

Conclusion

Of course, this is not the last word in the investigation and application of the Chebyshev approach to optimization problems in polynomial spaces. For instance, there can be further development along the following lines:

1. Describe the stratification of the sphere $\{\|P\|_E = 1\}$ in the space (2) by smooth faces. To our knowledge, this is an unsolved problem even in the case when E is a single interval.
2. Determine integers g, k, m_0, \dots, m_g corresponding to the solution of an extremum problem from the data of this problem.
3. Investigate the topology of the fibres of the period map $\Pi_-: \tilde{\mathcal{H}}_g^k \rightarrow \mathbb{R}^g$. We conjectured above that each component of a fiber is a cell.
4. Linear Poincaré theta series are poorly convergent in a neighbourhood of (a part of) the boundary of our moduli space. The question of improving the convergence arises. A more general question was put by Klein in 1923: regularize absolutely divergent theta series.
5. Develop global methods for solving equations (for instance, Abel's equations) in moduli spaces of curves. Newton's method works very well in a small neighbourhood of a solution, but its global dynamics is completely unpredictable.
6. Use other uniformizations of curves M for calculations. Fuchsian uniformization cannot be used because then linear Poincaré theta series will be divergent [149]. Another Schottky uniformization, not equivalent to the one presented here (and based on an incomplete dissection) was used in [37]; in this case Abel's equations and the parametric representation for polynomials get another form.
7. Use Riemann theta functions in the computations of the theory. The advantage of this approach is a higher convergence rate of series, particularly for large genera g . But it also has two deficiencies: (1) the Schottky problem of describing the period matrices of (hyperelliptic) Riemann surfaces in the Siegel upper half-

space [108] must be solved numerically; (2) the image of an algebraic curve in its Jacobian under the Abel–Jacobi map must be localized numerically.

8. Construct a compactification of the moduli space \mathcal{H}_g^k such that the cell decomposition of Chapter 4 becomes a cell complex.
9. Prove or disprove that the value L/n^2 for optimal stability polynomials converges for each $p = 2, 3, \dots$. Give the value of this limit in terms of Riemann surfaces.

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