Appendix A

A.1 Relevant Constants and Abbreviations

Speed of light	$c = 299792458\,\mathrm{m\,s^{-1}}$
Newton's gravitational constant	$G \simeq 6.674 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}s^{-2}}$
$\kappa = 8\pi G/c^2$	$\kappa \simeq 1.866 \times 10^{-26}\,\mathrm{m\,kg^{-1}}$
Planck constant	$h \simeq 6.62607 \times 10^{-34}\,\mathrm{J\,s}$
$\hbar = h/2\pi$	$\hbar \simeq 1.054572 \times 10^{-34}\,\mathrm{J\,s}$
Charge of a positron	$e \simeq 1.602177 \times 10^{-19}\,\mathrm{C}$
Electric fine structure constant	$\alpha \simeq 7.29735 \times 10^{-3} \simeq 1/137.04$
Permittivity of the vacuum	$\varepsilon_0 \simeq 8.85419 \times 10^{-12}\,\mathrm{C\ V^{-1}m^{-1}}$
	$= 8.85419 \times 10^{-12}\,\mathrm{C^2\,kg^{-1}m^{-3}\,s^2}$
Electron mass	$m_{\mathrm{e}} \simeq 0.510999\,\mathrm{MeV/c^2}$
Proton mass	$m_{\mathrm{p}} \simeq 938.272\,\mathrm{MeV/c^2}$
Neutron mass	$m_{\mathrm{n}} \simeq 939.565\,\mathrm{MeV/c^2}$
Lengths	$1\,\mathrm{ly} \simeq 0.9461 \times 10^{16}\,\mathrm{m}$
	$1\,\mathrm{pc} \simeq 3.08568 \times 10^{16}\,\mathrm{m}$
	$1\,\mathrm{nm} = 10^{-9}\,\mathrm{m}$
	$1\,\mathrm{Å} = 10^{-10}\,\mathrm{m}$
	$1\,\mathrm{fm} = 10^{-15}\,\mathrm{m}$
Energy and mass	$1\,\mathrm{J} = 1\,\mathrm{kg\,m^2\,s^{-2}}$
	$1\,\mathrm{eV} \simeq 1.602177 \times 10^{-19}\,\mathrm{J}$
	$1\,\mathrm{eV/c^2} \simeq 1.782662 \times 10^{-36}\,\mathrm{kg}$
Powers of Ten	$1\,\mathrm{k} = 10^3,\ 1\,\mathrm{M} = 10^6,$
	$1\,\mathrm{G} = 10^9,\ 1\mathrm{T} = 10^{12}.$

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A.2 Useful Internet Addresses

Satellite Experiments for the Measurement of the Cosmic Background Radiation:
WMAP: `map.gsfc.nasa.gov`
Planck: `www.rssd.esa.int/index.php?project=Planck`
Experiments for the Detection of Gravitational Waves:
GEO600: `www.geo600.org`
LIGO: `www.ligo-la.caltech.edu`
TAMA: `tamago.mtk.nao.ac.jp`
Virgo: `www.virgo.infn.it`
Particle Accelerators:
CERN (LEP, LHC): `www.cern.ch`
Fermilab (Tevatron): `www.fnal.gov`
Overview of the Properties of Known Elementary Particles:
Particle Data Group: `pdg.lbl.gov`

Solutions to Exercises

Chapter 1

1.1 We look for a maximum of the function $E_{\mathrm{binding}}(A, Z)$ (neglecting $\delta(N, Z)$, and replacing $N = A - Z$). Requiring the derivative of $E_{\mathrm{binding}}(A, Z)$ with respect to Z to vanish gives us the desired formula for $Z(A)$:

$$Z(A) = \frac{A}{2 + [a_c/(2a_a)]A^{2/3}}. \tag{A.1}$$

For $A = 238$ we obtain, using $a_c/a_a \simeq 0.030$, $Z(238) \simeq 92.4 \simeq 92$, which corresponds to the chemical element uranium.

