

## Chronological table

Fagnano, Conte G. C. di	1682–1766	Riemann, B.	1826–1866
Euler, L.	1707–1783	Dedekind, R.	1831–1916
Lagrange, J. L.	1736–1813	Weber, H.	1842–1913
Legendre, A. M.	1752–1833	Klein, F.	1849–1925
Fourier, J.	1768–1830	Hurwitz, A.	1859–1919
Gauss, C. F.	1777–1855	Minkowski, H.	1864–1909
Cauchy, A.	1789–1857	Hardy, G. H.	1877–1947
Abel, N. H.	1802–1829	Fueter, R.	1880–1950
Jacobi, C. G. J.	1804–1851	Littlewood, J. E.	1885–1977
Dirichlet, G. Lejeune	1805–1859	Weyl, H.	1885–1955
Liouville, J.	1809–1882	Hecke, E.	1887–1947
Weierstrass, K.	1815–1897	Ramanujan, S.	1887–1920
Hermite, C.	1822–1901	Mordell, L. J.	1888–1972
Eisenstein, G.	1823–1852	Vinogradov, I. M.	1891–1983
Kronecker, L.	1823–1891	Siegel, C. L.	1896–1981

The enduring is not a substitute for the transient,  
Neither the one for the other.

T. S. Eliot

# Subject index

- absolute invariant
  - of a biquadratic 44
  - of Felix Klein 95
- addition-theorem 17, 34, 46, 106, 120
- algebraic functional equation for  $J(\tau)$  96
- algebraic relation 39
- anharmonic group 110, 120
- approximate reciprocity formula 153
  
- basic periods 5, 8, 32, 99, 100
- biquadratic equation 41
  
- complementary modulus 99
- complete prime residue system 146
- complex multiplication 18, 44
- congruence properties
  - of  $c(n)$  97
  - of  $p(n)$  139
  - of  $\tau(n)$  139
- congruences, solution of 146
- congruent relative to the modular group 13
- connexion between  $J(z)$ ,  $\lambda(z)$ ,  $\eta(z)$  135
- cubic equation 40
  
- Dedekind's approach to modular functions 20
  - $\eta$ -function 80, 122, 136
  - sums 136, 137
  - differential equation for  $J(z)$  137
  - relation of  $J(z)$  and  $\lambda(z)$
  - transformation formula for  $\eta(z)$  126, 131
- degenerate forms of elliptic functions 46
- differential equation
  - for theta-functions 66
  - for Jacobian elliptic functions 99, 100, 103, 105, 121
  - for Dedekind's function  $w$  137
- Dirichlet's evaluation of Gaussian sums 152
  - evaluation of  $r_3(n)$  167
  - Gedächtnisrede on Jacobi 18
- discontinuous group 19
- discrete subgroup 5, 18
- discriminant of a biquadratic 95
  - of a cubic 33
- doubly periodic function 3, 47
- duplication formula for  $\wp$  45
  
- Eisenstein's approach to elliptic functions 56, 120
  - evaluation of  $r_k(n)$  167
  - series 96, 183
- elementary modular forms 34
- elliptic function 5
- elliptic integral 16, 17, 18, 79, 92, 98
- Euler's criterion 146
- Euler's theorems 123, 124
- expansion for  $J(\tau)$  82, 84
- expansions for theta-functions 65
  
- Fagnano's discovery 17
- Fermat's proposition 152
- field of elliptic functions 39
- Fourier and theta-functions 78
- Fueter on complex multiplication 18
- fundamental domain 15, 86, 114, 115, 117
- fundamental period-parallelogram 21
  
- Gauss's identity 125
- Gaussian integers 166
- Gaussian sums 141, 145
  - evaluation of 145
  - generalized 144
  - in number fields 153
- Gauss's law of quadratic reciprocity 141, 147, 153
- Gauss's lemma 153
- Gauss's sums 137
  
- Hardy's formula for  $r_k(n)$  167
- Hardy-Littlewood's circle method 167, 184
- Hardy-Ramanujan's formula 139
- Hasse's law of reciprocity 153
- Hecke's Gaussian sums in number fields 153
- Hecke's law of quadratic reciprocity 153
- Hecke's theorem on even integers 179
- Hermite on the formula for  $\theta'$  80
- Hermite's theta-functions 78
- Hurwitz's approach to modular functions 20
  - to Jacobian elliptic functions 119
  - proof of Jacobi's theorem 67, 79
  - solution of the problem of inversion 97, 98, 119
- hypergeometric differential equation 137

