Homotopy theory of infinite dimensional manifolds. *Topology* **5** (1966) 1-16.
89. Ramsey, J. R.: Extensions of Lusternik-Schnirelman Category theory to relative, equivariant and invariant Theories. Ph.D. Thesis, University of Wisconsin-Madison (1989).
90. Reeken, M.: Stability of critical points under small perturbations, Part I. *Manus. Math.* **7** (1972) 387-411.
91. Riahi, H.: Periodic orbits of  $n$ -body type problems. Ph.D. Thesis, Rutgers University, New Brunswick, N.J. (1992).
92. Riahi, H.: Periodic orbits of  $n$ -body type problems: the fixed period case. *Trans. Amer. Math. Soc.* **347** (1995) 4663-85.
93. Riahi, H.: Study of the critical points at infinity arising from the failure of the Palais-Smale condition for  $n$ -body type problems. *Amer. Math. Soc. Memoirs* **138**, No. 658 (1999).
94. Roos, J.E.: Homology of free loop spaces, cyclic homology, and non-rational Poincaré series in commutative algebra. *LNM* **1352** (1988) 173-89. Springer-Verlag, New York.
95. Rothenberg, M., and Steenrod, N.E.: The cohomology of classifying spaces of  $H$ -spaces. *Bull. Amer. Math. Soc.* **58** (1965) 872-75.
96. Samelson, H.: A connection between the Whitehead and the Pontryagin product. *Amer. J. Math.* **75** (1953) 744-52.
97. Serre, J-P.: Homologie singulière des espaces fibré. Applications. *Ann. of Math.* **54** (1951) 425-505.

98. Serre, J-P.: Lie algebras and Lie groups. 1964 Lectures at Harvard University. 2d ed. LNM **1500** (1992). Springer-Verlag, New York.
99. Shapiro, A.: Obstructions to the imbedding of a complex in a Euclidean space, I. The first obstruction. *Ann. of Math.* **66** (1957) 256-69.
100. Shnider, S., and Sternberg, S.: Quantum groups. From coalgebra to Drinfel'd algebras. A guided tour. Graduate Texts in Mathematical Physics, II. International Press, Cambridge, MA (1993)
101. Spanier, E. H.: Algebraic Topology. Springer-Verlag, New York (1966).
102. Stallings, J.: Centerless groups—an algebraic approach to Gottlieb's theorem. *Topology* **4** (1965) 129-34.
103. Steenrod, N.E.: The topology of fibre bundles. Princeton University Press, Princeton, NJ (1951).
104. Steenrod, N.E., and Whitehead, J.H.C.: Vector fields on the n-sphere. *Proc. Nat. Acad. Sci.* **37** (1951) 58-63.
105. Sullivan, D., and Vigué-Poirrier, M.: The homology theory of the closed geodesic problem. *J. Diff. Geom.* **11** (1976) 633-44.
106. Szulkin, A.: A relative category and applications to critical point theory for strongly indefinite functionals. *J. Nonlin. Anal.* **15** 725-39.
107. Thom, R.: La classification des immersions. Séminaire Bourbaki, 10e année: 1957-1958. *Textes de Conférences, Exposé* **157** (1957).
108. Vasiliev, V.A.: Complements of discriminants of smooth maps: topology and applications. *Trans. of Math. Monographs* **98**. Rev. ed. Amer. Math. Soc.(1994)
109. Vigué-Poirrier, M.: Homotopie rationnelle et croissance du nombre de géodésiques fermées. *Ann. scient. Éc. norm. sup., 4<sup>e</sup> série, t.* **17** (1984) 413-31.
110. Whitehead, G.W.: Elements of homotopy theory. Springer-Verlag, New York (1978).
111. Wu, W.-T.: Sur les classes caractéristiques des structures fibrés sphériques. *Actualités Sci. Indust.*, **1183** 1-89. Hermann, Paris (1952).
112. Wu, W.-T.: On the imbedding of manifolds in a Euclidean space. *Sci. Sinica* **13** (1964) 682-83.
113. Xicoténcatl Merino, M. (1997): Orbit configuration spaces, infinitesimal braid relations in homology and equivariant loop spaces. Ph.D. Thesis, University of Rochester, Rochester, NY (1997).