Chapter 2

2.1 First it is helpful to rewrite (2.6), (2.7), as we did in (2.8), in terms of the function $H(t) = \dot{a}(t)/a(t)$: Equation 2.6 becomes

$$3H^2(t) = \kappa c^2 \varrho(t), \tag{A.2}$$

and (2.7) becomes

$$2\dot{H}(t) + 3H^2(t) = -\kappa p(t) = -\kappa c^2 w \varrho(t), \tag{A.3}$$

owing to the assumed relation between $p(t)$ and $\varrho(t)$. After substituting $\varrho(t)$ from (A.2) in (A.3), we find (A.3) becomes

$$\dot{H}(t) = -\frac{3}{2}(1 + w)H^2(t). \tag{A.4}$$

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Now we have to distinguish the cases $w \neq -1$ and $w = -1$:

(a) $w \neq -1$:
The ansatz $H(t) = k/t$ (where k is a constant to be determined) leads to $k = 2/[3(1+w)]$. From $\dot{a}(t) = H(t)\,a(t)$ we obtain

$$a(t) = a_0 t^{2/(3(1+w))}, \tag{A.5}$$

and for $\varrho(t) = [3/(\kappa c^2)]H^2(t)$ (from (A.2))

$$\varrho(t) = \frac{4}{3\kappa c^2(1+w)^2 t^2}, \tag{A.6}$$

as well as

$$\mathrm{p}(t) = wc^2\varrho(t) = \frac{4w}{3\kappa(1+w)^2 t^2}. \tag{A.7}$$

(b) $w = -1$:
Equation (A.4) simplifies to $\dot{H}(t) = 0$ with the general solution $H(t) = k$. From $\dot{a}(t) = H(t)\,a(t)$ we obtain $a(t) = a_0 \mathrm{e}^{kt}$, and for $\varrho(t)$ we find $\varrho(t) = [3/(\kappa c^2)]k^2$, and furthermore $p(t) = -(3/\kappa)k^2$. Now $\varrho(t)$ and $p(t)$ are constant! It suffices to rewrite the constant k as $k = \sqrt{\kappa\Lambda/3}$, then we have $\varrho(t) = \Lambda/c^2$ and $p(t) = -\Lambda$. This corresponds to the original equations (2.6) and (2.7) with $\varrho(t) = p(t) = 0$, $\Lambda \neq 0$, and the solution for $a(t)$ corresponds to that in (2.20).

2.2 Let us assume that we have $7n$ protons and n neutrons at our disposal. (n is an arbitrary integer.) According to the assumption, all neutrons are used up in helium nuclei containing 2 neutrons; correspondingly $n/2$ helium nuclei are produced. However, these contain also 2 protons each, hence n protons disappear in helium nuclei and $6n$ protons (= hydrogen nuclei) are left over. Hence the ratio of hydrogen to helium nuclei is $Z_{\mathrm{H}} : Z_{\mathrm{He}} = 6n : (n/2) = 12 : 1$. However, we were asked for the ratio of the densities. A helium nucleus is about 4 times as heavy as a hydrogen nucleus, thus the ratio of densities is $\varrho_{\mathrm{H}} : \varrho_{\mathrm{He}} \simeq 3 : 1$ (i.e., 75% H, 25% He).

Chapter 3

3.1 We substitute the expressions from (3.7) for $\Delta t'$ and $\Delta x'$ in (3.8), and obtain

$$\begin{aligned}(\Delta\tau')^2 &= (\Delta t')^2 - \frac{1}{c^2}(\Delta x')^2\\ &= \gamma^2\left(\Delta t^2 - 2\frac{v_x}{c^2}\Delta t\Delta x + \frac{v_x^2}{c^2}\Delta x^2\right)\\ &\quad - \frac{\gamma^2}{c^2}\left(\Delta x^2 - 2v_x\Delta t\Delta x + v_x^2\Delta t^2\right)\end{aligned}$$

$$
\begin{aligned}
&= \Delta t^2 \gamma^2 \left(1 - \frac{v_x^2}{c^2}\right) - \frac{\Delta x^2}{c^2} \gamma^2 \left(1 - \frac{v_x^2}{c^2}\right) \\
&= \Delta t^2 - \frac{1}{c^2} \Delta x^2 = (\Delta \tau)^2, \qquad \text{(A.8)}
\end{aligned}
$$

using the definition of γ in (3.7).

