Glossary

Symbol	Definition	Equation	Section
a	minor radius of a torus	(3.53)	3.6.3
a	acceleration	(4.55)	4.4.2.1
a	radius of a dust particle		10.1.3.2
a_{WS}	Wigner-Seitz radius	(2.16)	2.1.3
A_{br}	coefficient for bremsstrahlung	(4.64)	4.4.2.3
A_{R}	coefficient in Richardson's law	(11.10)	11.1.5
b	impact parameter		4.2.5
b_{c}	impact parameter for charge collection	(10.5)	10.1.3
b_{90}	impact parameter for 90° deflection	(4.20)	4.2.5
B_{bc}, B_{cb}	Einstein coefficients		8.1.1
B_{m}	maximum magnetic field in a mirror		3.4.2
B_{p}	poloidal magnetic field	(3.55)	3.6.1
B_{r}	radial magnetic field		4.3.5
B_{t}	toroidal magnetic field	(3.52)	3.6
B_{tot}	total magnetic mirror field		3.4.2
$\mathbf{B}$	magnetic field vector		3.1
$\hat{\mathbf{B}}$	Fourier amplitude of **B**	(6.8)	6.1.2
$b_{\pi/2}$	impact parameter for 90° deflection		10.2.4
C	capacitance of a dust particle	(10.10)	10.1.3.2
C_0	capacitance of bulk plasma	(11.16)	11.2.1
C_{L}	sound speed of longitudinal mode	(10.93)	10.4.1.5
c_{s}	sound velocity	(6.79)	6.5.3
C_{s}	ion acoustic velocity	(6.78)	6.5.3
C_{T}	sound speed of transverse mode		10.4.1.5
D	diffusion coefficient	(4.31)	4.3.3
D	Stix parameter	(6.90)	6.6
D	spring constant	(10.87)	10.4.1.1
$D(\omega, \mathbf{k})$	dispersion function	(6.29)	6.2
D_{a}	ambipolar diffusion coefficient	(4.36)	4.3.3.1

A. Piel, *Plasma Physics*, DOI 10.1007/978-3-642-10491-6

d_0	equilibrium distance of dust particles	(10.68)	10.2.5
d_n	thickness of the normal cathode fall	(11.8)	11.1.3
$\hat{\mathbf{D}}_\omega$	Fourier coefficient of displacement vector	(6.12)	6.1.3
E_a	ambipolar electric field	(4.37)	4.3.3.1
E_p	electric field at grid point p	(9.81)	9.4.1
$\mathbf{E}$	electric field vector		3.1
$\hat{\mathbf{E}}$	Fourier amplitude of $\mathbf{E}$	(6.8)	6.1.2
f_b	burn fraction of a pellet	(4.74)	4.4.3.4
F_c	collection force	(10.46)	10.2.4
F_defl	deflection force		8
f_e1	perturbed electron distribution function	(9.35)	9.3.1
F_g	gravitational force on dust grain		10.2.1
F_i	force on i-th particle	(9.82)	9.4.1
f_m	resonance frequencies of a cavity	(6.58)	6.4.4
F_i	ion wind force	(10.45)	10.2.4
f_M	Maxwell distribution, general	(2.10)	2.1.1
$f_\mathrm{M}(v)$	Maxwell distribution of speeds	(4.6)	4.1
$f_\mathrm{M}(v_x)$	shifted Maxwellian	(5.5)	5.1.2
$f_\mathrm{M}^{(1)}$	1D Maxwell distribution	(4.1)	4.1
$f_\mathrm{M}^{(3)}$	3D Maxwell distribution	(4.5)	4.1
$F_\mathrm{M}(W)$	Maxwell distribution of energies	(4.9)	4.1.4
$\mathbf{F}_\mathrm{n}$	Epstein drag force	(10.41)	10.2.2
F_o	orbit force	(10.58)	10.2.4
F_p	ponderomotive force	(3.63)	3.7.1
F_rest	restoring force		8
$\mathbf{F}_\mathrm{tp}$	thermophoretic force	(10.43)	10.2.3
F_trap	force exerted by potential trap	(10.66)	10.2.5
F_Y	Yukawa interaction force	(10.64)	10.2.5
$f_+(x, v)$	part of distribution function	(9.25)	9.2.1
$f_-(x, v)$	part of distribution function	(9.27)	9.2.1
$F_<, F_>$	force from inside and outside	(10.75)	10.3.4
g	gravitational acceleration on Earth	(10.32)	10.2.1.1
g_L	Lunar gravitational acceleration		10.1.2.1
g_i	degeneracy factor of atomic level i		2.1.1
$g(T)$	used in calculation of burn fraction	(4.74)	4.4.3.4
H_p	poloidal magnetic field	(3.54)	3.6.1
H_t	toroidal magnetic field	(3.51)	3.6
H_0	scale height	(1.2)	
I	moment of inertia	(9.85)	9.4.2
$I_\mathrm{e,sat}$	electron saturation current of a probe	(7.29)	7.4.2
$I_\mathrm{i,sat}$	ion saturation current of a probe	(7.27)	7.4.1
I_p	probe current		7.4
I_N	particle flux	(5.14)	5.1.4
I_P	momentum flux	(5.16)	5.1.4

I_{ph}	Photoelectron current	(10.3)	10.1.2
j	current density	(4.29)	4.3.2
j_{e}	electron current density	(4.29)	4.3.2
j_{i}	ion current density	(4.29)	4.3.2
$j_{\mathrm{max,Pierce}}$	maximum current in a Pierce diode	(8.39)	
j_{t}	toroidal current density	(3.53)	3.6.3
$\hat{\mathbf{j}}$	Fourier amplitude of $\mathbf{j}$	(6.8)	6.1.2
J	longitudinal invariant		3.4.3
J_0	initial angular momentum		7.5.4
k	wavenumber	(6.8)	6.1.2
K	normalized wavenumber	(10.90)	10.4.1.2
k_{I}	spatial growth rate	(8.20)	8.1.5
k_{B}	Boltzmann's constant		2.1
$\mathbf{kk}$	tensor product of wave vecors	(6.27)	6.2
k_+, k_-	wavenumbers of beam modes	(8.26)	8.3.2
ℓ	length scale		2.2.2
L_{b}	inductance of bulk plasma	(11.17)	11.2.1
m_{a}	mass of an atom	(4.49)	4.4.1
m_{e}	electron mass		
m_{i}	ion mass		
m^*	effective mass describing collisions	(6.43)	6.3.2
m_{d}	dust mass	(10.32)	10.2.1.1
m_{e}^*	effective electron mass		6.3.2
$m_\odot$	solar mass	Table 1.1	1.2
M	Mach number	(7.18)	7.3.2
$\mathbf{M}$	magnetic dipole moment		3.1.3
M_{s}	total mass of spherical pellet	(4.75)	4.4.3.4
n	density		2.1
n_{co}	cut-off density	(6.50)	6.4
n_{e}	electron density		2.1
n_{e0}	unpertubed electron density		2.2.1
n_{i}	ion density		2.1
n_{i0}	unperturbed ion density		2.2.1
$n_{\mathrm{A+}}$	ion density		2.1.2
n_i	population density of atomic state i	(2.10)	2.1.1
N	number of particles		
$N(r)$	number of particles inside r	(10.72)	10.3.4
N_{a}	number of atomic targets	(4.13)	4.2.2
N_{De}	number of particles in electron Debye sphere	(2.33)	2.3.1
$N^{(\alpha)}$	total number of particles of species α	(9.5)	9.1.1
$\mathscr{N}$	refractive index	(6.24)	6.1.6
$\mathscr{N}_{\mathrm{L}}$	refractive index of L-mode	(6.96)	6.6.2
$\mathscr{N}_{\mathrm{R}}$	refractive index of R-mode	(6.95)	6.6.2

p	pressure		2.1
P	Stix parameter	(6.90)	6.6
P	Havnes parameter	(10.27)	10.1.6
$p(t)$	time-dependent probability	(4.44)	4.3.4
P_{br}	Power in bremsstrahlung	(4.59)	4.4.2.1
P_{DT}	Power from D–T reaction	(4.63)	4.4.2
P_{e}, P_{i}	probability of capturing an electron (ion)	(10.18)	10.1.5
P_{fus}	Power from fusion reaction		4.4.2.5
P_{ij}	stress tensor	(5.29)	5.1.5
P_{H}	heat loss rate	(4.62)	4.4.2
p_{mag}	magnetic pressure	(5.47)	5.2.2
P_x	x-component of momentum vector	(5.23)	5.1.4
Q	electric charge		2.2.1
Q	quality factor of a resonator		6.4.4
q_{d}	dust charge	(10.11)	10.1.3.2
Q_{DT}	fusion yield of D–T reaction		4.4.2.2
Q_{α}	energy of α-particles		4.4.2.4
r	radius		2.2.1
r_{L}	Larmor radius	(3.5)	3.1.2
R	major radius of a torus		3.6.3
R	equivalent resistance of charging characteristic	(10.15)	10.1.4
R_{b}	resistance of bulk plasma	(11.17)	11.2.1
r_{C}	Coulomb radius	(10.55)	10.2.4
$R(T)$	recombination rate		2.1.2
R_{c}	radius of curvature		5.2.2
R_{C}	radius of a Coulomb ball	(10.81)	10.3.4
R_{hs}	radius of hot spot	(4.70)	4.4.3.3
R_{m}	mirror ratio		3.4.2
R_1, R_2	electrical resistors		2.2.1
R_{total}	resistance of parallel circuit		2.2.1
s	distance		2.2.1
S	entropy	(2.3)	2.1.1.1
S	Stix parameter	(6.90)	6.6
s_1, s_2	sheath thickness	(11.18)	11.2.2
$S(\mathbf{k}, \omega)$	spectral energy density	(10.96)	10.4.2
$S(W_{\lambda})$	energy density of solar spectrum	(10.3)	10.1.2
S_{ion}	ionization rate	(4.18)	4.2.3
$S(T)$	ionization rate		2.1.2
T	temperature		
$\mathscr{T}$	tension		
T_{d}	dust temperature		10.4.3
T_{e}	electron temperature		2.1
T_{i}	ion temperature		2.1
Δt_k	variable time step for charge capture	(10.20)	10.1.5