# Index

- Adams-Hilton algebra, 207–210
- Adjoint bundle, 187–188
- Affine maps, 118, 131–132, 134
  - $\phi_{\underline{r}, \underline{s}}$ , 143
  - illustration, 133–134
- Algebra  $H^*(\mathbb{F}_k(\mathbb{R}^{n+1}))$ 
  - its structure, 103–105
- Algebra  $H^*(\mathbb{F}_k(\mathbb{R}^{n+1}; \mathbb{Z}))$ , 95
  - its module structure, 96–100
- Algebra  $H^*(\mathbb{F}_k(S^{n+1}; \mathbb{Z}))$ , 95
- Algebra  $H^*(\mathbb{F}_{k+1}(S^{m+1}); \mathbb{Z})$ 
  - $(n+1)$  even, 112–113
  - $(n+1)$  odd, 113–115
  
- Bowtie construction, 129
- Braid  $\sigma_i$ , 60
- Braid space  $\mathbb{F}_k(\mathbb{R}^2)/\Sigma_k$ , 59
- Braids, 57
- Braids  $\alpha_{rs}$ , 57
  
- Cell complex
  - $\simeq \mathbb{F}_k(M)$
  - $M = \mathbb{R}^{n+1}, S^{n+1}$ , 117–121
  - $\simeq \mathbb{F}_k(S^2)$ , 120–121, 151
  - $\simeq \mathbb{F}_k(S^{n+1})$ , 120–121
  - $\simeq \mathbb{F}_k(\mathbb{R}^2)$ , 120–121, 149–151
  - $\simeq \mathbb{F}_k(\mathbb{R}^{n+1})$ , 119–121
  - $\simeq \mathbb{F}_k(\mathbb{R}^{n+1})$ ,  $n > 1$ , 143–147
  - $\simeq \mathbb{F}_{k+1}(S^m)$ ,  $m$  even, 147–149
  - $\simeq \mathbb{F}_{k+1}(S^m)$ ,  $m$  odd, 147–149
- Cells
  - $e_{\underline{r}, \underline{s}}$ , 143–147
- Cellular chain algebra models, 207
- Cellular chain coalgebra, 153–155
- Cellular chain colagebra
  - for  $\mathbb{F}_k(\mathbb{R}^{n+1})$ , 155–157
  - for  $\mathbb{F}_k(S^{n+1})$ ,  $(n+1)$  odd, 157–161
  - for  $\mathbb{F}_k(S^{n+1})$ ,  $(n+1)$  even, 161–163
- Cellular chain models, 153
- Cellular spectral sequence, 204–206
- Chain coalgebras, 153
- cobar algebra, 207
  
- Cohomology classes  $\alpha_{rs}^*$ , 95–100
- Collision index, 293
- Configuration spaces of  $\mathbb{R}^2$ , 57–58
  - its natural filtration, 57
  - their asphericity, 58–59
  - their canonical fibrations, 81–84
  - f.h. triviality thereof, 58
- Configuration spaces of  $\mathbb{R}^{n+1}$ , 13–14, 29
  - their based loop space, 14
  - their canonical fibrations, 24–27
  - their free loop space, 14
  - their natural filtrations, 13–14
- Configuration spaces of  $S^2$ , 58
- Configuration spaces of  $S^{n+1}$ , 29–31
  - their canonical fibrations, 30
  - their f.h. triviality, 52–54
- Configuration spaces of manifolds
  - their canonical fibrations, 5–8
- Critical points, 296
  
- Diagonal map, 154
- Dirac class, 30
  - in  $H_*(\Omega(\mathbb{F}_{k+1}(S^{n+1})); \mathbb{Z})$ , 172
- Dirac class  $\Delta_{k+1}$ , 45–49
- Dirac class on  $k$  strands, 88
  
