A

Regressor Matrix and Parameter Vector of an *Object*

For a rigid body to which frame $\{\mathbf{A}\}$ is attached, the regressor matrix $\mathbf{Y}_{\mathbf{A}} \in \mathbb{R}^{6\times 13}$ and the parameter vector $\boldsymbol{\theta}_{\mathbf{A}} \in \mathbb{R}^{13}$ as appeared in (2.78) are expressed in this appendix.

The non-zero elements in $\mathbf{Y}_{\mathbf{A}} \in \mathbb{R}^{6\times 13}$ are listed as

$$y_{\mathbf{A}}(1,1) = \frac{d}{dt}\left({}^{\mathbf{A}}v_r\right)(1) + {}^{\mathbf{A}}v(5)\,{}^{\mathbf{A}}v_r(3) - {}^{\mathbf{A}}v(6)\,{}^{\mathbf{A}}v_r(2) + {}^{\mathbf{A}}g(1) \tag{A.1}$$

$$y_{\mathbf{A}}(1,2) = -{}^{\mathbf{A}}v(5)\,{}^{\mathbf{A}}v_r(5) - {}^{\mathbf{A}}v(6)\,{}^{\mathbf{A}}v_r(6) \tag{A.2}$$

$$y_{\mathbf{A}}(1,3) = -\frac{d}{dt}\left({}^{\mathbf{A}}v_r\right)(6) + {}^{\mathbf{A}}v(5)\,{}^{\mathbf{A}}v_r(4) \tag{A.3}$$

$$y_{\mathbf{A}}(1,4) = \frac{d}{dt}\left({}^{\mathbf{A}}v_r\right)(5) + {}^{\mathbf{A}}v(6)\,{}^{\mathbf{A}}v_r(4) \tag{A.4}$$

$$y_{\mathbf{A}}(2,1) = \frac{d}{dt}\left({}^{\mathbf{A}}v_r\right)(2) + {}^{\mathbf{A}}v(6)\,{}^{\mathbf{A}}v_r(1) - {}^{\mathbf{A}}v(4)\,{}^{\mathbf{A}}v_r(3) + {}^{\mathbf{A}}g(2) \tag{A.5}$$

$$y_{\mathbf{A}}(2,2) = \frac{d}{dt}\left({}^{\mathbf{A}}v_r\right)(6) + {}^{\mathbf{A}}v(4)\,{}^{\mathbf{A}}v_r(5) \tag{A.6}$$

$$y_{\mathbf{A}}(2,3) = -{}^{\mathbf{A}}v(4)\,{}^{\mathbf{A}}v_r(4) - {}^{\mathbf{A}}v(6)\,{}^{\mathbf{A}}v_r(6) \tag{A.7}$$

$$y_{\mathbf{A}}(2,4) = -\frac{d}{dt}\left({}^{\mathbf{A}}v_r\right)(4) + {}^{\mathbf{A}}v(6)\,{}^{\mathbf{A}}v_r(5) \tag{A.8}$$

$$y_{\mathbf{A}}(3,1) = \frac{d}{dt}\left({}^{\mathbf{A}}v_r\right)(3) + {}^{\mathbf{A}}v(4)\,{}^{\mathbf{A}}v_r(2) - {}^{\mathbf{A}}v(5)\,{}^{\mathbf{A}}v_r(1) + {}^{\mathbf{A}}g(3) \tag{A.9}$$

$$y_{\mathbf{A}}(3,2) = -\frac{d}{dt}\left({}^{\mathbf{A}}v_r\right)(5) + {}^{\mathbf{A}}v(4)\,{}^{\mathbf{A}}v_r(6) \tag{A.10}$$

$$y_{\mathbf{A}}(3,3) = \frac{d}{dt}\left({}^{\mathbf{A}}v_r\right)(4) + {}^{\mathbf{A}}v(5)\,{}^{\mathbf{A}}v_r(6) \tag{A.11}$$

$$y_{\mathbf{A}}(3,4) = -{}^{\mathbf{A}}v(4)\,{}^{\mathbf{A}}v_r(4) - {}^{\mathbf{A}}v(5)\,{}^{\mathbf{A}}v_r(5) \tag{A.12}$$

$$y_{\mathbf{A}}(4,3) = y_{\mathbf{A}}(3,1) \tag{A.13}$$

$$y_{\mathbf{A}}(4,4) = -y_{\mathbf{A}}(2,1) \tag{A.14}$$

$$y_{\mathbf{A}}(4,6) = y_{\mathbf{A}}(3,3) \tag{A.15}$$

$$y_{\mathbf{A}}(4,7) = -y_{\mathbf{A}}(2,4) \tag{A.16}$$

$$y_{\mathbf{A}}(4,8) = y_{\mathbf{A}}(3,2) \tag{A.17}$$
$$y_{\mathbf{A}}(4,9) = -y_{\mathbf{A}}(2,2) \tag{A.18}$$
$$y_{\mathbf{A}}(4,10) = {}^{\mathbf{A}}v(6){}^{\mathbf{A}}v_r(6) - {}^{\mathbf{A}}v(5){}^{\mathbf{A}}v_r(5) \tag{A.19}$$
$$y_{\mathbf{A}}(4,11) = \frac{d}{dt}\left({}^{\mathbf{A}}v_r\right)(4) + {}^{\mathbf{A}}v(5){}^{\mathbf{A}}v_r(6) - {}^{\mathbf{A}}v(6){}^{\mathbf{A}}v_r(5) \tag{A.20}$$
$$y_{\mathbf{A}}(4,12) = -{}^{\mathbf{A}}v(6){}^{\mathbf{A}}v_r(5) \tag{A.21}$$
$$y_{\mathbf{A}}(4,13) = {}^{\mathbf{A}}v(5){}^{\mathbf{A}}v_r(6) \tag{A.22}$$
$$y_{\mathbf{A}}(5,2) = -y_{\mathbf{A}}(3,1) \tag{A.23}$$
$$y_{\mathbf{A}}(5,4) = y_{\mathbf{A}}(1,1) \tag{A.24}$$
$$y_{\mathbf{A}}(5,5) = -y_{\mathbf{A}}(3,2) \tag{A.25}$$
$$y_{\mathbf{A}}(5,7) = y_{\mathbf{A}}(1,4) \tag{A.26}$$
$$y_{\mathbf{A}}(5,8) = -y_{\mathbf{A}}(3,3) \tag{A.27}$$
$$y_{\mathbf{A}}(5,9) = {}^{\mathbf{A}}v(4){}^{\mathbf{A}}v_r(4) - {}^{\mathbf{A}}v(6){}^{\mathbf{A}}v_r(6) \tag{A.28}$$
$$y_{\mathbf{A}}(5,10) = y_{\mathbf{A}}(1,3) \tag{A.29}$$
$$y_{\mathbf{A}}(5,11) = {}^{\mathbf{A}}v(6){}^{\mathbf{A}}v_r(4) \tag{A.30}$$
$$y_{\mathbf{A}}(5,12) = \frac{d}{dt}\left({}^{\mathbf{A}}v_r\right)(5) + {}^{\mathbf{A}}v(6){}^{\mathbf{A}}v_r(4) - {}^{\mathbf{A}}v(4){}^{\mathbf{A}}v_r(6) \tag{A.31}$$
$$y_{\mathbf{A}}(5,13) = -{}^{\mathbf{A}}v(4){}^{\mathbf{A}}v_r(6) \tag{A.32}$$
$$y_{\mathbf{A}}(6,2) = y_{\mathbf{A}}(2,1) \tag{A.33}$$
$$y_{\mathbf{A}}(6,3) = -y_{\mathbf{A}}(1,1) \tag{A.34}$$
$$y_{\mathbf{A}}(6,5) = y_{\mathbf{A}}(2,2) \tag{A.35}$$
$$y_{\mathbf{A}}(6,6) = -y_{\mathbf{A}}(1,3) \tag{A.36}$$
$$y_{\mathbf{A}}(6,8) = {}^{\mathbf{A}}v(5){}^{\mathbf{A}}v_r(5) - {}^{\mathbf{A}}v(4){}^{\mathbf{A}}v_r(4) \tag{A.37}$$
$$y_{\mathbf{A}}(6,9) = y_{\mathbf{A}}(2,4) \tag{A.38}$$
$$y_{\mathbf{A}}(6,10) = -y_{\mathbf{A}}(1,4) \tag{A.39}$$
$$y_{\mathbf{A}}(6,11) = -{}^{\mathbf{A}}v(5){}^{\mathbf{A}}v_r(4) \tag{A.40}$$
$$y_{\mathbf{A}}(6,12) = {}^{\mathbf{A}}v(4){}^{\mathbf{A}}v_r(5) \tag{A.41}$$
$$y_{\mathbf{A}}(6,13) = \frac{d}{dt}\left({}^{\mathbf{A}}v_r\right)(6) + {}^{\mathbf{A}}v(4){}^{\mathbf{A}}v_r(5) - {}^{\mathbf{A}}v(5){}^{\mathbf{A}}v_r(4) \tag{A.42}$$

where $y_{\mathbf{A}}(j,k) \in \mathbb{R}$ denotes an element of $\mathbf{Y_A} \in \mathbb{R}^{6\times 13}$ at row j and column k for $j \in \{1,6\}$ and $k \in \{1,13\}$; the three variables $\frac{d}{dt}\left({}^{\mathbf{A}}v_r\right)(j) \in \mathbb{R}$, ${}^{\mathbf{A}}v(j) \in \mathbb{R}$, and ${}^{\mathbf{A}}v_r(j) \in \mathbb{R}$ denote the jth elements of $\frac{d}{dt}\left({}^{\mathbf{A}}V_r\right) \in \mathbb{R}^6$, ${}^{\mathbf{A}}V \in \mathbb{R}^6$, and ${}^{\mathbf{A}}V_r \in \mathbb{R}^6$, respectively, for all $j \in \{1,6\}$; and ${}^{\mathbf{A}}g(j) \in \mathbb{R}$ denotes the jth element of ${}^{\mathbf{A}}\mathbf{R_I g} \in \mathbb{R}^3$ with $\mathbf{g} = [0,0,9.8]^T \in \mathbb{R}^3$ for all $j \in \{1,3\}$.

The 13 elements of $\boldsymbol{\theta}_{\mathbf{A}} \in \mathbb{R}^{13}$ are listed as

$$\theta_{\mathbf{A}1} = m_{\mathbf{A}} \tag{A.43}$$
$$\theta_{\mathbf{A}2} = m_{\mathbf{A}}{}^{\mathbf{A}}r_{mx} \tag{A.44}$$
$$\theta_{\mathbf{A}3} = m_{\mathbf{A}}{}^{\mathbf{A}}r_{my} \tag{A.45}$$
$$\theta_{\mathbf{A}4} = m_{\mathbf{A}}{}^{\mathbf{A}}r_{mz} \tag{A.46}$$
$$\theta_{\mathbf{A}5} = m_{\mathbf{A}}{}^{\mathbf{A}}r_{mx}^2 \tag{A.47}$$
$$\theta_{\mathbf{A}6} = m_{\mathbf{A}}{}^{\mathbf{A}}r_{my}^2 \tag{A.48}$$
$$\theta_{\mathbf{A}7} = m_{\mathbf{A}}{}^{\mathbf{A}}r_{mx}^2 \tag{A.49}$$
$$\theta_{\mathbf{A}8} = m_{\mathbf{A}}{}^{\mathbf{A}}r_{mx}{}^{\mathbf{A}}r_{my} - I_{\mathbf{A}xy} \tag{A.50}$$
$$\theta_{\mathbf{A}9} = m_{\mathbf{A}}{}^{\mathbf{A}}r_{mx}{}^{\mathbf{A}}r_{mz} - I_{\mathbf{A}xz} \tag{A.51}$$
$$\theta_{\mathbf{A}10} = m_{\mathbf{A}}{}^{\mathbf{A}}r_{my}{}^{\mathbf{A}}r_{mz} - I_{\mathbf{A}yz} \tag{A.52}$$
$$\theta_{\mathbf{A}11} = I_{\mathbf{A}xx} \tag{A.53}$$
$$\theta_{\mathbf{A}12} = I_{\mathbf{A}yy} \tag{A.54}$$
$$\theta_{\mathbf{A}13} = I_{\mathbf{A}zz} \tag{A.55}$$

where $\theta_{\mathbf{A}k}$ denotes the kth element of $\boldsymbol{\theta}_{\mathbf{A}} \in \mathbb{R}^{13}$ for all $k \in \{1, 13\}$; $m_{\mathbf{A}}$ is the mass; ${}^{\mathbf{A}}\mathbf{r}_m = [{}^{\mathbf{A}}r_{mx}, {}^{\mathbf{A}}r_{my}, {}^{\mathbf{A}}r_{mz}]^T \in \mathbb{R}^3$ denotes a vector pointing from the origin of frame **A** toward the mass center and expressed in frame **A** (it is equivalent to ${}^{\mathbf{A}}\mathbf{r}_{\mathbf{AB}}$ in (2.75)-(2.77)), and $I_{\mathbf{A}xx}$, $I_{\mathbf{A}yy}$, $I_{\mathbf{A}zz}$, $I_{\mathbf{A}xy}$, $I_{\mathbf{A}xz}$, and $I_{\mathbf{A}yz}$ are elements of $\mathbf{I}_{\mathbf{A}}$.

B

Mathematical Derivations

B.1 Proof of Lemma 4.1

Subtracting (4.24) from (4.32) yields

$$
\begin{aligned}
{}^{\mathbf{O}_i}F_r^* - {}^{\mathbf{O}_i}F^* = \mathbf{M}_{\mathbf{O}_i} \left[\frac{d}{dt}({}^{\mathbf{O}_i}V_r) - \frac{d}{dt}({}^{\mathbf{O}_i}V) \right] + \mathbf{C}_{\mathbf{O}_i}({}^{\mathbf{O}_i}\omega) \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right) \\
-\mathbf{Y}_{\mathbf{O}_i} \left(\boldsymbol{\theta}_{\mathbf{O}_i} - \hat{\boldsymbol{\theta}}_{\mathbf{O}_i} \right) + \mathbf{K}_{\mathbf{O}_i} \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right).
\end{aligned} \tag{B.1}
$$

In view of (B.1), the skew-symmetric property of $\mathbf{C}_{\mathbf{O}_i}({}^{\mathbf{O}_i}\omega)$, and Lemma 2.9, the time derivative of $\nu_{\mathbf{O}_i}$ in (4.40) can be written as

$$
\begin{aligned}
\dot{\nu}_{\mathbf{O}_i} &= \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right)^T \mathbf{M}_{\mathbf{O}_i} \frac{d}{dt} \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right) \\
&\quad - \sum_{\gamma=1}^{13} \left(\theta_{\mathbf{O}_i\gamma} - \hat{\theta}_{\mathbf{O}_i\gamma} \right) \dot{\hat{\theta}}_{\mathbf{O}_i\gamma} / \rho_{\mathbf{O}_i\gamma} \\
&= - \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right)^T \mathbf{C}_{\mathbf{O}_i}({}^{\mathbf{O}_i}\omega) \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right) \\
&\quad + \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right)^T \mathbf{Y}_{\mathbf{O}_i} \left(\boldsymbol{\theta}_{\mathbf{O}_i} - \hat{\boldsymbol{\theta}}_{\mathbf{O}_i} \right) \\
&\quad - \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right)^T \mathbf{K}_{\mathbf{O}_i} \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right) \\
&\quad + \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right)^T \left({}^{\mathbf{O}_i}F_r^* - {}^{\mathbf{O}_i}F^*\right) \\
&\quad - \sum_{\gamma=1}^{13} \left(\theta_{\mathbf{O}_i\gamma} - \hat{\theta}_{\mathbf{O}_i\gamma} \right) \dot{\hat{\theta}}_{\mathbf{O}_i\gamma} / \rho_{\mathbf{O}_i\gamma} \\
&= - \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right)^T \mathbf{K}_{\mathbf{O}_i} \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right) \\
&\quad + \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right)^T \left({}^{\mathbf{O}_i}F_r^* - {}^{\mathbf{O}_i}F^*\right) \\
&\quad + \sum_{\gamma=1}^{13} \Big\{ \left(\theta_{\mathbf{O}_i\gamma} - \hat{\theta}_{\mathbf{O}_i\gamma} \right)
\end{aligned}
$$

$$
\left.\left[s_{\mathbf{O}_i\gamma} - \frac{\dot{\hat{\theta}}_{\mathbf{O}_i\gamma}}{\rho_{\mathbf{O}_i\gamma}}\right]\right\}
$$

$$
\begin{aligned}
&\leqslant - \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right)^T \mathbf{K}_{\mathbf{O}_i} \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right) \\
&\quad + \left({}^{\mathbf{O}_i}V_r - {}^{\mathbf{O}_i}V\right)^T \left({}^{\mathbf{O}_i}F_r^* - {}^{\mathbf{O}_i}F^*\right).
\end{aligned} \tag{B.2}
$$

B.2 Derivation of (4.121)

Premultiplying $\mathcal{F}_b$ in (4.112) by $\mathcal{J}_b^T$ in (4.116) and using (4.25), (4.51)–(4.53), (4.73), (4.112), (4.113), and (4.117)–(4.119) yields

$$
\begin{aligned}
\mathcal{J}_b^T \mathcal{F}_b &= \sum_{i=1}^{n_o} \left(\mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}F^{*T}\right) + \sum_{j=1}^{n_c} \left(\mathcal{J}_{bj}^T \mathcal{F}_{bj}\right) \\
&= \sum_{i=1}^{n_o} \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}F^* + \sum_{j=1}^{n_c} \left(\mathcal{J}_{\mathbf{B}_j}^T {}^{\mathbf{B}_j}F^* + \sum_{k=1}^{l_j} \mathcal{J}_{\mathbf{B}_{jk}}^T {}^{\mathbf{B}_{jk}}F^*\right) \\
&= \sum_{i=1}^{n_o} \left(\sum_{\{\mathbf{T}_j\}\in\mathcal{T}_i} \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{T}_j} {}^{\mathbf{T}_j}F - \sum_{\{\mathbf{B}_j\}\in\mathcal{B}_i} \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{B}_j} {}^{\mathbf{B}_j}F - \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{S}_i} {}^{\mathbf{S}_i}F\right) \\
&\quad + \sum_{j=1}^{n_c} \Bigg[\mathcal{J}_{\mathbf{B}_j}^T {}^{\mathbf{B}_j}F - \mathcal{J}_{\mathbf{B}_j}^T {}^{\mathbf{B}_j}\mathbf{U}_{\mathbf{T}_{j1}} {}^{\mathbf{T}_{j1}}F \\
&\quad + \sum_{k=1}^{l_j-1} \left(\mathcal{J}_{\mathbf{B}_{jk}}^T {}^{\mathbf{B}_{jk}}F - \mathcal{J}_{\mathbf{B}_{jk}}^T {}^{\mathbf{B}_{jk}}\mathbf{U}_{\mathbf{T}_{j(k+1)}} {}^{\mathbf{T}_{j(k+1)}}F\right) \\
&\quad + \mathcal{J}_{\mathbf{B}_{jl_j}}^T {}^{\mathbf{B}_{jl_j}}F - \mathcal{J}_{\mathbf{B}_{jl_j}}^T {}^{\mathbf{B}_{jl_j}}\mathbf{U}_{\mathbf{T}_j} {}^{\mathbf{T}_j}F\Bigg] \\
&= \sum_{i=1}^{n_o} \left(\sum_{\{\mathbf{T}_j\}\in\mathcal{T}_i} \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{T}_j} {}^{\mathbf{T}_j}F - \sum_{\{\mathbf{B}_j\}\in\mathcal{B}_i} \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{B}_j} {}^{\mathbf{B}_j}F - \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{S}_i} {}^{\mathbf{S}_i}F\right) \\
&\quad + \sum_{j=1}^{n_c} \Bigg[\mathcal{J}_{\mathbf{B}_j}^T {}^{\mathbf{B}_j}F - \mathcal{J}_{\mathbf{B}_j}^T {}^{\mathbf{B}_j}\mathbf{U}_{\mathbf{B}_{j1}} {}^{\mathbf{B}_{j1}}F \\
&\quad + \sum_{k=1}^{l_j-1} \left(\mathcal{J}_{\mathbf{B}_{jk}}^T {}^{\mathbf{B}_{jk}}F - \mathcal{J}_{\mathbf{B}_{jk}}^T {}^{\mathbf{B}_{jk}}\mathbf{U}_{\mathbf{B}_{j(k+1)}} {}^{\mathbf{B}_{j(k+1)}}F\right) \\
&\quad + \mathcal{J}_{\mathbf{B}_{jl_j}}^T {}^{\mathbf{B}_{jl_j}}F - \mathcal{J}_{\mathbf{B}_{jl_j}}^T {}^{\mathbf{B}_{jl_j}}\mathbf{U}_{\mathbf{T}_j} {}^{\mathbf{T}_j}F\Bigg].
\end{aligned} \tag{B.3}
$$

For $\{\mathbf{T}_j\} \in \mathcal{T}_i$, it yields

$$
{}^{\mathbf{T}_j}V = {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{T}_j}^T \mathcal{J}_{\mathbf{O}_i} \mathcal{V} \tag{B.4}
$$

$$
{}^{\mathbf{T}_j}V = {}^{\mathbf{B}_{jl_j}}\mathbf{U}_{\mathbf{T}_j}^T \mathcal{J}_{\mathbf{B}_{jl_j}} \mathcal{V} \tag{B.5}
$$

from the definition of $\mathcal{J}_b$ in (4.116).

Since $\mathcal{V} \in \mathbb{R}^m$ is an independent vector, it follows from (B.4) and (B.5) that

$$ {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{T}_j}^T \mathcal{J}_{\mathbf{O}_i} = {}^{\mathbf{B}_{jl_j}}\mathbf{U}_{\mathbf{T}_j}^T \mathcal{J}_{\mathbf{B}_{jl_j}} \tag{B.6} $$

holds or equivalently

$$ \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{T}_j} = \mathcal{J}_{\mathbf{B}_{jl_j}}^T {}^{\mathbf{B}_{jl_j}}\mathbf{U}_{\mathbf{T}_j} \tag{B.7} $$

holds after performing the transpose operation.

Following the same procedure yields

$$ \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{B}_j} = \mathcal{J}_{\mathbf{B}_j}^T \tag{B.8} $$

for $\{\mathbf{B}_j\} \in \mathcal{B}_i$.

Meanwhile, for $k \in \mathcal{C}_{1j}$ and $j \in \{1, n_c\}$, re-write (4.73) as

$$ \begin{aligned} {}^{\mathbf{B}_{jk}}V &= \mathcal{J}_{\mathbf{B}_{jk}} \mathcal{V} \\ &= \mathbf{z}\dot{q}_{jk} + {}^{\mathbf{B}_{j(k-1)}}\mathbf{U}_{\mathbf{B}_{jk}}^T {}^{\mathbf{B}_{j(k-1)}}V \\ &= \mathbf{z}\mathcal{J}_{jk}\mathcal{V} + {}^{\mathbf{B}_{j(k-1)}}\mathbf{U}_{\mathbf{B}_{jk}}^T \mathcal{J}_{\mathbf{B}_{j(k-1)}} \mathcal{V}. \end{aligned} \tag{B.9} $$

Since $\mathcal{V} \in \mathbb{R}^m$ is an independent vector, it follows from (B.9) that

$$ \mathcal{J}_{\mathbf{B}_{jk}} = \mathbf{z}\mathcal{J}_{jk} + {}^{\mathbf{B}_{j(k-1)}}\mathbf{U}_{\mathbf{B}_{jk}}^T \mathcal{J}_{\mathbf{B}_{j(k-1)}} \tag{B.10} $$

holds. Performing a transpose operation yields

$$ \mathcal{J}_{\mathbf{B}_{jk}}^T = \mathcal{J}_{jk}^T \mathbf{z}^T + \mathcal{J}_{\mathbf{B}_{j(k-1)}}^T {}^{\mathbf{B}_{j(k-1)}}\mathbf{U}_{\mathbf{B}_{jk}} \tag{B.11} $$

with $\mathbf{B}_{j0} = \mathbf{B}_j$.

Accordingly, for $k \in \mathcal{C}_{3j}$ and $j \in \{1, n_c\}$, it follows that

$$ \mathcal{J}_{\mathbf{B}_{jk}}^T = \mathcal{J}_{jk}^T \mathbf{Z}^T + \mathcal{J}_{\mathbf{B}_{j(k-1)}}^T {}^{\mathbf{B}_{j(k-1)}}\mathbf{U}_{\mathbf{B}_{jk}} \tag{B.12} $$

holds with $\mathbf{B}_{j0} = \mathbf{B}_j$.

Combining (B.11) and (B.12) validates the following equation

$$ \begin{aligned} & - \sum_{k \in \mathcal{C}_{1j}} \mathcal{J}_{jk}^T \mathbf{z}^T {}^{\mathbf{B}_{jk}}F - \sum_{k \in \mathcal{C}_{3j}} \mathcal{J}_{jk}^T \mathbf{Z}^T {}^{\mathbf{B}_{jk}}F \\ & + \mathcal{J}_{\mathbf{B}_j}^T {}^{\mathbf{B}_j}F - \mathcal{J}_{\mathbf{B}_j}^T {}^{\mathbf{B}_j}\mathbf{U}_{\mathbf{B}_{j1}} {}^{\mathbf{B}_{j1}}F \\ & + \sum_{k=1}^{l_j - 1} \left(\mathcal{J}_{\mathbf{B}_{jk}}^T {}^{\mathbf{B}_{jk}}F - \mathcal{J}_{\mathbf{B}_{jk}}^T {}^{\mathbf{B}_{jk}}\mathbf{U}_{\mathbf{B}_{j(k+1)}} {}^{\mathbf{B}_{j(k+1)}}F \right) \\ & + \mathcal{J}_{\mathbf{B}_{jl_j}}^T {}^{\mathbf{B}_{jl_j}}F - \mathcal{J}_{\mathbf{B}_{jl_j}}^T {}^{\mathbf{B}_{jl_j}}\mathbf{U}_{\mathbf{T}_j} {}^{\mathbf{T}_j}F \\ = {} & \mathcal{J}_{\mathbf{B}_j}^T {}^{\mathbf{B}_j}F + \sum_{k=1}^{l_j} \left[\mathcal{J}_{\mathbf{B}_{jk}}^T - \mathcal{J}_{jk}^T \mathbf{z}_k^T - \mathcal{J}_{\mathbf{B}_{j(k-1)}}^T {}^{\mathbf{B}_{j(k-1)}}\mathbf{U}_{\mathbf{B}_{jk}} \right] {}^{\mathbf{B}_{jk}}F \\ & - \mathcal{J}_{\mathbf{B}_{jl_j}}^T {}^{\mathbf{B}_{jl_j}}\mathbf{U}_{\mathbf{T}_j} {}^{\mathbf{T}_j}F \\ = {} & \mathcal{J}_{\mathbf{B}_j}^T {}^{\mathbf{B}_j}F - \mathcal{J}_{\mathbf{B}_{jl_j}}^T {}^{\mathbf{B}_{jl_j}}\mathbf{U}_{\mathbf{T}_j} {}^{\mathbf{T}_j}F \end{aligned} \tag{B.13} $$

with $\mathbf{B}_{j0} = \mathbf{B}_j$ and

$$\mathbf{z}_k = \begin{cases} \mathbf{z} & \text{if} \quad k \in \mathcal{C}_{1j} \\ \mathbf{Z} & \text{if} \quad k \in \mathcal{C}_{3j} \end{cases} . \tag{B.14}$$

In view of (B.7), (B.8), (4.118), and (4.119), substituting (B.13) into (B.3) yields

$$\begin{aligned}
\mathcal{J}_b^T \mathcal{F}_b &= \sum_{i=1}^{n_o} \left(\sum_{\{\mathbf{T}_j\} \in \mathcal{T}_i} \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{T}_j} {}^{\mathbf{T}_j}F - \sum_{\{\mathbf{B}_j\} \in \mathcal{B}_i} \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{B}_j} {}^{\mathbf{B}_j}F - \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{S}_i} {}^{\mathbf{S}_i}F \right) \\
&\quad + \sum_{j=1}^{n_c} \Big(\mathcal{J}_{\mathbf{B}_j}^T {}^{\mathbf{B}_j}F - \mathcal{J}_{\mathbf{B}_{jl_j}}^T {}^{\mathbf{B}_{jl_j}}\mathbf{U}_{\mathbf{T}_j} {}^{\mathbf{T}_j}F \\
&\quad + \sum_{k \in \mathcal{C}_{1j}} \mathcal{J}_{jk}^T \mathbf{z}^{T\mathbf{B}_{jk}}F + \sum_{k \in \mathcal{C}_{3j}} \mathcal{J}_{jk}^T \mathbf{Z}^{T\mathbf{B}_{jk}}F \Big) \\
&= \left(\sum_{i=1}^{n_o} \sum_{\{\mathbf{T}_j\} \in \mathcal{T}_i} \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{T}_j} {}^{\mathbf{T}_j}F - \sum_{j=1}^{n_c} \mathcal{J}_{\mathbf{B}_{jl_j}}^T {}^{\mathbf{B}_{jl_j}}\mathbf{U}_{\mathbf{T}_j} {}^{\mathbf{T}_j}F \right) \\
&\quad + \left(- \sum_{i=1}^{n_o} \sum_{\{\mathbf{B}_j\} \in \mathcal{B}_i} \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{B}_j} {}^{\mathbf{B}_j}F + \sum_{j=1}^{n_c} \mathcal{J}_{\mathbf{B}_j}^T {}^{\mathbf{B}_j}F \right) \\
&\quad - \sum_{i=1}^{n_o} \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{S}_i} {}^{\mathbf{S}_i}F \\
&\quad + \sum_{j=1}^{n_c} \left(\sum_{k \in \mathcal{C}_{1j}} \mathcal{J}_{jk}^T \mathbf{z}^{T\mathbf{B}_{jk}}F + \sum_{k \in \mathcal{C}_{3j}} \mathcal{J}_{jk}^T \mathbf{Z}^{T\mathbf{B}_{jk}}F \right) \\
&= - \sum_{i=1}^{n_o} \mathcal{J}_{\mathbf{O}_i}^T {}^{\mathbf{O}_i}\mathbf{U}_{\mathbf{S}_i} {}^{\mathbf{S}_i}F \\
&\quad + \sum_{j=1}^{n_c} \left(\sum_{k \in \mathcal{C}_{1j}} \mathcal{J}_{jk}^T \mathbf{z}^{T\mathbf{B}_{jk}}F + \sum_{k \in \mathcal{C}_{3j}} \mathcal{J}_{jk}^T \mathbf{Z}^{T\mathbf{B}_{jk}}F \right) .
\end{aligned} \tag{B.15}$$

B.3 Proof of Lemma 6.1

With appropriate frame substitutions, it follows from (6.3)–(6.5), (6.16)–(6.24), and Lemma 4.1 that

$$\begin{aligned}
\dot{\nu}_{\mathbf{T}} \leqslant & - \left({}^{\mathbf{T}}V_r - {}^{\mathbf{T}}V \right)^T \mathbf{K}_{\mathbf{T}} \left({}^{\mathbf{T}}V_r - {}^{\mathbf{T}}V \right) \\
& + \left({}^{\mathbf{T}}V_r - {}^{\mathbf{T}}V \right)^T \left({}^{\mathbf{T}}F_r^* - {}^{\mathbf{T}}F^* \right)
\end{aligned} \tag{B.16}$$

$$\dot{\nu}_{\mathbf{A}} \leqslant -\left({}^{\mathbf{A}}V_r - {}^{\mathbf{A}}V\right)^T \mathbf{K}_{\mathbf{A}} \left({}^{\mathbf{A}}V_r - {}^{\mathbf{A}}V\right) + \left({}^{\mathbf{A}}V_r - {}^{\mathbf{A}}V\right)^T \left({}^{\mathbf{A}}F_r^* - {}^{\mathbf{A}}F^*\right) \tag{B.17}$$

$$\dot{\nu}_{\mathbf{B}} \leqslant -\left({}^{\mathbf{B}}V_r - {}^{\mathbf{B}}V\right)^T \mathbf{K}_{\mathbf{B}} \left({}^{\mathbf{B}}V_r - {}^{\mathbf{B}}V\right) + \left({}^{\mathbf{B}}V_r - {}^{\mathbf{B}}V\right)^T \left({}^{\mathbf{B}}F_r^* - {}^{\mathbf{B}}F^*\right) \tag{B.18}$$

hold.