T_{ph}	effective temperature of photoelectrons	(10.4)	10.1.2
$\mathbf{u}$	drift velocity		5.1.1
U_1, U_2	sheath voltage	(11.22)	11.2.2
u_{i}	ion streaming velocity		7.2
U_{b}	voltage drop in bulk plasma	(11.16)	11.2.1
U_{bd}	breakdown voltage	(11.6)	11.1.3.1
U_{dc}	self bias voltage	(11.27)	11.2.4
U_{ind}	induction voltage		5.1.1
U_{n}	normal cathode fall voltage		11.1.3
U_{p}	probe voltage		7.4
U_{rf}	radio-frequency voltage	(11.27)	11.2.4
v	velocity		
$\mathbf{v}$	velocity vector		3.1
v_x, v_y, v_z	components of the velocity vector		3.1
$\bar{v}_x, \bar{v}_y$	average velocities	(4.46)	4.3.4
v_{A}	Alfvén velocity	(5.83)	5.3.5
v_{b}	velocity of a beam electron	(8.11)	8.1.4
v_{c}	cut-off velocity	(9.24)	9.2.1
v_{B}	Bohm velocity	(7.17)	7.3.2
v_{d}	drift velocity	(4.27)	4.3.1
v_{D}	diamagnetic drift velocity	(5.50)	5.2.3
v_E	E×B drift velocity	(3.12)	3.1.4
v_{exhaust}	exhaust velocity		4.4.3.2
v_{g}	gravitational drift velocity	(3.14)	3.1.5
$\mathbf{v}_{\mathrm{gr}}$	group velocity vector	(6.23)	6.1.5
v_{m}	mass flow velocity	(5.56)	5.3.1
v_{p}	polarisation drift velocity	(3.47)	3.5.1
v_R	curvature drift velocity	(3.23)	3.2.3
v_{shell}	velocity of imploding shell	(4.69)	4.4.3.2
v_{th}	mean thermal speed	(4.7)	4.1.3
v_T	most probable speed	(4.4)	2.3.1
v_0	initial beam velocity	(8.3)	8.1.2
$v_+, v_-, \bar{v}$	contributions to the Pierce mode	(8.33)	8.3.3
$v_\perp$	perpendicular velocity		3.4.2
$v_{\nabla B}$	gradient drift velocity	(3.21)	3.2.2
$\mathbf{v}_\varphi$	phase velocity	(6.18)	6.1.4
W	energy		2.1.1
$W(x)$	charge assignment function	(9.79)	9.4.1
W_{b}	binding energy		
W_{ex}	excess energy		
W_0	initial energy		7.5.4
W_{ion}	ionisation energy		4.2.4
W_{kin}	kinetic energy	(4.8)	4.1.3
W_{m}	maximum energy in secondary emission	(10.2)	10.1.1
$\langle W_{\mathrm{kin}} \rangle$	mean beam kinetic energy	(8.17)	8.1.4

W_{pot}	potential energy	(9.84)	9.4.2
W_{R}	work function in Richardson's law	(11.10)	11.1.5
W_{tot}	total energy	(8.40), (9.86)	9.4.2
W_{Y}	energy of Yukawa interaction	(10.63)	10.2.5
$W_{\perp}$	energy of perpendicular motion	(3.49)	3.5.2
x_{min}	position of potential minimum in diode	(9.25)	9.2.1
Z	partition function		2.1.1
Z_{b}	impedance of bulk plasma	(11.16)	11.2.1
Z_{d}	dust charge number = $\lvert q_{\mathrm{d}}\rvert/e$	(10.13)	10.1.3.2
Z_j	charge number of species j		2.2.2
α	index denoting particle species	(6.80)	6.6
α	Townsend's electron multiplication factor	(11.1)	11.1.3.1
α_{b}	beam fraction in beam-plasma system	(8.3)	8.1.2
α_{P}	Pierce parameter	(8.37)	8.3.1
$\alpha(L)$	angle of Faraday rotation	(6.100)	6.6.2.1
β	ratio of kinetic and total pressure	(5.51)	5.2.3
γ	growth rate		8.1.3
γ	coefficient for secondary emission by ion impact	(11.4)	11.1.3.1
$\mathit{\Gamma}$	coupling parameter	(2.15)	2.1.3
$\mathit{\Gamma}_{\mathrm{a}}$	ambipolar flux	(4.34)	4.3.3.1
$\mathit{\Gamma}_{\mathrm{e,i}}$	flux of electrons (ions)	(4.31)	4.3.3
δ	accomodation coefficient for Epstein friction	(10.42)	10.2.2
δ_0	skin depth	(11.41)	11.3.1
δ_{cl}	collisionless skin depth	(11.43)	11.3.1
$\delta(W)$	secondary emission coefficient	(10.1)	10.1.1
δ_{m}	maximum of $\delta(W)$	(10.2)	10.1.1
ε	dielectric function for cold plasma	(6.45)	6.4
ε	... for beam-plasma system	(8.2)	8.1.2
ε	... for ion-acoustic wave	(6.75)	6.5.3
$\boldsymbol{\varepsilon}$	dielectric tensor	(6.89)	6.6
$\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}$	elements of the dielectric tensor	(6.34)	6.3.1
η	resistivity	(4.22)	4.2.5
η	efficiency of energy conversion	(4.63)	4.4.2
η_{c}	normalized cloud potential	(10.25)	10.1.6
η_{f}	normalized grain potential	(10.25)	10.1.6
η_{s}	Spitzer resistivity	(4.23)	4.2.5
θ	angle w.r.t. magnetic field direction		3.4.2
θ	normalized frequency for Pierce modes		8.3.3
θ_{c}	resonance cone angle	(6.105)	6.7
θ_{m}	loss cone angle	(3.56)	3.6.3
λ	wavelength		6.4
λ	Lagrange multiplier		2.1.1.1
$\ln(\mathit{\Lambda})$	Coulomb logarithm	(10.61)	10.2.4

λ_B	de Broglie wavelength	(2.36)	2.3.2
λ_D	Debye length		2.2.1
λ_De	Electron Debye length	(2.28)	2.2.1
λ_Di	Ion Debye length	(2.28)	2.2.1
λ_max	maximum of Planck curve	(2.14)	2.1.2
λ_min	shortest wavelength in crystal		10.4.1.1
λ_mfp	mean free path	(4.14)	4.2.2
λ_s	modified shielding length	(10.56)	10.2.4
μ	Lagrange multiplier		2.1.1.1
μ	magnetic moment	(3.34)	3.3.2
μ	ion-to-electron mass ratio		
μ	Mach cone angle	(10.94)	10.4.1.7
μ_e	electron mobility	(4.28)	4.3.1
μ_i	ion mobility	(4.28)	4.3.1
ν_coll	collision frequency	(4.15)	4.2.2
ν_DT	deuterium-tritium collision frequency	(4.60)	4.4.2.2
ν_ei	electron–ion collision frequency	(4.21)	4.2.5
ν_ion	ionization frequency	(4.17)	4.2.3
ν_m	momentum loss frequency	(4.25)	4.3.1
$\xi(W_\lambda)$	photoelectric efficiency	(10.3)	10.1.2
Ξ_d	normalized dust collision frequency	(10.90)	10.4.1.2
ρ	charge density	(5.1)	5.1.1 / 5.1.3
ρ_m	mass density in MHD	(5.38)	5.2
ρ_d	mass density of dust material		10.2.1
ρ_p	assigned charge to gridpoint p	(9.78)	9.4.1
ρ_0	initial mass density		4.4.3.4
ρR	density-radius product for a pellet		4.4.3.4
σ	cross section	(4.13)	4.2.2
σ	conductivity		4.3.2
σ_c	cross section for charge collection	(10.5)	10.1.3
$\sigma_\mathrm{e,i}$	electron (ion) conductivity	(4.30)	4.3.2
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$	elements of conductivity tensor	(6.32)	6.3.1
$\boldsymbol{\sigma}$	conductivity tensor	(4.47)	4.3.4
σ_ion	cross-section for ionization	(4.16)	4.2.3
τ	elapsed time	(9.20)	9.2.1
τ	relaxation time of dust charge	(10.16)	10.1.4
τ	temperature ratio $T_\mathrm{e}/T_\mathrm{i}$	(10.26)	10.1.6
τ_B	magnetic diffusion time	(5.62)	5.3.2
τ_c	inertial confinement time	(4.68)	4.4.3.1
τ_E	energy confinement time		4.4.2.5
φ	phase angle of a wave	(6.16)	6.1.4
$\varphi_\mathrm{air}, \varphi_\mathrm{p}$	phase angle in air (plasma)		6.4.3
φ_1, φ_2	phase angles of second harmonic		6.4.3
Φ	electric potential		2.2.1 / 7.2
Φ_f	floating potential (probe)	(7.34)	7.4.4

Φ_f	floating potential (dust grain)	(10.8)	10.1.3.1
Φ_p	potential at gridpoint *p*	(9.80)	9.4.1
Φ_p	plasma potential		7.4
Φ_m	magnetic flux		5.1.1
Φ_min	minimum potential in a diode	()9.28	9.2.2
χ	empirical constant for photoemission	(10.4)	10.1.2
χ	scattering angle	(10.57)	10.2.4
χ_b	permittivity of beam electrons	(8.3)	8.1.2
χ_d	permittivity of dust system	(10.99)	10.4.3
χ_e	permittivity of cold electrons	(8.21)	8.2
χ_i	permittivity of ions	(8.21)	8.2
χ_p	permittivity of plasma electrons	(8.2)	8.1.2
ψ	angle between **k** and **B**	(6.91)	6.6.1
ω	angular frequency		3.1 / 6.6.2
Ω	normalized frequency		10.4.1.2
ω_br	breathing frequency of dust cluster	(10.69)	10.2.5
ω_c	cyclotron frequency	(3.4)	3.1.2
ω_ce	electron cyclotron frequency		3.1.2
ω_ci	ion cyclotron frequency	(6.84)	6.6.1
ω_I	imaginary part = growth rate	(8.9)	8.1.2
Ω_L	normalized frequency of long. mode	(10.90)	10.4.1.2
ω_lh	lower hybrid frequency	(6.104)	6.6.3
ω_pd	dust plasma frequency	(10.98)	10.4.3
ω_pe	electron plasma frequency	(2.32)	2.2.3
ω_pi	ion plasma frequency	(6.75)	6.5.3
ω_R	real part of wave frequency	(8.8)	8.1.2
Ω_T	norm. frequency of transverse mode	(10.91)	10.4.1.2
ω_uh	upper hybrid frequency	(6.103)	6.6.3
ω_0	plasma frequency for strongly coupled system	(10.89)	10.4.1.1

Appendix: Constants and Formulas

> *"Reeling and Writhing, of course, to begin with," the Mock Turtle replied; "and then the different branches of Arithmetic—Ambition, Distraction, Uglification and Derision."*
>
> Lewis Carroll, Alice in Wonderland

1 Physical Constants

The physical constants are given here in SI-units with four digits accuracy. For problem solving in plasma physics, often two digits are sufficient in view of the uncertainty of measured plasma parameters.

Table 1 Physical constants

Electron mass	m_e	$= 9.109 \times 10^{-31}$	kg
Proton mass	m_p	$= 1.673 \times 10^{-27}$	kg
Proton-electron mass ratio	$m_\mathrm{p}/m_\mathrm{e}$	$= 1836$	
Elementary charge	e	$= 1.602 \times 10^{-19}$	A s
Specific charge of electron	e/m_e	$= 1.759 \times 10^{11}$	$\mathrm{C\,kg^{-1}}$
Speed of light in vacuum	c	$= 2.998 \times 10^{8}$	$\mathrm{m\,s^{-1}}$
Permittivity of free space	ε_0	$= 8.854 \times 10^{-12}$	$\mathrm{A\,s\,V^{-1}\,m^{-1}}$
	μ_0	$= 1.257 \times 10^{-6}$	$\mathrm{V\,s\,A^{-1}\,m^{-1}}$
Boltzmann constant	k_B	$= 1.381 \times 10^{-23}$	$\mathrm{J\,K^{-1}}$
Temperature associated with 1 eV	e/k_B	$= 11{,}600$	$\mathrm{K\,V^{-1}}$
Stefan-Boltzmann constant	σ	$= 5.670 \times 10^{-8}$	$\mathrm{Wm^{-2}K^{-4}}$
Planck's constant	h	$= 6.626 \times 10^{-34}$	Js
	$\hbar = h/2\pi$	$= 1.055 \times 10^{-34}$	Js
Avogadro's constant	N_A	$= 6.022 \times 10^{23}$	$\mathrm{mol^{-1}}$
Molar gas constant	R	$= 8.314$	$\mathrm{J\,K^{-1}\,mol^{-1}}$
Standard temperature		273.15	K
Earth mass	M_E	$= 5.974 \times 10^{24}$	kg
Mean Earth radius	R_E	$= 6.371 \times 10^{6}$	m
Universal gravitational constant	G	$= 6.673 \times 10^{-11}$	$\mathrm{Nm^2kg^{-2}}$
Gravitational acceleration	$g = GM_\mathrm{E}/R_\mathrm{E}^2$	$= 9.807$	$\mathrm{ms^{-2}}$

2 List of Useful Formulas

This compilation was inspired by the NRL plasma formulary, which is available free of charge at `http://wwwppd.nrl.navy.mil/nrlformulary/`. However, SI-units are used here for consistency with the rest of the book. Temperatures are given in eV, as indicated.