- integral quaternions 166  
 invariants  
   of a biquadratic 43, 95  
   of the  $\wp$ -function 32, 47, 53, 81, 122  
   special values 88  
   their relation to the periods 90  
 inversion problem  
   for  $J(\tau)$  97, 98  
   for  $\lambda(\tau)$  119
- Jacobian elliptic functions 25, 99, 101  
 Jacobi's differential equation  
   for theta-functions 66  
   for the elliptic functions sn, cn, dn 104  
 Jacobi's lemma 3, 4, 16  
   four theta-functions  $\theta, \theta_1, \theta_2, \theta_3$  63  
   formula for  $\theta'$  67, 155  
   infinite products for theta-functions 69  
   proof of Euler's theorem 123, 138  
   identity 125  
   theorem on  $r(n)$  156, 157
- Klein's approach to modular functions 20  
   absolute invariant 95, 98  
 Kronecker's approach to theta-functions 78  
   reciprocity formula for Gaussian sums 152
- Lagrange's theorem on congruences 146  
   formula for  $\binom{2}{-n}$  150  
   four squares theorem 155, 156  
   lattice 5, 26  
   lattice-point function  $r(n)$  156  
   its generalization  $r_k(n)$  166  
   lattice-points 156, 173  
   law of quadratic reciprocity 147  
   Legendre's relation 50  
   Legendre symbol 145, 152  
   Legendre's three squares theorem 166  
   lemma of Gauss 153  
   lemniscate integral 17  
   limiting case  
     of the  $\wp$ -function 46  
     of the  $\zeta$ -function and  $\sigma$ -function 57  
     of the theta transformation formula 141, 151  
   Lindelöf hypothesis 140  
   Liouville's methods of proof 25, 166  
   Lipschitz's formula 96
- Minkowski's arithmetical method 167, 184  
   evaluation of  $r_k(n)$  167  
   theorem on special matrices 178  
   Witt's proof 178, 179  
 modular form 20, 96, 140, 183
- modular function 11, 15, 20, 95, 99, 108, 116, 123  
 modular group 11, 77, 81, 114  
 modular transformation 11, 12, 13, 20, 109  
 modulus of an elliptic function 99  
 Mordell's method 167  
 multiple theta-series 173, 174, 183
- order of an elliptic function 23
- parity of integers 112  
 partition function 139  
 partitions 124  
 pentagonal numbers 124  
 periodic functions 1  
 period-lattice 5, 29, 30, 96  
 periods of meromorphic functions 1  
 Petersson's identity 96  
 Picard's theorem 116, 118, 121  
 plane algebraic curves 98  
 primitive period 2, 3, 6, 7, 8, 9  
 principal congruence subgroup 120  
 properly discontinuous group 19  
 properly unimodular transformation 6, 9, 10
- quadratic forms  
   positive-definite 170  
   inverse 172  
   indefinite 183, 184  
 quadratic non-residues 139, 145  
   residues 145
- Rademacher's identity 139  
 Ramanujan's  
    $\tau$ -function 139  
   conjecture 140  
 Reciprocity for Dedekind sums 137  
   for generalized Gaussian sums 144  
 reduced period 2, 3, 6  
 Riemann hypothesis 140  
 Riemann's Nachlass 80
- Schneider's theorems on transcendency 47, 57, 95, 96  
 Schwarz's account of Weierstrass's lectures 44  
 Siegel on Fagnano and Euler 17  
   on the  $\wp$ -function and transcendental numbers 47  
   on the functional equations for theta-functions 79  
   on the transformation formula for  $\eta(z)$  80  
   on the algebraic functional equation for  $J(\tau)$  96  
   on Euler's identity 132, 140

## Siegel

- on Lagrange's formula 153
- on Jacobi's theorem 159
- on sums of squares in number fields 168, 169
- on the theory of quadratic forms 183
- sigma-function of Weierstrass 48, 51
- sigma-functions  $\sigma, \sigma_1, \sigma_2, \sigma_3$  61, 62, 64
- simply periodic function 2, 47
- singular series 168
- special linear group  $SL(2; \mathbb{R})$  18
- square numbers 138
- Smith's evaluation of  $r_k(n)$  167
- sum of the zeros of an elliptic function 23
  - of the poles 23
- theta-functions  $\theta, \theta_1, \theta_2, \theta_3$  58, 63
  - relations with sigma-functions 60, 64
  - differential equation 66, 155
  - zeros 66
  - formula for  $\theta'$  67
  - infinite products 69, 119
  - functional equations 72
  - transformation formulae 73, 77
  - associated to positive-definite forms 176