3.2 From the formula (3.44) we obtain

$$r_{\mathrm{S}} \simeq 1.5 \times 10^{-27}\,\mathrm{m} \sim 10^{-17} r_{\mathrm{atom}} \sim 10^{-12} r_{\mathrm{nucleus}}. \qquad \text{(A.9)}$$

Chapter 4

4.1 Instead of (4.4) we now obtain

$$\left(-\omega^2 + c^2 k^2 + \frac{m^2 c^4}{\hbar^2}\right) \Phi(x, t) = 0, \qquad \text{(A.10)}$$

which is satisfied for all x and t only for $\omega^2 = c^2 k^2 + m^2 c^4 / \hbar^2$.

4.2 First we compute

$$\frac{\partial}{\partial x} \Phi(r) = \frac{\partial r}{\partial x} \frac{\partial}{\partial r} \Phi(r) = \left(-\frac{x}{r^3} - \frac{\lambda x}{r^2}\right) \Phi_0 \mathrm{e}^{-\lambda r}, \qquad \text{(A.11)}$$

then

$$\frac{\partial^2}{\partial x^2} \Phi(r) = \left(-\frac{\lambda}{r^2} + \frac{\lambda^2 x^2 - 1}{r^3} + \frac{3\lambda x^2}{r^4} + \frac{3x^2}{r^5}\right) \Phi_0 \mathrm{e}^{-\lambda r}. \qquad \text{(A.12)}$$

After a similar calculation with $x \to y$ and $x \to z$, it follows that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \Phi(r) = \frac{\lambda^2}{r} \Phi_0 \mathrm{e}^{-\lambda r} = \lambda^2 \Phi(r). \qquad \text{(A.13)}$$

Hence (4.11) is satisfied for $\lambda = mc/\hbar$.

Chapter 5

5.1 The conservation of total momentum in electron–electron scattering implies

$$\vec{p}_1{}^{\mathrm{a}} + \vec{p}_2{}^{\mathrm{a}} = \vec{p}_1{}^{\mathrm{b}} + \vec{p}_2{}^{\mathrm{b}}, \qquad \text{(A.14)}$$

and the conservation of total energy implies

$$E_1^{\mathrm{a}} + E_2^{\mathrm{a}} = E_1^{\mathrm{b}} + E_2^{\mathrm{b}}. \tag{A.15}$$

With $\vec{p}_2{}^{\mathrm{a}} = -\vec{p}_1{}^{\mathrm{a}}$ it follows from (A.14) that $\vec{p}_2{}^{\mathrm{b}} = -\vec{p}_1{}^{\mathrm{b}}$. Using $E = \sqrt{m_{\mathrm{e}}^2 c^4 + \vec{p}^{\,2} c^2}$ and replacing $\vec{p}_2{}^{\mathrm{a,b}}$ by $-\vec{p}_1{}^{\mathrm{a,b}}$, it follows from (A.15) that

$$2\sqrt{m_{\mathrm{e}}^2 c^4 + \vec{p}_1{}^{\mathrm{a}2} c^2} = 2\sqrt{m_{\mathrm{e}}^2 c^4 + \vec{p}_1{}^{\mathrm{b}2} c^2}, \tag{A.16}$$

hence $\left|\vec{p}_1{}^{\mathrm{b}}\right| = \left|\vec{p}_1{}^{\mathrm{a}}\right|$ and also $\left|\vec{p}_2{}^{\mathrm{b}}\right| = \left|\vec{p}_1{}^{\mathrm{a}}\right|$, $E_1^{\mathrm{b}} = E_1^{\mathrm{a}}$, and $E_2^{\mathrm{b}} = E_1^{\mathrm{a}} = E_2^{\mathrm{a}}$.

5.2 From (5.41) we deduce $E_{\mathrm{tot}}(n = 2) = -\frac{1}{4}E_{\mathrm{R}}$, $E_{\mathrm{tot}}(n = 1) = -E_{\mathrm{R}}$, hence the energy of the emitted photon is

$$E_{\mathrm{phot}} = E_{\mathrm{tot}}(n = 2) - E_{\mathrm{tot}}(n = 1) = \frac{3}{4}E_{\mathrm{R}}. \tag{A.17}$$

In order to calculate the Rydberg energy, we express it first in terms of the fine structure constant α (see 5.31): $E_{\mathrm{R}} = (m_{\mathrm{e}}/2)\alpha^2 c^2$. Thus we have

$$E_{\mathrm{phot}} = \frac{3}{8}\alpha^2 m_{\mathrm{e}} c^2 \simeq 10.2\,\mathrm{eV} = 16.3 \times 10^{-19}\,\mathrm{J}. \tag{A.18}$$

The wavelength of the photon follows from (4.7) and (4.8):

$$\lambda = \frac{c}{v} = \frac{hc}{E} \sim 1.22 \times 10^{-7}\,\mathrm{m} = 122\,\mathrm{nm}, \tag{A.19}$$

corresponding to ultraviolet radiation.