2.1 Lengths

The mass number of an ion is $\mu = m_\mathrm{i}/m_\mathrm{p}$.

- electron Debye length

$$\lambda_{\mathrm{De,Di}} = 7.43 \times 10^3\,\mathrm{m}\sqrt{\frac{T_{\mathrm{e,i}}(\mathrm{eV})}{n_{\mathrm{e,i}}(\mathrm{m}^{-3})}}$$

- thermal electron gyroradius

$$r_{\mathrm{Le}} = \frac{v_{\mathrm{Te}}}{\omega_{\mathrm{ce}}} = 3.37 \times 10^{-6}\,\mathrm{m}\,\frac{\sqrt{T_\mathrm{e}(\mathrm{eV})}}{B(\mathrm{T})}$$

- thermal ion gyroradius

$$r_{\mathrm{Li}} = \frac{v_{\mathrm{Ti}}}{\omega_{\mathrm{ci}}} = 1.45 \times 10^{-4}\,\mathrm{m}\,\frac{\sqrt{\mu T_\mathrm{i}(\mathrm{eV})}}{B(\mathrm{T})}$$

2.2 Frequencies

- electron plasma frequency

$$\omega_{\mathrm{pe}} = \sqrt{\frac{n_\mathrm{e} e^2}{\varepsilon_0 m_\mathrm{e}}} = 56.4\,\mathrm{s}^{-1}\sqrt{n_\mathrm{e}(\mathrm{m}^{-3})}$$

- ion plasma frequency

$$\omega_{\mathrm{pi}} = \sqrt{\frac{n_\mathrm{i} Z^2 e^2}{\varepsilon_0 m_\mathrm{i}}} = 1.32\,\mathrm{s}^{-1}\,Z\sqrt{\frac{n_\mathrm{i}(\mathrm{m}^{-3})}{\mu}}$$

- electron gyrofrequency

$$\omega_{\mathrm{ce}} = \frac{eB}{m_\mathrm{e}} = 1.76 \times 10^{11}\mathrm{s}^{-1}\,B(\mathrm{T})$$

- ion gyrofrequency

$$\omega_{\mathrm{ci}} = \frac{ZeB}{m_{\mathrm{i}}} = 9.58 \times 10^7 \mathrm{s}^{-1} \, \frac{Z}{\mu} \, B\,(\mathrm{T})$$

2.3 Velocities

- electron thermal speed

$$v_{\mathrm{Te}} = \sqrt{\frac{2k_{\mathrm{B}}T_{\mathrm{e}}}{m_{\mathrm{e}}}} = 5.93 \times 10^5 \,\mathrm{m\,s}^{-1} \sqrt{T_{\mathrm{e}}(\mathrm{eV})}$$

- ion thermal speed

$$v_{\mathrm{Ti}} = \sqrt{\frac{2k_{\mathrm{B}}T_{\mathrm{i}}}{m_{\mathrm{i}}}} = 1.38 \times 10^4 \,\mathrm{m\,s}^{-1} \sqrt{\frac{T_{\mathrm{i}}(\mathrm{eV})}{\mu}}$$

- ion sound speed

$$C_s = \sqrt{\frac{k_{\mathrm{B}}(T_{\mathrm{e}} + 3T_{\mathrm{i}})}{m_{\mathrm{i}}}} = 9.79 \times 10^3 \,\mathrm{m\,s}^{-1} \sqrt{\frac{(T_{\mathrm{e}} + 3T_{\mathrm{i}})(\mathrm{eV})}{\mu}}$$

- Alfvén speed

$$v_{\mathrm{A}} = \frac{B}{\sqrt{\mu_0 n_{\mathrm{i}} m_{\mathrm{i}}}} = 2.18 \times 10^{16} \,\mathrm{m\,s}^{-1} \frac{B(\mathrm{T})}{\sqrt{\mu \, n_{\mathrm{i}}(\mathrm{m}^{-3})}}$$

3 Useful Mathematics

3.1 Vector Relations

The definitions of dot product and vector product of two vectors can be summarized as follows:

$$\begin{aligned}
\mathbf{A}\cdot\mathbf{B} &= A_xB_x + A_yB_y + A_zB_z \\
\mathbf{A}\times\mathbf{B} &= \begin{vmatrix} \mathbf{e}_x & \mathbf{e}_y & \mathbf{e}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\
&= \mathbf{e}_x(A_yB_z - A_zB_y) + \mathbf{e}_y(A_zB_x - A_xB_z) + \mathbf{e}_z(A_xB_y - A_yB_x) \\
\mathbf{A}\cdot\mathbf{B} &= \mathbf{B}\cdot\mathbf{A} \\
\mathbf{A}\times\mathbf{B} &= -\mathbf{B}\times\mathbf{A}
\end{aligned}$$

The operator $\mathbf{A}\cdot\nabla$ is a scalar operator

$$\mathbf{A}\cdot\nabla = A_x\frac{\partial}{\partial x} + A_y\frac{\partial}{\partial y} + A_z\frac{\partial}{\partial z}\,.$$

The gradient of a vector function is defined as a tensor, which can be written as a matrix (see next paragraph)

$$\nabla\mathbf{A} = \begin{pmatrix} \frac{\partial A_x}{\partial x} & \frac{\partial A_x}{\partial y} & \frac{\partial A_x}{\partial z} \\ \frac{\partial A_y}{\partial x} & \frac{\partial A_y}{\partial y} & \frac{\partial A_y}{\partial z} \\ \frac{\partial A_z}{\partial x} & \frac{\partial A_z}{\partial y} & \frac{\partial A_z}{\partial z} \end{pmatrix}$$

The following list gives some standard rules for operations involving multiple dot products, vector products, and derivatives. Note that differential operators act only on the term on their r.h.s., A_c means that A is not affected by the differential operator. f is a scalar function.

$$\begin{aligned}
\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) &= \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B}) \\
\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) &= (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C} \\
\nabla(f\,g) &= f\nabla g + g\nabla f \\
\nabla\cdot(f\,\mathbf{A}) &= f\nabla\cdot\mathbf{A} + \mathbf{A}\cdot\nabla f \\
\nabla\times(f\,\mathbf{A}) &= f\nabla\times\mathbf{A} + \nabla f\times\mathbf{A}_c \\
\nabla\cdot(\mathbf{A}\times\mathbf{B}) &= \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B}) \\
\nabla\times(\mathbf{A}\times\mathbf{B}) &= \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A}) + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} \\
\nabla(\mathbf{A}\cdot\mathbf{B}) &= (\mathbf{A}\cdot\nabla)\,\mathbf{B} + (\mathbf{B}\cdot\nabla)\,\mathbf{A} + \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) \\
\mathbf{A}\times(\nabla\times\mathbf{B}) &= (\nabla\mathbf{B})\cdot\mathbf{A}_\mathrm{c} - (\mathbf{A}\cdot\nabla)\mathbf{B} \\
\Delta\mathbf{A} &= \nabla(\nabla\cdot\mathbf{A}) - \nabla\times(\nabla\times\mathbf{A}) \\
\nabla\cdot(\nabla\times\mathbf{A}) &= 0 \\
\nabla\times(\nabla f) &= 0
\end{aligned}$$

3.2 Matrices and Tensors

Matrices describe the mapping of a vector into a new vector, usually of different length and direction. A matrix is a special tensor of rank two. There are also tensors of higher rank, which we, however, omit from the discussion.

The product of a 3 × 3 matrix M and a vector $\mathbf{A}$ can be written as

$$\mathsf{M} \cdot \mathbf{A} = \begin{pmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{pmatrix} \cdot \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} M_{xx}A_x + M_{xy}A_y + M_{xz}A_z \\ M_{yx}A_x + M_{yy}A_y + M_{yz}A_z \\ M_{zx}A_x + M_{zy}A_y + M_{zz}A_z \end{pmatrix} .$$

The elements of the resulting vector are the dot product (scalar product) of a row of the matrix M with the vector $\mathbf{A}$.

Matrices can also be used to represent systems of linear equations. The following homogeneous system of equations

$$\begin{aligned} 3x + 5y - 2z &= 0 \\ 2x - 3y + z &= 0 \\ 5x + 2y - z &= 0 \end{aligned}$$

can be rewritten in matrix form as

$$\begin{pmatrix} 3 & 5 & -2 \\ 2 & -3 & 1 \\ 5 & 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 .$$

This system of equations has a non-zero solution, when the determinant of the matrix vanishes

$$\begin{vmatrix} 3 & 5 & -2 \\ 2 & -3 & 1 \\ 5 & 2 & -1 \end{vmatrix} = 3(3-2) + 5(5+2) - 2(4+15) = 0$$

The unit tensor is

$$\mathsf{I}_{\alpha\beta} = \delta_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

The dyadic product of two vectors $\mathbf{A}$ and $\mathbf{B}$ is a tensor of rank two containing the products of the vector components, $\mathsf{T}_{\alpha\beta} = A_\alpha B_\beta$,

$$\mathbf{A}\,\mathbf{B} = \mathsf{T} = \begin{pmatrix} A_xB_x & A_xB_y & A_xB_z \\ A_yB_x & A_yB_y & A_yB_z \\ A_zB_x & A_zB_y & A_zB_z \end{pmatrix} .$$

3.3 The Theorems of Gauss and Stokes

Assume that V is a volume bounded by a closed surface S, with dS positive outward from the enclosed volume, then

$$\oint_S \mathbf{A} \cdot \mathrm{d}S = \int_V (\nabla \cdot \mathbf{A})\,\mathrm{d}V\,.$$

If S is an open surface bounded by the contour C, of which the line element is ds, then

$$\oint_C \mathbf{A} \cdot \mathrm{d}s = \int_S (\nabla \times \mathbf{A}) \cdot \mathrm{d}S\,.$$

Solutions

"Forty-two!"