Furthermore, in view of (2.85), (6.1), (6.2), (6.6)–(6.12), (6.14), (6.15), and (6.25)–(6.29), it yields

$$\begin{aligned}
&\left({}^{\mathbf{T}}V_r - {}^{\mathbf{T}}V\right)^T \left({}^{\mathbf{T}}F_r^* - {}^{\mathbf{T}}F^*\right) \\
&\quad = \left({}^{\mathbf{T}}V_r - {}^{\mathbf{T}}V\right)^T \left[-\left({}^{\mathbf{T}}F_r - {}^{\mathbf{T}}F\right) + \left({}^{\mathbf{T}}F_{qr} - {}^{\mathbf{T}}F_q\right)\right] \\
&\quad = -\left({}^{\mathbf{T}}V_r - {}^{\mathbf{T}}V\right)^T \left({}^{\mathbf{T}}F_r - {}^{\mathbf{T}}F\right) \\
&\qquad + \left[\mathbf{z}_\tau \left(\dot{q}_r - \dot{q}\right) + {}^{\mathbf{B}}\mathbf{U}_{\mathbf{T}}^T \left({}^{\mathbf{B}}V_r - {}^{\mathbf{B}}V\right)\right]^T \left({}^{\mathbf{T}}F_{qr} - {}^{\mathbf{T}}F_q\right) \\
&\quad = -p_{\mathbf{T}} + \varrho\left(\dot{q}_r - \dot{q}\right)\left(\tau_{td} - \tau_t\right) - \varrho k_{vq}\left(\dot{q}_r - \dot{q}\right)^2 \\
&\qquad + \left(\dot{q}_r - \dot{q}\right)\mathbf{Y}_b(\dot{q})\left(\boldsymbol{\theta}_b - \hat{\boldsymbol{\theta}}_b\right) \\
&\qquad + ({}^{\mathbf{B}}V_r - {}^{\mathbf{B}}V)^T {}^{\mathbf{B}}\mathbf{U}_{\mathbf{T}}({}^{\mathbf{T}}F_{qr} - {}^{\mathbf{T}}F_q)
\end{aligned} \tag{B.19}$$

$$\begin{aligned}
&\left({}^{\mathbf{A}}V_r - {}^{\mathbf{A}}V\right)^T \left({}^{\mathbf{A}}F_r^* - {}^{\mathbf{A}}F^*\right) \\
&\quad = \left[\mathbf{z}_\tau \varrho(\dot{\phi}_r - \dot{\phi}) + {}^{\mathbf{B}}\mathbf{U}_{\mathbf{A}}^T({}^{\mathbf{B}}V_r - {}^{\mathbf{B}}V)\right]^T ({}^{\mathbf{A}}F_{\phi r} - {}^{\mathbf{A}}F_\phi) \\
&\quad = \varrho(\dot{\phi}_r - \dot{\phi})(\tau_d - \tau) - \varrho(\dot{\phi}_r - \dot{\phi})(\tau_{td} - \tau_t) \\
&\qquad - \varrho k_{v\phi}\left(\dot{\phi}_r - \dot{\phi}\right)^2 + \varrho(\dot{\phi}_r - \dot{\phi})\mathbf{Y}_m(\dot{\phi})\left(\boldsymbol{\theta}_m - \hat{\boldsymbol{\theta}}_m\right) \\
&\qquad + ({}^{\mathbf{B}}V_r - {}^{\mathbf{B}}V)^T {}^{\mathbf{B}}\mathbf{U}_{\mathbf{A}}({}^{\mathbf{A}}F_{\phi r} - {}^{\mathbf{A}}F_\phi)
\end{aligned} \tag{B.20}$$

$$\begin{aligned}
&\left({}^{\mathbf{B}}V_r - {}^{\mathbf{B}}V\right)^T \left({}^{\mathbf{B}}F_r^* - {}^{\mathbf{B}}F^*\right) \\
&\quad = \left({}^{\mathbf{B}}V_r - {}^{\mathbf{B}}V\right)^T \left[({}^{\mathbf{B}}F_r - {}^{\mathbf{B}}F)\right. \\
&\qquad\qquad \left. - {}^{\mathbf{B}}\mathbf{U}_{\mathbf{T}}({}^{\mathbf{T}}F_{qr} - {}^{\mathbf{T}}F_q) - {}^{\mathbf{B}}\mathbf{U}_{\mathbf{A}}({}^{\mathbf{A}}F_{\phi r} - {}^{\mathbf{A}}F_\phi)\right] \\
&\quad = p_{\mathbf{B}} - ({}^{\mathbf{B}}V_r - {}^{\mathbf{B}}V)^T {}^{\mathbf{B}}\mathbf{U}_{\mathbf{T}}({}^{\mathbf{T}}F_{qr} - {}^{\mathbf{T}}F_q) \\
&\qquad - ({}^{\mathbf{B}}V_r - {}^{\mathbf{B}}V)^T {}^{\mathbf{B}}\mathbf{U}_{\mathbf{A}}({}^{\mathbf{A}}F_{\phi r} - {}^{\mathbf{A}}F_\phi).
\end{aligned} \tag{B.21}$$

Moreover, it follows from (6.35)–(6.38) and Lemma 2.9 that

$$\begin{aligned}
&\frac{d}{dt}\left(\frac{1}{2}\sum_\gamma \left(\theta_{b\gamma} - \hat{\theta}_{b\gamma}\right)^2 / \rho_{b\gamma}\right) + \left(\dot{q}_r - \dot{q}\right)\mathbf{Y}_b(\dot{q})\left(\boldsymbol{\theta}_b - \hat{\boldsymbol{\theta}}_b\right) \\
&\quad = -\sum_\gamma \left(\theta_{b\gamma} - \hat{\theta}_{b\gamma}\right)\dot{\hat{\theta}}_{b\gamma}/\rho_{b\gamma} + \sum_\gamma s_{b\gamma}\left(\theta_{b\gamma} - \hat{\theta}_{b\gamma}\right) \\
&\quad = \sum_\gamma \left(\theta_{b\gamma} - \hat{\theta}_{b\gamma}\right)\left(s_{b\gamma} - \dot{\hat{\theta}}_{b\gamma}/\rho_{b\gamma}\right) \leqslant 0
\end{aligned} \tag{B.22}$$

$$\frac{d}{dt}\left(\frac{1}{2}\sum_{\gamma}\left(\theta_{m\gamma}-\hat{\theta}_{m\gamma}\right)^2/\rho_{m\gamma}\right)+\varrho(\dot{\phi}_r-\dot{\phi})\mathbf{Y}_m(\dot{\phi})\left(\boldsymbol{\theta}_m-\hat{\boldsymbol{\theta}}_m\right)$$
$$=-\sum_{\gamma}\left(\theta_{m\gamma}-\hat{\theta}_{m\gamma}\right)\dot{\hat{\theta}}_{m\gamma}/\rho_{m\gamma}+\sum_{\gamma}s_{m\gamma}\left(\theta_{m\gamma}-\hat{\theta}_{m\gamma}\right)$$
$$=\sum_{\gamma}\left(\theta_{m\gamma}-\hat{\theta}_{m\gamma}\right)\left(s_{m\gamma}-\dot{\hat{\theta}}_{m\gamma}/\rho_{m\gamma}\right)\leqslant 0 \tag{B.23}$$

hold.

Finally, substituting (B.19)–(B.21) into (B.16)–(B.18) and using (B.22) and (B.23) ensures that the time derivative of ν defined by (6.39) can be expressed as (6.43).

B.4 Proof of Lemma 7.3

When $\dot{x}=0$, re-write (7.137) as

$$\dot{f}_{ps}=\beta u\left[c_{p1}\upsilon\left(p_s-\frac{f_{ps}-xf_p}{s_a l_o}\right)-c_{n2}\upsilon\left(\frac{f_{ps}+(l_o-x)f_p}{s_b l_o}-p_r\right)\right]\varepsilon(u)$$
$$+\beta u\left[c_{n1}\upsilon\left(\frac{f_{ps}-xf_p}{s_a l_o}-p_r\right)-c_{p2}\upsilon\left(p_s-\frac{f_{ps}+(l_o-x)f_p}{s_b l_o}\right)\right]\varepsilon(-u). \tag{B.24}$$

The fact that function $\upsilon(x)$ is *monotonically increasing* will be used in the proof below.

With $u>0$, $f_{ps}\leqslant p_r s_b l_o+\gamma_p l_o$, $\delta_c\geqslant 1$, $p_r>0$, and $0<s_a<s_b$, it follows from (7.131), (7.138), (7.139), Assumption 5, and (B.24) that

$$\dot{f}_{ps}=\beta u\left[c_{p1}\upsilon\left(p_s-\frac{f_{ps}-xf_p}{s_a l_o}\right)-c_{n2}\upsilon\left(\frac{f_{ps}+(l_o-x)f_p}{s_b l_o}-p_r\right)\right]$$
$$>\beta u\left[c_{p1}\upsilon\left(2\delta_c^2\frac{2\gamma_p+s_b p_r}{s_a}-\frac{p_r s_b l_o+\gamma_p l_o-xf_p}{s_a l_o}\right)\right.$$
$$\left.-c_{n2}\upsilon\left(\frac{p_r s_b l_o+\gamma_p l_o+(l_o-x)f_p}{s_b l_o}-p_r\right)\right]$$
$$>\beta u\left[c_{p1}\upsilon\left(\delta_c^2\frac{4\gamma_p+2s_b p_r}{s_a}-\frac{2\gamma_p+p_r s_b}{s_a}\right)-c_{n2}\upsilon\left(\frac{2\gamma_p}{s_b}\right)\right]$$
$$\geqslant\beta u\left[c_{p1}\upsilon\left(\delta_c^2\frac{2\gamma_p+s_b p_r}{s_a}\right)-c_{n2}\upsilon\left(\frac{2\gamma_p}{s_b}\right)\right]$$
$$=\beta u\left[c_{p1}\delta_c\upsilon\left(\frac{2\gamma_p+s_b p_r}{s_a}\right)-c_{n2}\upsilon\left(\frac{2\gamma_p}{s_b}\right)\right]$$
$$>0 \tag{B.25}$$

holds.

With $u < 0$, $f_{ps} \leqslant p_r s_b l_o + \gamma_p l_o$, $\delta_c \geqslant 1$, $p_r > 0$, and $0 < s_a < s_b$, it follows from (7.131), (7.138), (7.139), Assumption 5, and (B.24) that

$$\begin{aligned}
\dot{f}_{ps} &= \beta u \left[c_{n1} \upsilon \left(\frac{f_{ps} - x f_p}{s_a l_o} - p_r \right) - c_{p2} \upsilon \left(p_s - \frac{f_{ps} + (l_o - x) f_p}{s_b l_o} \right) \right] \\
&> \beta u \left[c_{n1} \upsilon \left(\frac{p_r s_b l_o + \gamma_p l_o - x f_p}{s_a l_o} - p_r \right) \right. \\
&\qquad \left. - c_{p2} \upsilon \left(2\delta_c^2 \frac{2\gamma_p + s_b p_r}{s_a} - \frac{p_r s_b l_o + \gamma_p l_o + (l_o - x) f_p}{s_b l_o} \right) \right] \\
&> \beta u \left[c_{n1} \upsilon \left(\frac{2\gamma_p + p_r s_b}{s_a} \right) - c_{p2} \upsilon \left(\delta_c^2 \frac{4\gamma_p + 2 s_b p_r}{s_a} - \frac{2\gamma_p + p_r s_b}{s_b} \right) \right] \\
&> \beta u \left[c_{n1} \upsilon \left(\frac{2\gamma_p + p_r s_b}{s_a} \right) - c_{p2} \delta_c \upsilon \left(\frac{2\gamma_p + s_b p_r}{s_a} \right) \right] \\
&\geqslant 0
\end{aligned} \tag{B.26}$$

holds. The inequalities (B.25) and (B.26) prove (7.140).

With $u > 0$, $f_{ps} \geqslant p_s s_b l_o - \gamma_p l_o$, $\delta_c \geqslant 1$, $p_r > 0$, and $0 < s_a < s_b$, it follows from (7.131), (7.138), (7.139), Assumption 5, and (B.24) that

$$\begin{aligned}
\dot{f}_{ps} &= \beta u \left[c_{p1} \upsilon \left(p_s - \frac{f_{ps} - x f_p}{s_a l_o} \right) - c_{n2} \upsilon \left(\frac{f_{ps} + (l_o - x) f_p}{s_b l_o} - p_r \right) \right] \\
&\leqslant \beta u \left[c_{p1} \upsilon \left(p_s - \frac{p_s s_b l_o - \gamma_p l_o - x f_p}{s_a l_o} \right) \right. \\
&\qquad \left. - c_{n2} \upsilon \left(\frac{p_s s_b l_o - \gamma_p l_o + (l_o - x) f_p}{s_b l_o} - p_r \right) \right] \\
&< \beta u \left[c_{p1} \upsilon \left(p_s - \frac{p_s s_b - 2\gamma_p}{s_a} \right) - c_{n2} \upsilon \left(p_s - \frac{2\gamma_p}{s_b} - p_r \right) \right] \\
&< \beta u \left[c_{p1} \upsilon \left(\frac{2\gamma_p}{s_a} \right) - c_{n2} \upsilon \left(2\delta_c^2 \frac{2\gamma_p + s_b p_r}{s_a} - \frac{2\gamma_p}{s_b} - p_r \right) \right] \\
&< \beta u \left[c_{p1} \upsilon \left(\frac{2\gamma_p}{s_a} \right) - c_{n2} \delta_c \upsilon \left(\frac{2\gamma_p + s_b p_r}{s_a} \right) \right] \\
&< 0
\end{aligned} \tag{B.27}$$

holds.

With $u < 0$, $f_{ps} \geqslant p_s s_b l_o - \gamma_p l_o$, $\delta_c \geqslant 1$, $p_r > 0$, and $0 < s_a < s_b$, it follows from (7.131), (7.138), (7.139), Assumption 5, and (B.24) that

$$\begin{aligned}
\dot{f}_{ps} &= \beta u \left[c_{n1} \upsilon \left(\frac{f_{ps} - x f_p}{s_a l_o} - p_r \right) - c_{p2} \upsilon \left(p_s - \frac{f_{ps} + (l_o - x) f_p}{s_b l_o} \right) \right] \\
&\leqslant \beta u \left[c_{n1} \upsilon \left(\frac{p_s s_b l_o - \gamma_p l_o - x f_p}{s_a l_o} - p_r \right) \right. \\
&\qquad \left. - c_{p2} \upsilon \left(p_s - \frac{p_s s_b l_o - \gamma_p l_o + (l_o - x) f_p}{s_b l_o} \right) \right]
\end{aligned}$$

$$
\begin{aligned}
&< \beta u \left[c_{n1} \upsilon \left(\frac{p_s s_b - 2\gamma_p}{s_a} - p_r \right) - c_{p2} \upsilon \left(p_s - \frac{p_s s_b - 2\gamma_p}{s_b} \right) \right] \\
&< \beta u \left[c_{n1} \upsilon \left(2\delta_c^2 \frac{2\gamma_p + s_b p_r}{s_a} \frac{s_b}{s_a} - \frac{2\gamma_p}{s_a} - p_r \right) - c_{p2} \upsilon \left(\frac{2\gamma_p}{s_b} \right) \right] \\
&< \beta u \left[c_{n1} \delta_c \upsilon \left(\frac{2\gamma_p + s_b p_r}{s_a} \frac{s_b}{s_a} \right) - c_{p2} \upsilon \left(\frac{2\gamma_p}{s_b} \right) \right] \\
&< 0
\end{aligned}
\tag{B.28}
$$

holds. The inequalities (B.27) and (B.28) prove (7.141).

Equation (7.142) is obvious, in view of (B.24) with $u = 0$.

B.5 Proof of Lemma 7.4

Re-write (7.107) as

$$
\begin{aligned}
u &= -\frac{u_{fd}}{\frac{\hat{c}_{p1}\upsilon(p_s - p_a)}{l_o - x} + \frac{\hat{c}_{n2}\upsilon(p_b - p_r)}{x}} \varepsilon(u) - \frac{u_{fd}}{\frac{\hat{c}_{n1}\upsilon(p_a - p_r)}{l_o - x} + \frac{\hat{c}_{p2}\upsilon(p_s - p_b)}{x}} \varepsilon(-u) \\
&= -\frac{u_{fd}}{\left[\frac{\hat{c}_{p1}\upsilon(p_s - p_a)}{l_o - x} + \frac{\hat{c}_{n2}\upsilon(p_b - p_r)}{x} \right] \varepsilon(u) + \left[\frac{\hat{c}_{n1}\upsilon(p_a - p_r)}{l_o - x} + \frac{\hat{c}_{p2}\upsilon(p_s - p_b)}{x} \right] \varepsilon(-u)}
\end{aligned}
\tag{B.29}
$$

where $\varepsilon(u) = \varepsilon(-u_{fd})$ and $\varepsilon(-u) = \varepsilon(u_{fd})$ are used.

Substituting (7.106) into (B.29) yields

$$
u(t) = u_s(t) + k_u(t)\dot{x}(t) \tag{B.30}
$$

with $k_u(t) \in \mathbb{R}$ and $u_s(t) \in \mathbb{R}$ being defined by (7.145) and (7.149), respectively.

The following lemma will be used to complete the proof.

Lemma B.1. *Given three positive numbers, a, b, and c, the inequality*

$$
a \geqslant 2b + 2c^2 \tag{B.31}
$$

implies

$$
\sqrt{a} - \sqrt{b} - c \geqslant 0. \tag{B.32}
$$

Proof: Note that the inequality $(\sqrt{b} - c)^2 \geqslant 0$ leads to $b + c^2 \geqslant 2\sqrt{b}c$. Inequality (B.31) can be rewritten as

$$
\begin{aligned}
a &\geqslant 2b + 2c^2 \\
&= b + c^2 + b + c^2 \\
&\geqslant b + c^2 + 2\sqrt{b}c \\
&= (\sqrt{b} + c)^2.
\end{aligned}
$$

Thus, $a \geqslant 2b + 2c^2$ implies $a \geqslant (\sqrt{b} + c)^2$ and further implies $\sqrt{a} \geqslant \sqrt{b} + c$ and $\sqrt{a} - \sqrt{b} - c \geqslant 0$, since $\sqrt{a}$ is monotonically increasing for $a > 0$. ■

By using (B.30), re-write (7.137) as

$$\begin{aligned}\dot{f}_{ps} = \beta u \left[c_{p1} \upsilon \left(p_s - \frac{f_{ps} - x f_p}{s_a l_o} \right) - c_{n2} \upsilon \left(\frac{f_{ps} + (l_o - x) f_p}{s_b l_o} - p_r \right) \right] \varepsilon(u) \\ + \beta u \left[c_{n1} \upsilon \left(\frac{f_{ps} - x f_p}{s_a l_o} - p_r \right) - c_{p2} \upsilon \left(p_s - \frac{f_{ps} + (l_o - x) f_p}{s_b l_o} \right) \right] \varepsilon(-u) \\ + \left[\beta (s_a - s_b) + f_p \right] \frac{u - u_s}{k_u}. \qquad \text{(B.33)}\end{aligned}$$

The fact that function $\upsilon(x)$ is *monotonically increasing* will be used in the proof below.

With $u \geqslant \frac{1}{\alpha_s}|u_s|$, $f_{ps} \leqslant p_r s_b l_o + \gamma_p l_o$, $\delta_c \geqslant 1$, $p_r > 0$, and $0 < s_a < s_b$, it follows from (7.131), (7.138), (7.143), (B.33), Assumption 5, and Lemma B.1 that

$$\begin{aligned}\dot{f}_{ps} &= \beta u \left[c_{p1} \upsilon \left(p_s - \frac{f_{ps} - x f_p}{s_a l_o} \right) - c_{n2} \upsilon \left(\frac{f_{ps} + (l_o - x) f_p}{s_b l_o} - p_r \right) \right] \\ &\quad + \left[\beta (s_a - s_b) + f_p \right] \frac{u - u_s}{k_u} \\ &\geqslant \beta u \left[c_{p1} \upsilon \left(p_s - \frac{p_r s_b l_o + \gamma_p l_o - x f_p}{s_a l_o} \right) \right. \\ &\qquad \left. - c_{n2} \upsilon \left(\frac{p_r s_b l_o + \gamma_p l_o + (l_o - x) f_p}{s_b l_o} - p_r \right) \right] \\ &\quad - \left[\beta (s_b - s_a) + \gamma_p \right] \frac{|u| + |u_s|}{|k_u|} \\ &> \beta u \left[c_{p1} \upsilon \left(p_s - \frac{p_r s_b + 2\gamma_p}{s_a} \right) - c_{n2} \upsilon \left(\frac{p_r s_b + 2\gamma_p}{s_b} - p_r \right) \right] \\ &\quad - \left[\beta (s_b - s_a) + \gamma_p \right] \frac{(1 + \alpha_s)|u|}{|k_u|} \\ &> \beta u \left[c_{p1} \upsilon \left(2\delta_c^2 \frac{2\gamma_p + s_b p_r}{s_a} + 2 \left\{ \frac{\left[\beta (s_b - s_a) + \gamma_p \right] (1 + \alpha_s)}{|k_u| \beta \min\{c_{p1}, c_{n1}, c_{p2}, c_{n2}\}} \right\}^2 \right) \right. \\ &\qquad \left. - c_{n2} \upsilon \left(\frac{2\gamma_p}{s_b} \right) \right] - \left[\beta (s_b - s_a) + \gamma_p \right] \frac{(1 + \alpha_s)|u|}{|k_u|} \\ &= \beta u \left\{ c_{p1} \delta_c \upsilon \left(2 \frac{2\gamma_p + s_b p_r}{s_a} + 2 \left\{ \frac{\left[\beta (s_b - s_a) + \gamma_p \right] (1 + \alpha_s)}{|k_u| \beta \max\{c_{p1}, c_{n1}, c_{p2}, c_{n2}\}} \right\}^2 \right) \right. \\ &\qquad \left. - c_{n2} \upsilon \left(\frac{2\gamma_p}{s_b} \right) - \left[\beta (s_b - s_a) + \gamma_p \right] \frac{(1 + \alpha_s)}{|k_u| \beta} \right\} \\ &\geqslant \beta u c_{p1} \delta_c \left\{ \upsilon \left(2 \frac{2\gamma_p + s_b p_r}{s_a} + 2 \left\{ \frac{\left[\beta (s_b - s_a) + \gamma_p \right] (1 + \alpha_s)}{|k_u| \beta \max\{c_{p1}, c_{n1}, c_{p2}, c_{n2}\}} \right\}^2 \right) \right.\end{aligned}$$

$$
\begin{aligned}
&\qquad -\upsilon\left(\frac{2\gamma_p}{s_b}\right)-[\beta(s_b-s_a)+\gamma_p]\frac{(1+\alpha_s)}{|k_u|\beta c_{p1}\delta_c}\Bigg\} \\
&> \beta u c_{p1}\delta_c\left\{\upsilon\left(2\frac{2\gamma_p+s_b p_r}{s_a}+2\left\{\frac{[\beta(s_b-s_a)+\gamma_p](1+\alpha_s)}{|k_u|\beta\max\{c_{p1},c_{n1},c_{p2},c_{n2}\}}\right\}^2\right)\right. \\
&\qquad \left.-\upsilon\left(\frac{2\gamma_p+s_b p_r}{s_a}\right)-[\beta(s_b-s_a)+\gamma_p]\frac{(1+\alpha_s)}{|k_u|\beta c_{p1}\delta_c}\right\} \\
&\geqslant 0
\end{aligned}
\tag{B.34}
$$

holds. The last inequality uses (7.91) and Lemma B.1 after equalizing

$$
a = 2\frac{2\gamma_p+s_b p_r}{s_a}+2\left\{\frac{[\beta(s_b-s_a)+\gamma_p](1+\alpha_s)}{|k_u|\beta\max\{c_{p1},c_{n1},c_{p2},c_{n2}\}}\right\}^2 \tag{B.35}
$$

$$
b = \frac{2\gamma_p+s_b p_r}{s_a} \tag{B.36}
$$

$$
c = [\beta(s_b-s_a)+\gamma_p]\frac{(1+\alpha_s)}{|k_u|\beta c_{p1}\delta_c}. \tag{B.37}
$$

With $u \leqslant -\frac{1}{\alpha_s}|u_s|$, $f_{ps} \leqslant p_r s_b l_o + \gamma_p l_o$, $\delta_c \geqslant 1$, $p_r > 0$, and $0 < s_a < s_b$, it follows from (7.131), (7.138), (7.143), (B.33), Assumption 5, and Lemma B.1 that

$$
\begin{aligned}
\dot{f}_{ps} &= \beta u\left[c_{n1}\upsilon\left(\frac{f_{ps}-xf_p}{s_a l_o}-p_r\right)-c_{p2}\upsilon\left(p_s-\frac{f_{ps}+(l_o-x)f_p}{s_b l_o}\right)\right] \\
&\quad +[\beta(s_a-s_b)+f_p]\frac{u-u_s}{k_u} \\
&\geqslant \beta u\left[c_{n1}\upsilon\left(\frac{\gamma_p l_o+p_r s_b l_o-xf_p}{s_a l_o}-p_r\right)\right. \\
&\qquad \left.-c_{p2}\upsilon\left(p_s-\frac{\gamma_p l_o+p_r s_b l_o+(l_o-x)f_p}{s_b l_o}\right)\right] \\
&\quad -[\beta(s_b-s_a)+\gamma_p]\frac{|u|+|u_s|}{|k_u|} \\
&> \beta u\left[c_{n1}\upsilon\left(\frac{2\gamma_p+p_r s_b}{s_a}-p_r\right)-c_{p2}\upsilon\left(p_s-\frac{2\gamma_p+p_r s_b}{s_b}\right)\right] \\
&\quad -[\beta(s_b-s_a)+\gamma_p]\frac{(1+\alpha_s)|u|}{|k_u|} \\
&> \beta u\left[c_{n1}\upsilon\left(\frac{2\gamma_p+p_r s_b}{s_a}-p_r\right)\right. \\
&\qquad \left.-c_{p2}\upsilon\left(2\delta_c^2\frac{2\gamma_p+s_b p_r}{s_a}+2\left\{\frac{[\beta(s_b-s_a)+\gamma_p](1+\alpha_s)}{|k_u|\beta\min\{c_{p1},c_{n1},c_{p2},c_{n2}\}}\right\}^2\right)\right] \\
&\quad -[\beta(s_b-s_a)+\gamma_p]\frac{(1+\alpha_s)|u|}{|k_u|}
\end{aligned}
$$

$$
\begin{aligned}
&= -\beta u \left\{ -c_{n1}\upsilon \left(\frac{2\gamma_p + p_r s_b}{s_a} - p_r \right) - [\beta(s_b - s_a) + \gamma_p] \frac{(1+\alpha_s)}{|k_u|\beta} \right. \\
&\qquad \left. + c_{p2}\delta_c \upsilon \left(2\frac{2\gamma_p + s_b p_r}{s_a} + 2\left\{ \frac{[\beta(s_b - s_a) + \gamma_p](1+\alpha_s)}{|k_u|\beta \max\{c_{p1}, c_{n1}, c_{p2}, c_{n2}\}} \right\}^2 \right) \right\} \\
&\geqslant -\beta u c_{p2}\delta_c \left\{ -\upsilon \left(\frac{2\gamma_p + p_r s_b}{s_a} - p_r \right) - [\beta(s_b - s_a) + \gamma_p] \frac{(1+\alpha_s)}{|k_u|\beta c_{p2}\delta_c} \right. \\
&\qquad \left. + \upsilon \left(2\frac{2\gamma_p + s_b p_r}{s_a} + 2\left\{ \frac{[\beta(s_b - s_a) + \gamma_p](1+\alpha_s)}{|k_u|\beta \max\{c_{p1}, c_{n1}, c_{p2}, c_{n2}\}} \right\}^2 \right) \right\} \\
&> -\beta u c_{p2}\delta_c \left\{ -\upsilon \left(\frac{2\gamma_p + p_r s_b}{s_a} \right) - [\beta(s_b - s_a) + \gamma_p] \frac{(1+\alpha_s)}{|k_u|\beta c_{p2}\delta_c} \right. \\
&\qquad \left. + \upsilon \left(2\frac{2\gamma_p + s_b p_r}{s_a} + 2\left\{ \frac{[\beta(s_b - s_a) + \gamma_p](1+\alpha_s)}{|k_u|\beta \max\{c_{p1}, c_{n1}, c_{p2}, c_{n2}\}} \right\}^2 \right) \right\} \\
&\geqslant 0
\end{aligned} \tag{B.38}
$$

holds. The last inequality uses (7.91), (B.35)-(B.37), and Lemma B.1. This proves (7.147).