- Eilenberg-Moore spectral sequence, 188, 207
  - a geometric realization, 205
- Ends
  - analytic, 294, 296
  - topological, 293–294, 296
- Exact sequence
  - of graded augmented algebras
  - its definition, 103
  
- Factor table, 118, 132, 134, 140, 153, 155
- Factor tables
  - illustration, 133–134
- Factors

- of a  $p$ -fold twisted product, 130
- Fundamental class
  - of  $S_1, \dots, m$ , 143
- Fundamental fiber sequence
  - of  $\mathbb{F}_k(\mathbb{R}^{n+1})$ , 15
  - of  $\mathbb{F}_k(M)$ , 28
  - of  $\mathbb{F}_{k+1}(S^{n+1})$ , 31
- $\Gamma$ -category, 293–294
- Hilton's Theorem, 14, 176
- $H_*(\Omega(\mathbb{F}_k(M)))$ 
  - as a Pontryagin algebra, 171–172
- $H_*(\Omega(\mathbb{F}_{k+1}(S^{n+1})); \mathbb{Z})$ 
  - as a Hopf algebra
    - $(n+1)$  even, 172
    - $(n+1)$  odd, 171
- $H_*(\Omega(\mathbb{F}_{k-r,r}); \mathbb{Z})$ 
  - as a Hopf algebra, 171
- $H_*(\Omega(\bigvee_{j=1}^r S_{r+1j}); \mathbb{Q})$ 
  - as a Pontryagin algebra, 22
- Homotopy class  $\delta_{k+1}$ , 29–31
- Homotopy classes  $\alpha_{r,s}$ , 14, 29–31
  - as generators, 13–14, 16–17
  - their definition, 16
  - as homology classes, 95
- Homotopy classes  $\beta_{r,s}$ , 29–31
  - as generators, 34–36
- Homotopy classes  $\gamma_{r,s}$ , 29–31
- Homotopy inverse, 187
  - for  $M(X)$ , 194–196
- Hopf algebra, 22
- Hopf map, 29, 35
- Infinitesimal braid relations, 20
- J-homomorphism, 30, 42
- $k$ -body problems, 294
- Kronecker product, 117
- Leray-Hirsch Theorem, 111, 113, 114, 118, 126, 138
- Loop spaces
  - based  $\Omega(X)$ , 187–193
  - free  $\Lambda(X)$ , 196–200
- Milnor's simplicial model, 194–196, 201
- Moore paths, 188
- Multispherical cycles, 119, 134–137, 141
- Palais-Smale condition, 299
- Path spaces
  - based  $\mathcal{P}(X)$ , 187–193
- Perturbation
  - illustration, 133–134
  - of  $\alpha_{r,s}$  by  $\alpha_{t,u}$ , 118, 127–132
  - $\pi_*(\mathbb{F}_k(\mathbb{R}^{n+1}))$ 
    - as a graded Lie algebra, 13–14
    - its graded Lie algebra structure, 21–24
  - $\pi_*(\mathbb{F}_r(S^{n+1}))$ ,  $n > 1$ 
    - its Lie algebra structure, 49–50
  - $\pi_*(\mathbb{F}_{k+1}(S^{n+1}))$ 
    - $(n+1)$  even, 30
    - $(n+1)$  odd, 30
    - as a graded Lie algebra, 30
    - as a graded Lie algebra, 30
  - $\pi_*(\mathbb{F}_{k+1}(S^{n+1}))$ ,  $n > 1$ 
    - its filtration, 31–33
  - $\pi_1(\mathbb{F}_{k+1}(S^2), q^e)$ 
    - a presentation thereof, 84–89
- Pontryagin algebra, 171
- Primitive elements
  - as a graded Lie algebra, 171
  - in  $H_*(\Omega(\mathbb{F}_{k-r,r}))$ , 171, 174–177
  - in  $H_*(\Omega(\bigvee_{j=1}^r S_{r+1j}); \mathbb{Q})$ , 22
- Principal bundle, 194–196
  - Milnor's simplicial model, 197–200
- Problems of  $k$ -body type, 293–294
- Pure braid group  $\pi_1(\mathbb{F}_k(\mathbb{R}^2), q)$ 
  - a presentation, 73–81
  - a presentation thereof, 57–58
  - generators thereof, 59–63
- Quasi fibration, 146, 194–196
  - principal, 188–193, 204
- Quasi-fibration, 58
- Quaternionic space, 30
- Rational homotopy type, 14
- $RPT$ -model for  $\Omega(\prod_{i=1}^m S_i)$ , 210–215
- $RPT$ -monoid
  - generated by  $X$ , 208–210
- $RPT$ -constructions, 187–188
- $RPT$ -model
  - for  $\Lambda(X)$ , 196–200
  - for  $\Omega(X)$ , 188–193
  - for  $\mathcal{P}(X)$ , 188–193
- $RPT$ -model for  $\Lambda_\sigma(X)$ , 201–204
- $RPT$ -monoid
  - generated by  $X$ , 187
- Simplicial
  - complex, 193
  - loops, 193
  - Milnor's simplicial model, 193