Douglas Adams, The Hitchhikers Guide to the Galaxy

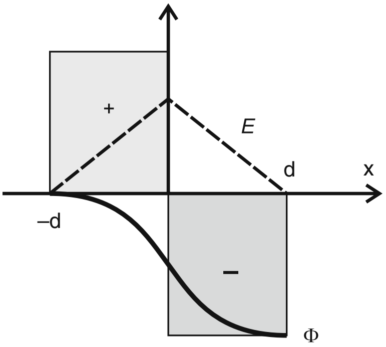
Problems of Chapter 2

2.1

$$\lambda_{\mathrm{De}} = \sqrt{\frac{\varepsilon_0 k_{\mathrm{B}} T_{\mathrm{e}}}{n_{\mathrm{e}} e^2}} = \sqrt{\frac{\varepsilon_0 k_{\mathrm{B}}}{e^2}} \sqrt{\frac{T_{\mathrm{e}}}{n_{\mathrm{e}}}} = 69.0\,\mathrm{m} \sqrt{\frac{T_{\mathrm{e}}(\mathrm{K})}{n_{\mathrm{e}}(\mathrm{m}^{-3})}}$$

2.2 (a) $\lambda_{\mathrm{De}} = \lambda_{\mathrm{Di}} = 69\sqrt{3000 \times 10^{-12}}\,\mathrm{m} = 3.8 \times 10^{-3}\,\mathrm{m}$.
(b) Note that $3\,\mathrm{eV} \mathrel{\widehat{=}} 3 \times 11600\,\mathrm{K}$. $\lambda_{\mathrm{De}} = 1.3 \times 10^{-4}\,\mathrm{m}$, $\lambda_{\mathrm{Di}} = 1.2 \times 10^{-5}\,\mathrm{m}$.

2.3 Poisson's equation states that the curvature of the potential is proportional to the (negative) space charge. $\Phi'' = -e n_{\mathrm{i}}/\varepsilon_0$. (a) The electric field increases linearly in the positive space charge region from $E(-d) = 0$ to $E(0) = E_{\max}$ and decreases in the negative space charge region to $E(d) = 0$. Hence, there is no electric field at the edges of the quasineutral plasma. The potential decreases in the positive space charge region as $\Phi(x) = -\frac{1}{2} n_{\mathrm{i}} e (x + d)^2/\varepsilon_0$ and reaches $\Phi(0) = -9.0 \times 10^3\,\mathrm{V}$. In the negative space charge region, $\Phi(x)$ has the opposite curvature and reaches a final value $\Phi(d) = -18.0 \times 10^3\,\mathrm{V}$.



2.4 Calculate the derivatives

$$\Phi(r) = \frac{a}{r} f(r)\,, \quad \Phi' = -\frac{a}{r^2} f(r) + \frac{a}{r} f'\,, \quad \Phi'' = \frac{2a}{r^3} f - \frac{2a}{r^2} f' + \frac{a}{r} f''$$

Inserting into (2.21) gives

$$\frac{a}{r}\left[f'' - \frac{1}{\lambda_\mathrm{D}^2} f \right] = 0\,.$$

2.5 Inserting the Wigner-Seitz radius $a_\mathrm{WS} = [3/(4\pi n_\mathrm{i})]^{1/3}$ into (2.15) gives

$$\Gamma_\mathrm{i} = \frac{(4\pi/3)^{1/3}}{4\pi} \frac{e^2 n_\mathrm{i}^{1/3}}{\varepsilon_0 k_\mathrm{B} T_\mathrm{i}} = \frac{1}{3}\left(\frac{4\pi}{3} n_\mathrm{i} \lambda_\mathrm{Di}^3 \right)^{-2/3} = \frac{1}{3} N_\mathrm{Di}^{-2/3}$$

2.6 Starting from the definitions $S = -k_\mathrm{B} \sum_i n_i \ln n_i$ and $U = \sum_i n_i W_i$ we use the thermodynamic definition of temperature $1/T = \partial S/\partial U$.

$$\frac{1}{T} = \frac{\partial S}{\partial \lambda} \frac{\partial \lambda}{\partial U} = \frac{-k_\mathrm{B} \sum\limits_i \left[\frac{\partial n_i}{\partial \lambda}(\lambda W_i - \ln Z) + \frac{\partial n_i}{\partial \lambda} \right]}{\sum\limits_i \frac{\partial n_i}{\partial \lambda} W_i}$$

$$= \frac{-k_\mathrm{B} \lambda \sum\limits_i \frac{\partial n_i}{\partial \lambda} W_i + k_\mathrm{B} (\ln Z - 1) \sum\limits_i \frac{\partial n_i}{\partial \lambda}}{\sum\limits_i \frac{\partial n_i}{\partial \lambda} W_i} = k_\mathrm{B} \lambda\,.$$

Using $\sum_i n_i = 1$, we obtain the result $\lambda = (k_\mathrm{B} T)^{-1}$.

Problems of Chapter 3

3.1 Ampere's law states that the integral of the magnetic field strength H along a closed path equals the current flowing though the area bounded by this path, $\oint \mathbf{H} \cdot \mathrm{d}\mathbf{s} = I$. Choose a circle of radius r centered at the axis of the wire for the path. Then, for any $r < a$ the current flow through this circle is the fraction $I\, r^2/a^2$ and we obtain

$$2\pi r H_\varphi = I \frac{r^2}{a^2} \quad \Rightarrow \quad H_\varphi = \frac{I\, r}{2\pi a^2}\,,$$

which increases linearly to a maximum $H_\varphi = I/(2\pi a)$ at $r = a$. For $r > a$ the encircled current is always I and $H_\varphi = I/(2\pi r)$ becomes a decreasing function of radius.

3.2 The current $I(r)$ flowing through a circle of radius $r < a$ determines the magnetic field

$$I(r) = 2\pi \int_0^r r' j(r') \mathrm{d}r' = 2\pi j_0 \left(\frac{r^2}{2} - \frac{r^4}{4a^2} \right) \quad \Rightarrow \quad H_\varphi = j_0 \left(\frac{r}{2} - \frac{r^3}{4a^2} \right) .$$

3.3 $B = 49.4\,\mu\mathrm{T}$. (a) $\omega_{\mathrm{ce}} = 8.7 \times 10^6 \mathrm{s}^{-1}$. (b) $r_{\mathrm{ce}} = 0.22\,\mathrm{m}$.

3.4 (a) Use $\mathbf{M} \cdot \mathbf{r} = Mr \cos\theta$ to define the angle θ. The unit vector $\mathbf{e}_r = \mathbf{r}/r$.

$$\begin{aligned} B_r &= \mathbf{B} \cdot \mathbf{e}_r = \frac{\mu_0}{4\pi} \frac{3r^2(\mathbf{r} \cdot \mathbf{M}) - r^2(\mathbf{r} \cdot \mathbf{M})}{r^6} = \frac{\mu_0 M}{4\pi} \frac{2\cos\theta}{r^3} \\ B_\theta &= |\mathbf{e}_r \times \mathbf{B}| = \frac{\mu_0 M}{4\pi} \frac{\sin\theta}{r^3} . \end{aligned}$$

(b) Solve the last line for M, insert $\theta = 90°$, $B_\theta = 30\,\mu\mathrm{T}$ and use the Earth radius plus about (100–200) km altitude for the ionosphere to obtain $M \approx 8 \times 10^{22}\,\mathrm{A\,m^{-2}}$.

3.5 At the magnetic equator, $\theta = 90°$ and the magnetic field has only a θ-component $B_\theta = (\mu_0 M/4\pi) r^{-3}$. (a) Hence, $\mathrm{d}B_\theta/\mathrm{d}r = -3(\mu_0 M/4\pi) r^{-4}$ and $R_{\mathrm{c}} = r/3 = (R_{\mathrm{E}} + 500\,\mathrm{km})/3 = 2{,}290\,\mathrm{km}$. (b) $v_R = 2v_{\nabla B} = 6W/(qrB)$. At $H = 500\,\mathrm{km}$ altitude, the equatorial magnetic field of $30\,\mu\mathrm{T}$ has decreased by a factor $[R_{\mathrm{E}}/(R_{\mathrm{E}} + H]^3 = 0.80$ to $24\,\mu\mathrm{T}$. Then $v_R = 0.11\,\mathrm{m\,s^{-1}}$.

3.6 Starting from the equations of motion

$$\dot{v}_x = -\frac{e}{m_{\mathrm{e}}} E - \omega_{\mathrm{ce}} v_y \quad \text{and} \quad \dot{v}_y = \omega_{\mathrm{ce}} v_x$$

we see that the electron velocity performs harmonic oscillations at the frequency ω_{ce} and $v_x = v_\perp \sin(\omega_{\mathrm{ce}} t)$. There is no cosine term because $v_x(0) = 0$. Then, $v_y = \omega_{\mathrm{ce}} \int v_x \,\mathrm{d}t + C = v_\perp [1 - \cos(\omega_{\mathrm{ce}} t)]$ to fulfill $v_y(0) = 0$. From $\dot{v}_x(0) = 0$ we obtain $v_\perp = -E/B$. Hence,

$$\begin{aligned} v_x &= -\frac{E}{B} \sin(\omega_{\mathrm{ce}} t) , & v_y &= \frac{E}{B} [1 - \cos(\omega_{\mathrm{ce}} t)] \\ x &= \frac{E}{B\omega_{\mathrm{ce}}} [\cos(\omega_{\mathrm{ce}} t) - 1] , & y &= -\frac{E}{B} t + \frac{E}{B\omega_{\mathrm{ce}}} \sin(\omega_{\mathrm{ce}} t) \end{aligned}$$

3.7 The differential equation for a field line with the dipole source at the origin reads

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{2}{3} \frac{z}{x} - \frac{1}{3} \frac{x}{z} .$$

Set $z = wx$ to obtain $\mathrm{d}z/\mathrm{d}x = w + x(\mathrm{d}w/\mathrm{d}x)$. Inserting into the differential equation

$$x\frac{\mathrm{d}w}{\mathrm{d}x} = -\frac{1}{3}w - \frac{1}{3}\frac{1}{w} \quad \Rightarrow \frac{w\mathrm{d}w}{w^2+1} = -\frac{1}{3}\frac{\mathrm{d}x}{x} \quad \Rightarrow \frac{1}{2}\ln(w^2+1) = -\frac{1}{3}\ln(x) + C\,.$$

This results in $w = (-1+c_1x^{-2/3})^{1/2}$ and $z = x(-1+c_1x^{-2/3})^{1/2}$. The maximum distance x_{max} of the field line is determined by $z = 0$, which defines $c_1 = x_{\mathrm{max}}^{2/3}$, hence the shape of the field line becomes

$$z = \sqrt{x_{\mathrm{max}}^{2/3}x^{4/3} - x^2}\,.$$

Problems of Chapter 4

4.1 Let A be the normalizing factor of the velocity distribution $f_{\mathrm{M}}(|v|)$. Then

$$0 = A\frac{\mathrm{d}}{\mathrm{d}v}v^2\exp\left(-\frac{mv^2}{2k_{\mathrm{B}}T}\right) = A\left(2v - v^2\frac{mv}{k_{\mathrm{B}}T}\right)\exp\left(-\frac{mv^2}{2k_{\mathrm{B}}T}\right),$$

which requires that the expression in parantheses vanishes. This gives the desired result $v_T = (2k_{\mathrm{B}}T/m)^{1/2}$.