With $u \geqslant \frac{1}{\alpha_s}|u_s|$, $f_{ps} \geqslant p_s s_b l_o - \gamma_p l_o$, $\delta_c \geqslant 1$, $p_r > 0$, and $0 < s_a < s_b$, it follows from (7.131), (7.138), (7.143), (B.33), Assumption 5, and Lemma B.1 that

$$
\begin{aligned}
\dot{f}_{ps} &= \beta u \left[c_{p1}\upsilon \left(p_s - \frac{f_{ps} - x f_p}{s_a l_o} \right) - c_{n2}\upsilon \left(\frac{f_{ps} + (l_o - x) f_p}{s_b l_o} - p_r \right) \right] \\
&\quad + [\beta(s_a - s_b) + f_p] \frac{u - u_s}{k_u} \\
&\leqslant \beta u \left[c_{p1}\upsilon \left(p_s - \frac{p_s s_b l_o - \gamma_p l_o - x f_p}{s_a l_o} \right) \right. \\
&\qquad \left. - c_{n2}\upsilon \left(\frac{p_s s_b l_o - \gamma_p l_o + (l_o - x) f_p}{s_b l_o} - p_r \right) \right] \\
&\quad + [\beta(s_b - s_a) + \gamma_p] \frac{|u| + |u_s|}{|k_u|} \\
&< \beta u \left[c_{p1}\upsilon \left(\frac{2\gamma_p}{s_a} \right) - c_{n2}\upsilon \left(p_s - \frac{2\gamma_p}{s_b} - p_r \right) \right] \\
&\quad + [\beta(s_b - s_a) + \gamma_p] \frac{(1+\alpha_s)|u|}{|k_u|} \\
&< \beta u \left[c_{p1}\upsilon \left(\frac{2\gamma_p}{s_a} \right) \right. \\
&\qquad \left. - c_{n2}\upsilon \left(2\delta_c^2 \frac{2\gamma_p + s_b p_r}{s_a} + 2\left\{ \frac{[\beta(s_b - s_a) + \gamma_p](1+\alpha_s)}{|k_u|\beta \min\{c_{p1}, c_{n1}, c_{p2}, c_{n2}\}} \right\}^2 \right) \right] \\
&\quad + [\beta(s_b - s_a) + \gamma_p] \frac{(1+\alpha_s)|u|}{|k_u|}
\end{aligned}
$$

$$
\begin{aligned}
&= \beta u \left\{ c_{p1} \upsilon \left(\frac{2\gamma_p}{s_a} \right) + [\beta(s_b - s_a) + \gamma_p] \frac{(1+\alpha_s)}{|k_u|\beta} \right. \\
&\qquad \left. - c_{n2}\delta_c \upsilon \left(2\frac{2\gamma_p + s_b p_r}{s_a} + 2\left\{ \frac{[\beta(s_b - s_a) + \gamma_p](1+\alpha_s)}{|k_u|\beta \max\{c_{p1}, c_{n1}, c_{p2}, c_{n2}\}} \right\}^2 \right) \right\} \\
&\leqslant -\beta u c_{n2}\delta_c \left\{ -\upsilon \left(\frac{2\gamma_p}{s_a} \right) - [\beta(s_b - s_a) + \gamma_p] \frac{(1+\alpha_s)}{|k_u|\beta c_{n2}\delta_c} \right. \\
&\qquad \left. + \upsilon \left(2\frac{2\gamma_p + s_b p_r}{s_a} + 2\left\{ \frac{[\beta(s_b - s_a) + \gamma_p](1+\alpha_s)}{|k_u|\beta \max\{c_{p1}, c_{n1}, c_{p2}, c_{n2}\}} \right\}^2 \right) \right\} \\
&< -\beta u c_{n2}\delta_c \left\{ -\upsilon \left(\frac{2\gamma_p + s_b p_r}{s_a} \right) - [\beta(s_b - s_a) + \gamma_p] \frac{(1+\alpha_s)}{|k_u|\beta c_{n2}\delta_c} \right. \\
&\qquad \left. + \upsilon \left(2\frac{2\gamma_p + s_b p_r}{s_a} + 2\left\{ \frac{[\beta(s_b - s_a) + \gamma_p](1+\alpha_s)}{|k_u|\beta \max\{c_{p1}, c_{n1}, c_{p2}, c_{n2}\}} \right\}^2 \right) \right\} \\
&\leqslant 0
\end{aligned}
\tag{B.39}
$$

holds. The last inequality uses (7.91), (B.35)–(B.37), and Lemma B.1.

With $u \leqslant -\frac{1}{\alpha_s}|u_s|$, $f_{ps} \geqslant p_s s_b l_o - \gamma_p l_o$, $\delta_c \geqslant 1$, $p_r > 0$, and $0 < s_a < s_b$, it follows from (7.131), (7.138), (7.143), (B.33), Assumption 5, and Lemma B.1 that

$$
\begin{aligned}
\dot{f}_{ps} &= \beta u \left[c_{n1} \upsilon \left(\frac{f_{ps} - x f_p}{s_a l_o} - p_r \right) - c_{p2} \upsilon \left(p_s - \frac{f_{ps} + (l_o - x) f_p}{s_b l_o} \right) \right] \\
&\quad + [\beta(s_a - s_b) + f_p] \frac{u - u_s}{k_u} \\
&\leqslant \beta u \left[c_{n1} \upsilon \left(\frac{p_s s_b l_o - \gamma_p l_o - x f_p}{s_a l_o} - p_r \right) \right. \\
&\qquad \left. - c_{p2} \upsilon \left(p_s - \frac{p_s s_b l_o - \gamma_p l_o + (l_o - x) f_p}{s_b l_o} \right) \right] \\
&\quad + [\beta(s_b - s_a) + \gamma_p] \frac{|u| + |u_s|}{|k_u|} \\
&< \beta u \left[c_{n1} \upsilon \left(p_s - \frac{2\gamma_p}{s_a} - p_r \right) - c_{p2} \upsilon \left(\frac{2\gamma_p}{s_b} \right) \right] \\
&\quad + [\beta(s_b - s_a) + \gamma_p] \frac{(1+\alpha_s)|u|}{|k_u|} \\
&< \beta u \left[c_{n1} \upsilon \left(2\delta_c^2 \frac{2\gamma_p + s_b p_r}{s_a} + 2\left\{ \frac{[\beta(s_b - s_a) + \gamma_p](1+\alpha_s)}{|k_u|\beta \min\{c_{p1}, c_{n1}, c_{p2}, c_{n2}\}} \right\}^2 \right) \right. \\
&\qquad \left. - c_{p2} \upsilon \left(\frac{2\gamma_p}{s_b} \right) \right] + [\beta(s_b - s_a) + \gamma_p] \frac{(1+\alpha_s)|u|}{|k_u|} \\
&= \beta u \left\{ c_{n1}\delta_c \upsilon \left(2\frac{2\gamma_p + s_b p_r}{s_a} + 2\left\{ \frac{[\beta(s_b - s_a) + \gamma_p](1+\alpha_s)}{|k_u|\beta \max\{c_{p1}, c_{n1}, c_{p2}, c_{n2}\}} \right\}^2 \right) \right.
\end{aligned}
$$

$$
\begin{aligned}
&-c_{p2}\upsilon\left(\frac{2\gamma_p}{s_b}\right)-\left[\beta(s_b-s_a)+\gamma_p\right]\frac{(1+\alpha_s)}{|k_u|\beta}\Bigg\}\\
\leqslant{}& \beta u c_{n1}\delta_c\left\{\upsilon\left(2\frac{2\gamma_p+s_bp_r}{s_a}+2\left\{\frac{\left[\beta(s_b-s_a)+\gamma_p\right](1+\alpha_s)}{|k_u|\beta\max\{c_{p1},c_{n1},c_{p2},c_{n2}\}}\right\}^2\right)\right.\\
&-\upsilon\left(\frac{2\gamma_p}{s_b}\right)-\left[\beta(s_b-s_a)+\gamma_p\right]\frac{(1+\alpha_s)}{|k_u|\beta c_{n1}\delta_c}\Bigg\}\\
<{}& \beta u c_{n1}\delta_c\left\{\upsilon\left(2\frac{2\gamma_p+s_bp_r}{s_a}+2\left\{\frac{\left[\beta(s_b-s_a)+\gamma_p\right](1+\alpha_s)}{|k_u|\beta\max\{c_{p1},c_{n1},c_{p2},c_{n2}\}}\right\}^2\right)\right.\\
&-\upsilon\left(\frac{2\gamma_p+s_bp_r}{s_a}\right)-\left[\beta(s_b-s_a)+\gamma_p\right]\frac{(1+\alpha_s)}{|k_u|\beta c_{n1}\delta_c}\Bigg\}\\
\leqslant{}& 0
\end{aligned}
\tag{B.40}
$$

holds. The last inequality uses (7.91), (B.35)-(B.37), and Lemma B.1. This proves (7.148).

B.6 Derivation of (10.72) and (10.73)

In view of (2.85), (10.41)–(10.44), (10.47)–(10.55), and (10.62)–(10.66), it yields

$$
\begin{aligned}
&\left({}^{\mathbf{B}_{LT}}V_r-{}^{\mathbf{B}_{LT}}V\right)^T\left({}^{\mathbf{B}_{LT}}F_r^*-{}^{\mathbf{B}_{LT}}F^*\right)\\
&\quad=\left({}^{\mathbf{B}_{LT}}V_r-{}^{\mathbf{B}_{LT}}V\right)^T\left[\left({}^{\mathbf{B}_{LT}}F_r-{}^{\mathbf{B}_{LT}}F\right)-{}^{\mathbf{B}_{LT}}\mathbf{U}_{\mathbf{B}_T}\left({}^{\mathbf{B}_{T1}}F_r-{}^{\mathbf{B}_{T1}}F\right)\right]\\
&\quad=p_{\mathbf{B}_{LT}}-\left({}^{\mathbf{B}_{LT}}V_r-{}^{\mathbf{B}_{LT}}V\right)^T{}^{\mathbf{B}_{LT}}\mathbf{U}_{\mathbf{B}_T}\left({}^{\mathbf{B}_{T1}}F_r-{}^{\mathbf{B}_{T1}}F\right)\\
&\quad=p_{\mathbf{B}_{LT}}-\left[{}^{\mathbf{B}_{LT}}\mathbf{U}_{\mathbf{B}_T}^T\left({}^{\mathbf{B}_{LT}}V_r-{}^{\mathbf{B}_{LT}}V\right)\right]^T\left({}^{\mathbf{B}_{T1}}F_r-{}^{\mathbf{B}_{T1}}F\right)\\
&\quad=p_{\mathbf{B}_{LT}}-\left[\left({}^{\mathbf{B}_{T1}}V_r-{}^{\mathbf{B}_{T1}}V\right)-\mathbf{z}\left((\dot q_{Tr}-\dot q_T)-(\dot q_{LTr}-\dot q_{LT})\right)\right]^T\\
&\qquad\qquad\times\left({}^{\mathbf{B}_{T1}}F_r-{}^{\mathbf{B}_{T1}}F\right)\\
&\quad=p_{\mathbf{B}_{LT}}-p_{\mathbf{B}_{T1}}+\left((\dot q_{Tr}-\dot q_T)-(\dot q_{LTr}-\dot q_{LT})\right)\mathbf{z}^T\left({}^{\mathbf{B}_{T1}}F_r-{}^{\mathbf{B}_{T1}}F\right)\\
&\quad=p_{\mathbf{B}_{LT}}-p_{\mathbf{B}_{T1}}+\left((\dot q_{Tr}-\dot q_T)-(\dot q_{LTr}-\dot q_{LT})\right)\left(\mathbf{z}^{T\,\mathbf{B}_{T1}}F_r-\tau_{LT}\right)\\
&\quad=p_{\mathbf{B}_{LT}}-p_{\mathbf{B}_{T1}}-k_{LT}\left((\dot q_{Tr}-\dot q_T)-(\dot q_{LTr}-\dot q_{LT})\right)^2
\end{aligned}
\tag{B.41}
$$

$$
\begin{aligned}
&\left({}^{\mathbf{B}_{LS}}V_r-{}^{\mathbf{B}_{LS}}V\right)^T\left({}^{\mathbf{B}_{LS}}F_r^*-{}^{\mathbf{B}_{LS}}F^*\right)\\
&\quad=\left({}^{\mathbf{B}_{LS}}V_r-{}^{\mathbf{B}_{LS}}V\right)^T\left[\left({}^{\mathbf{B}_{LS}}F_r-{}^{\mathbf{B}_{LS}}F\right)-{}^{\mathbf{B}_{LS}}\mathbf{U}_{\mathbf{B}_{LT}}\left({}^{\mathbf{B}_{LT}}F_r-{}^{\mathbf{B}_{LT}}F\right)\right]\\
&\quad=(\dot q_{LSr}-\dot q_{LS})\,\mathbf{z}^T\left({}^{\mathbf{B}_{LS}}F_r-{}^{\mathbf{B}_{LS}}F\right)\\
&\qquad-\left[{}^{\mathbf{B}_{LS}}\mathbf{U}_{\mathbf{B}_{LT}}^{-T}\left(\left({}^{\mathbf{B}_{LT}}V_r-{}^{\mathbf{B}_{LT}}V\right)-\mathbf{z}\left(\dot q_{LNr}-\dot q_{LN}\right)\right)\right]^T\\
&\qquad\qquad\times{}^{\mathbf{B}_{LS}}\mathbf{U}_{\mathbf{B}_{LT}}\left({}^{\mathbf{B}_{LT}}F_r-{}^{\mathbf{B}_{LT}}F\right)\\
&\quad=(\dot q_{LSr}-\dot q_{LS})\left(\mathbf{z}^{T\,\mathbf{B}_{LS}}F_r-\tau_{LS}\right)\\
&\qquad-\left[\left({}^{\mathbf{B}_{LT}}V_r-{}^{\mathbf{B}_{LT}}V\right)-\mathbf{z}\left(\dot q_{LNr}-\dot q_{LN}\right)\right]^T\left({}^{\mathbf{B}_{LT}}F_r-{}^{\mathbf{B}_{LT}}F\right)
\end{aligned}
$$

$$
\begin{aligned}
&= (\dot{q}_{LSr} - \dot{q}_{LS})\, \mathbf{z}^{T\,\mathbf{B}_{LS}}F_r - p_{\mathbf{B}_{LT}} \\
&\quad + (\dot{q}_{LNr} - \dot{q}_{LN})\, \mathbf{z}^T \left({}^{\mathbf{B}_{LT}}F_r - {}^{\mathbf{B}_{LT}}F\right) \\
&= -k_{LS} (\dot{q}_{LSr} - \dot{q}_{LS})^2 - p_{\mathbf{B}_{LT}} + (\dot{q}_{LNr} - \dot{q}_{LN}) \left(\mathbf{z}^{T\,\mathbf{B}_{LT}}F_r - \tau_{LN}\right) \\
&= -p_{\mathbf{B}_{LT}} - k_{LS} (\dot{q}_{LSr} - \dot{q}_{LS})^2 - k_{LN} (\dot{q}_{LNr} - \dot{q}_{LN})^2 . \qquad \text{(B.42)}
\end{aligned}
$$

B.7 Derivation of (10.106) and (10.107)

In view of (2.85), (10.75)–(10.78), (10.81)–(10.89), and (10.96)–(10.100), it yields

$$
\begin{aligned}
&\left({}^{\mathbf{B}_{RT}}V_r - {}^{\mathbf{B}_{RT}}V\right)^T \left({}^{\mathbf{B}_{RT}}F_r^* - {}^{\mathbf{B}_{RT}}F^*\right) \\
&\quad = \left({}^{\mathbf{B}_{RT}}V_r - {}^{\mathbf{B}_{RT}}V\right)^T \left[\left({}^{\mathbf{B}_{RT}}F_r - {}^{\mathbf{B}_{RT}}F\right) - {}^{\mathbf{B}_{RT}}\mathbf{U}_{\mathbf{B}_T} \left({}^{\mathbf{B}_{T2}}F_r - {}^{\mathbf{B}_{T2}}F\right)\right] \\
&\quad = p_{\mathbf{B}_{RT}} - \left({}^{\mathbf{B}_{RT}}V_r - {}^{\mathbf{B}_{RT}}V\right)^T {}^{\mathbf{B}_{RT}}\mathbf{U}_{\mathbf{B}_T} \left({}^{\mathbf{B}_{T2}}F_r - {}^{\mathbf{B}_{T2}}F\right) \\
&\quad = p_{\mathbf{B}_{RT}} - \left[{}^{\mathbf{B}_{RT}}\mathbf{U}_{\mathbf{B}_T}^T \left({}^{\mathbf{B}_{RT}}V_r - {}^{\mathbf{B}_{RT}}V\right)\right]^T \left({}^{\mathbf{B}_{T2}}F_r - {}^{\mathbf{B}_{T2}}F\right) \\
&\quad = p_{\mathbf{B}_{RT}} - \left[\left({}^{\mathbf{B}_{T2}}V_r - {}^{\mathbf{B}_{T2}}V\right) - \mathbf{z}\left((\dot{q}_{Tr} - \dot{q}_T) - (\dot{q}_{RTr} - \dot{q}_{RT})\right)\right]^T \\
&\qquad\qquad \times \left({}^{\mathbf{B}_{T2}}F_r - {}^{\mathbf{B}_{T2}}F\right) \\
&\quad = p_{\mathbf{B}_{RT}} - p_{\mathbf{B}_{T2}} + \left((\dot{q}_{Tr} - \dot{q}_T) - (\dot{q}_{RTr} - \dot{q}_{RT})\right) \mathbf{z}^T \left({}^{\mathbf{B}_{T2}}F_r - {}^{\mathbf{B}_{T2}}F\right) \\
&\quad = p_{\mathbf{B}_{RT}} - p_{\mathbf{B}_{T2}} + \left((\dot{q}_{Tr} - \dot{q}_T) - (\dot{q}_{RTr} - \dot{q}_{RT})\right) \left(\mathbf{z}^{T\,\mathbf{B}_{T2}}F_r - \tau_{RT}\right) \\
&\quad = p_{\mathbf{B}_{RT}} - p_{\mathbf{B}_{T2}} - k_{RT} \left((\dot{q}_{Tr} - \dot{q}_T) - (\dot{q}_{RTr} - \dot{q}_{RT})\right)^2 \qquad \text{(B.43)} \\
&\left({}^{\mathbf{B}_{RS}}V_r - {}^{\mathbf{B}_{RS}}V\right)^T \left({}^{\mathbf{B}_{RS}}F_r^* - {}^{\mathbf{B}_{RS}}F^*\right) \\
&\quad = \left({}^{\mathbf{B}_{RS}}V_r - {}^{\mathbf{B}_{RS}}V\right)^T \\
&\qquad\quad \times \left[\left({}^{\mathbf{B}_{RS}}F_r - {}^{\mathbf{B}_{RS}}F\right) - {}^{\mathbf{B}_{RS}}\mathbf{U}_{\mathbf{B}_{RT}} \left({}^{\mathbf{B}_{RT}}F_r - {}^{\mathbf{B}_{RT}}F\right)\right] \\
&\quad = (\dot{q}_{RSr} - \dot{q}_{RS})\, \mathbf{z}^T \left({}^{\mathbf{B}_{RS}}F_r - {}^{\mathbf{B}_{RS}}F\right) \\
&\qquad - \left[{}^{\mathbf{B}_{RS}}\mathbf{U}_{\mathbf{B}_{RT}}^{-T} \left(\left({}^{\mathbf{B}_{RT}}V_r - {}^{\mathbf{B}_{RT}}V\right) - \mathbf{z}(\dot{q}_{RNr} - \dot{q}_{RN})\right)\right]^T \\
&\qquad\qquad \times {}^{\mathbf{B}_{RS}}\mathbf{U}_{\mathbf{B}_{RT}} \left({}^{\mathbf{B}_{RT}}F_r - {}^{\mathbf{B}_{RT}}F\right) \\
&\quad = (\dot{q}_{RSr} - \dot{q}_{RS}) \left(\mathbf{z}^{T\,\mathbf{B}_{RS}}F_r - \tau_{RS}\right) \\
&\qquad - \left[\left({}^{\mathbf{B}_{RT}}V_r - {}^{\mathbf{B}_{RT}}V\right) - \mathbf{z}(\dot{q}_{RNr} - \dot{q}_{RN})\right]^T \left({}^{\mathbf{B}_{RT}}F_r - {}^{\mathbf{B}_{RT}}F\right) \\
&\quad = (\dot{q}_{RSr} - \dot{q}_{RS})\, \mathbf{z}^{T\,\mathbf{B}_{RS}}F_r \\
&\qquad - p_{\mathbf{B}_{RT}} + (\dot{q}_{RNr} - \dot{q}_{RN})\, \mathbf{z}^T \left({}^{\mathbf{B}_{RT}}F_r - {}^{\mathbf{B}_{RT}}F\right) \\
&\quad = -k_{RS} (\dot{q}_{RSr} - \dot{q}_{RS})^2 - p_{\mathbf{B}_{RT}} + (\dot{q}_{RNr} - \dot{q}_{RN}) \left(\mathbf{z}^{T\,\mathbf{B}_{RT}}F_r - \tau_{RN}\right) \\
&\quad = -p_{\mathbf{B}_{RT}} - k_{RS} (\dot{q}_{RSr} - \dot{q}_{RS})^2 - k_{RN} (\dot{q}_{RNr} - \dot{q}_{RN})^2 . \qquad \text{(B.44)}
\end{aligned}
$$

B.8 Derivation of (10.211)

In view of (2.85), (10.76), (10.82), (10.84), (10.87), (10.97), and (10.99), it yields

$$
\begin{aligned}
&\left({}^{\mathbf{B}_{RS}}V_r - {}^{\mathbf{B}_{RS}}V\right)^T \left({}^{\mathbf{B}_{RS}}F_r^* - {}^{\mathbf{B}_{RS}}F^*\right) \\
&\quad = \left({}^{\mathbf{B}_{RS}}V_r - {}^{\mathbf{B}_{RS}}V\right)^T \\
&\qquad\qquad \times \left[\left({}^{\mathbf{B}_{RS}}F_r - {}^{\mathbf{B}_{RS}}F\right) - {}^{\mathbf{B}_{RS}}\mathbf{U}_{\mathbf{B}_{RT}} \left({}^{\mathbf{B}_{RT}}F_r - {}^{\mathbf{B}_{RT}}F\right)\right] \\
&\quad = p_{\mathbf{B}_{RS}} - \left[{}^{\mathbf{B}_{RS}}\mathbf{U}_{\mathbf{B}_{RT}}^{-T} \left(\left({}^{\mathbf{B}_{RT}}V_r - {}^{\mathbf{B}_{RT}}V\right) - \mathbf{z}\left(\dot{q}_{RNr} - \dot{q}_{RN}\right)\right)\right]^T \\
&\qquad\qquad \times {}^{\mathbf{B}_{RS}}\mathbf{U}_{\mathbf{B}_{RT}} \left({}^{\mathbf{B}_{RT}}F_r - {}^{\mathbf{B}_{RT}}F\right) \\
&\quad = p_{\mathbf{B}_{RS}} - \left[\left({}^{\mathbf{B}_{RT}}V_r - {}^{\mathbf{B}_{RT}}V\right) - \mathbf{z}\left(\dot{q}_{RNr} - \dot{q}_{RN}\right)\right]^T \left({}^{\mathbf{B}_{RT}}F_r - {}^{\mathbf{B}_{RT}}F\right) \\
&\quad = p_{\mathbf{B}_{RS}} - p_{\mathbf{B}_{RT}} + \left(\dot{q}_{RNr} - \dot{q}_{RN}\right)\mathbf{z}^T \left({}^{\mathbf{B}_{RT}}F_r - {}^{\mathbf{B}_{RT}}F\right) \\
&\quad = p_{\mathbf{B}_{RS}} - p_{\mathbf{B}_{RT}} - k_{RN}\left(\dot{q}_{RNr} - \dot{q}_{RN}\right)^2 .
\end{aligned} \tag{B.45}
$$