- topological group thereof, 193–196, 201
- paths, 193
- Special complex, 154
- Sphere  $S^n$
- its tangent bundle, 5, 11, 25, 29
- Spherical map, 142
  - $\phi_{\beta_r, \underline{s}}$ , 147
- stable, 6–8, 79–81
- Stereographic projection, 31, 33, 34
- Stiefel manifold, 30, 95, 112, 157, 161
- Stiefel Manifolds, 55
- Stiefel manifolds, 5, 10, 11, 119, 153
- strongly stable, 6–8, 79–81
- Symmetric group  $\Sigma_k$ , 13–14, 29–31
  - its action, 18–19
  - its effect on  $\alpha_{rs}^*$ , 95, 100–101
  - its effect on  $\delta_{k+1}$ , 38
  - its effect on  $\pi_1(\mathbb{F}_k(\mathbb{R}^2), q)$ , 58, 63–69
- Symmetric group  $\Sigma_{k+1}$ 
  - its action on  $\mathbb{F}_{k+1}(S^{n+1})$ , 33
  - its action on  $\pi_n(\mathbb{F}_{k+1}(S^{n+1}))$ , 34–36
- Twisted Product, 171
- Twisted product, 21, 33, 95, 96
  - implications in cohomology, 111–112
  - in  $H_*(\mathbb{F}_k(\mathbb{R}^{n+1}))$ , 117–121, 127–132, 134–137, 155
  - in  $H_*(\mathbb{F}_{k+1}(S^m))$ , 119–121
  - in  $H_*(\mathbb{F}_{k+1}(S^m))$ ,  $m$  even, 141–143
  - in  $H_*(\mathbb{F}_{k+1}(S^m))$ ,  $m$  odd, 137–141
  - in  $H_*(\mathbb{F}_{k+1}(S^m); \mathbb{Z})$ ,  $m$  odd, 161
  - in  $H_*(\mathbb{F}_{k-r, r})$ , 126–127
- Whitehead product, 13, 14, 20–21
  - basic products, 21, 175, 176
- Yang-Baxter relations
  - in  $\pi_*(\mathbb{F}_k(\mathbb{R}^{n+1}))$ , 13
  - their proof, 20
  - in  $\pi_*(\mathbb{F}_{k+1}(S^3))$ , 45
  - in  $\pi_*(\mathbb{F}_{k+1}(S^{n+1}))$ , 34
  - $(n+1)$  odd, 30
  - in  $\pi_*(\mathbb{F}_{k+1}(S^{n+1}))$ ,  $n > 2$ , 42–45
  - in  $\pi_1(\mathbb{F}_k(\mathbb{R}^2), q)$ , 57–58, 69–73
  - in  $H_*(\Omega(\mathbb{F}_k(\mathbb{R}^{n+1})))$
  - the quadratic nature thereof, 207
  - in  $H_*(\Omega(\mathbb{F}_{k-r, r}); \mathbb{Z})$ , 171
  - in cohomology, 95, 101–103