4.2 The mean thermal velocity is the first moment of the distribution of speeds

$$v_{\mathrm{th}} = 4\pi\left(\frac{m}{2\pi k_{\mathrm{B}}T}\right)^{3/2}\int_0^\infty v^3\exp\left(-\frac{mv^2}{2k_{\mathrm{B}}T}\right)\mathrm{d}v = \left(\frac{8k_{\mathrm{B}}T}{\pi m}\right)^{1/2}\underbrace{\int_0^\infty y\mathrm{e}^{-y}\mathrm{d}y}_{=1}$$

4.3 Reduce the degree of the velocity moment by partial integration

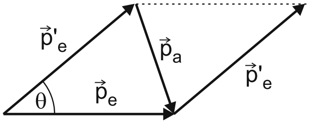
$$\begin{aligned}\int_0^\infty v^4\mathrm{e}^{-av^2}\mathrm{d}v &= -\frac{1}{2a}\int_0^\infty v^3(-2av\mathrm{e}^{-av^2})\mathrm{d}v\\ &= -\frac{1}{2a}\underbrace{\left[v^3\mathrm{e}^{-av^2}\right]_0^\infty}_{=0} + \frac{3}{2a}\int_0^\infty v^2\mathrm{e}^{-av^2}\mathrm{d}v\\ &= -\frac{3}{4a^2}\underbrace{\left[v\mathrm{e}^{-av^2}\right]_0^\infty}_{=0} + \frac{3}{4a^2}\int_0^\infty \mathrm{e}^{-av^2}\mathrm{d}v = \frac{3}{8a^2}\sqrt{\frac{\pi}{a}}\,.\end{aligned}$$

Set $a = m/2k_{\mathrm{B}}T$, then the mean square velocity becomes

$$\langle v^2 \rangle = 4\pi \left(\frac{m}{2\pi k_\mathrm{B} T} \right)^{3/2} \frac{3\sqrt{\pi}}{8} \left(\frac{2k_\mathrm{B} T}{m} \right)^{5/2} = 3\frac{k_\mathrm{B} T}{m} \, .$$

Finally: $\frac{m}{2}\langle v^2 \rangle = \frac{3}{2}k_\mathrm{B} T$.

4.4 Let the electron momentum before and after the collision be $\mathbf{p}_e$ and $\mathbf{p'}_e$. The scattering angle is θ and the momentum transfered to the atom $\mathbf{p}_a = \mathbf{p}_e - \mathbf{p'}_e$.



Then the kinetic energy transfered to the atom, which equals the energy loss of the electron, is (for $|\mathbf{p'}_e| \approx |\mathbf{p}_e|$)

$$\Delta W = \frac{\mathbf{p}_e^2 - 2\mathbf{p}_e \cdot \mathbf{p'}_e + \mathbf{p'}_e^2}{2m_\mathrm{a}} \approx \frac{\mathbf{p}_e^2}{2m_\mathrm{e}} 2\frac{m_\mathrm{e}}{m_\mathrm{a}} \left[1 - \cos(\theta)\right] .$$

4.5 The average fractional energy loss is

$$\left\langle \frac{\Delta W}{W} \right\rangle = 2\frac{m_\mathrm{e}}{m_\mathrm{a}} \frac{1}{4\pi} \int_0^{2\pi} \mathrm{d}\varphi \int_0^{\pi} \mathrm{d}\theta \sin\theta [1 - \cos\theta] = 2\frac{m_\mathrm{e}}{m_\mathrm{a}} \frac{1}{2} \int_{-1}^{1} (1 - x)\mathrm{d}x = 2\frac{m_\mathrm{e}}{m_\mathrm{a}} \, .$$

4.6 The average velocity $\bar{v}_x$ is given by

$$\begin{aligned}
\bar{v}_x &= \frac{E_x \nu_{\mathrm{m,i}}^2}{B_z \omega_\mathrm{ci}} \int_0^\infty [1 - \cos(\omega_\mathrm{ci} t)] \mathrm{e}^{-\nu_{\mathrm{m,i}} t} \mathrm{d}t = \frac{E_x \nu_\mathrm{m,i}}{B_z \omega_\mathrm{ci}} \left[1 - \nu_\mathrm{m,i} \int_0^\infty \cos(\omega_\mathrm{ci} t) \mathrm{e}^{-\nu_{\mathrm{m,i}} t} \mathrm{d}t \right] \\
&= \frac{E_x \nu_\mathrm{m,i}}{B_z \omega_\mathrm{ci}} \left[1 - \frac{1}{2}\nu_\mathrm{m,i} \int_0^\infty \left(\mathrm{e}^{(\mathrm{i}\omega_\mathrm{ci} - \nu_\mathrm{m,i})t} + \mathrm{e}^{(-\mathrm{i}\omega_\mathrm{ci} - \nu_\mathrm{m,i})t} \right) \mathrm{d}t \right] \\
&= \frac{E_x \nu_\mathrm{m,i}}{B_z \omega_\mathrm{ci}} \left[1 + \frac{1}{2}\nu_\mathrm{m,i} \left(\frac{1}{\mathrm{i}\omega_\mathrm{ci} - \nu_\mathrm{m,i}} + \frac{1}{-\mathrm{i}\omega_\mathrm{ci} - \nu_\mathrm{m,i}} \right) \right] \\
&= \frac{E_x \nu_\mathrm{m,i}}{B_z \omega_\mathrm{ci}} \frac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2 + \nu_\mathrm{m,i}^2} = \frac{E_x}{B_z} \frac{\omega_\mathrm{ci}/\nu_\mathrm{m,i}}{1 + (\omega_\mathrm{ci}/\nu_\mathrm{m,i})^2}
\end{aligned}$$

The solution for $\bar{v}_y$ is analogous, except for $\sin(\alpha) = (1/2\mathrm{i})(\mathrm{e}^{\mathrm{i}\alpha} - \mathrm{e}^{-\mathrm{i}\alpha})$.

4.7 The equation of motion for an "average ion" reads

$$m_\mathrm{i} \nu_\mathrm{m,i} \mathbf{v}_\mathrm{i} = e(\mathbf{E} + \mathbf{v}_\mathrm{i} \times \mathbf{B}) \, .$$

Introducing the Hall paramater $h = \omega_{\mathrm{ci}}/\nu_{\mathrm{m,i}}$ and using the usual convention $\mathbf{E} = (E_x, 0, 0)$ and $\mathbf{B} = (0, 0, B_z)$, we have

$$v_{\mathrm{ix}} = \mu_{\mathrm{i}} E_x + h v_{\mathrm{iy}} \quad \text{and} \quad v_{\mathrm{iy}} = -h v_{\mathrm{ix}}\,,$$

from which we easily obtain

$$v_{\mathrm{ix}} = \frac{\mu_{\mathrm{i}} E_x}{1+h^2} \quad \text{and} \quad v_{\mathrm{iy}} = -\frac{E_x}{B_z}\frac{h^2}{1+h^2}\,.$$

4.8 The total current is given by the sum of all partial currents in ring segments $\mathrm{d}I \propto n(r) 2\pi\, \mathrm{d}r$. Hence, the equivalent density for a parabolic density profile $n(r) = n_0(1 - r^2/a^2)$ becomes

$$\bar{n} = \frac{1}{\pi a^2} 2\pi \int_0^a n(r) r \mathrm{d}r = \frac{2n_0}{a^2} \int_0^a \left(r - \frac{r^3}{a^2} \right) \mathrm{d}r = \frac{1}{2} n_0\,.$$

4.9 Ignition occurs, when the α-heating balances the losses by Bremsstrahlung and finite particle confinement time, $P_\alpha = P_{\mathrm{br}} + P_{\mathrm{H}}$. Noting that at each DT-reaction one α-particle is generated, we have

$$P_\alpha = \frac{\eta}{1-\eta} P_{\mathrm{DT}} \quad \Rightarrow \quad \frac{\eta}{1-\eta} = \frac{Q_\alpha}{Q_{\mathrm{DT}}} \quad \Rightarrow \quad \eta = 0.154\,.$$

Problems of Chapter 5

5.1 Start from $H_\varphi = I/(2\pi a)$ and $p_{\mathrm{mag}} = B^2/(2\mu_0)$. (a) Then

$$p_{\mathrm{mag}} = \frac{\mu_0 I^2}{8\pi^2 a^2} = 1.6 \times 10^8\,\mathrm{Pa}\,.$$

Set 1 atm ≈ 1 bar, then $p_{\mathrm{mag}} = 1600\,\mathrm{bar}$. (b) Note that the magnetic pressure has to balance the sum of electron and ion pressure

$$n k_{\mathrm{B}}(T_{\mathrm{e}} + T_{\mathrm{i}}) = 1.6 \times 10^8\,\mathrm{Pa} \quad \rightarrow \quad \frac{k_{\mathrm{B}} T_{\mathrm{e}}}{e} = 500\,\mathrm{V}\,.$$

The plasma temperature reaches 500 eV in the compressed pinch state.

5.2 $p_{\mathrm{mag}} = B^2/(2\mu_0)$ and $\beta = p_{\mathrm{kin}}(0)/p_{\mathrm{total}}$. Then $B = (2\mu_0 \beta^{-1} 2n k_{\mathrm{B}} T)^{1/2} = 3.6\,\mathrm{T}$.

5.3 The Alfvén velocity becomes

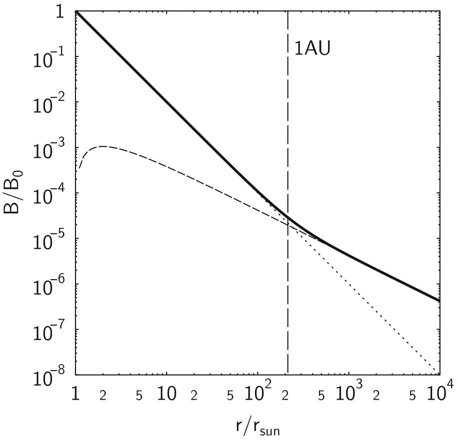
$$v_\mathrm{A} = 2.18 \times 10^{16}\,\mathrm{m\,s^{-1}}\,\frac{3}{\sqrt{2\times 10^{20}}}\,. = 4.6\times 10^{6}\,\mathrm{m\,s^{-1}}\,.$$

5.4 The comparison of Alfvén velocity and sound velocity gives

$$v_\mathrm{A} = 2.18 \times 10^{16}\,\mathrm{m\,s^{-1}}\,\frac{3\times 10^{-5}}{\sqrt{16\times 10^{12}}} = 1.6\times 10^{5}\,\mathrm{m\,s^{-1}}$$

$$C_s = 9.79\times 10^{3}\sqrt{\frac{0.256 * 4}{16}}\,\mathrm{m\,s^{-1}} = 2.5\times 10^{3}\,\mathrm{m\,s^{-1}}\,.$$

5.5 Use $\omega_\odot = 2.7\times 10^{-6}\,\mathrm{s^{-1}}$, $r_\odot = 7\times 10^{8}\,\mathrm{m}$ and $u_r = 400\,\mathrm{km\,s^{-1}}$. The figure displays $B(r)/B_0$ (heavy line), B_r/B_0 (dotted line) and B_φ/B_0 (short-dashed line). It is found that the magnetic field in the Parker spiral decays $\propto r^{-2}$ for $r < 1\,\mathrm{AU}$ and $\propto r^{-1}$ for $r > 1\,\mathrm{AU}$.



5.6 The total change in magnetic flux is given by the integral

$$\begin{aligned}\Delta\Phi_\mathrm{m} &= \int_0^\infty [B_0(r) - B_0]2\pi r\,\mathrm{d}r\\ &= 2\pi B_0\int_0^\infty \left[\left(1 - \frac{2\mu_0 k_\mathrm{B} T_\mathrm{e} n_0}{B_0^2}\,\mathrm{e}^{-(r/a)^2}\right)^{1/2} - 1\right] r\,\mathrm{d}r\\ &\approx -B_0\pi a^2 n_0\beta\int_0^\infty x\mathrm{e}^{-x^2}\,\mathrm{d}x = -\frac{1}{2}B_0\pi a^2 n_0\beta\,.\end{aligned}$$

Problems of Chapter 6

6.1 The phase and group velocities are

$$\text{(a) } v_\varphi = \frac{\omega}{k} = \frac{\omega_{\text{pi}}\lambda_{\text{De}}}{\sqrt{1+k^2\lambda_{\text{De}}^2}} \quad \text{and} \quad v_{\text{gr}} = \frac{\mathrm{d}\omega}{\mathrm{d}k} = v_\varphi \frac{1}{1+k^2\lambda_{\text{De}}^2}\,.$$

(b) For $k^2\lambda_{\text{De}}^2 \ll 1$, the phase velocity equals the group velocity, $v_\varphi \approx v_{\text{gr}}$, and is independent of k. There is no dispersion of waves of different frequency.