B.9 Proof of Lemma 11.1

For constant σ_f and σ_c, it follows from (11.1)–(11.6), (11.12)–(11.20), (11.24), and Lemma 2.9 that

$$
\begin{aligned}
\int_0^\infty p_{\mathbf{S}_s}(t)dt &= \int_0^\infty ({}^{\mathbf{S}_s}V_r - {}^{\mathbf{S}_s}V)^T({}^{\mathbf{S}_s}F_r - {}^{\mathbf{S}_s}F)dt \\
&= \int_0^\infty (\mathbf{v}_{sfr} - \mathbf{v}_{sf})^T \sigma_f(\mathbf{f}_{sfd} - \mathbf{f}_{sf})dt \\
&\quad + \int_0^\infty [\mathbf{v}_{scr} - (1-\sigma_c)\mathbf{v}_{sc}]^T \sigma_c(\mathbf{f}_{scd} - \mathbf{f}_{sc})dt \\
&= \int_0^\infty \sigma_f(\mathbf{v}_{sfr} - \mathbf{v}_{sf})^T \mathbf{M}_f(\dot{\mathbf{v}}_{sfr} - \dot{\mathbf{v}}_{sf})dt \\
&\quad - \int_0^\infty \sigma_f(\mathbf{v}_{sfr} - \mathbf{v}_{sf})^T \mathbf{Y}_{sf}(\boldsymbol{\theta}_{sf} - \hat{\boldsymbol{\theta}}_{sf})dt \\
&\quad + \int_0^\infty \sigma_c \mathbf{v}_{scr}^T(\mathbf{f}_{scd} - \mathbf{f}_{sc})dt \\
&= \int_0^\infty \sigma_f(\mathbf{v}_{sfr} - \mathbf{v}_{sf})^T \mathbf{M}_f(\dot{\mathbf{v}}_{sfr} - \dot{\mathbf{v}}_{sf})dt \\
&\quad + \int_0^\infty \sum_\gamma (\theta_{sf\gamma} - \hat{\theta}_{sf\gamma})(-\dot{\hat{\theta}}_{sf\gamma}/\rho_{sf\gamma})dt \\
&\quad - \int_0^\infty \sum_\gamma (\theta_{sf\gamma} - \hat{\theta}_{sf\gamma})(s_{sf\gamma} - \dot{\hat{\theta}}_{sf\gamma}/\rho_{sf\gamma})dt \\
&\quad + \int_0^\infty \sigma_c\left(\mathbf{v}_{scd} - \mathbf{A}_c\tilde{\mathbf{f}}_{sc}\right)^T \\
&\qquad\quad [\mathbf{A}_c^{-1}(\mathbf{C}_c^{-1}\dot{\mathbf{v}}_{scd} + \mathbf{v}_{scd}) - (\mathbf{C}_c^{-1}\dot{\tilde{\mathbf{f}}}_{sc} + \tilde{\mathbf{f}}_{sc})]dt \\
&\geqslant \int_0^\infty \sigma_f(\mathbf{v}_{sfr} - \mathbf{v}_{sf})^T \mathbf{M}_f(\dot{\mathbf{v}}_{sfr} - \dot{\mathbf{v}}_{sf})dt
\end{aligned}
$$

$$
\begin{aligned}
&+\int_0^\infty \sum_\gamma (\theta_{sf\gamma} - \hat{\theta}_{sf\gamma})(-\dot{\hat{\theta}}_{sf\gamma}/\rho_{sf\gamma})dt \\
&+\int_0^\infty \sigma_c \left(\mathbf{v}_{scd} - \mathbf{A}_c\tilde{\mathbf{f}}_{sc}\right)^T \mathbf{A}_c^{-1}\mathbf{C}_c^{-1} \left(\dot{\mathbf{v}}_{scd} - \mathbf{A}_c\dot{\tilde{\mathbf{f}}}_{sc}\right) dt \\
&+\int_0^\infty \sigma_c \left(\mathbf{v}_{scd} - \mathbf{A}_c\tilde{\mathbf{f}}_{sc}\right)^T \mathbf{A}_c^{-1} \left(\mathbf{v}_{scd} - \mathbf{A}_c\tilde{\mathbf{f}}_{sc}\right) dt \\
\geqslant &-\frac{1}{2}\sigma_f(\mathbf{v}_{sfr}(0) - \mathbf{v}_{sf}(0))^T\mathbf{M}_f(\mathbf{v}_{sfr}(0) - \mathbf{v}_{sf}(0)) \\
&-\frac{1}{2}\sum_\gamma (\theta_{sf\gamma} - \hat{\theta}_{sf\gamma}(0))^2/\rho_{sf\gamma} \\
&-\frac{1}{2}\sigma_c(\mathbf{v}_{scd}(0) - \mathbf{A}_c\tilde{\mathbf{f}}_{sc}(0))^T\mathbf{A}_c^{-1}\mathbf{C}_c^{-1}(\mathbf{v}_{scd}(0) - \mathbf{A}_c\tilde{\mathbf{f}}_{sc}(0))
\end{aligned}
\tag{B.46}
$$

holds. This proves the lemma by equalizing

$$
\begin{aligned}
\gamma_s = &\frac{1}{2}\sigma_f(\mathbf{v}_{sfr}(0) - \mathbf{v}_{sf}(0))^T\mathbf{M}_f(\mathbf{v}_{sfr}(0) - \mathbf{v}_{sf}(0)) \\
&+\frac{1}{2}\sum_\gamma (\theta_{sf\gamma} - \hat{\theta}_{sf\gamma}(0))^2/\rho_{sf\gamma} \\
&+\frac{1}{2}\sigma_c(\mathbf{v}_{scd}(0) - \mathbf{A}_c\tilde{\mathbf{f}}_{sc}(0))^T\mathbf{A}_c^{-1}\mathbf{C}_c^{-1}(\mathbf{v}_{scd}(0) - \mathbf{A}_c\tilde{\mathbf{f}}_{sc}(0)).
\end{aligned}
\tag{B.47}
$$

B.10 Proof of Lemma 11.2

It follows from (11.28)–(11.31), (11.34)–(11.36), (11.41)–(11.46), (11.50), and Lemma 2.9 that

$$
\begin{aligned}
\int_0^\infty p_{\mathbf{S}_m}(t)dt &= \int_0^\infty ({}^{\mathbf{S}_m}V_r - {}^{\mathbf{S}_m}V)^T({}^{\mathbf{S}_m}F_r - {}^{\mathbf{S}_m}F)dt \\
&= \int_0^\infty (\mathbf{v}_{mr} - \mathbf{v}_m)^T(\mathbf{f}_{md} - \mathbf{f}_m)dt \\
&= \int_0^\infty (\mathbf{v}_{mr} - \mathbf{v}_m)^T\mathbf{M}_h(\dot{\mathbf{v}}_{mr} - \dot{\mathbf{v}}_m)dt \\
&\quad -\int_0^\infty (\mathbf{v}_{mr} - \mathbf{v}_m)^T\mathbf{Y}_m(\boldsymbol{\theta}_m - \hat{\boldsymbol{\theta}}_m)dt \\
&\quad +\int_0^\infty (\mathbf{v}_{mr} - \mathbf{v}_m)^T[\alpha_h\mathrm{sign}(\mathbf{v}_{mr} - \mathbf{v}_m) - \mathbf{f}_h^*]dt \\
&= \int_0^\infty (\mathbf{v}_{mr} - \mathbf{v}_m)^T\mathbf{M}_h(\dot{\mathbf{v}}_{mr} - \dot{\mathbf{v}}_m)dt \\
&\quad +\int_0^\infty \sum_\gamma (\theta_{m\gamma} - \hat{\theta}_{m\gamma})(-\dot{\hat{\theta}}_{m\gamma}/\rho_{m\gamma})dt
\end{aligned}
$$

$$
\begin{aligned}
&-\int_0^\infty \sum_\gamma (\boldsymbol{\theta}_m - \hat{\boldsymbol{\theta}}_m)(s_{m\gamma} - \dot{\hat{\theta}}_{m\gamma}/\rho_{m\gamma})dt \\
&+\int_0^\infty (\mathbf{v}_{mr} - \mathbf{v}_m)^T[\alpha_h \mathrm{sign}(\mathbf{v}_{mr} - \mathbf{v}_m) - \mathbf{f}_h^*]dt \\
\geqslant &-\frac{1}{2}(\mathbf{v}_{mr}(0) - \mathbf{v}_m(0))^T \mathbf{M}_h (\mathbf{v}_{mr}(0) - \mathbf{v}_m(0)) \\
&-\frac{1}{2}\sum_\gamma (\theta_{m\gamma} - \hat{\theta}_{m\gamma}(0))^2/\rho_{m\gamma} \qquad \text{(B.48)}
\end{aligned}
$$

holds. This proves the lemma by equalizing

$$
\begin{aligned}
\gamma_m = &\frac{1}{2}(\mathbf{v}_{mr}(0) - \mathbf{v}_m(0))^T \mathbf{M}_h (\mathbf{v}_{mr}(0) - \mathbf{v}_m(0)) \\
&+\frac{1}{2}\sum_\gamma (\theta_{m\gamma} - \hat{\theta}_{m\gamma}(0))^2/\rho_{m\gamma}. \qquad \text{(B.49)}
\end{aligned}
$$

B.11 Proof of Theorem 13.2

Subtracting (13.7) from (13.34) yields

$$
\begin{aligned}
&\rho\left[\ddot{y}_r(x,t) - \ddot{y}(x,t)\right] + EI\left[y_r''''(x,t) - y''''(x,t)\right] \\
&+k_v\left[\dot{y}_r(x,t) - \dot{y}(x,t)\right] - \left[\rho - \hat{\rho}(t)\right]\ddot{y}_r(x,t) - \left[EI - \widehat{EI}(t)\right] y_r''''(x,t) = 0. \qquad \text{(B.50)}
\end{aligned}
$$

With integration by parts, it follows from (B.50) and (13.19) that

$$
\begin{aligned}
\dot{\nu}_K &= \int_0^l \rho\left[\dot{y}_r(x,t) - \dot{y}(x,t)\right]\left[\ddot{y}_r(x,t) - \ddot{y}(x,t)\right]dx \\
&= -\int_0^l \left[\dot{y}_r(x,t) - \dot{y}(x,t)\right] EI\left[y_r''''(x,t) - y''''(x,t)\right]dx \\
&\quad -\int_0^l k_v\left[\dot{y}_r(x,t) - \dot{y}(x,t)\right]^2 dx \\
&\quad +\int_0^l \left[\dot{y}_r(x,t) - \dot{y}(x,t)\right]\left[\rho - \hat{\rho}(t)\right]\ddot{y}_r(x,t)dx \\
&\quad +\int_0^l \left[\dot{y}_r(x,t) - \dot{y}(x,t)\right]\left[EI - \widehat{EI}(t)\right] y_r''''(x,t)dx \\
&= -\left[\dot{y}_r(x,t) - \dot{y}(x,t)\right] EI\left[y_r'''(x,t) - y'''(x,t)\right]\Big|_0^l \\
&\quad +\int_0^l \left[\dot{y}_r'(x,t) - \dot{y}'(x,t)\right] EI\left[y_r'''(x,t) - y'''(x,t)\right]dx \\
&\quad -\int_0^l k_v\left[\dot{y}_r(x,t) - \dot{y}(x,t)\right]^2 dx
\end{aligned}
$$

$$
\begin{aligned}
&+ [\rho - \hat{\rho}(t)] \int_0^l [\dot{y}_r(x,t) - \dot{y}(x,t)] \, \ddot{y}_r(x,t) dx \\
&+ \left[EI - \widehat{EI}(t)\right] \int_0^l [\dot{y}_r(x,t) - \dot{y}(x,t)] \, y_r''''(x,t) dx \\
= &- [\dot{y}_r(x,t) - \dot{y}(x,t)] \, EI \, [y_r'''(x,t) - y'''(x,t)] \, |_0^l \\
&+ [\dot{y}_r'(x,t) - \dot{y}'(x,t)] \, EI \, [y_r''(x,t) - y''(x,t)] \, |_0^l \\
&- \int_0^l [\dot{y}_r''(x,t) - \dot{y}''(x,t)] \, EI \, [y_r''(x,t) - y''(x,t)] \, dx \\
&- \int_0^l k_v \, [\dot{y}_r(x,t) - \dot{y}(x,t)]^2 \, dx \\
&+ [\rho - \hat{\rho}(t)] \int_0^l [\dot{y}_r(x,t) - \dot{y}(x,t)] \, \ddot{y}_r(x,t) dx \\
&+ \left[EI - \widehat{EI}(t)\right] \int_0^l [\dot{y}_r(x,t) - \dot{y}(x,t)] \, y_r''''(x,t) dx \\
= &- [\dot{y}_r(l,t) - \dot{y}(l,t)] \, EI \, [y_r'''(l,t) - y'''(l,t)] \\
&+ [\dot{y}_r(0,t) - \dot{y}(0,t)] \, EI \, [y_r'''(0,t) - y'''(0,t)] \\
&+ [\dot{y}_r'(l,t) - \dot{y}'(l,t)] \, EI \, [y_r''(l,t) - y''(l,t)] \\
&- [\dot{y}_r'(0,t) - \dot{y}'(0,t)] \, EI \, [y_r''(0,t) - y''(0,t)] \\
&- \int_0^l [\dot{y}_r''(x,t) - \dot{y}''(x,t)] \, EI \, [y_r''(x,t) - y''(x,t)] \, dx \\
&- \int_0^l k_v \, [\dot{y}_r(x,t) - \dot{y}(x,t)]^2 \, dx \\
&+ [\rho - \hat{\rho}(t)] \int_0^l [\dot{y}_r(x,t) - \dot{y}(x,t)] \, \ddot{y}_r(x,t) dx \\
&+ \left[EI - \widehat{EI}(t)\right] \int_0^l [\dot{y}_r(x,t) - \dot{y}(x,t)] \, y_r''''(x,t) dx \\
= &- [\dot{y}_r(l,t) - \dot{y}(l,t)] \left[\widehat{EI} y_r'''(l,t) - EI y'''(l,t)\right] \\
&+ [\dot{y}_r(0,t) - \dot{y}(0,t)] \left[\widehat{EI} y_r'''(0,t) - EI y'''(0,t)\right] \\
&+ [\dot{y}_r'(l,t) - \dot{y}'(l,t)] \left[\widehat{EI} y_r''(l,t) - EI y''(l,t)\right] \\
&- [\dot{y}_r'(0,t) - \dot{y}'(0,t)] \left[\widehat{EI} y_r''(0,t) - EI y''(0,t)\right] \\
&- \int_0^l [\dot{y}_r''(x,t) - \dot{y}''(x,t)] \, EI \, [y_r''(x,t) - y''(x,t)] \, dx \\
&- \int_0^l k_v \, [\dot{y}_r(x,t) - \dot{y}(x,t)]^2 \, dx \\
&+ [\rho - \hat{\rho}(t)] \int_0^l [\dot{y}_r(x,t) - \dot{y}(x,t)] \, \ddot{y}_r(x,t) dx
\end{aligned}
$$

$$
\begin{aligned}
&+\left[EI-\widehat{EI}(t)\right]\int_0^l\left[\dot{y}_r(x,t)-\dot{y}(x,t)\right]y_r''''(x,t)dx\\
&-\left[EI-\widehat{EI}(t)\right]\left[\dot{y}_r(l,t)-\dot{y}(l,t)\right]y_r'''(l,t)\\
&+\left[EI-\widehat{EI}(t)\right]\left[\dot{y}_r(0,t)-\dot{y}(0,t)\right]y_r'''(0,t)\\
&+\left[EI-\widehat{EI}(t)\right]\left[\dot{y}_r'(l,t)-\dot{y}'(l,t)\right]y_r''(l,t)\\
&-\left[EI-\widehat{EI}(t)\right]\left[\dot{y}_r'(0,t)-\dot{y}'(0,t)\right]y_r''(0,t)
\end{aligned}
\tag{B.51}
$$

holds.

Re-define the non-negative accompanying function ν as

$$
\nu=\nu_K+\nu_V+\frac{1}{2\rho_\rho}\left[\rho-\hat{\rho}(t)\right]^2+\frac{1}{2\rho_{EI}}\left[EI-\widehat{EI}(t)\right]^2. \tag{B.52}
$$

Substituting (B.51) and the time derivative of (13.20) into the time derivative of (B.52) and using (13.41), (13.42), and Lemma 2.9 yields

$$
\begin{aligned}
\dot{\nu}=&-\int_0^l k_v\left[\dot{y}_r(x,t)-\dot{y}(x,t)\right]^2dx\\
&+\left[\rho-\hat{\rho}(t)\right]\left[s_\rho-\frac{\dot{\hat{\rho}}(t)}{\rho_\rho}\right]+\left[EI-\widehat{EI}(t)\right]\left[s_{EI}-\frac{\dot{\widehat{EI}}(t)}{\rho_{EI}}\right]\\
&+p_B-p_T\\
\leqslant&-\int_0^l k_v\left[\dot{y}_r(x,t)-\dot{y}(x,t)\right]^2dx\\
&+p_B-p_T
\end{aligned}
\tag{B.53}
$$

with

$$
\begin{aligned}
p_B=&\left[\dot{y}_r(0,t)-\dot{y}(0,t)\right]\left[\widehat{EI}y_r'''(0,t)-EIy'''(0,t)\right]\\
&-\left[\dot{y}_r'(0,t)-\dot{y}'(0,t)\right]\left[\widehat{EI}y_r''(0,t)-EIy''(0,t)\right]\\
=&\left[\dot{y}_r(0,t)-\dot{y}(0,t)\right](f_{Br}-f_B)\\
&+\left[\dot{y}_r'(0,t)-\dot{y}'(0,t)\right](m_{Br}-m_B)
\end{aligned}
\tag{B.54}
$$

$$
\begin{aligned}
p_T=&\left[\dot{y}_r(l,t)-\dot{y}(l,t)\right]\left[\widehat{EI}y_r'''(l,t)-EIy'''(l,t)\right]\\
&-\left[\dot{y}_r'(l,t)-\dot{y}'(l,t)\right]\left[\widehat{EI}y_r''(l,t)-EIy''(l,t)\right]\\
=&\left[\dot{y}_r(l,t)-\dot{y}(l,t)\right](f_{Tr}-f_T)\\
&+\left[\dot{y}_r'(l,t)-\dot{y}'(l,t)\right](m_{Tr}-m_T)
\end{aligned}
\tag{B.55}
$$

in view of (13.8)–(13.11) and (13.35)–(13.38).

Consider the fact that the flexible beam has one *driving cutting point* associated with point T and one *driven cutting point* associated with point B. Using (13.19), (13.20), (B.52), (B.53), and Definition 2.17 completes the proof.

B.12 Derivation of (13.83)–(13.88)

Substituting (13.64), (13.74), and (13.75) into (13.4) yields

$$
\begin{aligned}
&\int_{t_1}^{t_2} (\delta e_K - \delta e_P + \delta \overline{w_E})\, dt \\
&\quad = - \int_0^l \int_{t_1}^{t_2} \rho \left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}} \frac{\partial^2}{\partial t^2} \mathbf{r}_b(x,t) \right)^T {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}} \delta \mathbf{r}_b(x,t) dt dx \\
&\qquad - \int_0^l \int_{t_1}^{t_2} \left\{ \frac{\rho}{S_x} \mathrm{diag}(J_x, I_y, I_z) {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}} \frac{\partial}{\partial t} \boldsymbol{\omega}_b + \frac{\rho}{S_x} \begin{bmatrix} (I_z - I_y) {}^{\mathbf{B}_b}\omega_y {}^{\mathbf{B}_b}\omega_z \\ I_y {}^{\mathbf{B}_b}\omega_x {}^{\mathbf{B}_b}\omega_z \\ -I_z {}^{\mathbf{B}_b}\omega_x {}^{\mathbf{B}_b}\omega_y \end{bmatrix} \right\}^T \\
&\qquad\qquad \times {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}} \delta \mathbf{q}_b dt dx \\
&\qquad - \int_{t_1}^{t_2} ES_x \left(\frac{\partial u_x(x,t)}{\partial x} \right) \delta u_x(x,t) \Big|_0^l dt \\
&\qquad + \int_{t_1}^{t_2} \int_0^l ES_x \left(\frac{\partial^2 u_x(x,t)}{\partial x^2} \right) \delta u_x(x,t) dx dt \\
&\qquad - \int_{t_1}^{t_2} EI_z \left(\frac{\partial^2 y_b(x,t)}{\partial x^2} \right) \delta \left(\frac{\partial y_b(x,t)}{\partial x} \right) \Big|_0^l dt \\
&\qquad + \int_{t_1}^{t_2} \int_0^l \alpha_z(x,t) EI_z \left(\frac{\partial^3 y_b(x,t)}{\partial x^3} \right) \delta \left(\frac{\partial y_b(x,t)}{\partial x} \right) dx dt \\
&\qquad + \int_{t_1}^{t_2} (1 - \alpha_z(x,t)) EI_z \left(\frac{\partial^3 y_b(x,t)}{\partial x^3} \right) \delta y_b(x,t) \Big|_0^l dt \\
&\qquad - \int_{t_1}^{t_2} \int_0^l EI_z \frac{\partial}{\partial x} \left((1 - \alpha_z(x,t)) \frac{\partial^3 y_b(x,t)}{\partial x^3} \right) \delta y_b(x,t) dx dt \\
&\qquad - \int_{t_1}^{t_2} EI_y \left(\frac{\partial^2 z_b(x,t)}{\partial x^2} \right) \delta \left(\frac{\partial z_b(x,t)}{\partial x} \right) \Big|_0^l dt \\
&\qquad + \int_{t_1}^{t_2} \int_0^l \alpha_y(x,t) EI_y \left(\frac{\partial^3 z_b(x,t)}{\partial x^3} \right) \delta \left(\frac{\partial z_b(x,t)}{\partial x} \right) dx dt \\
&\qquad + \int_{t_1}^{t_2} (1 - \alpha_y(x,t)) EI_y \left(\frac{\partial^3 z_b(x,t)}{\partial x^3} \right) (\delta z_b(x,t)) \Big|_0^l dt \\
&\qquad - \int_{t_1}^{t_2} \int_0^l EI_y \frac{\partial}{\partial x} \left((1 - \alpha_y(x,t)) \frac{\partial^3 z_b(x,t)}{\partial x^3} \right) \delta z_b(x,t) dx dt \\
&\qquad - \int_{t_1}^{t_2} GJ_x \left(\frac{\partial \Theta_x(x,t)}{\partial x} \right) \delta \Theta_x(x,t) \Big|_0^l dt \\
&\qquad + \int_{t_1}^{t_2} \int_0^l GJ_x \left(\frac{\partial^2 \Theta_x(x,t)}{\partial x^2} \right) \delta \Theta_x(x,t) dx dt
\end{aligned}
$$

$$
\begin{aligned}
&+\int_{t_1}^{t_2}\left[({}^{\mathbf{B}}\mathbf{R_I}\mathbf{f_B})^T({}^{\mathbf{B}}\mathbf{R_I}\delta\mathbf{r_B})+({}^{\mathbf{B}}\mathbf{R_I}\mathbf{m_B})^T({}^{\mathbf{B}}\mathbf{R_I}\delta\mathbf{q_B})\right.\\
&\left.-({}^{\mathbf{T}}\mathbf{R_I}\mathbf{f_T})^T({}^{\mathbf{T}}\mathbf{R_I}\delta\mathbf{r_T})-({}^{\mathbf{T}}\mathbf{R_I}\mathbf{m_T})^T({}^{\mathbf{T}}\mathbf{R_I}\delta\mathbf{q_T})\right]dt\\
=&-\int_{t_1}^{t_2}\int_0^l\rho\left({}^{\mathbf{B}_b}\mathbf{R_I}\frac{\partial^2}{\partial t^2}\mathbf{r}_b(x,t)\right)^T{}^{\mathbf{B}_b}\mathbf{R_I}\delta\mathbf{r}_b(x,t)dxdt\\
&-\int_{t_1}^{t_2}\int_0^l\left\{\frac{\rho}{S_x}\mathrm{diag}(J_x,I_y,I_z){}^{\mathbf{B}_b}\mathbf{R_I}\frac{\partial}{\partial t}\boldsymbol{\omega}_b+\frac{\rho}{S_x}\begin{bmatrix}(I_z-I_y){}^{\mathbf{B}_b}\omega_y{}^{\mathbf{B}_b}\omega_z\\ I_y{}^{\mathbf{B}_b}\omega_x{}^{\mathbf{B}_b}\omega_z\\ -I_z{}^{\mathbf{B}_b}\omega_x{}^{\mathbf{B}_b}\omega_y\end{bmatrix}\right\}^T\\
&\qquad\times{}^{\mathbf{B}_b}\mathbf{R_I}\delta\mathbf{q}_b dxdt\\
&+\int_{t_1}^{t_2}\int_0^l ES_x\left(\frac{\partial^2u_x(x,t)}{\partial x^2}\right)\delta u_x(x,t)dxdt\\
&-\int_{t_1}^{t_2}\int_0^l EI_z\frac{\partial}{\partial x}\left((1-\alpha_z(x,t))\frac{\partial^3y_b(x,t)}{\partial x^3}\right)\delta y_b(x,t)dxdt\\
&-\int_{t_1}^{t_2}\int_0^l EI_y\frac{\partial}{\partial x}\left((1-\alpha_y(x,t))\frac{\partial^3z_b(x,t)}{\partial x^3}\right)\delta z_b(x,t)dxdt\\
&+\int_{t_1}^{t_2}\int_0^l GJ_x\left(\frac{\partial^2\Theta_x(x,t)}{\partial x^2}\right)\delta\Theta_x(x,t)dxdt\\
&+\int_{t_1}^{t_2}\int_0^l \alpha_y(x,t)EI_y\left(\frac{\partial^3z_b(x,t)}{\partial x^3}\right)\delta\left(\frac{\partial z_b(x,t)}{\partial x}\right)dxdt\\
&+\int_{t_1}^{t_2}\int_0^l \alpha_z(x,t)EI_z\left(\frac{\partial^3y_b(x,t)}{\partial x^3}\right)\delta\left(\frac{\partial y_b(x,t)}{\partial x}\right)dxdt\\
&-\int_{t_1}^{t_2}ES_x\left(\frac{\partial u_x(x,t)}{\partial x}\right)\delta u_x(x,t)\Big|_{x=l}dt\\
&+\int_{t_1}^{t_2}(1-\alpha_z(x,t))EI_z\left(\frac{\partial^3y_b(x,t)}{\partial x^3}\right)\delta y_b(x,t)\Big|_{x=l}dt\\
&+\int_{t_1}^{t_2}(1-\alpha_y(x,t))EI_y\left(\frac{\partial^3z_b(x,t)}{\partial x^3}\right)\delta z_b(x,t)\Big|_{x=l}dt\\
&-\int_{t_1}^{t_2}GJ_x\left(\frac{\partial\Theta_x(x,t)}{\partial x}\right)\delta\Theta_x(x,t)\Big|_{x=l}dt\\
&-\int_{t_1}^{t_2}EI_y\left(\frac{\partial^2z_b(x,t)}{\partial x^2}\right)\delta\left(\frac{\partial z_b(x,t)}{\partial x}\right)\Big|_{x=l}dt\\
&-\int_{t_1}^{t_2}EI_z\left(\frac{\partial^2y_b(x,t)}{\partial x^2}\right)\delta\left(\frac{\partial y_b(x,t)}{\partial x}\right)\Big|_{x=l}dt\\
&+\int_{t_1}^{t_2}ES_x\left(\frac{\partial u_x(x,t)}{\partial x}\right)\delta u_x(x,t)\Big|_{x=0}dt\\
&-\int_{t_1}^{t_2}(1-\alpha_z(x,t))EI_z\left(\frac{\partial^3y_b(x,t)}{\partial x^3}\right)\delta y_b(x,t)\Big|_{x=0}dt
\end{aligned}
$$