Chapter 6

6.1 We already know the quark content of neutrons and protons, and use the u, d, and s quark masses and charges from Table 6.1, according to which the s quark is about $0.2\,\mathrm{GeV}/c^2$ heavier than the u and d quarks. The baryons Λ^0 and Σ^+, Σ^0, Σ^- are 0.18–0.26 GeV/c^2 heavier than a neutron or a proton, and contain, accordingly, one s quark. The nature of the two additional quarks follows from the electric charges, with the result:
Λ^0 and $\Sigma^0 \sim$ (uds), $\Sigma^+ \sim$ (uus), $\Sigma^- \sim$ (dds).

The baryons Ξ^0 and Ξ^- are about $0.38\,\mathrm{GeV}/c^2$ heavier than a neutron or a proton, and contain accordingly two s quarks:
$\Xi^0 \sim$ (uss), $\Xi^- \sim$ (dss).

Chapter 7

7.1 An $\bar{\mathrm{s}}$ quark with charge $+\frac{1}{3}e$ can turn into a $\bar{\mathrm{u}}$ quark by the emission of a virtual W^+ boson. The virtual W^+ boson can decay into the following quarks or leptons (with masses smaller than the $\bar{\mathrm{s}}$ mass): $(\mathrm{u}\bar{\mathrm{d}})$, $(\mathrm{e}^+\nu_\mathrm{e})$, $(\mu^+\nu_\mu)$. Thus we obtain the three possibilities $\bar{\mathrm{s}} \rightarrow \bar{\mathrm{u}} + \mathrm{u} + \bar{\mathrm{d}}$, $\bar{\mathrm{s}} \rightarrow \bar{\mathrm{u}} + \mathrm{e}^+ + \nu_\mathrm{e}$, $\bar{\mathrm{s}} \rightarrow \bar{\mathrm{u}} + \mu^+ + \nu_\mu$. (Even though the sum of the quark masses $\bar{\mathrm{u}}, \mathrm{u}$, and $\bar{\mathrm{d}}$ is larger than the $\bar{\mathrm{s}}$ mass, these quarks can subsequently form relatively light pions.)

7.2 First, the three decay possibilities of the $\bar{\mathrm{s}}$ quark lead to the following three decay possibilities of a K^+ meson consisting of a $\mathrm{u}\bar{\mathrm{s}}$ pair:

$$\mathrm{K}^+ \rightarrow \mathrm{u} + \bar{\mathrm{u}} + \mathrm{u} + \bar{\mathrm{d}} \rightarrow \pi^0 + \pi^+, \tag{A.20}$$

$$\mathrm{K}^+ \rightarrow \mathrm{u} + \bar{\mathrm{u}} + \mathrm{e}^+ + \nu_\mathrm{e} \rightarrow \pi^0 + \mathrm{e}^+ + \nu_\mathrm{e},$$

$$\mathrm{K}^+ \rightarrow \mathrm{u} + \bar{\mathrm{u}} + \mu^+ + \nu_\mu \rightarrow \pi^0 + \mu^+ + \nu_\mu. \tag{A.21}$$

In addition, a quark can emit a gluon (or even two gluons), which can decay, in turn, into a $\mathrm{u}\bar{\mathrm{u}}$ or a $\mathrm{d}\bar{\mathrm{d}}$ pair, leading to additional pions. (According to (6.10), pions are relatively light.) The eight additional decay possibilities are

$$\mathrm{K}^+ \rightarrow \pi^0 + \pi^0 + \pi^+, \quad \pi^+ + \pi^+ + \pi^-, \tag{A.22}$$

$$\pi^0 + \pi^0 + \mathrm{e}^+ + \nu_\mathrm{e}, \quad \pi^+ + \pi^- + \mathrm{e}^+ + \nu_\mathrm{e},$$

$$\pi^0 + \pi^0 + \mu^+ + \nu_\mu, \quad \pi^+ + \pi^- + \mu^+ + \nu_\mu,$$

$$\pi^0 + \pi^0 + \pi^0 + \mathrm{e}^+ + \nu_\mathrm{e},$$

$$\pi^0 + \pi^+ + \pi^- + \mathrm{e}^+ + \nu_\mathrm{e}. \tag{A.23}$$

(The latter two of these decays, for which we expect a very low probability, have not yet been detected.)