6.2 Start from $v_\varphi v_{\text{gr}} = c^2$. Then: $\omega\,\mathrm{d}\omega = c^2 k\,\mathrm{d}k$, which after integrating yields $\frac{1}{2}\omega^2 = \frac{1}{2}c^2k^2 + D$. Since ω is real, $\omega^2 > 0$, hence the integration constant must be positive and can be chosen as $D = \frac{1}{2}c^2k_0^2 > 0$. Then, $\omega^2 = c^2(k^2 + k_0^2)$. This means, there can be a cut-off frequency $\omega_{\text{co}} = k_0 c$.

6.3 (a) Start from (6.95) for the refractive index of the R-wave. The ion term can be neglected because of $\omega_{\text{pi}}^2 \ll \omega_{\text{pe}}^2$. In the electron term we use $\omega \ll \omega_{\text{ce}}$ to arrive at

$$\mathcal{N}_{\text{R}} \approx \left(1 + \frac{\omega_{\text{pe}}^2}{\omega\omega_{\text{ce}}}\right)^{1/2},$$

from which the desired limit is obtained when $\omega_{\text{pe}}^2 \gg \omega\omega_{\text{ce}}$. (b) Now, use the definition $\mathcal{N} = kc/\omega$ and solve for ω to arrive at

$$\omega = \frac{k^2c^2\omega_{\text{ce}}}{\omega_{\text{pe}}^2} \quad \text{and} \quad \frac{\mathrm{d}\omega}{\mathrm{d}k} = 2\frac{\omega}{k}\,.$$

6.4 Use the definition $n_{\text{co}} = \varepsilon_0 m_{\text{e}}\omega^2/e^2$ and $\omega = 2\pi c/\lambda$ to obtain $n_{\text{co}} = 2.8 \times 10^{27}\,\text{m}^{-3}$.

6.5 The cut-off is defined by $\mathcal{N} = 0$. The we have

$$0 = \mathcal{N}^2 = \varepsilon = 1 - \frac{\omega_{\text{pe}}^2}{\omega^2} - \frac{\omega_{\text{pp}}^2}{\omega^2}$$

resulting in $\omega_{\text{co}} = 2^{1/2}\omega_{\text{pe}}$.

6.6 $f_{\text{ce}} = 1.4\,\text{MHz}$, $f_{\text{pe}} = 12.7\,\text{MHz}$, $f_{\text{uh}} = (f_{\text{ce}}^2 + f_{\text{pe}}^2)^{1/2} = 12.8\,\text{MHz}$.

6.7

$$\frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{\omega}{k} \Rightarrow \frac{\mathrm{d}\omega}{\omega} = \frac{\mathrm{d}k}{k} \Rightarrow \ln\omega = \ln k + c \Rightarrow \omega = v_\varphi k\,.$$

Problems of Chapter 7

7.1 Equate the electron retardation current (7.31) with the ion saturation current (7.27) and set $\Phi_\mathrm{p} = 0$ for convenience. This balance defines the floating potential Φ_f. Solving for the floating potential and using the quasineutrality of the unperturbed plasma $n_\mathrm{e0} = n_\mathrm{i0}$ gives

$$\Phi_\mathrm{f} = \frac{k_\mathrm{B} T_\mathrm{e}}{e} \ln \left[0.61 (2\pi)^{1/2} \left(\frac{m_\mathrm{e}}{m_\mathrm{i}} \right)^{1/2} \right] = \begin{cases} -3.3 k_\mathrm{B} T_\mathrm{e}/e & (\mathrm{H}^+) \\ -5.2 k_\mathrm{B} T_\mathrm{e}/e & (\mathrm{Ar}^+) \end{cases}$$

7.2 The one-dimensional electron current to the probe is

$$j_\mathrm{e} = -e \int\limits_{v_\mathrm{min}}^{\infty} v_z f(v_z) \mathrm{d}v_z \quad \text{and} \quad v_\mathrm{min} = \left(\frac{2eU_\mathrm{p}}{m_\mathrm{e}} \right)^{1/2} .$$

Then the derivative of the probe characteristic can be written as

$$\frac{\mathrm{d}j_\mathrm{e}}{\mathrm{d}U_\mathrm{p}} = \frac{\mathrm{d}j_\mathrm{e}}{\mathrm{d}v_\mathrm{min}} \frac{\mathrm{d}v_\mathrm{min}}{\mathrm{d}U_\mathrm{p}} = +e v_\mathrm{min} f(v_\mathrm{min}) \frac{e/m_\mathrm{e}}{\sqrt{2eU_\mathrm{p}/m_\mathrm{e}}} = \frac{e^2}{m_\mathrm{e}} f[v(U_\mathrm{p})] .$$

7.3 Use $U_2 = U_1 + U_p$ to rewrite (7.46) as

$$-I_\mathrm{p} = I_\mathrm{i0} + I_\mathrm{e0} \exp\left(\frac{e(U_1 - \phi_\mathrm{p})}{k_\mathrm{B} T_\mathrm{e}} \right) \exp\left(\frac{eU_\mathrm{p}}{k_\mathrm{B} T_\mathrm{e}} \right)$$

Then eliminate the exponential containing $U_2 - \phi_\mathrm{p}$ by means of (7.46) yielding

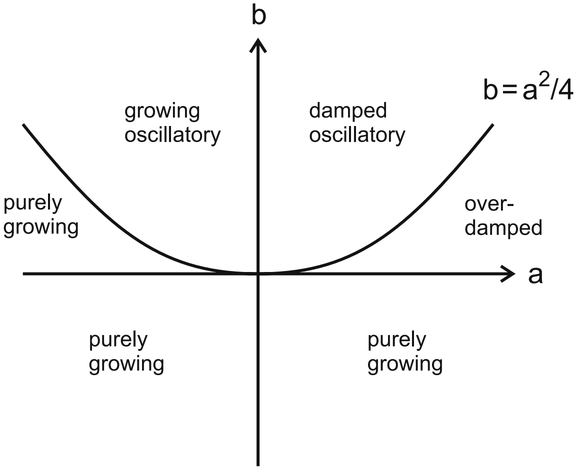
$$-I_\mathrm{p} = I_\mathrm{i0} + (I_\mathrm{p} - I_\mathrm{i0}) \exp\left(\frac{eU_\mathrm{p}}{k_\mathrm{B} T_\mathrm{e}} \right) ,$$

which by simple rearrangement gives the tanh-shape of the double probe characteristic.

7.4 The slope of the double probe characteristic in the origin is $\mathrm{d}I_\mathrm{p}/\mathrm{d}U_\mathrm{p} = eI_\mathrm{i0}/(2k_\mathrm{B} T_\mathrm{e})$. Then a straight line through the origin with this slope intersects the asymptote $I = I_\mathrm{i0}$ at $U_\mathrm{p} = 2k_\mathrm{B} T_\mathrm{e}/e$.

Problems of Chapter 8

8.1 Set $x = \hat{x} \exp(-\mathrm{i}\omega t) = \hat{x} \exp(-\mathrm{i}\omega_R t) \exp(\omega_I t)$, which has unstable solutions for $\omega_I > 0$. Overdamped or purely growing modes have $\omega_R = 0$. The characteristic equation for the differential equation then becomes $\omega^2 + \mathrm{i}\omega a - b = 0$ with the solution

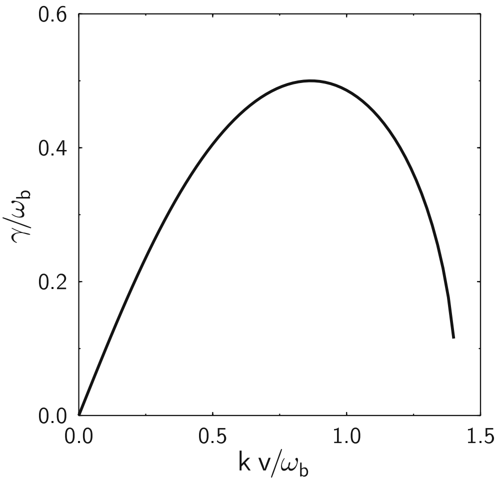


$$\omega_{1,2} = -\frac{\mathrm{i}}{2}a \pm \sqrt{b - \frac{a^2}{4}} \, .$$

8.2 The dielectric function reads

$$\varepsilon = 1 - \frac{\omega_{\mathrm{b}}^2}{(\omega - kv)^2} - \frac{\omega_{\mathrm{b}}^2}{(\omega + kv)^2} \, ,$$

which is equivalent to a quartic equation with real or pairs of complex conjugate roots. The roots are given by



$$\omega = \pm \left[k^2 v^2 + \omega_b^2 \pm \left(\omega_b^4 + 4k^2 v^2 \omega_b^2 \right)^{1/2} \right]^{1/2} .$$

Two of these roots are purely imaginary and the one with positive imaginary part is unstable.

8.3 Insert $\omega = \omega_R + i\omega_I$ in (8.22) and require that the imaginary terms cancel:

$$0 = \omega_I - \frac{\omega_R \omega_I}{|\omega|^4} \omega_{pe} \omega_{pi}^2 .$$

Noting that $\omega_R = |\omega| \cos(\theta)$ yields (8.23). Now, set $\omega_I = |\omega| \sin(\theta)$ with $|\omega|$ from (8.23) and calculate $d\omega_I / d\theta = 0$ yielding $\tan(\theta) = \sqrt{3}$ and $\theta = \pi/3$. From $\omega = [\omega_{pi}^2 \omega_{pe} \cos(\theta)]^{1/3} \exp(i\pi/3)$ we obtain (8.24).

Problems of Chapter 9

9.1 Note that the total energy W_{tot} is a constant of motion. Let $g(W_{tot}) = g(\frac{1}{2}mv^2 + q\Phi)$ be the distribution function. Then the Vlasov equation can be written as

$$\frac{dg}{dW_{tot}} \left(\underbrace{\frac{\partial W_{tot}}{\partial t}}_{=0} + v \frac{\partial W_{tot}}{\partial x} - \frac{q}{m} \Phi' \frac{\partial W_{tot}}{\partial v} \right) = \frac{dg}{dW_{tot}} \left(+q\Phi' v - mv \frac{q}{m} \Phi' \right) = 0 .$$

9.2

$$\frac{1}{n_{e0}} \int_0^\infty v f_M^{(1)}(v) dv = \left(\frac{m_e}{2\pi k_B T_e} \right)^{1/2} \frac{k_B T_e}{m_e} \int_0^\infty e^{-y} \, dy = \left(\frac{k_B T_e}{2\pi m_e} \right)^{1/2} .$$

9.3 The integral is of the same type as in the previous problem, but now the lower bound of the integral is $v_c = (2e(\Phi - \Phi_{min})/m)^{1/2}$.