$$
\begin{aligned}
& - \int_{t_1}^{t_2} (1-\alpha_y(x,t)) EI_y \left(\frac{\partial^3 z_b(x,t)}{\partial x^3} \right) \delta z_b(x,t) \bigg|_{x=0} dt \\
& + \int_{t_1}^{t_2} GJ_x \left(\frac{\partial \Theta_x(x,t)}{\partial x} \right) \delta \Theta_x(x,t) \bigg|_{x=0} dt \\
& + \int_{t_1}^{t_2} EI_y \left(\frac{\partial^2 z_b(x,t)}{\partial x^2} \right) \delta \left(\frac{\partial z_b(x,t)}{\partial x} \right) \bigg|_{x=0} dt \\
& + \int_{t_1}^{t_2} EI_z \left(\frac{\partial^2 y_b(x,t)}{\partial x^2} \right) \delta \left(\frac{\partial y_b(x,t)}{\partial x} \right) \bigg|_{x=0} dt \\
& + \int_{t_1}^{t_2} \left[({}^{\mathbf{B}}\mathbf{R_I f_B})^T ({}^{\mathbf{B}}\mathbf{R_I} \delta \mathbf{r_B}) + ({}^{\mathbf{B}}\mathbf{R_I m_B})^T ({}^{\mathbf{B}}\mathbf{R_I} \delta \mathbf{q_B}) \right. \\
& \left. - ({}^{\mathbf{T}}\mathbf{R_I f_T})^T ({}^{\mathbf{T}}\mathbf{R_I} \delta \mathbf{r_T}) - ({}^{\mathbf{T}}\mathbf{R_I m_T})^T ({}^{\mathbf{T}}\mathbf{R_I} \delta \mathbf{q_T}) \right] dt \\
= & - \int_{t_1}^{t_2} \int_0^l \rho \left({}^{\mathbf{B}_b}\mathbf{R_I} \frac{\partial^2}{\partial t^2} \mathbf{r}_b(x,t) \right)^T {}^{\mathbf{B}_b}\mathbf{R_I} \delta \mathbf{r}_b(x,t) dx dt \\
& - \int_{t_1}^{t_2} \int_0^l \begin{bmatrix} -ES_x \left(\frac{\partial^2 u_x(x,t)}{\partial x^2} \right) \\ EI_z \frac{\partial}{\partial x} \left((1-\alpha_z(x,t)) \frac{\partial^3 y_b(x,t)}{\partial x^3} \right) \\ EI_y \frac{\partial}{\partial x} \left((1-\alpha_y(x,t)) \frac{\partial^3 z_b(x,t)}{\partial x^3} \right) \end{bmatrix}^T \begin{bmatrix} \delta u_x(x,t) \\ \delta y_b(x,t) \\ \delta z_b(x,t) \end{bmatrix} dx dt \\
& - \int_{t_1}^{t_2} \int_0^l \left\{ \frac{\rho}{S_x} \mathrm{diag}(J_x, I_y, I_z) {}^{\mathbf{B}_b}\mathbf{R_I} \frac{\partial}{\partial t} \boldsymbol{\omega}_b + \frac{\rho}{S_x} \begin{bmatrix} (I_z - I_y) {}^{\mathbf{B}_b}\omega_y {}^{\mathbf{B}_b}\omega_z \\ I_y {}^{\mathbf{B}_b}\omega_x {}^{\mathbf{B}_b}\omega_z \\ -I_z {}^{\mathbf{B}_b}\omega_x {}^{\mathbf{B}_b}\omega_y \end{bmatrix} \right\}^T \\
& \qquad \times {}^{\mathbf{B}_b}\mathbf{R_I} \delta \mathbf{q}_b dx dt \\
& + \int_{t_1}^{t_2} \int_0^l \begin{bmatrix} GJ_x \left(\frac{\partial^2 \Theta_x(x,t)}{\partial x^2} \right) \\ -\alpha_y(x,t) EI_y \left(\frac{\partial^3 z_b(x,t)}{\partial x^3} \right) \\ \alpha_z(x,t) EI_z \left(\frac{\partial^3 y_b(x,t)}{\partial x^3} \right) \end{bmatrix}^T \begin{bmatrix} \delta \Theta_x(x,t) \\ -\delta \left(\frac{\partial z_b(x,t)}{\partial x} \right) \\ \delta \left(\frac{\partial y_b(x,t)}{\partial x} \right) \end{bmatrix} dx dt \\
& + \int_{t_1}^{t_2} \begin{bmatrix} - ES_x \left(\frac{\partial u_x(x,t)}{\partial x} \right) \Big|_{x=l} \\ (1-\alpha_z(x,t)) EI_z \left(\frac{\partial^3 y_b(x,t)}{\partial x^3} \right) \Big|_{x=l} \\ (1-\alpha_y(x,t)) EI_y \left(\frac{\partial^3 z_b(x,t)}{\partial x^3} \right) \Big|_{x=l} \end{bmatrix}^T \begin{bmatrix} \delta u_x(x,t)|_{x=l} \\ \delta y_b(x,t)|_{x=l} \\ \delta z_b(x,t)|_{x=l} \end{bmatrix} dt \\
& + \int_{t_1}^{t_2} \begin{bmatrix} -GJ_x \left(\frac{\partial \Theta_x(x,t)}{\partial x} \right) \Big|_{x=l} \\ EI_y \left(\frac{\partial^2 z_b(x,t)}{\partial x^2} \right) \Big|_{x=l} \\ -EI_z \left(\frac{\partial^2 y_b(x,t)}{\partial x^2} \right) \Big|_{x=l} \end{bmatrix}^T \begin{bmatrix} \delta \Theta_x(x,t)|_{x=l} \\ -\delta \left(\frac{\partial z_b(x,t)}{\partial x} \right) |_{x=l} \\ \delta \left(\frac{\partial y_b(x,t)}{\partial x} \right) |_{x=l} \end{bmatrix} dt
\end{aligned}
$$

$$
\begin{aligned}
&-\int_{t_1}^{t_2}\begin{bmatrix} -ES_x\left(\frac{\partial u_x(x,t)}{\partial x}\right)\Big|_{x=0} \\ (1-\alpha_z(x,t))EI_z\left(\frac{\partial^3 y_b(x,t)}{\partial x^3}\right)\Big|_{x=0} \\ (1-\alpha_y(x,t))EI_y\left(\frac{\partial^3 z_b(x,t)}{\partial x^3}\right)\Big|_{x=0}\end{bmatrix}^T \begin{bmatrix}\delta u_x(x,t)|_{x=0} \\ \delta y_b(x,t)|_{x=0} \\ \delta z_b(x,t)|_{x=0}\end{bmatrix} dt \\
&-\int_{t_1}^{t_2}\begin{bmatrix} -GJ_x\left(\frac{\partial \Theta_x(x,t)}{\partial x}\right)\Big|_{x=0} \\ EI_y\left(\frac{\partial^2 z_b(x,t)}{\partial x^2}\right)\Big|_{x=0} \\ -EI_z\left(\frac{\partial^2 y_b(x,t)}{\partial x^2}\right)\Big|_{x=0}\end{bmatrix}^T \begin{bmatrix}\delta \Theta_x(x,t)|_{x=0} \\ -\delta\left(\frac{\partial z_b(x,t)}{\partial x}\right)\Big|_{x=0} \\ \delta\left(\frac{\partial y_b(x,t)}{\partial x}\right)\Big|_{x=0}\end{bmatrix} dt \\
&+\int_{t_1}^{t_2}\left[({}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\mathbf{f}_{\mathbf{B}})^T({}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\delta\mathbf{r}_{\mathbf{B}}) + ({}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\mathbf{m}_{\mathbf{B}})^T({}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\delta\mathbf{q}_{\mathbf{B}})\right. \\
&\left.-({}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}}\mathbf{f}_{\mathbf{T}})^T({}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}}\delta\mathbf{r}_{\mathbf{T}}) - ({}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}}\mathbf{m}_{\mathbf{T}})^T({}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}}\delta\mathbf{q}_{\mathbf{T}})\right] dt \\
&= 0. \qquad \text{(B.56)}
\end{aligned}
$$

It leads to (13.83)–(13.88) from given (13.77)–(13.82).

B.13 Proof of Theorem 13.3

A non-negative accompany function

$$
\nu = \nu_a + \nu_a + \nu_c \qquad \text{(B.57)}
$$

with

$$
\nu_a = \frac{1}{2}\int_0^l \rho\frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t) - \mathbf{r}_b(x,t)\right)^T \frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t) - \mathbf{r}_b(x,t)\right) dx \qquad \text{(B.58)}
$$

$$
\begin{aligned}
\nu_b = \frac{1}{2}\int_0^l \frac{\rho}{S_x}&\left[{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\left(\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b\right)\right]^T \mathrm{diag}(J_x, I_y, I_z) \\
&\times\left[{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\left(\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b\right)\right] dx \qquad \text{(B.59)}
\end{aligned}
$$

$$
\begin{aligned}
\nu_c = &\frac{1}{2}\int_0^l ES_x\left[\frac{\partial}{\partial x}\left(u_{xr}(x,t) - u_x(x,t)\right)\right]^2 dx \\
&+\frac{1}{2}\int_0^l EI_z\left[\frac{\partial^2}{\partial x^2}\left(y_{br}(x,t) - y_b(x,t)\right)\right]^2 dx \\
&+\frac{1}{2}\int_0^l EI_y\left[\frac{\partial^2}{\partial x^2}\left(z_{br}(x,t) - z_b(x,t)\right)\right]^2 dx \\
&+\frac{1}{2}\int_0^l GJ_x\left[\frac{\partial}{\partial x}\left(\Theta_{xr}(x,t) - \Theta_x(x,t)\right)\right]^2 dx \qquad \text{(B.60)}
\end{aligned}
$$

is assigned.

Subtracting (13.83) and (13.84) from (13.96) and (13.97) yields

$$\begin{aligned}
&\rho^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\frac{\partial^2}{\partial t^2}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right)\\
&\quad=-\begin{bmatrix} -ES_x\frac{\partial^2}{\partial x^2}\left(u_{xr}(x,t)-u_x(x,t)\right)\\ EI_z\frac{\partial}{\partial x}\left((1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3}-(1-\alpha_z(x,t))\frac{\partial^3 y_b(x,t)}{\partial x^3}\right)\\ EI_y\frac{\partial}{\partial x}\left((1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3}-(1-\alpha_y(x,t))\frac{\partial^3 z_b(x,t)}{\partial x^3}\right)\end{bmatrix}\\
&\qquad-\mathbf{K}_p{}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\left(\frac{\partial}{\partial t}\mathbf{r}_{br}(x,t)-\frac{\partial}{\partial t}\mathbf{r}_b(x,t)\right)
\end{aligned}\tag{B.61}$$

$$\begin{aligned}
&\frac{\rho}{S_x}\mathrm{diag}(J_x,I_y,I_z)^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\left[\left(\frac{\partial}{\partial t}\boldsymbol{\omega}_{br}-\frac{\partial}{\partial t}\boldsymbol{\omega}_b\right)-(\boldsymbol{\omega}_b\times)\left(\boldsymbol{\omega}_{br}-\boldsymbol{\omega}_b\right)\right]\\
&\quad=-\frac{\rho}{S_x}\begin{bmatrix} I_z{}^{\mathbf{B}_b}\omega_y\left({}^{\mathbf{B}_b}\omega_{zr}-{}^{\mathbf{B}_b}\omega_z\right)-I_y{}^{\mathbf{B}_b}\omega_z\left({}^{\mathbf{B}_b}\omega_{yr}-{}^{\mathbf{B}_b}\omega_y\right)\\ I_y{}^{\mathbf{B}_b}\omega_z\left({}^{\mathbf{B}_b}\omega_{xr}-{}^{\mathbf{B}_b}\omega_x\right)\\ -I_z{}^{\mathbf{B}_b}\omega_y\left({}^{\mathbf{B}_b}\omega_{xr}-{}^{\mathbf{B}_b}\omega_x\right)\end{bmatrix}\\
&\qquad+\begin{bmatrix} GJ_x\frac{\partial^2}{\partial x^2}\left(\Theta_{xr}(x,t)-\Theta_x(x,t)\right)\\ -EI_y\left(\alpha_{yr}(x,t)\frac{\partial^3 z_{br}(x,t)}{\partial x^3}-\alpha_y(x,t)\frac{\partial^3 z_b(x,t)}{\partial x^3}\right)\\ EI_z\left(\alpha_{zr}(x,t)\frac{\partial^3 y_{br}(x,t)}{\partial x^3}-\alpha_z(x,t)\frac{\partial^3 y_b(x,t)}{\partial x^3}\right)\end{bmatrix}\\
&\qquad-\mathbf{K}_\omega{}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\left(\boldsymbol{\omega}_{br}-\boldsymbol{\omega}_b\right).
\end{aligned}\tag{B.62}$$

In view of (13.49), (13.92), (B.61), and

$$\frac{d}{dt}\left({}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\right)=-{}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}(\boldsymbol{\omega}_b\times)\tag{B.63}$$

the time-derivative of (B.58) can be expressed as

$$\begin{aligned}
\dot{\nu}_a&=\int_0^l\rho\frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right)^T\frac{\partial^2}{\partial t^2}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right)dx\\
&=\int_0^l\rho\left[{}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right)\right]^T\\
&\qquad\times\left[{}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\frac{\partial^2}{\partial t^2}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right)\right]dx\\
&=-\int_0^l\begin{bmatrix}{}^{\mathbf{B}_b}v_{bxr}-{}^{\mathbf{B}_b}v_{bx}\\ {}^{\mathbf{B}_b}v_{byr}-{}^{\mathbf{B}_b}v_{by}\\ {}^{\mathbf{B}_b}v_{bzr}-{}^{\mathbf{B}_b}v_{bz}\end{bmatrix}^T\\
&\qquad\times\begin{bmatrix} -ES_x\frac{\partial^2}{\partial x^2}\left(u_{xr}(x,t)-u_x(x,t)\right)\\ EI_z\frac{\partial}{\partial x}\left((1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3}-(1-\alpha_z(x,t))\frac{\partial^3 y_b(x,t)}{\partial x^3}\right)\\ EI_y\frac{\partial}{\partial x}\left((1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3}-(1-\alpha_y(x,t))\frac{\partial^3 z_b(x,t)}{\partial x^3}\right)\end{bmatrix}dx
\end{aligned}$$

$$
\begin{aligned}
&-\int_0^l \left[{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right)\right]^T \\
&\qquad\times\mathbf{K}_p{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right)dx \\
&=\int_0^l \left({}^{\mathbf{B}_b}v_{bxr}-{}^{\mathbf{B}_b}v_{bx}\right)ES_x\frac{\partial^2}{\partial x^2}\left(u_{xr}(x,t)-u_x(x,t)\right)dx \\
&\quad-\int_0^l \left({}^{\mathbf{B}_b}v_{byr}-{}^{\mathbf{B}_b}v_{by}\right)EI_z \\
&\qquad\times\frac{\partial}{\partial x}\left[(1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3}-(1-\alpha_z(x,t))\frac{\partial^3 y_b(x,t)}{\partial x^3}\right]dx \\
&\quad-\int_0^l \left({}^{\mathbf{B}_b}v_{bzr}-{}^{\mathbf{B}_b}v_{bz}\right)EI_y \\
&\qquad\times\frac{\partial}{\partial x}\left[(1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3}-(1-\alpha_y(x,t))\frac{\partial^3 z_b(x,t)}{\partial x^3}\right]dx \\
&\quad-\int_0^l \frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right)^T\left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T\mathbf{K}_p{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\right) \\
&\qquad\times\frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right)dx.
\end{aligned}
\tag{B.64}
$$

The first to third terms in the right hand side of (B.64) can be further expressed by

$$
\begin{aligned}
&\int_0^l \left({}^{\mathbf{B}_b}v_{bxr}-{}^{\mathbf{B}_b}v_{bx}\right)ES_x\frac{\partial^2}{\partial x^2}\left(u_{xr}(x,t)-u_x(x,t)\right)dx \\
&\quad=\left({}^{\mathbf{B}_b}v_{bxr}-{}^{\mathbf{B}_b}v_{bx}\right)ES_x\frac{\partial}{\partial x}\left(u_{xr}(x,t)-u_x(x,t)\right)\Big|_0^l \\
&\qquad-\int_0^l \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{bxr}-{}^{\mathbf{B}_b}v_{bx}\right)ES_x\frac{\partial}{\partial x}\left(u_{xr}(x,t)-u_x(x,t)\right)dx
\end{aligned}
\tag{B.65}
$$

$$
\begin{aligned}
&-\int_0^l \left({}^{\mathbf{B}_b}v_{byr}-{}^{\mathbf{B}_b}v_{by}\right)EI_z \\
&\qquad\times\frac{\partial}{\partial x}\left[(1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3}-(1-\alpha_z(x,t))\frac{\partial^3 y_b(x,t)}{\partial x^3}\right]dx \\
&\quad=-\left({}^{\mathbf{B}_b}v_{byr}-{}^{\mathbf{B}_b}v_{by}\right)EI_z \\
&\qquad\qquad\times\left[(1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3}-(1-\alpha_z(x,t))\frac{\partial^3 y_b(x,t)}{\partial x^3}\right]\Bigg|_0^l \\
&\qquad+\int_0^l \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{byr}-{}^{\mathbf{B}_b}v_{by}\right)EI_z \\
&\qquad\qquad\times\left[(1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3}-(1-\alpha_z(x,t))\frac{\partial^3 y_b(x,t)}{\partial x^3}\right]dx
\end{aligned}
\tag{B.66}
$$

$$
\begin{aligned}
&-\int_0^l \left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right) EI_y \\
&\qquad \times \frac{\partial}{\partial x}\left[(1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3} - (1-\alpha_y(x,t))\frac{\partial^3 z_b(x,t)}{\partial x^3}\right] dx \\
&\quad = -\left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right) EI_y \\
&\qquad\qquad \times \left[(1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3} - (1-\alpha_y(x,t))\frac{\partial^3 z_b(x,t)}{\partial x^3}\right]\Bigg|_0^l \\
&\qquad + \int_0^l \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right) EI_y \\
&\qquad\qquad \times \left[(1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3} - (1-\alpha_y(x,t))\frac{\partial^3 z_b(x,t)}{\partial x^3}\right] dx.
\end{aligned}
\tag{B.67}
$$

Then, in view of (13.48), (13.91), (B.62), and (B.63), the time-derivative of (B.59) can be expressed by

$$
\begin{aligned}
\dot{\nu}_b &= \int_0^l \frac{\rho}{S_x}\left[{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\left(\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b\right)\right]^T \mathrm{diag}(J_x, I_y, I_z) \\
&\qquad \times \left[{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\left(\frac{\partial}{\partial t}\boldsymbol{\omega}_{br} - \frac{\partial}{\partial t}\boldsymbol{\omega}_b\right) - {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}(\boldsymbol{\omega}_b\times)\left(\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b\right)\right] dx \\
&= -\int_0^l \frac{\rho}{S_x}\begin{bmatrix} {}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x \\ {}^{\mathbf{B}_b}\omega_{yr} - {}^{\mathbf{B}_b}\omega_y \\ {}^{\mathbf{B}_b}\omega_{zr} - {}^{\mathbf{B}_b}\omega_z \end{bmatrix}^T \\
&\qquad \times \begin{bmatrix} (I_z {}^{\mathbf{B}_b}\omega_y\left({}^{\mathbf{B}_b}\omega_{zr} - {}^{\mathbf{B}_b}\omega_z\right) - I_y {}^{\mathbf{B}_b}\omega_z\left({}^{\mathbf{B}_b}\omega_{yr} - {}^{\mathbf{B}_b}\omega_y\right) \\ I_y {}^{\mathbf{B}_b}\omega_z\left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x\right) \\ -I_z {}^{\mathbf{B}_b}\omega_y\left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x\right) \end{bmatrix} dx \\
&\quad + \int_0^l \begin{bmatrix} {}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x \\ {}^{\mathbf{B}_b}\omega_{yr} - {}^{\mathbf{B}_b}\omega_y \\ {}^{\mathbf{B}_b}\omega_{zr} - {}^{\mathbf{B}_b}\omega_z \end{bmatrix}^T \\
&\qquad \times \begin{bmatrix} GJ_x \frac{\partial^2}{\partial x^2}\left(\Theta_{xr}(x,t) - \Theta_x(x,t)\right) \\ -EI_y\left(\alpha_{yr}(x,t)\frac{\partial^3 z_{br}(x,t)}{\partial x^3} - \alpha_y(x,t)\frac{\partial^3 z_b(x,t)}{\partial x^3}\right) \\ EI_z\left(\alpha_{zr}(x,t)\frac{\partial^3 y_{br}(x,t)}{\partial x^3} - \alpha_z(x,t)\frac{\partial^3 y_b(x,t)}{\partial x^3}\right) \end{bmatrix} dx \\
&\quad - \int_0^l \left[{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\left(\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b\right)\right]^T \mathbf{K}_\omega \left[{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\left(\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b\right)\right] dx \\
&= \int_0^l \left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x\right) GJ_x \frac{\partial^2}{\partial x^2}\left(\Theta_{xr}(x,t) - \Theta_x(x,t)\right) dx \\
&\quad - \int_0^l \left({}^{\mathbf{B}_b}\omega_{yr} - {}^{\mathbf{B}_b}\omega_y\right) EI_y
\end{aligned}
$$

$$\times \left(\alpha_{yr}(x,t) \frac{\partial^3 z_{br}(x,t)}{\partial x^3} - \alpha_y(x,t) \frac{\partial^3 z_b(x,t)}{\partial x^3} \right) dx$$
$$+ \int_0^l \left({}^{\mathbf{B}_b}\omega_{zr} - {}^{\mathbf{B}_b}\omega_z \right) EI_z$$
$$\times \left(\alpha_{zr}(x,t) \frac{\partial^3 y_{br}(x,t)}{\partial x^3} - \alpha_z(x,t) \frac{\partial^3 y_b(x,t)}{\partial x^3} \right) dx$$
$$- \int_0^l (\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b)^T \left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T \mathbf{K}_\omega {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}} \right) (\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b)\, dx. \tag{B.68}$$

Subtracting (13.50) and (13.51) from (13.93) and (13.94), respectively, yields

$$ {}^{\mathbf{B}_b}\omega_{yr} - {}^{\mathbf{B}_b}\omega_y = -\frac{\partial}{\partial x} \left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz} \right) \tag{B.69}$$
$$ {}^{\mathbf{B}_b}\omega_{zr} - {}^{\mathbf{B}_b}\omega_z = \frac{\partial}{\partial x} \left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by} \right). \tag{B.70}$$

By using (13.48), (13.49), (13.91), (13.92), (B.69), and (B.70), it follows from (13.85)–(13.88), (13.98)–(13.101), and (B.64)–(B.68) that

$$\dot{\nu}_a + \dot{\nu}_b = \left({}^{\mathbf{B}_b}v_{bxr} - {}^{\mathbf{B}_b}v_{bx} \right) ES_x \frac{\partial}{\partial x} \left(u_{xr}(x,t) - u_x(x,t) \right) \Big|_0^l$$
$$- \int_0^l \frac{\partial}{\partial x} \left({}^{\mathbf{B}_b}v_{bxr} - {}^{\mathbf{B}_b}v_{bx} \right) ES_x \frac{\partial}{\partial x} \left(u_{xr}(x,t) - u_x(x,t) \right) dx$$
$$- \left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by} \right) EI_z$$
$$\times \left[(1 - \alpha_{zr}(x,t)) \frac{\partial^3 y_{br}(x,t)}{\partial x^3} - (1 - \alpha_z(x,t)) \frac{\partial^3 y_b(x,t)}{\partial x^3} \right] \Big|_0^l$$
$$+ \int_0^l \frac{\partial}{\partial x} \left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by} \right) EI_z \left(\frac{\partial^3 y_{br}(x,t)}{\partial x^3} - \frac{\partial^3 y_b(x,t)}{\partial x^3} \right) dx$$
$$- \left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz} \right) EI_y$$
$$\times \left[(1 - \alpha_{yr}(x,t)) \frac{\partial^3 z_{br}(x,t)}{\partial x^3} - (1 - \alpha_y(x,t)) \frac{\partial^3 z_b(x,t)}{\partial x^3} \right] \Big|_0^l$$
$$+ \int_0^l \frac{\partial}{\partial x} \left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz} \right) EI_y \left(\frac{\partial^3 z_{br}(x,t)}{\partial x^3} - \frac{\partial^3 z_b(x,t)}{\partial x^3} \right) dx$$
$$- \int_0^l \frac{\partial}{\partial t} \left(\mathbf{r}_{br}(x,t) - \mathbf{r}_b(x,t) \right)^T \left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T \mathbf{K}_p {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}} \right)$$
$$\times \frac{\partial}{\partial t} \left(\mathbf{r}_{br}(x,t) - \mathbf{r}_b(x,t) \right) dx$$
$$+ \int_0^l \left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x \right) GJ_x \frac{\partial^2}{\partial x^2} \left(\Theta_{xr}(x,t) - \Theta_x(x,t) \right) dx$$
$$- \int_0^l (\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b)^T \left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T \mathbf{K}_\omega {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}} \right) (\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b)\, dx$$

$$
\begin{aligned}
= & -\int_0^l \frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right)^T \left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T \mathbf{K}_p {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\right) \\
& \qquad \times \frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right) dx \\
& -\int_0^l \left(\boldsymbol{\omega}_{br}-\boldsymbol{\omega}_b\right)^T \left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T \mathbf{K}_\omega {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\right)\left(\boldsymbol{\omega}_{br}-\boldsymbol{\omega}_b\right) dx \\
& + \left({}^{\mathbf{B}_b}v_{bxr} - {}^{\mathbf{B}_b}v_{bx}\right) ES_x \frac{\partial}{\partial x}\left(u_{xr}(x,t)-u_x(x,t)\right)\Big|_0^l \\
& - \left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right) EI_z \\
& \qquad \times \left[(1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3} - (1-\alpha_z(x,t))\frac{\partial^3 y_b(x,t)}{\partial x^3}\right]\Bigg|_0^l \\
& - \left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right) EI_y \\
& \qquad \times \left[(1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3} - (1-\alpha_y(x,t))\frac{\partial^3 z_b(x,t)}{\partial x^3}\right]\Bigg|_0^l \\
& + \left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x\right) GJ_x \frac{\partial}{\partial x}\left(\Theta_{xr}(x,t)-\Theta_x(x,t)\right)\Big|_0^l \\
& - \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right)\left[-EI_y\left(\frac{\partial^2 z_{br}(x,t)}{\partial x^2} - \frac{\partial^2 z_b(x,t)}{\partial x^2}\right)\right]\Bigg|_0^l \\
& + \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right) EI_z \left(\frac{\partial^2 y_{br}(x,t)}{\partial x^2} - \frac{\partial^2 y_b(x,t)}{\partial x^2}\right)\Bigg|_0^l \\
& - \int_0^l \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{bxr} - {}^{\mathbf{B}_b}v_{bx}\right) ES_x \frac{\partial}{\partial x}\left(u_{xr}(x,t)-u_x(x,t)\right) dx \\
& - \int_0^l \frac{\partial^2}{\partial x^2}\left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right) EI_z \frac{\partial^2}{\partial x^2}\left(y_{br}(x,t)-y_b(x,t)\right) dx \\
& - \int_0^l \frac{\partial^2}{\partial x^2}\left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right) EI_y \frac{\partial^2}{\partial x^2}\left(z_{br}(x,t)-z_b(x,t)\right) dx \\
& - \int_0^l \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x\right) GJ_x \frac{\partial}{\partial x}\left(\Theta_{xr}(x,t)-\Theta_x(x,t)\right) dx \\
= & -\int_0^l \frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right)^T \left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T \mathbf{K}_p {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\right) \\
& \qquad \times \frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right) dx \\
& -\int_0^l \left(\boldsymbol{\omega}_{br}-\boldsymbol{\omega}_b\right)^T \left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T \mathbf{K}_\omega {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\right)\left(\boldsymbol{\omega}_{br}-\boldsymbol{\omega}_b\right) dx \\
& + \left[{}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\left(\dot{\mathbf{r}}_{\mathbf{B}r}-\dot{\mathbf{r}}_{\mathbf{B}}\right)\right]^T {}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\left(\mathbf{f}_{\mathbf{B}r}-\mathbf{f}_{\mathbf{B}}\right) \\
& + \left[{}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\left(\boldsymbol{\omega}_{\mathbf{B}r}-\boldsymbol{\omega}_{\mathbf{B}}\right)\right]^T {}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\left(\mathbf{m}_{\mathbf{Br}}-\mathbf{m}_{\mathbf{B}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&- \left[{}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}} \left(\dot{\mathbf{r}}_{\mathbf{T}r} - \dot{\mathbf{r}}_{\mathbf{T}}\right)\right]^T {}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}} \left(\mathbf{f}_{\mathbf{T}r} - \mathbf{f}_{\mathbf{T}}\right) \\
&- \left[{}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}} \left(\boldsymbol{\omega}_{\mathbf{T}r} - \boldsymbol{\omega}_{\mathbf{T}}\right)\right]^T {}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}} \left(\mathbf{m}_{\mathbf{T}r} - \mathbf{m}_{\mathbf{T}}\right) \\
&- \int_0^l \frac{\partial}{\partial x} \left({}^{\mathbf{B}_b}v_{bxr} - {}^{\mathbf{B}_b}v_{bx}\right) ES_x \frac{\partial}{\partial x} \left(u_{xr}(x,t) - u_x(x,t)\right) dx \\
&- \int_0^l \frac{\partial^2}{\partial x^2} \left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right) EI_z \frac{\partial^2}{\partial x^2} \left(y_{br}(x,t) - y_b(x,t)\right) dx \\
&- \int_0^l \frac{\partial^2}{\partial x^2} \left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right) EI_y \frac{\partial^2}{\partial x^2} \left(z_{br}(x,t) - z_b(x,t)\right) dx \\
&- \int_0^l \frac{\partial}{\partial x} \left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_{x}\right) GJ_x \frac{\partial}{\partial x} \left(\Theta_{xr}(x,t) - \Theta_x(x,t)\right) dx \qquad \text{(B.71)}
\end{aligned}
$$

holds with

$$
\begin{aligned}
\dot{\mathbf{r}}_{\mathbf{B}r} &= \left.\frac{\partial}{\partial t}\mathbf{r}_{br}(x,t)\right|_{x=0} \\
\dot{\mathbf{r}}_{\mathbf{B}} &= \left.\frac{\partial}{\partial t}\mathbf{r}_{b}(x,t)\right|_{x=0} \\
\boldsymbol{\omega}_{\mathbf{B}r} &= \left.\boldsymbol{\omega}_{br}(x,t)\right|_{x=0} \\
\boldsymbol{\omega}_{\mathbf{B}} &= \left.\boldsymbol{\omega}_{b}(x,t)\right|_{x=0} \\
\dot{\mathbf{r}}_{\mathbf{T}r} &= \left.\frac{\partial}{\partial t}\mathbf{r}_{br}(x,t)\right|_{x=l} \\
\dot{\mathbf{r}}_{\mathbf{T}} &= \left.\frac{\partial}{\partial t}\mathbf{r}_{b}(x,t)\right|_{x=l} \\
\boldsymbol{\omega}_{\mathbf{T}r} &= \left.\boldsymbol{\omega}_{br}(x,t)\right|_{x=l} \\
\boldsymbol{\omega}_{\mathbf{T}} &= \left.\boldsymbol{\omega}_{b}(x,t)\right|_{x=l} .
\end{aligned}
$$