theta-series 170, 183

triangular numbers 138

uniformization 98

unimodular transformation 5, 6, 9, 10, 87, 121

univalent function 97

Vinogradov's method 168, 169

Waring's problem 168, 169

Weierstrassian elliptic functions 25

Weierstrass's  $\wp$ -function 26

its zeros 47

sigma-function  $\sigma$  48, 51zeta-function  $\zeta$  48

product 56

normal form 98

Weyl on Siegel's theory of quadratic forms 184

Witt's proof of Minkowski's theorem 178

zeta-function of Riemann 82

mean-value theorems 140

zeta-function of Weierstrass 48

# Grundlehren der mathematischen Wissenschaften

*A Series of Comprehensive Studies in Mathematics*

---

## *A Selection*

190. Faith: Algebra: Rings, Modules, and Categories I
191. Faith: Algebra II, Ring Theory
192. Mal'cev: Algebraic Systems
193. Pólya/Szegő: Problems and Theorems in Analysis I
194. Igusa: Theta Functions
195. Berberian: Baer\*-Rings
196. Athreya/Ney: Branching Processes
197. Benz: Vorlesungen über Geometrie der Algebren
198. Gaal: Linear Analysis and Representation Theory
199. Nitsche: Vorlesungen über Minimalflächen
200. Dold: Lectures on Algebraic Topology
201. Beck: Continuous Flows in the Plane
202. Schmetterer: Introduction to Mathematical Statistics
203. Schoeneberg: Elliptic Modular Functions
204. Popov: Hyperstability of Control Systems
205. Nikol'skiĭ: Approximation of Functions of Several Variables and Imbedding Theorems
206. André: Homologie des Algèbres Commutatives
207. Donoghue: Monotone Matrix Functions and Analytic Continuation
208. Lacey: The Isometric Theory of Classical Banach Spaces
209. Ringel: Map Color Theorem
210. Gihman/Skorohod: The Theory of Stochastic Processes I
211. Comfort/Negreponis: The Theory of Ultrafilters
212. Switzer: Algebraic Topology – Homotopy and Homology
215. Schaefer: Banach Lattices and Positive Operators
217. Stenström: Rings of Quotients
218. Gihman/Skorohod: The Theory of Stochastic Processes II
219. Duvant/Lions: Inequalities in Mechanics and Physics
220. Kirillov: Elements of the Theory of Representations
221. Mumford: Algebraic Geometry I: Complex Projective Varieties
222. Lang: Introduction to Modular Forms
223. Bergh/Löfström: Interpolation Spaces. An Introduction
224. Gilbarg/Trudinger: Elliptic Partial Differential Equations of Second Order
225. Schütte: Proof Theory
226. Karoubi: K-Theory. An Introduction
227. Grauert/Remmert: Theorie der Steinschen Räume
228. Segal/Kunze: Integrals and Operators
229. Hasse: Number Theory
230. Klingenberg: Lectures on Closed Geodesics
231. Lang: Elliptic Curves: Diophantine Analysis
232. Gihman/Skorohod: The Theory of Stochastic Processes III
233. Stroock/Varadhan: Multidimensional Diffusion Processes
234. Aigner: Combinatorial Theory
235. Dynkin/Yushkevich: Controlled Markov Processes
236. Grauert/Remmert: Theory of Stein Spaces
237. Köthe: Topological Vector Spaces II

238. Graham/McGehee: Essays in Commutative Harmonic Analysis  
 239. Elliott: Probabilistic Number Theory I  
 240. Elliott: Probabilistic Number Theory II  
 241. Rudin: Function Theory in the Unit Ball of  $C^n$   
 242. Huppert/Blackburn: Finite Groups II  
 243. Huppert/Blackburn: Finite Groups III  
 244. Kubert/Lang: Modular Units  
 245. Cornfeld/Fomin/Sinai: Ergodic Theory  
 246. Naimark/Stern: Theory of Group Representations  
 247. Suzuki: Group Theory I  
 248. Suzuki: Group Theory II  
 249. Chung: Lectures from Markov Processes to Brownian Motion  
 250. Arnold: Geometrical Methods in the Theory of Ordinary Differential Equations  
 251. Chow/Hale: Methods of Bifurcation Theory  
 252. Aubin: Nonlinear Analysis on Manifolds. Monge-Ampère Equations  
 253. Dwork: Lectures on  $p$ -adic Differential Equations  
 254. Freitag: Siegelsche Modulfunktionen  
 255. Lang: Complex Multiplication  
 256. Hörmander: The Analysis of Linear Partial Differential Operators I  
 257. Hörmander: The Analysis of Linear Partial Differential Operators II  
 258. Smoller: Shock Waves and Reaction-Diffusion Equations  
 259. Duren: Univalent Functions  
 260. Freidlin/Wentzell: Random Perturbations of Dynamical Systems  
 261. Bosch/Güntzer/Remmert: Non Archimedean Analysis – A Systematic Approach to Rigid Geometry  
 262. Doob: Classical Potential Theory and Its Probabilistic Counterpart  
 263. Krasnosel'skiĭ/Zabreĭko: Geometrical Methods of Nonlinear Analysis  
 264. Aubin/Cellina: Differential Inclusions  
 265. Grauert/Remmert: Coherent Analytic Sheaves  
 266. de Rham: Differentiable Manifolds  
 267. Arbarello/Cornalba/Griffiths/Harris: Geometry of Algebraic Curves, Vol. I  
 268. Arbarello/Cornalba/Griffiths/Harris: Geometry of Algebraic Curves, Vol. II  
 269. Schapira: Microdifferential Systems in the Complex Domain  
 270. Scharlau: Quadratic and Hermitian Forms  
 271. Ellis: Entropy, Large Deviations, and Statistical Mechanics  
 272. Elliott: Arithmetic Functions and Integer Products  
 274. Hörmander: The Analysis of Linear Partial Differential Operators III  
 275. Hörmander: The Analysis of Linear Partial Differential Operators IV  
 276. Liggett: Interacting Particle Systems  
 277. Fulton/Lang: Riemann–Roch Algebra  
 278. Barr/Wells: Toposes, Triples and Theories  
 279. Bishop/Bridges: Constructive Analysis  
 280. Neukirch: Class Field Theory



Springer-Verlag Berlin Heidelberg New York Tokyo