9.4 Follow the advice preceding (9.55).

9.5 The unstable mode has a frequency difference from the plasma frequency given by (8.8). Then the difference between the phase velocity and the beam velocity is

$$v_0 - v_\varphi = \frac{\Delta\omega}{k} = \frac{\omega_{pe}}{k} \frac{1}{2} \left(\frac{\alpha_b}{2} \right)^{1/3} .$$

With $\hat{\Phi}_t = m(v_0 - v_\varphi)^2/4$ and $\omega_{pe}/k = v_0$ we obtain the desired result.

9.6 The trapping condition reads $m\Delta v^2 = 4e\hat{\Phi}$ for a difference between phase velocity and beam velocity $\Delta v = (\omega_{pe} - \omega)/k = \Delta\omega/k$. The fastest growing mode has $\Delta\omega = 2^{-4/3}\alpha_{\mathrm{b}}^{1/3}\omega_{\mathrm{pe}}$. Then the energy density becomes

$$W_E = \frac{\varepsilon_0}{2}\langle E^2\rangle = \frac{\varepsilon_0}{4}k^2\hat{\Phi}^2 = \frac{\varepsilon_0}{4}k^2\left(\frac{m}{4e}\right)^2\Delta v^4\,.$$

Eliminating $k = \omega_{\mathrm{pe}}/v_0$ and using $\alpha_{\mathrm{b}} = n_{\mathrm{b}}/n_{\mathrm{p}}$ gives the desired result.

Problems of Chapter 10

10.1 The dust charge is $q_{\mathrm{d}} = 4\pi\varepsilon_0 a\Phi = 4.45\times10^{-18}\,\mathrm{C}$ which is $Z_{\mathrm{d}} = 27.8$. This gives an initial (electric) potential energy $W_{\mathrm{pot}} = 1.78\times10^{-17}\,\mathrm{J}$ corresponding to 111 eV. From energy conservation, $W_{\mathrm{pot}} = mg_L h$, a maximum height of $h = 883\,\mathrm{m}$ is obtained.

10.2 For $P\to\infty$, (10.25) requires $\eta_{\mathrm{f}} = \eta_{\mathrm{c}}$. Therefore, (10.26) reduces to

$$(\mu\tau)^{-1/2} = \mathrm{e}^{-(\eta_{\mathrm{f}}+\tau\eta_{\mathrm{c}})} \quad \text{or} \quad \eta_{\mathrm{f}} = \frac{1}{2(1+\tau)}\ln(\mu\tau) = 0.078\,.$$

The normalized ion density becomes $n_{\mathrm{i}}/n_\infty = \mathrm{e}^{\tau\eta_{\mathrm{c}}} = \mathrm{e}^{7.8} = 2440$, a tremendously high value, which demonstrates the inadequacy of assuming a Boltzmann response for the ions.

10.3 The electron Debye length is $\lambda_{\mathrm{De}} = 410\,\mu\mathrm{m}$. At the Bohm velocity $v_{\mathrm{B}} = k_{\mathrm{B}}T_{\mathrm{e}}/m_{\mathrm{i}}$, an ion has the kinetic energy $m_{\mathrm{i}}v_{\mathrm{B}}^2/2 = k_{\mathrm{B}}T_{\mathrm{e}}/2$. The normalized floating potential of a sphere is $e\Phi_{\mathrm{f}}/k_{\mathrm{B}}T_{\mathrm{e}} = 2.41$. The collection radius then becomes

$$b_{\mathrm{c}} = a\left(1+\frac{2.41\,k_{\mathrm{B}}T_{\mathrm{e}}}{0.5k_{\mathrm{B}}T_{\mathrm{e}}}\right)^{1/2} = 2.41\,a = 12.2\,\mu\mathrm{m}\,.$$

The dust charge is $q_{\mathrm{d}} = (1675\times5\times3)e = 4.03\times10^{-15}\,\mathrm{C}$. This gives a Coulomb radius $r_{\mathrm{C}} = 12\,\mu\mathrm{m}$. Because the collection radius is much smaller than the electron Debye length, the ion wind force is dominated by the orbit force. Since the Coulomb radius equals the collection radius, the grazing trajectory touches the grain at the "midnight" position, i.e., exactly opposite to the illumination direction.

10.4 At $r_{\min}$ and for $\dot{r} = 0$ the energy equation (10.53) reads

$$\frac{m_{\mathrm{i}}}{2}v_0^2 = \frac{m_{\mathrm{i}}}{2}\frac{(m_{\mathrm{i}}v_0 b)^2}{m_{\mathrm{i}}^2 r_{\min}^2} - \frac{q_{\mathrm{d}}e}{4\pi\varepsilon_0 r_{\min}}$$
$$0 = r_{\min}^2 - b^2 + 2r_{\min}r_{\mathrm{C}}\,,$$

which gives the desired result.

10.5 Consider a symmetric displacement of both particles by δr from their equilibrium positions. Expand the restoring force into a Taylor series, which gives a first order term

$$\delta F_{\mathrm{r}} = -m\omega_0^2 \delta r - \frac{2q_{\mathrm{d}}^2}{4\pi\epsilon_0 d_0^3}(2\delta r)\,.$$

Using the definition (10.68) for d_0, we obtain the equation of motion $\delta\ddot{r} + 3\omega_0^2\delta r = 0$, which yields the frequency of the breathing mode $\omega_{\mathrm{br}} = \sqrt{3}\omega_0$.

10.6 Define the spring constant D_n for an interaction with a neighbor at distance $n\Delta$. Then,

$$D_n = \frac{q_{\mathrm{d}}^2}{4\pi\varepsilon_0 n^3 \Delta^3}\left(2 + 2n\kappa + n^2\kappa^2\right)\mathrm{e}^{-n\kappa}\,,$$

which gives the desired r.h.s. of the dispersion relation after summing over all pairs of neighbors at $\pm n\Delta$.

10.7 The deflecting force for the i-th particle is given by

$$F_i = F_\Delta\left(\frac{\eta_i - \eta_{i-1}}{\Delta} + \frac{\eta_i - \eta_{i+1}}{\Delta}\right).$$

The distance between particles i and $i+1$ increases as

$$s = \left[\Delta^2 + (\eta_i - \eta_{i+1})^2\right]^{1/2} \approx \Delta\left[1 + \frac{1}{2}\left(\frac{\eta_i - \eta_{i+1}}{\Delta}\right)^2\right],$$

which contains only a second-order correction to Δ that can be neglected. Assuming a wave-like perturbation $\eta(x,t) = \hat{\eta}\mathrm{e}^{\mathrm{i}(kx-\omega t)}$ we have

$$-\omega^2 m = \frac{F_\Delta}{\Delta}\left(2 - \mathrm{e}^{-\mathrm{i}k\Delta} - \mathrm{e}^{+\mathrm{i}k\Delta}\right) = \frac{4F_\Delta}{\Delta}\sin^2\left(\frac{k\Delta}{2}\right),$$

which gives a purely imaginary ω that attains a maximum at $k\Delta/2 = \pi/2$ or $\Delta = \lambda/2$, i.e., a zig-zag arrangement of nearest neighbors.

10.8 The force between infinitely large charged planes is independent of the distance between the planes. Point-like particles in a linear chain interact via a three-dimensional force law, which may be Coulomb or Yukawa and decays as r^{-2} or faster.

Problems of Chapter 11

11.1 A charge in a parallel-plate capacitor causes image charges on the plates. As the charge moves, the image charges change and a displacement current flows in the outer circuit. The exponentially growing electron avalanche generates a current waveform of exponential shape. When the electrons move at a constant drift velocity $v_\mathrm{d} = -\mu_\mathrm{e} E$, the current rises as $I \propto \mathrm{e}^{\alpha v_\mathrm{d} t}$. The current is zero again when the last electron has reached the electrode.

11.2 (a) Inserting the separation ansatz into the diffusion equation, and dividing by $R(r)T(t)$, we have

$$\frac{1}{T(t)}\frac{\mathrm{d}T(t)}{\mathrm{d}t} - \frac{D_\mathrm{a}}{R(r)}\left(\frac{\mathrm{d}^2R(r)}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}R(r)}{\mathrm{d}r}\right) = 0\,.$$

Because each of these two terms is only dependent on t or on r, respectively, the terms must be a constant (with the dimension of a reciprocal time), say $-1/\tau$. Then $T(t) \propto \exp -t/\tau$.

(b) The constancy of the term containing $R(t)$ can be rewritten as

$$\frac{\mathrm{d}^2R}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}R}{\mathrm{d}r} + \frac{R}{D_\mathrm{a}\tau} = 0$$

$$\frac{\mathrm{d}^2R}{\mathrm{d}x^2} + \frac{1}{x}\frac{\mathrm{d}R}{\mathrm{d}x} + R = 0, \text{ with } x = r(D_\mathrm{a}\tau)^{-1/2}\,.$$

This is Bessels differential equation for $J_0(x)$.

(c) From $R(a) = 0$ we obtain $2.405 = a(D_\mathrm{a}\tau)^{-1/2}$ with the first zero $J_0(2.405) = 0$. and $\tau = D_\mathrm{a}^{-1}(a/2.405)^2$.

11.3 Multiplying (11.17) by the applied voltage $\hat{U}$ gives the total current $\hat{I} = \hat{U}/Z_\mathrm{b}$ in the two branches. Then the ratio of the current in the capacitor to that in the RL-branch is

$$\frac{\hat{I}_C}{\hat{I}_{R+L}} = \frac{\mathrm{i}\omega(\nu_\mathrm{m} + \mathrm{i}\omega)}{\omega_\mathrm{pe}^2}\,,$$

which is a small quantity in the given limit.

11.4 The collisionless skin depth is $\delta_\mathrm{cl} = c/\omega_\mathrm{pe}$. At $n_\mathrm{e} = 10^{17}\,\mathrm{m}^{-3}$ we have $\omega_\mathrm{pe} = 1.78 \times 10^{10}\,\mathrm{s}^{-1}$ and obtain $\delta_\mathrm{cl} = 0.017\,\mathrm{m}$.