Substituting (13.95) into (B.71) and using (13.89), (13.90), and (B.60) yields

$$
\begin{aligned}
\dot{\nu} &= \dot{\nu}_a + \dot{\nu}_b + \dot{\nu}_c \\
&= -\int_0^l \frac{\partial}{\partial t}\left[\mathbf{r}_{br}(x,t) - \mathbf{r}_b(x,t)\right]^T \left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T \mathbf{K}_p {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\right) \\
&\qquad \times \frac{\partial}{\partial t}\left[\mathbf{r}_{br}(x,t) - \mathbf{r}_b(x,t)\right] dx \\
&\quad - \int_0^l \left(\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b\right)^T \left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T \mathbf{K}_\omega {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\right) \left(\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b\right) dx \\
&\quad - ES_x\lambda_x \int_0^l \left[\frac{\partial}{\partial x}\left(u_{xr}(x,t) - u_x(x,t)\right)\right]^2 dx \\
&\quad - EI_z\lambda_y \int_0^l \left[\frac{\partial^2}{\partial x^2}\left(y_{br}(x,t) - y_b(x,t)\right)\right]^2 dx \\
&\quad - EI_y\lambda_z \int_0^l \left[\frac{\partial^2}{\partial x^2}\left(z_{br}(x,t) - z_b(x,t)\right)\right]^2 dx
\end{aligned}
$$

$$
-GJ_x\lambda_\Theta \int_0^l \left[\frac{\partial}{\partial x}\left(\Theta_{xr}(x,t) - \Theta_x(x,t)\right)\right]^2 dx
$$
$$
+p_{\mathbf{B}} - p_{\mathbf{T}} \tag{B.72}
$$

where

$$
\begin{aligned}
p_{\mathbf{B}} &= \left[{}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\left(\dot{\mathbf{r}}_{\mathbf{B}r} - \dot{\mathbf{r}}_{\mathbf{B}}\right)\right]^T {}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\left(\mathbf{f}_{\mathbf{B}r} - \mathbf{f}_{\mathbf{B}}\right) \\
&\quad + \left[{}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\left(\boldsymbol{\omega}_{\mathbf{B}r} - \boldsymbol{\omega}_{\mathbf{B}}\right)\right]^T {}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\left(\mathbf{m}_{\mathbf{Br}} - \mathbf{m}_{\mathbf{B}}\right) \\
&= -\left({}^{\mathbf{B}_b}v_{bxr} - {}^{\mathbf{B}_b}v_{bx}\right) ES_x \frac{\partial}{\partial x}\left(u_{xr}(x,t) - u_x(x,t)\right)\Big|_{x=0} \\
&\quad + \left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right) EI_z \\
&\qquad \times \left[(1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3} - (1-\alpha_z(x,t))\frac{\partial^3 y_b(x,t)}{\partial x^3}\right]\Bigg|_{x=0} \\
&\quad + \left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right) EI_y \\
&\qquad \times \left[(1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3} - (1-\alpha_y(x,t))\frac{\partial^3 z_b(x,t)}{\partial x^3}\right]\Bigg|_{x=0} \\
&\quad + \left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x\right)\left[-GJ_x\frac{\partial}{\partial x}\left(\Theta_{xr}(x,t) - \Theta_x(x,t)\right)\right]\Bigg|_{x=0} \\
&\quad - \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right)\left[EI_y\left(\frac{\partial^2 z_{br}(x,t)}{\partial x^2} - \frac{\partial^2 z_b(x,t)}{\partial x^2}\right)\right]\Bigg|_{x=0} \\
&\quad + \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right)\left[-EI_z\left(\frac{\partial^2 y_{br}(x,t)}{\partial x^2} - \frac{\partial^2 y_b(x,t)}{\partial x^2}\right)\right]\Bigg|_{x=0}
\end{aligned} \tag{B.73}
$$

$$
\begin{aligned}
p_{\mathbf{T}} &= \left[{}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}}\left(\dot{\mathbf{r}}_{\mathbf{T}r} - \dot{\mathbf{r}}_{\mathbf{T}}\right)\right]^T {}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}}\left(\mathbf{f}_{\mathbf{T}r} - \mathbf{f}_{\mathbf{T}}\right) \\
&\quad + \left[{}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}}\left(\boldsymbol{\omega}_{\mathbf{T}r} - \boldsymbol{\omega}_{\mathbf{T}}\right)\right]^T {}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}}\left(\mathbf{m}_{\mathbf{T}r} - \mathbf{m}_{\mathbf{T}}\right) \\
&= -\left({}^{\mathbf{B}_b}v_{bxr} - {}^{\mathbf{B}_b}v_{bx}\right) ES_x \frac{\partial}{\partial x}\left(u_{xr}(x,t) - u_x(x,t)\right)\Big|_{x=l} \\
&\quad + \left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right) EI_z \\
&\qquad \times \left[(1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3} - (1-\alpha_z(x,t))\frac{\partial^3 y_b(x,t)}{\partial x^3}\right]\Bigg|_{x=l} \\
&\quad + \left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right) EI_y \\
&\qquad \times \left[(1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3} - (1-\alpha_y(x,t))\frac{\partial^3 z_b(x,t)}{\partial x^3}\right]\Bigg|_{x=l} \\
&\quad + \left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x\right)\left[-GJ_x\frac{\partial}{\partial x}\left(\Theta_{xr}(x,t) - \Theta_x(x,t)\right)\right]\Bigg|_{x=l} \\
&\quad - \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right)\left[EI_y\left(\frac{\partial^2 z_{br}(x,t)}{\partial x^2} - \frac{\partial^2 z_b(x,t)}{\partial x^2}\right)\right]\Bigg|_{x=l} \\
&\quad + \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right)\left[-EI_z\left(\frac{\partial^2 y_{br}(x,t)}{\partial x^2} - \frac{\partial^2 y_b(x,t)}{\partial x^2}\right)\right]\Bigg|_{x=l}
\end{aligned} \tag{B.74}
$$

denote the *virtual power flows* at the two ends of the flexible beam.

B.14 Proof of Theorem 13.4

Subtracting (13.83) and (13.84) from (13.107) and (13.108) yields

$$
\begin{aligned}
&\rho^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\frac{\partial^2}{\partial t^2}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right)\\
&\quad= -\mathbf{K}_p{}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\left(\frac{\partial}{\partial t}\mathbf{r}_{br}(x,t)-\frac{\partial}{\partial t}\mathbf{r}_b(x,t)\right)\\
&\qquad-\begin{bmatrix} -ES_x\frac{\partial^2}{\partial x^2}\left(u_{xr}(x,t)-u_x(x,t)\right)\\ EI_z\frac{\partial}{\partial x}\left((1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3}-(1-\alpha_z(x,t))\frac{\partial^3 y_b(x,t)}{\partial x^3}\right)\\ EI_y\frac{\partial}{\partial x}\left((1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3}-(1-\alpha_y(x,t))\frac{\partial^3 z_b(x,t)}{\partial x^3}\right)\end{bmatrix}\\
&\qquad+(\rho-\hat{\rho})\,{}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\frac{\partial^2}{\partial t^2}\mathbf{r}_{br}(x,t)\\
&\qquad+\begin{bmatrix} -\left(ES_x-\widehat{ES_x}\right)\frac{\partial^2}{\partial x^2}u_{xr}(x,t)\\ \left(EI_z-\widehat{EI_z}\right)\frac{\partial}{\partial x}\left((1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3}\right)\\ \left(EI_y-\widehat{EI_y}\right)\frac{\partial}{\partial x}\left((1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3}\right)\end{bmatrix}
\end{aligned}\tag{B.75}
$$

$$
\begin{aligned}
&\frac{\rho}{S_x}\mathrm{diag}(J_x,I_y,I_z)^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\left[\left(\frac{\partial}{\partial t}\boldsymbol{\omega}_{br}-\frac{\partial}{\partial t}\boldsymbol{\omega}_b\right)-(\boldsymbol{\omega}_b\times)(\boldsymbol{\omega}_{br}-\boldsymbol{\omega}_b)\right]\\
&\quad= -\frac{\rho}{S_x}\begin{bmatrix}(I_z{}^{\mathbf{B}_b}\omega_y\left({}^{\mathbf{B}_b}\omega_{zr}-{}^{\mathbf{B}_b}\omega_z\right)-I_y{}^{\mathbf{B}_b}\omega_z\left({}^{\mathbf{B}_b}\omega_{yr}-{}^{\mathbf{B}_b}\omega_y\right)\\ I_y{}^{\mathbf{B}_b}\omega_z\left({}^{\mathbf{B}_b}\omega_{xr}-{}^{\mathbf{B}_b}\omega_x\right)\\ -I_z{}^{\mathbf{B}_b}\omega_y\left({}^{\mathbf{B}_b}\omega_{xr}-{}^{\mathbf{B}_b}\omega_x\right)\end{bmatrix}\\
&\qquad-\mathbf{K}_\omega{}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}(\boldsymbol{\omega}_{br}-\boldsymbol{\omega}_b)\\
&\qquad+\begin{bmatrix} GJ_x\frac{\partial^2}{\partial x^2}\left(\Theta_{xr}(x,t)-\Theta_x(x,t)\right)\\ -EI_y\left(\alpha_{yr}(x,t)\frac{\partial^3 z_{br}(x,t)}{\partial x^3}-\alpha_y(x,t)\frac{\partial^3 z_b(x,t)}{\partial x^3}\right)\\ EI_z\left(\alpha_{zr}(x,t)\frac{\partial^3 y_{br}(x,t)}{\partial x^3}-\alpha_z(x,t)\frac{\partial^3 y_b(x,t)}{\partial x^3}\right)\end{bmatrix}\\
&\qquad+\mathrm{diag}\left(\left(\frac{\rho}{S_x}J_x-\widehat{\frac{\rho}{S_x}J_x}\right),\left(\frac{\rho}{S_x}I_y-\widehat{\frac{\rho}{S_x}I_y}\right),\left(\frac{\rho}{S_x}I_z-\widehat{\frac{\rho}{S_x}I_z}\right)\right)\\
&\qquad\quad\times\left({}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\frac{\partial}{\partial t}\boldsymbol{\omega}_{br}-{}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}(\boldsymbol{\omega}_b\times)\boldsymbol{\omega}_{br}\right)\\
&\qquad+\begin{bmatrix}\left(\frac{\rho}{S_x}I_z-\widehat{\frac{\rho}{S_x}I_z}\right){}^{\mathbf{B}_b}\omega_y{}^{\mathbf{B}_b}\omega_{zr}-\left(\frac{\rho}{S_x}I_y-\widehat{\frac{\rho}{S_x}I_y}\right){}^{\mathbf{B}_b}\omega_z{}^{\mathbf{B}_b}\omega_{yr}\\ \left(\frac{\rho}{S_x}I_y-\widehat{\frac{\rho}{S_x}I_y}\right){}^{\mathbf{B}_b}\omega_z{}^{\mathbf{B}_b}\omega_{xr}\\ -\left(\frac{\rho}{S_x}I_z-\widehat{\frac{\rho}{S_x}I_z}\right){}^{\mathbf{B}_b}\omega_y{}^{\mathbf{B}_b}\omega_{xr}\end{bmatrix}
\end{aligned}
$$

$$-\begin{bmatrix} \left(GJ_x - \widehat{GJ_x}\right) \frac{\partial^2}{\partial x^2} \Theta_{xr}(x,t) \\ -\left(EI_y - \widehat{EI_y}\right) \alpha_{yr}(x,t) \frac{\partial^3 z_{br}(x,t)}{\partial x^3} \\ \left(EI_z - \widehat{EI_z}\right) \alpha_{zr}(x,t) \frac{\partial^3 y_{br}(x,t)}{\partial x^3} \end{bmatrix}. \tag{B.76}$$

With the same non-negative accompanying function defined by (B.57)–(B.60), replacing (B.61) and (B.62) by (B.75) and (B.76) and using (13.48), (13.49), (13.91), (13.92), and (B.72)–(B.74) yields

$$\begin{aligned}
\dot{\nu} = &-\int_0^l \frac{\partial}{\partial t} \left[\mathbf{r}_{br}(x,t) - \mathbf{r}_b(x,t)\right]^T \left({}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}^T \mathbf{K}_p {}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\right) \\
&\qquad \times \frac{\partial}{\partial t} \left[\mathbf{r}_{br}(x,t) - \mathbf{r}_b(x,t)\right] dx \\
&-\int_0^l \left(\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b\right)^T \left({}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}^T \mathbf{K}_\omega {}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\right) \left(\boldsymbol{\omega}_{br} - \boldsymbol{\omega}_b\right) dx \\
&-ES_x\lambda_x \int_0^l \left[\frac{\partial}{\partial x}\left(u_{xr}(x,t) - u_x(x,t)\right)\right]^2 dx \\
&-EI_z\lambda_y \int_0^l \left[\frac{\partial^2}{\partial x^2}\left(y_{br}(x,t) - y_b(x,t)\right)\right]^2 dx \\
&-EI_y\lambda_z \int_0^l \left[\frac{\partial^2}{\partial x^2}\left(z_{br}(x,t) - z_b(x,t)\right)\right]^2 dx \\
&-GJ_x\lambda_\Theta \int_0^l \left[\frac{\partial}{\partial x}\left(\Theta_{xr}(x,t) - \Theta_x(x,t)\right)\right]^2 dx \\
&+\left({}^{\mathbf{B}_b}v_{bxr} - {}^{\mathbf{B}_b}v_{bx}\right) ES_x \frac{\partial}{\partial x}\left(u_{xr}(x,t) - u_x(x,t)\right)\Big|_0^l \\
&-\left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right) EI_z \\
&\qquad \times \left[\left(1 - \alpha_{zr}(x,t)\right)\frac{\partial^3 y_{br}(x,t)}{\partial x^3} - \left(1 - \alpha_z(x,t)\right)\frac{\partial^3 y_b(x,t)}{\partial x^3}\right]\Bigg|_0^l \\
&-\left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right) EI_y \\
&\qquad \times \left[\left(1 - \alpha_{yr}(x,t)\right)\frac{\partial^3 z_{br}(x,t)}{\partial x^3} - \left(1 - \alpha_y(x,t)\right)\frac{\partial^3 z_b(x,t)}{\partial x^3}\right]\Bigg|_0^l \\
&+\left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x\right) GJ_x \frac{\partial}{\partial x}\left(\Theta_{xr}(x,t) - \Theta_x(x,t)\right)\Big|_0^l \\
&-\frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right)\left[-EI_y\left(\frac{\partial^2 z_{br}(x,t)}{\partial x^2} - \frac{\partial^2 z_b(x,t)}{\partial x^2}\right)\right]\Bigg|_0^l \\
&+\frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right) EI_z \left(\frac{\partial^2 y_{br}(x,t)}{\partial x^2} - \frac{\partial^2 y_b(x,t)}{\partial x^2}\right)\Bigg|_0^l
\end{aligned}$$

$$+\int_0^l \left[{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right)\right]^T(\rho-\hat{\rho})\,{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\frac{\partial^2}{\partial t^2}\mathbf{r}_{br}(x,t)dx$$

$$+\int_0^l \begin{bmatrix} {}^{\mathbf{B}_b}v_{bxr}-{}^{\mathbf{B}_b}v_{bx} \\ {}^{\mathbf{B}_b}v_{byr}-{}^{\mathbf{B}_b}v_{by} \\ {}^{\mathbf{B}_b}v_{bzr}-{}^{\mathbf{B}_b}v_{bz} \end{bmatrix}^T \times \begin{bmatrix} -\left(ES_x-\widehat{ES_x}\right)\frac{\partial^2}{\partial x^2}u_{xr}(x,t) \\ \left(EI_z-\widehat{EI_z}\right)\frac{\partial}{\partial x}\left((1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3}\right) \\ \left(EI_y-\widehat{EI_y}\right)\frac{\partial}{\partial x}\left((1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3}\right) \end{bmatrix} dx$$

$$+\int_0^l \begin{bmatrix} \left({}^{\mathbf{B}_b}\omega_{xr}-{}^{\mathbf{B}_b}\omega_x\right)\left(\frac{\rho}{S_x}J_x-\widehat{\frac{\rho}{S_x}J_x}\right) \\ \left({}^{\mathbf{B}_b}\omega_{yr}-{}^{\mathbf{B}_b}\omega_y\right)\left(\frac{\rho}{S_x}I_y-\widehat{\frac{\rho}{S_x}I_y}\right) \\ \left({}^{\mathbf{B}_b}\omega_{zr}-{}^{\mathbf{B}_b}\omega_z\right)\left(\frac{\rho}{S_x}I_z-\widehat{\frac{\rho}{S_x}I_z}\right) \end{bmatrix}^T \times\left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\frac{\partial}{\partial t}\boldsymbol{\omega}_{br}-{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}(\boldsymbol{\omega}_b\times)\boldsymbol{\omega}_{br}\right)dx$$

$$+\int_0^l \begin{bmatrix} {}^{\mathbf{B}_b}\omega_{xr}-{}^{\mathbf{B}_b}\omega_x \\ {}^{\mathbf{B}_b}\omega_{yr}-{}^{\mathbf{B}_b}\omega_y \\ {}^{\mathbf{B}_b}\omega_{zr}-{}^{\mathbf{B}_b}\omega_z \end{bmatrix}^T \times \begin{bmatrix} \left(\frac{\rho}{S_x}I_z-\widehat{\frac{\rho}{S_x}I_z}\right){}^{\mathbf{B}_b}\omega_y{}^{\mathbf{B}_b}\omega_{zr}-\left(\frac{\rho}{S_x}I_y-\widehat{\frac{\rho}{S_x}I_y}\right){}^{\mathbf{B}_b}\omega_z{}^{\mathbf{B}_b}\omega_{yr} \\ \left(\frac{\rho}{S_x}I_y-\widehat{\frac{\rho}{S_x}I_y}\right){}^{\mathbf{B}_b}\omega_z{}^{\mathbf{B}_b}\omega_{xr} \\ -\left(\frac{\rho}{S_x}I_z-\widehat{\frac{\rho}{S_x}I_z}\right){}^{\mathbf{B}_b}\omega_y{}^{\mathbf{B}_b}\omega_{xr} \end{bmatrix} dx$$

$$-\int_0^l \begin{bmatrix} {}^{\mathbf{B}_b}\omega_{xr}-{}^{\mathbf{B}_b}\omega_x \\ {}^{\mathbf{B}_b}\omega_{yr}-{}^{\mathbf{B}_b}\omega_y \\ {}^{\mathbf{B}_b}\omega_{zr}-{}^{\mathbf{B}_b}\omega_z \end{bmatrix}^T \begin{bmatrix} \left(GJ_x-\widehat{GJ_x}\right)\frac{\partial^2}{\partial x^2}\Theta_{xr}(x,t) \\ -\left(EI_y-\widehat{EI_y}\right)\alpha_{yr}(x,t)\frac{\partial^3 z_{br}(x,t)}{\partial x^3} \\ \left(EI_z-\widehat{EI_z}\right)\alpha_{zr}(x,t)\frac{\partial^3 y_{br}(x,t)}{\partial x^3} \end{bmatrix} dx$$

$$=-\int_0^l \frac{\partial}{\partial t}\left[\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right]^T\left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T\mathbf{K}_p{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\right) \times\frac{\partial}{\partial t}\left[\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right]dx$$

$$-\int_0^l \left(\boldsymbol{\omega}_{br}-\boldsymbol{\omega}_b\right)^T\left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T\mathbf{K}_\omega{}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\right)\left(\boldsymbol{\omega}_{br}-\boldsymbol{\omega}_b\right)dx$$

$$-ES_x\lambda_x\int_0^l \left[\frac{\partial}{\partial x}\left(u_{xr}(x,t)-u_x(x,t)\right)\right]^2dx$$

$$-EI_z\lambda_y\int_0^l \left[\frac{\partial^2}{\partial x^2}\left(y_{br}(x,t)-y_b(x,t)\right)\right]^2dx$$

$$-EI_y\lambda_z \int_0^l \left[\frac{\partial^2}{\partial x^2}\left(z_{br}(x,t) - z_b(x,t)\right)\right]^2 dx$$

$$-GJ_x\lambda_\Theta \int_0^l \left[\frac{\partial}{\partial x}\left(\Theta_{xr}(x,t) - \Theta_x(x,t)\right)\right]^2 dx$$

$$+ \left({}^{\mathbf{B}_b}v_{bxr} - {}^{\mathbf{B}_b}v_{bx}\right)\left(\widehat{ES_x}\frac{\partial}{\partial x}u_{xr}(x,t) - ES_x\frac{\partial}{\partial x}u_x(x,t)\right)\Bigg|_0^l$$

$$- \left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right)\Bigg[\widehat{EI_z}(1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3}$$

$$-EI_z(1-\alpha_z(x,t))\frac{\partial^3 y_b(x,t)}{\partial x^3}\Bigg]\Bigg|_0^l$$

$$- \left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right)\Bigg[\widehat{EI_y}(1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3}$$

$$-EI_y(1-\alpha_y(x,t))\frac{\partial^3 z_b(x,t)}{\partial x^3}\Bigg]\Bigg|_0^l$$

$$+ \left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_{x}\right)\left(\widehat{GJ_x}\frac{\partial}{\partial x}\Theta_{xr}(x,t) - GJ_x\frac{\partial}{\partial x}\Theta_x(x,t)\right)\Bigg|_0^l$$

$$- \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right)\left(-\widehat{EI_y}\frac{\partial^2 z_{br}(x,t)}{\partial x^2} - (-EI_y)\frac{\partial^2 z_b(x,t)}{\partial x^2}\right)\Bigg|_0^l$$

$$+ \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right)\left(\widehat{EI_z}\frac{\partial^2 y_{br}(x,t)}{\partial x^2} - EI_z\frac{\partial^2 y_b(x,t)}{\partial x^2}\right)\Bigg|_0^l$$

$$+ \left(ES_x - \widehat{ES_x}\right)\left({}^{\mathbf{B}_b}v_{bxr} - {}^{\mathbf{B}_b}v_{bx}\right)\frac{\partial}{\partial x}u_{xr}(x,t)\Bigg|_0^l$$

$$- \left(EI_z - \widehat{EI_z}\right)\left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right)(1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3}\Bigg|_0^l$$

$$- \left(EI_y - \widehat{EI_y}\right)\left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right)(1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3}\Bigg|_0^l$$

$$+ \left(GJ_x - \widehat{GJ_x}\right)\left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_{x}\right)\frac{\partial}{\partial x}\Theta_{xr}(x,t)\Bigg|_0^l$$

$$+ \left(EI_y - \widehat{EI_y}\right)\frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right)\frac{\partial^2 z_{br}(x,t)}{\partial x^2}\Bigg|_0^l$$

$$+ \left(EI_z - \widehat{EI_z}\right)\frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right)\frac{\partial^2 y_{br}(x,t)}{\partial x^2}\Bigg|_0^l$$

$$+ \int_0^l \left[{}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\frac{\partial}{\partial t}\left(\mathbf{r}_{br}(x,t) - \mathbf{r}_b(x,t)\right)\right]^T (\rho - \hat{\rho})\,{}^{\mathbf{B}_b}\mathbf{R}_\mathbf{I}\frac{\partial^2}{\partial t^2}\mathbf{r}_{br}(x,t)dx$$

$$
\begin{aligned}
&+\int_0^l \begin{bmatrix} {}^{\mathbf{B}_b}v_{bxr} - {}^{\mathbf{B}_b}v_{bx} \\ {}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by} \\ {}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz} \end{bmatrix}^T \\
&\quad\times \begin{bmatrix} -\left(ES_x - \widehat{ES_x}\right)\frac{\partial^2}{\partial x^2}u_{xr}(x,t) \\ \left(EI_z - \widehat{EI_z}\right)\frac{\partial}{\partial x}\left((1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3}\right) \\ \left(EI_y - \widehat{EI_y}\right)\frac{\partial}{\partial x}\left((1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3}\right) \end{bmatrix} dx \\
&+\int_0^l \begin{bmatrix} \left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x\right)\left(\frac{\rho}{S_x}J_x - \widehat{\frac{\rho}{S_x}J_x}\right) \\ \left({}^{\mathbf{B}_b}\omega_{yr} - {}^{\mathbf{B}_b}\omega_y\right)\left(\frac{\rho}{S_x}I_y - \widehat{\frac{\rho}{S_x}I_y}\right) \\ \left({}^{\mathbf{B}_b}\omega_{zr} - {}^{\mathbf{B}_b}\omega_z\right)\left(\frac{\rho}{S_x}I_z - \widehat{\frac{\rho}{S_x}I_z}\right) \end{bmatrix}^T \\
&\quad\times \left({}^{\mathbf{B}_b}\mathbf{R_I}\frac{\partial}{\partial t}\boldsymbol{\omega}_{br} - {}^{\mathbf{B}_b}\mathbf{R_I}(\boldsymbol{\omega}_b\times)\boldsymbol{\omega}_{br}\right) dx \\
&+\int_0^l \begin{bmatrix} {}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x \\ {}^{\mathbf{B}_b}\omega_{yr} - {}^{\mathbf{B}_b}\omega_y \\ {}^{\mathbf{B}_b}\omega_{zr} - {}^{\mathbf{B}_b}\omega_z \end{bmatrix}^T \\
&\quad\times \begin{bmatrix} \left(\frac{\rho}{S_x}I_z - \widehat{\frac{\rho}{S_x}I_z}\right){}^{\mathbf{B}_b}\omega_y{}^{\mathbf{B}_b}\omega_{zr} - \left(\frac{\rho}{S_x}I_y - \widehat{\frac{\rho}{S_x}I_y}\right){}^{\mathbf{B}_b}\omega_z{}^{\mathbf{B}_b}\omega_{yr} \\ \left(\frac{\rho}{S_x}I_y - \widehat{\frac{\rho}{S_x}I_y}\right){}^{\mathbf{B}_b}\omega_z{}^{\mathbf{B}_b}\omega_{xr} \\ -\left(\frac{\rho}{S_x}I_z - \widehat{\frac{\rho}{S_x}I_z}\right){}^{\mathbf{B}_b}\omega_y{}^{\mathbf{B}_b}\omega_{xr} \end{bmatrix} dx \\
&-\int_0^l \begin{bmatrix} {}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x \\ {}^{\mathbf{B}_b}\omega_{yr} - {}^{\mathbf{B}_b}\omega_y \\ {}^{\mathbf{B}_b}\omega_{zr} - {}^{\mathbf{B}_b}\omega_z \end{bmatrix}^T \begin{bmatrix} \left(GJ_x - \widehat{GJ_x}\right)\frac{\partial^2}{\partial x^2}\Theta_{xr}(x,t) \\ -\left(EI_y - \widehat{EI_y}\right)\alpha_{yr}(x,t)\frac{\partial^3 z_{br}(x,t)}{\partial x^3} \\ \left(EI_z - \widehat{EI_z}\right)\alpha_{zr}(x,t)\frac{\partial^3 y_{br}(x,t)}{\partial x^3} \end{bmatrix} dx.
\end{aligned}
\tag{B.77}
$$

Finally, define a non-negative accompanying function for the three-dimensional flexible beam with adaptive control as

$$
\begin{aligned}
\nu_p = \nu &+ \frac{1}{2}\rho_\rho(\rho - \hat{\rho}(t))^2 + \frac{1}{2}\rho_{ES_x}(ES_x - \widehat{ES_x}(t))^2 \\
&+\frac{1}{2}\rho_{EI_z}(EI_z - \widehat{EI_z}(t))^2 + \frac{1}{2}\rho_{EI_y}(EI_y - \widehat{EI_y}(t))^2 \\
&+\frac{1}{2}\rho_{\frac{\rho}{S_x}J_x}(\frac{\rho}{S_x}J_x - \widehat{\frac{\rho}{S_x}J_x}(t))^2 + \frac{1}{2}\rho_{\frac{\rho}{S_x}I_z}(\frac{\rho}{S_x}I_z - \widehat{\frac{\rho}{S_x}I_z}(t))^2 \\
&+\frac{1}{2}\rho_{\frac{\rho}{S_x}I_y}(\frac{\rho}{S_x}I_y - \widehat{\frac{\rho}{S_x}I_y}(t))^2 + \frac{1}{2}\rho_{GJ_x}(GJ_x - \widehat{GJ_x}(t))^2.
\end{aligned}
\tag{B.78}
$$