Finally, the $\mathrm{u}\bar{\mathrm{s}}$ pair can annihilate into a virtual W^+ boson, which can decay either into a $\mathrm{u}\bar{\mathrm{d}}$ pair (which, after the emission of gluons, can form the final states $\pi^0\pi^+$, $\pi^0\pi^0\pi^+$ and $\pi^+\pi^+\pi^-$ already listed above), or into purely leptonic $\mathrm{e}^+\nu_\mathrm{e}$ or $\mu^+\nu_\mu$ pairs. The latter lead to additional possible K^+ decays

$$\mathrm{K}^+ \rightarrow \mathrm{e}^+ + \nu_\mathrm{e}, \quad \mathrm{K}^+ \rightarrow \mu^+ + \nu_\mu. \tag{A.24}$$

The decays (A.20) and (A.22) are denoted as *hadronic decays*, those in (A.24) as *leptonic decays*, and those in (A.21) and (A.23) as *semileptonic*.

Chapter 8

8.1

(a) With $R = 27\,\text{km}/2\pi \simeq 4300$ m we obtain for the coefficient $ce^2/(6\pi\varepsilon_0 R^2)$ in (8.6)

$$\frac{ce^2}{6\pi\varepsilon_0 R^2} \simeq 2.5 \times 10^{-27}\,\text{kg}\,\text{m}^2\,\text{s}^{-3} = 2.5 \times 10^{-27}\,\text{W}. \tag{A.25}$$

For an electron with mass $m_\text{e} \simeq 5.11 \times 10^{-4}\,\text{GeV}/c^2$ and an energy $E = 104\,\text{GeV}$ we obtain $E/(m_\text{e}c^2) \simeq 2.035 \times 10^5$. Then formula (8.6) gives $P \simeq 4.3 \times 10^{-6}\,\text{W} \simeq 2.7 \times 10^{13}\,\text{eV}$ per second. Hence an electron emits an energy of $2.7 \times 10^{13}\,\text{eV} = 27000\,\text{GeV}$ per second, about 260 times its total energy—this energy loss has to be compensated by an energy feed by corresponding electric fields.

(b) For a proton with mass $m_\text{p} \simeq 0.938\,\text{GeV}/c^2$ and an energy $E = 7 \times 10^3\,\text{GeV}$ we obtain $E/(m_\text{p}c^2) \simeq 7.46 \times 10^3$. The factor $ce^2/(6\pi\varepsilon_0 R^2)$ is the same as in (A.25), hence (8.6) gives $P \simeq 7.7 \times 10^{-12}\,\text{W} \simeq 4.8 \times 10^7\,\text{eV}$ per second. Thus a proton radiates an energy of $4.8 \times 10^7\,\text{eV} = 0.048\,\text{GeV}$ per second, only a small fraction of its total energy.

Chapter 9

9.1

(a) Derivation of the hermiticity of A_{ij} from the unitarity of U_{ij}: we have omitted terms of the order A_{ij}^2 in the series expansion $\text{U}_{ij} = \delta_{ij} + \text{i}A_{ij} + \ldots$, and likewise we can neglect terms of this order in the condition for unitarity:

$$\begin{aligned}\delta_{ij} &= \sum_{j=1}^{N} \text{U}_{ij}\text{U}_{kj}^* \simeq \sum_{j=1}^{N} \left(\delta_{ij} + \text{i}A_{ij}\right)\left(\delta_{jk} - \text{i}A_{jk}^*\right) \\ &\simeq \delta_{ik} + \text{i}A_{ik} - \text{i}A_{ki}^* + \mathcal{O}\left(A^2\right).\end{aligned} \tag{A.26}$$

It follows that $A_{ki}^* = A_{ik}$.