It follows from (13.113)–(13.128), (B.77), (B.78), and Lemma 2.9 that

$$\begin{aligned}
\dot{\nu}_p \leqslant & -\int_0^l \frac{\partial}{\partial t}\left[\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right]^T \left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T \mathbf{K}_p {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\right) \\
& \qquad \times \frac{\partial}{\partial t}\left[\mathbf{r}_{br}(x,t)-\mathbf{r}_b(x,t)\right] dx \\
& -\int_0^l \left(\boldsymbol{\omega}_{br}-\boldsymbol{\omega}_b\right)^T \left({}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}^T \mathbf{K}_\omega {}^{\mathbf{B}_b}\mathbf{R}_{\mathbf{I}}\right)\left(\boldsymbol{\omega}_{br}-\boldsymbol{\omega}_b\right) dx \\
& -ES_x\lambda_x \int_0^l \left[\frac{\partial}{\partial x}\left(u_{xr}(x,t)-u_x(x,t)\right)\right]^2 dx \\
& -EI_z\lambda_y \int_0^l \left[\frac{\partial^2}{\partial x^2}\left(y_{br}(x,t)-y_b(x,t)\right)\right]^2 dx \\
& -EI_y\lambda_z \int_0^l \left[\frac{\partial^2}{\partial x^2}\left(z_{br}(x,t)-z_b(x,t)\right)\right]^2 dx \\
& -GJ_x\lambda_\Theta \int_0^l \left[\frac{\partial}{\partial x}\left(\Theta_{xr}(x,t)-\Theta_x(x,t)\right)\right]^2 dx \\
& +p_{\mathbf{B}}-p_{\mathbf{T}}
\end{aligned} \tag{B.79}$$

holds with

$$\begin{aligned}
p_{\mathbf{B}} = & \left[{}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\left(\dot{\mathbf{r}}_{\mathbf{B}r}-\dot{\mathbf{r}}_{\mathbf{B}}\right)\right]^T {}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\left(\mathbf{f}_{\mathbf{B}r}-\mathbf{f}_{\mathbf{B}}\right) \\
& +\left[{}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\left(\boldsymbol{\omega}_{\mathbf{B}r}-\boldsymbol{\omega}_{\mathbf{B}}\right)\right]^T {}^{\mathbf{B}}\mathbf{R}_{\mathbf{I}}\left(\mathbf{m}_{\mathbf{Br}}-\mathbf{m}_{\mathbf{B}}\right) \\
= & -\left({}^{\mathbf{B}_b}v_{bxr}-{}^{\mathbf{B}_b}v_{bx}\right)\left(\widehat{ES}_x\frac{\partial}{\partial x}u_{xr}(x,t)-ES_x\frac{\partial}{\partial x}u_x(x,t)\right)\Bigg|_{x=0} \\
& +\left({}^{\mathbf{B}_b}v_{byr}-{}^{\mathbf{B}_b}v_{by}\right)\Bigg[\widehat{EI}_z(1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3} \\
& \qquad -EI_z(1-\alpha_z(x,t))\frac{\partial^3 y_b(x,t)}{\partial x^3}\Bigg]\Bigg|_{x=0} \\
& +\left({}^{\mathbf{B}_b}v_{bzr}-{}^{\mathbf{B}_b}v_{bz}\right)\Bigg[\widehat{EI}_y(1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3} \\
& \qquad -EI_y(1-\alpha_y(x,t))\frac{\partial^3 z_b(x,t)}{\partial x^3}\Bigg]\Bigg|_{x=0} \\
& +\left({}^{\mathbf{B}_b}\omega_{xr}-{}^{\mathbf{B}_b}\omega_x\right)\left[-\left(\widehat{GJ}_x\frac{\partial}{\partial x}\Theta_{xr}(x,t)-GJ_x\frac{\partial}{\partial x}\Theta_x(x,t)\right)\right]\Bigg|_{x=0} \\
& -\frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{bzr}-{}^{\mathbf{B}_b}v_{bz}\right)\left(\widehat{EI}_y\frac{\partial^2 z_{br}(x,t)}{\partial x^2}-EI_y\frac{\partial^2 z_b(x,t)}{\partial x^2}\right)\Bigg|_{x=0} \\
& +\frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{byr}-{}^{\mathbf{B}_b}v_{by}\right)\left[-\left(\widehat{EI}_z\frac{\partial^2 y_{br}(x,t)}{\partial x^2}-EI_z\frac{\partial^2 y_b(x,t)}{\partial x^2}\right)\right]\Bigg|_{x=0}
\end{aligned} \tag{B.80}$$

$$\begin{aligned}
p_{\mathbf{T}} = & \left[{}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}}\left(\dot{\mathbf{r}}_{\mathbf{T}r}-\dot{\mathbf{r}}_{\mathbf{T}}\right)\right]^T {}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}}\left(\mathbf{f}_{\mathbf{T}r}-\mathbf{f}_{\mathbf{T}}\right) \\
& +\left[{}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}}\left(\boldsymbol{\omega}_{\mathbf{T}r}-\boldsymbol{\omega}_{\mathbf{T}}\right)\right]^T {}^{\mathbf{T}}\mathbf{R}_{\mathbf{I}}\left(\mathbf{m}_{\mathbf{T}r}-\mathbf{m}_{\mathbf{T}}\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\left({}^{\mathbf{B}_b}v_{bxr} - {}^{\mathbf{B}_b}v_{bx}\right)\left(\widehat{ES_x}\frac{\partial}{\partial x}u_{xr}(x,t) - ES_x\frac{\partial}{\partial x}u_x(x,t)\right)\bigg|_{x=l} \\
&\quad + \left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right)\left[\widehat{EI_z}(1-\alpha_{zr}(x,t))\frac{\partial^3 y_{br}(x,t)}{\partial x^3}\right. \\
&\qquad\qquad \left. - EI_z(1-\alpha_z(x,t))\frac{\partial^3 y_b(x,t)}{\partial x^3}\right]\bigg|_{x=l} \\
&\quad + \left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right)\left[\widehat{EI_y}(1-\alpha_{yr}(x,t))\frac{\partial^3 z_{br}(x,t)}{\partial x^3}\right. \\
&\qquad\qquad \left. - EI_y(1-\alpha_y(x,t))\frac{\partial^3 z_b(x,t)}{\partial x^3}\right]\bigg|_{x=l} \\
&\quad + \left({}^{\mathbf{B}_b}\omega_{xr} - {}^{\mathbf{B}_b}\omega_x\right)\left[-\left(\widehat{GJ_x}\frac{\partial}{\partial x}\Theta_{xr}(x,t) - GJ_x\frac{\partial}{\partial x}\Theta_x(x,t)\right)\right]\bigg|_{x=l} \\
&\quad - \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{bzr} - {}^{\mathbf{B}_b}v_{bz}\right)\left(\widehat{EI_y}\frac{\partial^2 z_{br}(x,t)}{\partial x^2} - EI_y\frac{\partial^2 z_b(x,t)}{\partial x^2}\right)\bigg|_{x=l} \\
&\quad + \frac{\partial}{\partial x}\left({}^{\mathbf{B}_b}v_{byr} - {}^{\mathbf{B}_b}v_{by}\right)\left[-\left(\widehat{EI_z}\frac{\partial^2 y_{br}(x,t)}{\partial x^2} - EI_z\frac{\partial^2 y_b(x,t)}{\partial x^2}\right)\right]\bigg|_{x=l}.
\end{aligned}
\tag{B.81}
$$

Consider the fact that the flexible beam has one *driving cutting point* associated with frame $\{\mathbf{T}\}$ and one *driven cutting point* associated with frame $\{\mathbf{B}\}$. Using (B.78), (B.79), and Definition 2.17 completes the proof.

References

1. Abbati-Marescotti, A., Bonivento, C., Melchiorri, C.: On the invariance of the hybrid position/force control. J. Intell. Robot. Syst. 3, 233–250 (1990)
2. Alleyne, A., Liu, R.: On the limitations of force tracking control for hydraulic servosystems. ASME J. Dynamic Syst. Measure. Contr. 121(2), 184–190 (1999)
3. Albu-Schaffer, A., Hirzinger, G.: Parameter identification and passivity based joint control for a 7 DOF torque controlled light weight robot. In: Proc. of 2001 IEEE Int. Conf. Robot. Automat., Seoul, Korea, pp. 2852–2858 (2001)
4. An, C.H., Atkeson, C.G., Hollerbach, J.: Model-based control of a robot manipulator. MIT Press, Cambridge (1988)
5. Anderson, R.J., Spong, M.W.: Bilateral control of teleoperators with time delay. IEEE Trans. Automat. Contr. 34(5), 494–501 (1989)
6. Anderson, R.J., Spong, M.W.: Asymptotic stability for force reflecting teleoperators with time delay. Int. J. Robot. Res. 11(2), 135–149 (1992)
7. Angeles, J.: Fundamentals of robotic mechanical systems: theory, methods, and algorithms. Springer, Heidelberg (2003)
8. Antonelli, G.: Underwater robots: motion and force control of vehicle-manipulator. Springer, Heidelberg (2003)
9. Arimoto, S., Miyazaki, F., Kawamura, S.: Cooperative motion control of multiple robot arms or fingers. In: Proc. of 1987 IEEE Int. Conf. Robot. Automat., pp. 1407–1412 (1987)
10. Arimoto, S.: Fundamental problems of robot control: part 1: innovation in the realm of robot servo-loops, and part 2: a nonlinear circuit theory toward an understanding of dexterous motions. Robotica 13, 19–27, 111–122 (1995)
11. Asada, H., Slotine, J.J.E.: Robot analysis and control. John Wiley & Sons, Chichester (1986)
12. Bejczy, A.K.: Teleoperators. IEEE Industrial Electronics Society Newsletter, 4–12 (1996)
13. Bessonnet, G., Chesse, S., Sardain, P.: Optimal gait synthesis of a seven-link planar biped. Int. J. Robot. Res. 23(10-11), 1059–1073 (2004)
14. Bonitz, R.G., Hsia, T.C.: Internal force-based impedance control for cooperating manipulators. IEEE Trans. Robot. Automat. 12(1), 78–89 (1996)
15. Bridges, M.M., Dawson, D.M., Qu, Z., Martindale, S.C.: Robust control of rigid-link flexible-joint robots with redundant joint actuators. IEEE Trans. Syst. Man Cybern. 24(7), 961–970 (1994)

16. Brogliato, B., Ortega, R., Lozano, R.: Global tracking controllers for flexible-joint manipulators: a comparative study. Automatica 31(7), 941–956 (1995)
17. Bu, F., Yao, B.: Desired compensation adaptive robust control of single-rod electro-hydraulic actuator. In: Proc. of 2001 American Control Conference, Washington, DC, pp. 3926–3931 (2001)
18. Butler, Z., Kotay, K., Rus, D., Tomita, K.: Generic decentralized control for a class of self-reconfigurable robots. In: Proc. of 2002 IEEE Int. Conf. Robot. Automat., pp. 809–816 (2002)
19. Caballero, R., Armada, M.A., Akinfiev, T.: Robust cascade controller for nonlinearly actuated biped robots: experimental evaluation. Int. J. Robot. Res. 23(10-11), 1075–1095 (2004)
20. Caccavale, F., Chiacchio, P., Chiaverini, S.: Stability analysis of a joint space control law for a two-manipulator system. IEEE Trans. Automat. Contr. 44(1), 85–88 (1999)
21. Canudas de Wit, C., Siciliano, B., Bastin, G.: Theory of robot control. Springer, Berlin (1996)
22. Canudas de Wit, C., Brogliato, B.: Direct adaptive impedance control including transition phases. Automatica 33(4), 643–649 (1997)
23. Chang, Y., Daniel, R.W.: On the adaptive control of flexible joint robots. Automatica 28(5), 969–974 (1992)
24. Chartrand, G.: Directed graphs as mathematical models. In: Introductory graph theory, pp. 16–19. Courier Dover Publications, New York (1985)
25. Chen, I.M., Burdick, J.W.: Determining task optimal modular robot assembly configurations. In: Proc. of 1995 IEEE Int. Conf. Robot. Automat., pp. 132–137 (1995)
26. Cheng, F.T., Orin, D.: Optimal force distribution in multiple-chain robotic systems. IEEE Trans. Syst. Man Cybern. 21, 13–24 (1991)
27. Chevallereau, C., Aoustin, Y.: Optimal reference trajectories for walking and running a biped robot. Robotica 19(5), 557–569 (2001)
28. Chevallereau, C., Abba, G., Aoustin, Y., Plestan, F., Westervelt, E.R., Canudas de Wit, C., Grizzle, J.W.: RABBIT: a testbed for advanced control theory. IEEE Control Systems Magazine 23, 57–79 (2003)
29. Chevallereau, C.: Time-scaling control for an underactuated biped robot. IEEE Trans. Robot. Automat. 19(2), 362–368 (2003)
30. Chiacchio, P., Chiaverini, S., Sciavicco, L., Siciliano, B.: Global task space manipulability ellipsoids for multiple-arm systems. IEEE Trans. Robot. Automat. 7, 678–685 (1991)
31. Chiaverini, S., Sciavicco, L.: The parallel approach to force/position control of robotic manipulators. IEEE Trans. Robot. Automat. 9(4), 361–373 (1993)
32. Chou, J.C.: Quaternion kinematic and dynamic differential equations. IEEE Trans. Robot. Automat. 8(1), 53–64 (1992)
33. Chirikjian, G.S., Zhou, Y., Suthakorn, J.: Self-replicating robots for lunar development. IEEE Trans. Mechatronics 7(4), 462–472 (2002)
34. Christensen, D.J., Stoy, K.: Selecting a meta-module to shape-change the ATRON self-reconfigurable robot. In: Proc. of 2006 Int. Conf. Robot. Automat., Orlando, FL, pp. 2532–2538 (2006)
35. Chung, W.K., Han, J., Youm, Y., Kim, S.H.: Task based design of modular robot manipulator using efficient genetic algorithm. In: Proc. of 1997 IEEE Int. Conf. Robot. Automat., Albuquerque, NM, pp. 507–512 (1997)
36. Colgate, J.E.: Robust impedance shaping telemanipulation. IEEE Trans. Robot. Automat. 9(4), 374–384 (1993)

37. Collins, S.H., Wisse, M., Ruina, A.: A three-dimensional passive-dynamic walking robot with two legs and knees. Int. J. Robot. Res. 20(7), 607–615 (2001)
38. Cooke, J.D.: Dependence of human arm movements on limb mechanical properties. Brain Res. 165, 366–369 (1979)
39. Craig, J.J.: Introduction to robotics: mechanics and control. Addison-Wesley, Reading (1986)
40. Dauchez, P., Fournier, A., Jourdan, R.: Hybrid control of a two-arm robot for complex tasks. Robot. Autonomous Syst. 5, 323–332 (1989)
41. Dawson, D.M., Qu, Z., Carrol, J.J.: Tracking control of rigid-link electrically-driven robot manipulators. Int. J. Control. 56, 991–1006 (1992)
42. De Luca, A., Siciliano, B.: Inversion-based nonlinear control of robot arms with flexible links. AIAA J. Guidance Control and Dynamics 16(6), 1169–1176 (1993)
43. De Luca, A., Manes, C.: Modeling of robots in contact with a dynamic environment. IEEE Trans. Robot. Automat. 10(3), 542–548 (1994)
44. De Luca, A., Farina, R., Lucibello, P.: On the control of robots with visco-elastic joints. In: Proc. of 2005 IEEE Int. Conf. Robot. Automat., Barcelona, Spain, pp. 4308–4313 (2005)
45. De Queiroz, M.S., Dawson, D.M., Agarwal, M., Zhang, F.: Adaptive nonlinear boundary control of a flexible link robot arm. IEEE Trans. Robot. Automat. 15(4), 779–787 (1999)
46. De Schutter, J., Van Brussel, H.: Compliant robot motion. Int. J. Robot. Res. 7(4), 3–33 (1988)
47. De Schutter, J., Bruyninckx, H., Demey, S., Dutre, S., De Geeter, J., Katupitiya, J.: Force control. In: Melchiorri, C., Tornambe, A. (eds.) Modelling and control of mechanisms and robots, pp. 227–264. World Scientific, Singapore (1996)
48. De Schutter, J., Torfs, D., Bruyninckx, H., Dutre, S.: Invariant hybrid force/position control of a velocity controlled robot with compliant end effector using modal decoupling. Int. J. Robot. Res. 16(3), 340–356 (1997)
49. Dhaouadi, R., Ghorbel, F.H., Gandhi, P.S.: A new dynamic model of hysteresis in harmonic drives. IEEE Trans. Ind. Electron. 50(6), 1165–1171 (2003)
50. Dickson, W.C., Cannon Jr., R.H.: Experimental results of two free-flying robots capturing and manipulating a free-flying object. In: Proc. of IEEE/IROS 1995, pp. 51–58 (1995)
51. Dubowsky, S., Papadopoulos, E.: The kinematics, dynamics, and control of free-flying and free-floating space robotic systems. IEEE Trans. Robot. Automat. 9(5), 531–543 (1993)
52. Egeland, O., Sagli, J.R.: Coordination of motion in a spacecraft/manipulator system. Int. J. Rob. Res. 12(4), 366–379 (1993)
53. Eryilmaz, B., Wilson, B.H.: Improved tracking control of hydraulic systems. ASME J. Dynamic Syst. Measure. Contr. 123(3), 457–462 (2001)
54. Fu, K.S., Gonzalez, R.C., Lee, C.S.G.: Robotics: control, sensing, vision, and intelligence. McGraw-Hill, New York (1987)
55. Fukuda, T., Kawauchi, Y.: Cellular robotic system (CEBOT) as one of the realization of self-organizing intelligent universal manipulator. In: Proc. of 1990 IEEE Int. Conf. Robot. Automat., pp. 662–667 (1990)
56. Funda, J., Taylor, R.H., Eldridge, B., Gomory, S., Gruben, K.G.: Constrained Cartesian motion control for teleoperated surgical robots. IEEE Trans. Robot. Automat. 12(3), 453–465 (1996)
57. Godler, I., Horiuchi, M., Hashimoto, M., Ninomiya, T.: Accuracy improvement of built-in torque sensing for harmonic drives. IEEE/ASME Trans. Mechatronics 5(4), 360–366 (2000)

58. Goldenberg, A.A., Wiercienski, J., Kuzan, P., Szymczyk, C., Fenton, R.G., Shaver, B.: A remote manipulator for forestry operation. IEEE Trans. Robot. Automat. 11(2), 185–197 (1995)
59. Good, M.C., Sweet, L.M., Strbel, K.L.: Dynamic models for control system design of integrated robot and drive systems. ASME J. Dynamic Syst. Measure. Contr. 107(1), 53–59 (1985)
60. Gorinevsky, D.M., Formalsky, A.M., Schneider, A.Y.: Force control of robotics systems. CRC Press, Boca Raton (1997)
61. Gray, A.: Euclidean spaces. In: Modern differential geometry of curves and surfaces with mathematica, 2nd edn., pp. 2–5. CRC Press, Boca Raton (1997)
62. Grizzle, J.W., Abba, G., Plestan, F.: Asymptotically stable walking for biped robots: analysis via systems with impulse effects. IEEE Trans. Automat. Contr. 46(1), 51–64 (2001)
63. Gupta, K.C.: Mechanics and control of robots. Springer, New York (1997)
64. Hannaford, B.: A design framework for teleoperators with kinesthetic feedback. IEEE Trans. Robot. Automat. 5(4), 426–434 (1989)
65. Hannaford, B., Wood, L., McAffee, D.A., Zak, H.: Performance evaluation of a six-axis generalized force-reflecting teleoperator. IEEE Trans. Syst. Man Cybern. 21(3), 620–633 (1991)
66. Hashimoto, M.: Robot motion control based on joint torque sensing. In: Proc. of 1989 IEEE Int. Conf. Robot. Automat., pp. 256–261 (1989)
67. Hashimoto, M., Kiyosawa, Y., Paul, R.P.: A torque sensing technique for robots with harmonic drives. IEEE Trans. Robot. Automat. 9(1), 108–116 (1993)
68. Hayati, S.: Hybrid position/force control of multi-arm cooperating robots. In: Proc. of 1986 IEEE Int. Conf. Robot. Automat., pp. 82–89 (1986)
69. Hirai, K., Hirose, M., Haikawa, Y., Takenaka, T.: The development of Honda humanoid robot. In: Proc. of 1998 IEEE Int. Conf. Robot. Automat., pp. 1321–1326 (1998)
70. Hirzinger, G., Brunner, B., Dietrich, J., Heindl, J.: Sensor-based space robotics – ROTEX and its telerobotic features. IEEE Trans. Robot. Automat. 9(5), 649–663 (1993)
71. Hogan, N.: Impedance control: an approach to manipulation, parts 1-3. ASME J. Dynamic Syst. Measure. Contr. 107, 1–24 (1985)
72. Honegger, M., Corke, P.: Model-based control of hydraulically actuated manipulators. In: Proc. of 2001 IEEE Int. Conf. Robot. Automat., Seoul, Korea, pp. 2553–2559 (2001)
73. Hu, Y.R., Goldenberg, A.A.: Dynamic control of coordinated redundant robots with torque optimization. Automatica 29(6), 1411–1424 (1993)
74. Hu, Y.R., Goldenberg, A.A.: An adaptive approach to motion and force control of multiple coordinated robots. ASME J. Dynamic Syst. Measure. Contr. 115(1), 60–69 (1993)
75. Hsu, P.: Coordinated control of multiple manipulator systems. IEEE Trans. Robot. Automat. 9(4), 400–410 (1993)
76. Inaba, N., Oda, M.: Autonomous satellite capture by a space robot. In: Proc. of 2000 Int. Conf. Robot. Automat., San Francisco, CA, pp. 1169–1174 (2000)
77. Jain, A., Rodriguez, G.: An analysis of the kinematics and dynamics of underactuated manipulators. IEEE Trans. Robot. Automat. 9(5), 411–422 (1993)
78. Jankowski, K.P., Van Brussel, H.: An approach to discrete inverse dynamics control of flexible joint robots. IEEE Trans. Robot. Automat. 8(5), 651–658 (1992)
79. Jean, J.H., Fu, L.C.: Adaptive hybrid control strategies for constrained robots. IEEE Trans. Automat. Contr. 38(4), 598–603 (1993)

80. Jean, J.H., Fu, L.C.: An adaptive control scheme for coordinated multi-manipulator systems. IEEE Trans. Robot. Automat. 9, 226–231 (1993)
81. Junkins, J., Kim, Y.: Introduction to dynamics and control of flexible structure. In: AIAA Education Series, Washington, DC (1993)
82. Kanoh, H., Tzafestas, S., Lee, H.G., Kalat, J.: Modelling and control of flexible robot arms. In: Proc. of 25th IEEE Conference on Decision and Control, pp. 1866–1870 (1986)
83. Kazerooni, H., Tsay, T.I., Hollerbach, K.: A controller design framework for telerobotic systems. IEEE Trans. Control Systems Technology 1(1), 50–62 (1993)
84. Kazerooni, H., Her, M.G.: The dynamics and control of a haptic interface device. IEEE Trans. Robot. Automat. 10(4), 453–464 (1994)
85. Kelly, R., Ortega, R., Ailon, A., Loria, A.: Global regulation of flexible joint robots using approximate differentiation. IEEE Trans. Automat. Contr. 39(6), 1222–1224 (1994)
86. Khadem, S.E., Pirmohammadi, A.A.: Analytical development of dynamic equations of motion for a three-dimensional flexible link manipulator with revolute and prismatic joints. IEEE Trans. Syst. Man Cybern. B 33(2), 237–249 (2003)
87. Khalil, H.K.: Nonlinear systems, 2nd edn. Prentice Hall, Englewood Cliffs (1996)
88. Khatib, O.: A unified approach for motion and force control of robot manipulators: the operational space formulation. IEEE J. of Robot. Automat. 3(1), 43–53 (1987)
89. Khorasani, K.: Adaptive control of flexible joint robots. In: Proc. of 1991 IEEE Int. Conf. Robot. Automat., pp. 2127–2134 (1991)
90. Kircanski, N.M., Goldenberg, A.A.: An experimental study of nonlinear stiffness, hysteresis, and friction effects in robot joints with harmonic drives and torque sensors. Int. J. Robot. Res. 16(2), 214–239 (1997)
91. Koga, M., Kosuge, K., Furuta, K., Nosaki, K.: Coordinated motion control of robot arms based on the virtual internal model. IEEE Trans. Robot. Automat. 8, 77–85 (1992)
92. Koivo, A.J.: Fundamentals for control of robot manipulators. John Wiley & Sons, Chichester (1989)
93. Koivo, A.J., Unseren, M.A.: Reduced order model and decoupled control architecture for two manipulators holding a rigid object. ASME J. Dynamic Syst. Measure. Contr. 113, 646–654 (1991)
94. Kosuge, K., Ishikawa, J., Furuta, K., Sakai, M.: Control of single-master multi-slave manipulator system using VIM. In: Proc. of 1990 IEEE Int. Conf. Robot. Automat., pp. 1172–1177 (1990)
95. Kosuge, K., Takeuchi, H., Furuta, K.: Motion control of a robot arm using joint torque sensors. IEEE Trans. Robot. Automat. 6(2), 258–263 (1990)
96. Kreutz, K., Lokshin, A.: Load balancing and closed chain multiple arm control. In: Proc. of 1988 American Contr. Conf., pp. 2148–2155 (1988)
97. Kuo, A.D.: Stabilization of lateral motion in passive dynamic walking. Int. J. Robot. Res. 18(9), 917–930 (1999)
98. Lawrence, D.A.: Stability and transparency in bilateral teleoperation. IEEE Trans. Robot. Automat. 9(5), 624–637 (1993)
99. Lee, S., Lee, H.S.: Modeling, design, and evaluation of advanced teleoperator control systems with short time delay. IEEE Trans. Robot. Automat. 9(5), 607–623 (1993)
100. Lewis, F.L., Dawson, D.M., Abdallah, C.T.: Robot manipulator control: theory and practice. Marcel Dekker Publishing Company, New York (2004)

101. Leung, G.M.H., Francis, B.A., Apkarian, J.: Bilateral controller for teleoperators with time delay via μ-synthesis. IEEE Trans. Robot. Automat. 11(1), 105–116 (1995)
102. Liu, G., Goldenberg, A.A.: Robust control of robot manipulators incorporating motor dynamics. In: Proc. of IEEE/IROS 1993, pp. 68–75 (1993)
103. Liu, G., Abdul, S., Goldenberg, A.A.: Distributed control of modular and reconfigurable robot with torque sensing. Robotica 26(1), 75–84 (2008)
104. Liu, R., Alleyne, A.: Nonlinear force/pressure tracking of an electro-hydraulic actuator. ASME J. Dynamic Syst. Measure. Contr. 122(1), 232–237 (2000)
105. Liu, Y.H., Arimoto, S., Kitagaki, K.: Adaptive control for holonomically constrained robots: time-invariant and time-variant cases. In: Proc. of 1995 IEEE Int. Conf. Robot. Automat., pp. 905–912 (1995)
106. Liu, Y.H., Xu, Y., Bergerman, M.: Cooperation control of multiple manipulators with passive joints. IEEE Trans. Robot. Automat. 15(2), 258–267 (1999)
107. Lohmeier, S., Buschmann, T., Ulbrich, H., Pfeiffer, F.: Modular joint design for performance enhanced humanoid robot LOLA. In: Proc. of 2006 IEEE Int. Conf. Robot. Automat., Orlando, FL, pp. 88–93 (2006)
108. Longman, R.W., Lindberg, R.E., Zedd, M.F.: Satellite-mounted robot manipulators - new kinematics and reaction moment compensation. Int. J. Robot. Res. 6(3), 87–103 (1987)
109. Lozano, R., Brogliato, B.: Adaptive control of robot manipulators with flexible joints. IEEE Trans. Automat. Contr. 37(2), 174–181 (1992)
110. Luh, J.Y.S., Walker, M.W., Paul, R.P.C.: On-line computational scheme for mechanical manipulators. ASME J. Dynamic Syst. Measure. Contr. 102, 69–76 (1980)
111. Luh, J.Y.S., Zheng, Y.F.: Constrained relations between two coordinated industrial robots for motion control. Int. J. Robot. Res. 6(3), 60–70 (1987)
112. Luo, Z.H.: Direct strain feedback control of flexible robot arms: new theoretical and experimental results. IEEE Trans. Automat. Contr. 38(11), 1610–1622 (1993)
113. Luo, Z.H., Guo, B.Z., Morgul, O.: Stability and stabilization of infinite dimensional systems with applications. Springer, London (1999)
114. Macchelli, A., Melchiorri, C., Stramigioli, S.: Port-based modeling of a flexible link. IEEE Trans. Robot. Automat. 23(4), 650–660 (2007)
115. Marino, R., Nicosia, S.: Singular perturbation techniques in the adaptive control of elastic robots. In: Proc. of First IFAC Symp. Robot Control (1985)
116. Mattila, J., Virvalo, T.: Energy-efficient motion control of a hydraulic manipulator. In: Proc. 2000 IEEE Int. Conf. Robot. Automat., San Francisco, CA, pp. 3000–3006 (2000)
117. McClamroch, N.H., Wang, D.W.: Feedback stabilization and tracking of constrained robots. IEEE Trans. Automat. Contr. 33, 419–426 (1988)
118. McGeer, T.: Passive dynamic walking. Int. J. Robot. Res. 9(2), 62–82 (1990)
119. Meirovitch, L.: Principles and techniques of vibrations. Prentice-Hall, Englewood Cliffs (1997)
120. Merritt, H.E.: Hydraulic control systems. Wiley, New York (1976)
121. Miles, E.S., Cannon Jr., R.H.: Utilizing human vision and computer vision to direct a robot in a semi-structured environment via task-level commands. In: Proc. of IEEE/IROS 1995, pp. 366–371 (1995)
122. Mrad, F., Ahmad, S.: Control of flexible joint robots. In: Proc. of 1991 IEEE Int. Conf. Robot. Automat., pp. 2832–2837 (1991)