(b) Derivation of the vanishing trace of A_{ij} from $\det(\text{U}) = 1$: it is most convenient to use the generally valid formula log(det(U)) = Tr(log(U)) together with the series expansion $\log(1+\varepsilon) \simeq \varepsilon$:

$$\begin{aligned}1 &= \det(\text{U}) = \text{e}^{\log(\det(\text{U}))} = \text{e}^{\text{Tr}(\log(\text{U}))} \\ &\simeq \text{e}^{\text{Tr}(\log(\delta_{ij}+\text{i}A_{ij}))} \simeq \text{e}^{\text{Tr}(\text{i}A_{ij})},\end{aligned} \tag{A.27}$$

which is satisfied only for $\mathrm{Tr}\left(A_{ij}\right) = \sum_{i=1}^{N} A_{ii} = 0$.

9.2 First, complex $N \times N$ matrices contain $2N^2$ real parameters (two for each complex matrix element). The N^2 conditions $A_{ij} = A_{ji}^*$ reduce the number of free parameters to N^2, and the condition of a vanishing trace by one more. Hence we are left with $N^2 - 1$ free real parameters, corresponding to $N^2 - 1$ linearly independent matrices. For $N = 2$ this number is equal to 3, and for $N = 3$ we find 8 independent matrices (accordingly there exist 8 independent gluons).

Chapter 11

11.1 The relation

$$\alpha_{\mathrm{s,measured}}(E) = \frac{\alpha_{\mathrm{s}}}{1 + b_{\mathrm{s}}\alpha_{\mathrm{s}} \ln\left(\Lambda^2/E^2\right)} = \frac{1}{b_{\mathrm{s}} \ln(\Lambda_{\mathrm{QCD}}^2/E^2)} \tag{A.28}$$

can be written in the form

$$\frac{1}{\alpha_{\mathrm{s}}} + b_{\mathrm{s}} \ln\left(\Lambda^2/E^2\right) = b_{\mathrm{s}} \ln\left(\Lambda_{\mathrm{QCD}}^2/E^2\right), \tag{A.29}$$

which gives, solved for Λ_{QCD}^2,

$$\Lambda_{\mathrm{QCD}}^2 = \Lambda^2 \mathrm{e}^{1/(\alpha_{\mathrm{s}} b_{\mathrm{s}})}. \tag{A.30}$$

From (A.28) we obtain

$$\Lambda_{\mathrm{QCD}}^2 = E^2 \mathrm{e}^{1/\left(\alpha_{\mathrm{s,measured}}(E) b_{\mathrm{s}}\right)}. \tag{A.31}$$

This formula is valid for all E. Using (A.28) for $E = E_1$, and substituting (A.31) for Λ_{QCD}^2 with $E = E_2$, we obtain after some calculation

$$\alpha_{\mathrm{s,measured}}(E_1) = \frac{\alpha_{\mathrm{s,measured}}(E_2)}{1 + b_{\mathrm{s}}\alpha_{\mathrm{s,measured}}(E_2) \ln\left(E_2^2/E_1^2\right)}. \tag{A.32}$$

From (A.32) we obtain for $E_1 = 22\,\mathrm{GeV}$, $E_2 = 91\,\mathrm{GeV}$ and $\alpha_{\mathrm{s,measured}}$ $(91\,\mathrm{GeV}) \simeq 0.12$ together with $b_{\mathrm{s}} = -23/12\pi$,

$$\alpha_{\mathrm{s,measured}}(22\,\mathrm{GeV}) \simeq 0.15, \tag{A.33}$$

which agrees with Fig. 11.5 within the error bars.

Chapter 12

12.1 The sought-after mass M satisfies

$$\frac{e^2}{4\pi\varepsilon_0} = GM^2. \tag{A.34}$$

It follows that

$$M \simeq 2 \times 10^{-9}\,\mathrm{kg} \simeq 1.1 \times 10^{18}\,\mathrm{GeV}/c^2 \simeq \frac{1}{2} M_{\mathrm{Planck}}. \tag{A.35}$$

Comment: For elementary particles of mass about the Planck mass, the magnitude of the gravitational force would be of the same order as the magnitude of the forces originating from the interactions of the Standard Model; in this sense, all four interactions would be “unified”.

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