123. Murata, S., Yoshida, E., Kamimura, A., Kurokawa, H., Tomita, K., Kokaji, S.: M-TRAN: self-reconfigurable modular robotic system. IEEE/ASME Trans. Mechatronics 7(4), 431–441 (2002)
124. Murphy, S.H., Wen, J.T., Saridis, G.N.: Simulation of cooperating robot manipulators on a mobile platform. IEEE Trans. Robot. Automat. 7(4), 468–477 (1991)
125. Nakamura, Y., Nagai, K., Yoshikawa, T.: Dynamics and stability in coordination of multiple robotic mechanisms. Int. J. Robot. Res. 8(2), 44–61 (1989)
126. Nakamura, Y., Mukherjee, R.: Exploiting nonholonomic redundancy of free-flying space robots. IEEE Trans. Robot. Automat. 9(4), 499–506 (1993)
127. Namvar, M., Aghili, F.: A combined scheme for identification and robust torque control of hydraulic actuators. ASME J. Dynamic Syst. Measure. Contr. 125(4), 595–606 (2003)
128. Natale, C.: Interaction control of robot manipulators - six-degrees-of-freedom tasks. Springer, Heidelberg (2003)
129. Nicosia, S., Tomei, P.: A tracking controller for flexible joint robots using only link position feedback. IEEE Trans. Automat. Contr. 40(5), 885–890 (1995)
130. Niksefat, N., Sepehri, N.: Robust force controller design for a hydraulic actuator based on experimental input-output data. In: Proc. of 1999 American Contr. Conf., San Diego, CA, pp. 3718–3722 (1999)
131. Niksefat, N., Wu, C.Q., Sepehri, N.: Design of a Lyapunov controller for an electro-hydraulic actuator during contact tasks. ASME J. Dynamic Syst. Measure. Contr. 123(2), 299–307 (2001)
132. Nof, S.Y. (ed.): Handbook of industrial robotics, 2nd edn. John Wiley & Sons, Chichester (1999)
133. Oda, M.: Coordinated control of spacecraft attitude and its manipulator. In: Proc. of 1996 IEEE Int. Conf. Robot. Automat., pp. 732–738 (1996)
134. Orin, D.E., Oh, S.Y.: Control of force distribution in robotic mechanisms containing closed kinematic chains. ASME J. Dynamic Syst. Measure. Contr. 102, 134–141 (1981)
135. Papadopoulos, E., Dubowsky, S.: On the nature of control algorithms for free-floating space manipulators. IEEE Trans. Robot. Automat. 7(6), 750–758 (1991)
136. Papadopoulos, E., Dubowsky, S.: Dynamic singularities in free-floating space manipulators. ASME J. Dynamic Syst. Measure. Contr. 115, 44–52 (1993)
137. Paredis, C.J.J., Khosla, P.K.: Kinematic design of serial link manipulators from task specifications. Int. J. Robot. Res. 12(3), 274–287 (1993)
138. Paul, R.P.: Robot manipulator: mathematics, programming, and control. MIT Press, Cambridge (1981)
139. Qu, Z.: Input-output robust tracking control design for flexible joint robots. IEEE Trans. Automat. Contr. 40(1), 78–83 (1995)
140. Qu, Z., Dawson, D.M.: Robust tracking control of robot manipulators. IEEE Press, Los Alamitos (1996)
141. Raibert, M.H., Craig, J.J.: Hybrid position/force control of manipulators. ASME J. Dynamic Syst. Measure. Contr. 102(2), 126–133 (1981)
142. Royden, H.: Real analysis. Prentice-Hall, Englewood Cliffs (1988)
143. Rus, D., Vona, M.: Crystalline robots: self-reconfiguration with compressible unit modules. Autonomous Robots 10(1), 107–124 (2001)
144. Schneider, S.A., Cannon Jr., R.H.: Object impedance control for cooperative manipulation: theory and experimental. IEEE Trans. Robot. Automat. 8, 383–394 (1992)
145. Sciavicco, L., Siciliano, B.: Modelling and control of robot manipulators. Springer, Heidelberg (2000)

146. Shen, W.M., Salemi, B., Will, P.: Hormone-inspired adaptive communication and distributed control for CONRO self-reconfigurable robots. IEEE Trans. Robot. Automat. 18(5), 700–712 (2002)
147. Seraji, H., Colbaugh, R.: Force tracking in impedance control. Int. J. Robot. Res. 16(1), 97–117 (1997)
148. Sheridan, T.B.: Telerobotics. Automatica 25(4), 487–507 (1989)
149. Siciliano, B., Villani, L.: Robot force control. Kluwer Academic Publishers, Dordrecht (2000)
150. Siciliano, B., Khatib, O.: Springer handbook of robotics. Springer, Heidelberg (2008)
151. Sirouspour, M.R., Salcudean, S.E.: Nonlinear control of hydraulic robots. IEEE Trans. Robot. Automat. 17(2), 173–182 (2001)
152. Skogestad, S., Postlethwaite, I.: Multivariable feedback control - analysis and design. John Wiley & Sons, Chichester (1996)
153. Slotine, J.J.E., Li, W.P.: On the adaptive control of robot manipulators. Int. J. Robot. Res. 6(3), 49–59 (1987)
154. Slotine, J.J.E., Li, W.P.: Adaptive manipulator control: a case study. IEEE Trans. Automat. Contr. 33(11), 995–1003 (1988)
155. Slotine, J.J.E., Li, W.P.: Applied nonlinear control. Prentice Hall, Englewood Cliffs (1991)
156. Sohl, G.A., Bobrow, J.E.: Experiments and simulations on the nonlinear control of a hydraulic servosystem. IEEE Trans. Control Systems Technology 7(2), 238–247 (1999)
157. Somolinos, J.A., Feliu, V., Sanchez, L.: Design, dynamic modelling and experimental validation of a new three-degree-of-freedom flexible arm. Mechatronics 12(7), 919–948 (2002)
158. Spofford, J.R., Akin, D.L.: Redundancy control of a free-flying telerobot. AIAA J. Guidance Control and Dynamics 12(3), 515–523 (1990)
159. Spong, M.W.: Modeling and control of elastic joint robots. ASME J. Dynamic Syst. Measure. Contr. 109, 310–319 (1987)
160. Spong, M.W., Khorasani, K., Kokotovic, P.V.: An integral manifold approach to the feedback control of flexible joint robots. IEEE Trans. Robot. Automat. 3(4), 291–300 (1987)
161. Spong, M.W., Vidyasagar, M.: Robot dynamics and control. John Wiley & Sons, Chichester (1989)
162. Spong, M.W.: On the force control problem for flexible joint manipulators. IEEE Trans. Automat. Contr. 34(1), 107–111 (1989)
163. Su, C.Y., Stepanenko, Y.: Hybrid adaptive/robust motion control of rigid-link electrically-driven robot manipulators. IEEE Trans. Robot. Automat. 11(3), 426–432 (1995)
164. Sun, Z., Tsao, T.C.: Adaptive control with asymptotic tracking performance and its application to an electro-hydraulic servo system. ASME J. Dynamic Syst. Measure. Contr. 122(1), 188–195 (2000)
165. Tafazoli, S., Lawrence, P.D., Salcudean, S.E.: Identification of inertial and friction parameters for excavator arms. IEEE Trans. Robot. Automat. 15(5), 966–971 (1999)
166. Takegaki, M., Arimoto, S.: A new feedback method for dynamic control of manipulators. ASME J. Dynamic Syst. Measure. Contr. 103, 119–125 (1981)
167. Tao, J.M., Luh, J.Y.S., Zheng, Y.F.: Compliant coordination control of two moving industrial robots. IEEE Trans. Robot. Automat. 6, 322–330 (1990)

168. Tao, G.: A simple alternative to the Barbălat lemma. IEEE Trans. Automat. Contr. 42(5), 698 (1997)
169. Tarn, T.J., Bejczy, A.K., Yun, X.: Design of dynamic control of two cooperating robot arms: closed chain formulation. In: Proc. of 1987 IEEE Int. Conf. Robot. Automat., pp. 7–13 (1987)
170. Tarn, T.J., Bejczy, A.K., Yun, X., Li, Z.: Effect of motor dynamics on nonlinear feedback robot arm control. IEEE Trans. Robot. Automat. 7(1), 114–122 (1991)
171. Tomei, P.: A simple PD controller for robots with elastic joints. IEEE Trans. Automat. Contr. 36(10), 1208–1213 (1991)
172. Tomei, P.: Tracking control of flexible joint robots with uncertain parameters and disturbances. IEEE Trans. Automat. Contr. 39(5), 1067–1072 (1994)
173. Turetta, A., Casalino, G., Sorbara, A.: Distributed control architecture for self-reconfigurable manipulators. Int. J. Robot. Res. 27(3-4), 481–504 (2008)
174. Tuttle, T.D., Seering, W.P.: A nonlinear model of a harmonic drive gear transmission. IEEE Trans. Robot. Automat. 12(3), 368–374 (1996)
175. Uchiyama, M., Dauchez, P.: A symmetric hybrid position/force control scheme for the coordination of two robots. In: Proc. of 1988 IEEE Int. Conf. Robot. Automat., pp. 350–356 (1988)
176. Umetani, Y., Yoshida, K.: Resolved motion rate control of space manipulators with generalized Jacobian matrix. IEEE Trans. Robot. Automat. 5(3), 303–314 (1989)
177. Unseren, M.A.: A rigid body model and decoupled control architecture for two manipulators holding a complex object. Robot. Autonomous Syst. 10, 115–131 (1992)
178. Van der Schaft, A.: L_2-gain and passivity techniques in nonlinear control. Springer, Heidelberg (2000)
179. Vukobratovic, M., Borovac, B., Surla, D., Stokic, D.: Biped locomotion - dynamics, stability, control and application. Springer, Heidelberg (1990)
180. Walker, I.D., Freeman, R.A., Marcus, S.I.: Internal object loading for multiple cooperating robot manipulators. In: Proc. of 1989 IEEE Int. Conf. Robot. Automat., pp. 606–611 (1989)
181. Walker, M.W., Kim, D., Dionise, J.: Adaptive coordinated motion control of two manipulator arms. In: Proc. of 1989 IEEE Int. Conf. Robot. Automat., pp. 1084–1090 (1989)
182. Walker, M.W.: Adaptive control of manipulators containing closed kinematic loops. IEEE Trans. Robot. Automat. 6, 10–19 (1990)
183. Walker, M.W., Wee, L.B.: Adaptive control of space-based robot manipulators. IEEE Trans. Robot. Automat. 7(6), 828–835 (1991)
184. Wang, D., Vidyasagar, M.: Modeling a class of multilink manipulators with the last link flexible. IEEE Trans. Robot. Automat. 8(1), 33–41 (1992)
185. Wen, J.T., Kreutz-Delgado, K.: The attitude control problem. IEEE Trans. Automat. Contr. 36(10), 1148–1162 (1991)
186. Wen, J.T., Kreutz-Delgado, K.: Motion and force control of multiple robotic manipulators. Automatica 28(4), 729–743 (1992)
187. Westervelt, E.R., Grizzle, J.W., Koditschek, D.E.: Hybrid zero dynamics of planar biped walkers. IEEE Trans. Automat. Contr. 48(1), 42–56 (2003)
188. Xi, N., Tarn, T.J., Bejczy, A.K.: Intelligent planning and control for multirobot coordination: an event-based approach. IEEE Trans. Robot. Automat. 12(3), 439–452 (1996)
189. Xu, Y., Kanade, T.: Space robotics: dynamics and control. Kluwer Academic Publishers, Dordrecht (1992)

190. Xu, Y., Shum, H.Y., Kanade, T., Lee, J.J.: Parameterization and adaptive control of space robot systems. IEEE Trans. Aerospace Electron. Syst. 30(2), 435–451 (1994)
191. Yabuta, T.: Nonlinear basic stability concept of the hybrid position/force control scheme for robot manipulators. IEEE Trans. Robot. Automat. 8(5), 663–670 (1992)
192. Yan, J., Salcudean, S.E.: Teleoperation controller design using H_∞-optimization with application to motion-scaling. IEEE Trans. Contr. Syst. Technology 4(3), 244–258 (1996)
193. Yao, B., Gao, W.B., Chan, S.P., Cheng, M.: VSC coordinated control of two manipulator arms in the presence of environmental constraints. IEEE Trans. Automat. Contr. 37, 1806–1812 (1992)
194. Yao, B., Bu, F., Reedy, J., Chiu, G.: Adaptive robust motion control of single-rod hydraulic actuators: theory and experiments. IEEE/ASME Trans. Mechatronics 5(1), 79–91 (2000)
195. Yim, M., Duff, D.G., Roufas, K.D.: Polybot: a modular reconfigurable robot. In: Proc. of 2000 IEEE Int. Conf. Robot. Automat., San Francisco, CA, pp. 514–520 (2000)
196. Yim, M., Shen, W.M., Salemi, B., Rus, D., Moll, M., Lipson, H., Klavins, E., Chirikjian, G.S.: Modular self-reconfigurable robot systems. IEEE Robot. Automat. Magazine 14(1), 43–52 (2007)
197. Yokokohji, Y., Toyoshima, T., Yoshikawa, T.: Efficient computational algorithm for trajectory control of free-flying space robots with multiple arms. IEEE Trans. Robot. Automat. 9(5), 571–579 (1993)
198. Yokokohji, Y., Yoshikawa, T.: Bilateral control of master-slave manipulators for ideal kinesthetic coupling – formulation and experiment. IEEE Trans. Robot. Automat. 10(5), 605–620 (1994)
199. Yoshida, K., Kurazume, R., Umetani, Y.: Dual arm coordination in space free-flying robot. In: Proc. of 1991 IEEE Int. Conf. Robot. Automat., pp. 2516–2521 (1991)
200. Yoshikawa, T.: Foundations of robotics: analysis and control. MIT Press, Cambridge (1990)
201. Yoshikawa, T., Zheng, X.Z.: Coordinated dynamic hybrid position/force control for multiple robot manipulators handling one constrained object. Int. J. Robot. Res. 12(3), 219–230 (1993)
202. Yuan, J., Stepanenko, Y.: Composite adaptive control of flexible joint robots. Automatica 29(3), 609–619 (1993)
203. Yuan, J.: Composite adaptive control of constrained robots. IEEE Trans. Robot. Automat. 12(4), 640–645 (1996)
204. Yun, X.P., Kumar, V.R.: An approach to simultaneous control of trajectory and interaction forces in dual-arm configurations. IEEE Trans. Robot. Automat. 7(5), 618–625 (1991)
205. Yun, X.P.: Modeling and control of two constrained manipulators. J. Intell. Robot. Syst. 4, 363–377 (1991)
206. Zhen, R.R., Goldenberg, A.A.: An adaptive approach to constrained robot motion control. In: Proc. of 1995 IEEE Int. Conf. Robot. Automat., pp. 1833–1838 (1995)
207. Zheng, Y.F., Hemami, H.: Impact effects of biped contact with the environment. IEEE Trans. Syst. Man Cybern. 14(3), 437–443 (1984)
208. Zheng, Y.F., Luh, J.Y.S.: Control of two coordinated robots in motion. In: Proc. of 1985 IEEE Int. Conf. Robot. Automat., pp. 1761–1766 (1985)

209. Zhu, W.H., Xi, Y.G., Zhang, Z.J.: Coordinative control of two spacerobots based on virtual decomposition. In: Proc. of 3rd IEEE Conf. Control Application., Glasgow, Scotland, pp. 327–332 (1994)
210. Zhu, M., Salcudean, S.E.: Achieving transparency for teleoperator systems under position and rate control. In: Proc. of IEEE/IROS 1995, vol. 2, pp. 7–12 (1995)
211. Zhu, W.H., Xi, Y.G., Zhang, Z.J., Bien, Z., De Schutter, J.: Virtual decomposition based control for generalized high dimensional robotic systems with complicated structure. IEEE Trans. Robot. Automat. 13(3), 411–436 (1997)
212. Zhu, W.H., Bien, Z., De Schutter, J.: Adaptive motion/force control of coordinated multiple manipulators with joint flexibility based on virtual decomposition. IEEE Trans. Automat. Contr. 43(1), 46–60 (1998)
213. Zhu, W.H., De Schutter, J.: Experiment with two industrial robot manipulators rigidly holding an egg. In: Proc. of 1998 IEEE Int. Conf. Robot. Automat., Leuven, Belgium, pp. 1534–1539 (1998)
214. Zhu, W.H., De Schutter, J.: Adaptive control of mixed rigid/flexible joint robot manipulators based on virtual decomposition. IEEE Trans. Robot. Automat. 15(2), 310–317 (1999)
215. Zhu, W.H., De Schutter, J.: Control of two industrial manipulators rigidly holding an egg. IEEE Control Systems Magazine 19(2), 24–30 (1999)
216. Zhu, W.H., De Schutter, J.: Adaptive control of electrically driven space robots based on virtual decomposition. AIAA J. Guidance Control and Dynamics 22(2), 329–339 (1999)
217. Zhu, W.H., Salcudean, S.E.: Stability guaranteed teleoperation: an adaptive motion/force control approach. IEEE Trans. Automat. Contr. 45(11), 1951–1969 (2000)
218. Zhu, W.H., De Schutter, J.: Experimental verifications of virtual decomposition based motion/force control. IEEE Trans. Robot. Automat. 18(3), 379–386 (2002)
219. Zhu, W.H., Piedboeuf, J.C., Gonthier, Y.: Emulation of a space robot using a hydraulic manipulator on ground. In: Proc. of 2002 IEEE Int. Conf. Robot. Automat., Washington, DC, pp. 2315–2320 (2002)
220. Zhu, W.H., De Schutter, J.: Virtual decomposition based motion/force control of an industrial manipulator KUKA361. In: Preprint of 15th IFAC World Congress, Barcelona, Spain, pp. 1902–1907 (2002)
221. Zhu, W.H., Doyon, M.: Adaptive control of harmonic drives. In: Proc. of 43rd IEEE Conference on Decision and Control, The Atlantis, Bahamas, pp. 2602–2608 (2004)
222. Zhu, W.H., Piedboeuf, J.C.: Adaptive output force tracking control of hydraulic cylinders with applications to robot manipulators. ASME J. Dynamic Syst. Measure. Contr. 127(2), 206–217 (2005)
223. Zhu, W.H.: On adaptive synchronization control of coordinated multirobots with flexible/rigid constraints. IEEE Trans. Robotics 21(3), 520–525 (2005)
224. Zhu, W.H., Dupuis, E., Doyon, M., Piedboeuf, J.C.: Adaptive control of harmonic drives based on virtual decomposition. IEEE/ASME Trans. Mechatronics 11(5), 604–614 (2006)
225. Zhu, W.H., Dupuis, E., Doyon, M.: Adaptive control of harmonic drives. ASME J. Dynamic Syst. Measure. Contr. 129, 182–193 (2007)
226. Zhu, W.H., Lamarche, T.: Modular robot manipulators based on virtual decomposition control. In: Proc. of 2007 IEEE Int. Conf. Robot. Automat., pp. 2235–2240 (2007)
227. Zhu, W.H.: Dynamics of general constrained robots derived from rigid bodies. ASME J. Applied Mechanics 75(3) (2008)

228. Zhu, W.H., Lange, C., Callot, M.: Virtual decomposition control of a planar flexible-link robot. In: Preprint of 17th IFAC World Congress, Seoul, Korea, pp. 1697–1702 (2008)
229. Ziaei, K., Sepehri, N.: Design of a nonlinear adaptive controller for an electro-hydraulic actuator. ASME J. Dynamic Syst. Measure. Contr. 123(3), 449–456 (2001)

Index

Springer Tracts in Advanced Robotics

Edited by B. Siciliano, O. Khatib and F. Groen

Further volumes of this series can be found on our homepage: springer.com

Vol. 59: Otake, M.
Electroactive Polymer Gel Robots – Modelling and Control of Artificial Muscles
238 p. 2010 [978-3-540-23955-0]

Vol. 58: Kröger, T.
On-Line Trajectory Generation in Robotic Systems – Basic Concepts for Instantaneous Reactions to Unforeseen (Sensor) Events
230 p. 2010 [978-3-642-05174-6]

Vol. 57: Chirikjian, G.S.; Choset, H.; Morales, M., Murphey, T. (Eds.)
Algorithmic Foundations of Robotics VIII – Selected Contributions of the Eighth International Workshop on the Algorithmic Foundations of Robotics
680 p. 2010 [978-3-642-00311-0]

Vol. 56: Buehler, M.; Iagnemma, K.; Singh S. (Eds.)
The DARPA Urban Challenge – Autonomous Vehicles in City Traffic
625 p. 2009 [978-3-642-03990-4]

Vol. 55: Stachniss, C.
Robotic Mapping and Exploration
196 p. 2009 [978-3-642-01096-5]

Vol. 54: Khatib, O.; Kumar, V.; Pappas, G.J. (Eds.)
Experimental Robotics: The Eleventh International Symposium
579 p. 2009 [978-3-642-00195-6]

Vol. 53: Duindam, V.; Stramigioli, S.
Modeling and Control for Efficient Bipedal Walking Robots
211 p. 2009 [978-3-540-89917-4]

Vol. 52: Nüchter, A.
3D Robotic Mapping
201 p. 2009 [978-3-540-89883-2]

Vol. 51: Song, D.
Sharing a Vision
186 p. 2009 [978-3-540-88064-6]

Vol. 50: Alterovitz, R.; Goldberg, K.
Motion Planning in Medicine: Optimization and Simulation Algorithms for Image-Guided Procedures
153 p. 2008 [978-3-540-69257-7]

Vol. 49: Ott, C.
Cartesian Impedance Control of Redundant and Flexible-Joint Robots
190 p. 2008 [978-3-540-69253-9]

Vol. 48: Wolter, D.
Spatial Representation and Reasoning for Robot Mapping
185 p. 2008 [978-3-540-69011-5]

Vol. 47: Akella, S.; Amato, N.; Huang, W.; Mishra, B.; (Eds.)
Algorithmic Foundation of Robotics VII
524 p. 2008 [978-3-540-68404-6]

Vol. 46: Bessière, P.; Laugier, C.; Siegwart R. (Eds.)
Probabilistic Reasoning and Decision Making in Sensory-Motor Systems
375 p. 2008 [978-3-540-79006-8]

Vol. 45: Bicchi, A.; Buss, M.; Ernst, M.O.; Peer A. (Eds.)
The Sense of Touch and Its Rendering
281 p. 2008 [978-3-540-79034-1]

Vol. 44: Bruyninckx, H.; Přeučil, L.; Kulich, M. (Eds.)
European Robotics Symposium 2008
356 p. 2008 [978-3-540-78315-2]

Vol. 43: Lamon, P.
3D-Position Tracking and Control for All-Terrain Robots
105 p. 2008 [978-3-540-78286-5]

Vol. 42: Laugier, C.; Siegwart, R. (Eds.)
Field and Service Robotics
597 p. 2008 [978-3-540-75403-9]

Vol. 41: Milford, M.J.
Robot Navigation from Nature
194 p. 2008 [978-3-540-77519-5]

Vol. 40: Birglen, L.; Laliberté, T.; Gosselin, C.
Underactuated Robotic Hands
241 p. 2008 [978-3-540-77458-7]

Vol. 39: Khatib, O.; Kumar, V.; Rus, D. (Eds.)
Experimental Robotics
563 p. 2008 [978-3-540-77456-3]

Vol. 38: Jefferies, M.E.; Yeap, W.-K. (Eds.)
Robotics and Cognitive Approaches to
Spatial Mapping
328 p. 2008 [978-3-540-75386-5]

Vol. 37: Ollero, A.; Maza, I. (Eds.)
Multiple Heterogeneous Unmanned Aerial
Vehicles
233 p. 2007 [978-3-540-73957-9]

Vol. 36: Buehler, M.; Iagnemma, K.;
Singh, S. (Eds.)
The 2005 DARPA Grand Challenge – The Great
Robot Race
520 p. 2007 [978-3-540-73428-4]

Vol. 35: Laugier, C.; Chatila, R. (Eds.)
Autonomous Navigation in Dynamic
Environments
169 p. 2007 [978-3-540-73421-5]

Vol. 34: Wisse, M.; van der Linde, R.Q.
Delft Pneumatic Bipeds
136 p. 2007 [978-3-540-72807-8]

Vol. 33: Kong, X.; Gosselin, C.
Type Synthesis of Parallel
Mechanisms
272 p. 2007 [978-3-540-71989-2]

Vol. 32: Milutinović, D.; Lima, P.
Cells and Robots – Modeling and Control of
Large-Size Agent Populations
130 p. 2007 [978-3-540-71981-6]

Vol. 31: Ferre, M.; Buss, M.; Aracil, R.;
Melchiorri, C.; Balaguer C. (Eds.)
Advances in Telerobotics
500 p. 2007 [978-3-540-71363-0]

Vol. 30: Brugali, D. (Ed.)
Software Engineering for Experimental Robotics
490 p. 2007 [978-3-540-68949-2]

Vol. 29: Secchi, C.; Stramigioli, S.; Fantuzzi, C.
Control of Interactive Robotic Interfaces – A
Port-Hamiltonian Approach
225 p. 2007 [978-3-540-49712-7]

Vol. 28: Thrun, S.; Brooks, R.;
Durrant-Whyte, H. (Eds.)
Robotics Research – Results of the 12th
International Symposium ISRR
602 p. 2007 [978-3-540-48110-2]

Vol. 27: Montemerlo, M.; Thrun, S.
FastSLAM – A Scalable Method for the
Simultaneous Localization and Mapping
Problem in Robotics
120 p. 2007 [978-3-540-46399-3]

Vol. 26: Taylor, G.; Kleeman, L.
Visual Perception and Robotic Manipulation – 3D
Object Recognition, Tracking and Hand-Eye
Coordination
218 p. 2007 [978-3-540-33454-5]

Vol. 25: Corke, P.; Sukkarieh, S. (Eds.)
Field and Service Robotics – Results of the 5th
International Conference
580 p. 2006 [978-3-540-33452-1]

Vol. 24: Yuta, S.; Asama, H.; Thrun, S.;
Prassler, E.; Tsubouchi, T. (Eds.)
Field and Service Robotics – Recent Advances in
Research and Applications
550 p. 2006 [978-3-540-32801-8]

Vol. 23: Andrade-Cetto, J,; Sanfeliu, A.
Environment Learning for Indoor Mobile Robots
– A Stochastic State Estimation Approach
to Simultaneous Localization and Map Building
130 p. 2006 [978-3-540-32795-0]

Vol. 22: Christensen, H.I. (Ed.)
European Robotics Symposium 2006
209 p. 2006 [978-3-540-32688-5]

Vol. 21: Ang Jr., H.; Khatib, O. (Eds.)
Experimental Robotics IX – The 9th International
Symposium on Experimental Robotics
618 p. 2006 [978-3-540-28816-9]

Vol. 20: Xu, Y.; Ou, Y.
Control of Single Wheel Robots
188 p. 2005 [978-3-540-28184-9]

Vol. 19: Lefebvre, T.; Bruyninckx, H.;
De Schutter, J. Nonlinear Kalman Filtering
for Force-Controlled Robot Tasks
280 p. 2005 [978-3-540-28023-1]

Vol. 18: Barbagli, F.; Prattichizzo, D.;
Salisbury, K. (Eds.)
Multi-point Interaction with Real
and Virtual Objects
281 p. 2005 [978-3-540-26036-3]