References

1. R.N. Franklin, N.S.J. Braithwaite, Plasma Sources Sci. Technol. **18**, 010201 (2009)
2. H.M. Mott-Smith, Nature **233**, 219 (1971)
3. D.A. Frank-Kamenezki, *Plasma – Der Vierte Aggregatzustand* (Verlag Progress, Moskau, 1963)
4. F. Fraunberger, *Illustrierte Geschichte der Elektrizität* (Aulis, Köln, 1985)
5. C.G. Suits, *The collected works of Irving Langmuir* (Macmillan, Pergamon, New York, 1961)
6. W. Schottky, J.V. Issendorff, Z. Phys. **31**, 163 (1925)
7. A.V. Engel, M. Steenbeck, *Elektrische Gasentladungen*, Vol. 1& 2 (Springer, Berlin, 1934)
8. R. Seeliger, *Einführung in die Physik der Gasentladungen* (Barth, Leipzig, 1934)
9. M.J. Druyvesteyn, F.M. Penning, Rev. Mod. Phys. **12**, 87 (1940)
10. E.V. Appleton, J. Inst. Elec. Engrs. **71**, 642 (1932)
11. S. Chapman, *Solar Plasma, Geomagnetism and Aurora* (Blackie, London, 1964)
12. J.A. Ratcliffe, *The Magnetoionic Theory and its Application to the Ionosphere* (Cambridge, New-York, 1959)
13. K.G. Budden, *Radio Waves in the Ionosphere* (Cambridge University Press, New York, 1961)
14. K. Rawer, *Die Ionosphäre* (Noordhoff, Groningen, 1953)
15. A. Unsöld, *Die Physik der Sternatmosphären* (Springer, Berlin, 1968)
16. P. Sweet, *IAU Symposium* **6**, 123 (1958)
17. E.N. Parker, Phys. Rev. **107**, 830 (1957)
18. K. Birkeland, *The Norwegian Aurora Polaris Expedition 1902–1903*, Vol. I (J.A. Barth, Leipzig, 1908), p. 98
19. L. Biermann, Z. Astrophys. **29**, 274 (1951)
20. E.N. Parker, Ap. J. **128**, 664 (1958)
21. K.I. Gringauz, V.V. Bezrukikh, V.D. Ozerov, R.E. Rybchinskii, Planet. Space Sci. **9**, 97 (1962)
22. R.G. Marsden, Space Sci. Rev. **78**, 67 (1996)
23. D.J. McComas, B.L. Barraclough, H.O. Funsten, J.T. Gosling, E. Santiago-Munoz, R.M. Skoug, B.E. Goldstein, M. Neugebauer, P. Riley, A. Balogh, J. Geophys. Res. **105**, 10,419 (2000)
24. W. Feldman, B. Barraclaugh, J. Phillips, Y. Wang, Astronom. Astrophys. **316**, 350 (1996)
25. J. van Allen, L.A. Frank, Nature **183**, 430 (1959)
26. D. Bilitza, B. Reinisch, Adv. Space Res. **42**, 599 (2008)
27. B. Jacob, Lighting Res, Technol. **41**, 219 (2009)
28. Department of the Environment and Heritage, *Final Report on: Public Lighting in Australia - Energy Efficiency Challenges and Opportunities* (Australian Government, 2005)
29. M. Born, Plasma Sources Sci. Technol. **11**, A55 (2002)
30. B.A. Smith, et al., Science **215**, 504 (1982)
31. C.J. Mitchell, M. Horanyi, O. Havnes, C.C. Porco, Science **311**, 1587 (2006)
32. J.H. Chu, L.I, Phys. Rev. Lett. **72**, 4009 (1994)

33. Y. Hayashi, K. Tachibana, Jpn. J. Appl. Phys. **33**, L804 (1994)
34. H. Thomas, G.E. Morfill, V. Demmel, J. Goree, B. Feuerbacher, D. Möhlmann, Phys. Rev. Lett. **73**, 652 (1994)
35. J. Pieper, J. Goree, R. Quinn, Phys. Rev. E **54**, 5636 (1996)
36. O. Arp, D. Block, A. Piel, Phys. Rev. Lett. **93**, 165004 (2004)
37. T.K. Fowler, *The Fusion Quest* (Johns Hopkins University Press, Baltimore MD, 1997)
38. H.S. Bosch, G.M. Hale, Nucl. Fusion **32**, 611 (1992)
39. H.S. Bosch, G.M. Hale, Nucl. Fusion **32**, 1919 (1992)
40. J.H. Nuckolls, L. Wood, A. Thiessen, G.B. Zimmermann, Nature **239**, 139 (1972)
41. R.C. Arnold, J.M. ter Vehn, Rep. Prog. Phys. **50**, 559 (1987)
42. J. Ongena, A.M. Messiaen, Trans. Fusion Sci. Technol. **49**, 425 (2006)
43. F. Wagner, et al., Phys. Rev. Lett. **49**, 1408 (1982)
44. F. Wagner, U. Stroth, Plasma Phys. Control. Fusion **35**, 1321 (1993)
45. F. Wagner, Plasma Phys. Control. Fusion **49**, B1 (2007)
46. P.H. Rebut, The JET-Team, Plasma Phys. Control. Fusion **34**, 1749 (1992)
47. R.J. Hawryluk, Rev. Mod. Phys. **70**, 537 (1998)
48. R.J. Hawryluk, S. Batha, W. Blanchard, M. Beer, et al., Phys. Plasmas **5**, 1577 (1998)
49. P.H. Rebut, Plasma Phys. Control. Fusion **48**, B1 (2006)
50. K. Walter, *LLNL Science & Technology Report*, 4–14 September 2003
51. J. Lindl, Phys. Plasmas **2**, 3933 (1995)
52. J.D. Lindl, P. Amendt, R.L. Berger, S.G. Glendinning, S.H. Glenzer, S.W. Haan, R.L. Kauffman, O.L. Landen, L.J. Suter, Phys. Plasmas **11**, 339 (2004)
53. National Research Council, *Plasma Science—Advancing Knowledge in the National Interest* (NAS, Washington D.C., 2007)
54. M.N. Saha, Phil. Mag. **40**, 472 (1920)
55. P. Debye, E. Hückel, Phys. Z. **24**, 185 (1923)
56. D.H.E. Dubin, T.M. O'Neil, Rev. Mod. Phys. **71**, 87 (1999)
57. J.J. Bollinger, D.J. Wineland, D.H.E. Dubin, Phys. Plasmas **1**, 1403 (1994)
58. G. Baur, G. Boero, S. Brauksiepe, et al., Phys. Lett. B **368**, 251 (1996)
59. G. Blanford, D.C. Christian, K. Gollwitzer, M. Mandelkern, C.T. Munger, J. Schultz, G. Zioulas, Phys. Rev. Lett. **80**, 3037 (1998)
60. M. Amoretti, C. Amsler, G. Bonomi, A. Bouchta, P. Bowe, C. Carraro, C.L. Cesar, et al., Nature **419**, 456 (2002)
61. V. Erckmann, U. Gasparino, Plasma Phys. Control. Fusion **36**, 1869 (1994)
62. R. Prater, Phys. Plasmas **11**, 2349 (2004)
63. H. Wobig, talk on the occasion of the retirement of G. Grieger (2000)
64. R. Jaenicke, E. Ascasibar, P. Grigull, I. Lakicevic, A. Weller, M. Zippe, Nucl. Fusion **33**, 687 (1993)
65. R.B. Brode, Rev. Mod. Phys. **5**, 257 (1933)
66. R. Rejoub, B.G. Lindsay, R.F. Stebbings, Phys. Rev. A **65**, 042713 (2002)
67. L. Spitzer, R. Harm, Phys. Rev. **89**, 977 (1953)
68. Y.P. Raizer, *Gas Discharge Physics* (Springer, Berlin, 1991)
69. A. Bouchoule, J.P. Boeuf, A. heron, O. Duchemin, Plasma Phys. Control. Fusion **46**, B407 (2004)
70. G.D. Racca, A. Marini, L. Stagnaro, J. van Dooren, L. di Napoli, B.H. Foing, R. Lumb, J. Volp, et al., Planet. Space Sci. **50**, 1323 (2002)
71. D.M. Di Cara, D. Estublier, Acta Astronaut. **57**, 250 (2005)
72. A. Lazurenko, V. Krasnoselskikh, A. Bouchoule, IEEE Trans. Plasma Sci. **36**, 1977 (2008)
73. D. Gawron, S. Mazouffre, N. Sadeghi, A. Héron, Plasma Sources Sci. Technol. **17**, 025001 (2008)
74. W. Panofsky, M. Phillips, *Classical Electricity and Magnetism* (Addison-Wesley, Reading MA, 1969)

75. S. Glasstone, R.H. Lovberg, *Controlled Thermonuclear Reactions* (van Nostrand, Princeton NJ, 1960)
76. J.D. Lawson, Proc. Phys. Soc. (Lond.) **B70**, 6 (1957)
77. J. Lister, H. Weisen, Europhys. News **36**, 47 (2005)
78. S. Nakai, H. Takabe, Rep. Prog. Phys. **59**, 1071 (1996)
79. M.D. Rosen, Phys. Plasmas **6**, 1690 (1999)
80. C. Truesdell, Phys. Rev. **78**, 823 (1950)
81. W.H. Bennett, Phys. Rev. **45**, 890 (1934)
82. R.B. Spielman, C. Deeney, G.A. Chandler, M.R. Douglas, et al., Phys. Plasmas **5**, 2105 (1998)
83. M.E. Cuneo, D.B. Sinars, E.M. Waisman, D.E. Bliss, et al., Phys. Plasmas **15**, 056318 (2006)
84. H. Alfvén, Nature **150**, 405 (1942)
85. H. Alfvén, C.G. Fälthammar, *Cosmical Electrodynamics* (Clarendon, Oxford, 1950)
86. S. Lundquist, Nature **164**, 145 (1949)
87. B. Lehnert, Phys. Rev. **94**, 815 (1954)
88. W.H. Bostick, M.A. Levine, Phys. Rev. **87**, 671 (1952)
89. D.F. Jephcott, Nature **183**, 1652 (1959)
90. J.M. Wilcox, F.I. Boley, A.W. De Silva, Phys. Fluids **3**, 15 (1960)
91. H. Rishbeth, J. Atmos. Solar-Terr. Phys. **63**, 1883 (2001)
92. O. Heaviside, *Telegraphy*, vol. 33 (Encyclopedia Britannica, London, 1902)
93. A.E. Kennelly, Elect. World Eng. **6**, 473 (1902)
94. G. Breit, M.A. Tuve, Nature **116**, 357 (1925)
95. G. Breit, M.A. Tuve, Phys. Rev. **28**, 554 (1926)
96. E.V. Appleton, J.A. Ratcliffe, Proc. Roy. Soc. **A117**, 576 (1928)
97. I. Langmuir, L. Tonks, Science **68**, 598 (1928)
98. H.G. Booker, *Cold Plasma Waves* (Martinus Nijhoff Dordrecht, 1984)
99. D.G. Swanson, *Plasma Waves* (Academic New York, 1989)
100. T.H. Stix, *Waves in Plasmas* (AIP New York, 1992)
101. M. Brambilla, *Kinetic Theory of Plasma Waves* (Oxford University Press, Oxford, 1998)
102. H.G. Adler, A. Piel, J. Quant. Spectrosc. Radiat. Transf. **45**, 11 (1991)
103. F.A. Hopf, A. Tomita, G. Al-Jumaily, Opt. Lett. **5**, 386 (1988)
104. F.A. Hopf, A.Q. Tomita, G. Al-Jumaily, M. Cervantes, T. Liepmann, Opti. Comm. **36**, 487 (1981)
105. F. A.Hopf, M. Cervantes, Appl. Opti. **21**, 668 (1982)
106. T.W. Liepmann, F.A. Hopf, Appl. Optics **24**, 1485 (1985)
107. N. Bretz, F. Jobes, J. Irby, Rev. Sci. Instrum. **68**, 713 (1997)
108. F.C. Jobes, N.L. Bretz, Rev. Sci. Instrum. **68**, 709 (1997)
109. E. Stoffels, W.W. Stoffels, D. Vender, M. Kando, G.M.W. Kroesen, F.J. de Hoog, Phys. Rev. E **51**, 2425 (1995)
110. D. Bohm, E.P. Gross, Phys. Rev. **75**, 1851 (1949)
111. D. Bohm, E.P. Gross, Phys. Rev. **75**, 1864 (1949)
112. T.H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New-York, 1962)
113. J.A. Eilek, F.N. Owen, Ap. J. **567**, 202 (2002)
114. J.V. Hollweg, M.K. Bird, H. Volland, P. Edenhofer, C.T. Stelzried, B.L. Seidel, J. Geophys. Res. **87**, 1 (1982)
115. S. Mancuso, S.R. Spangler, Ap. J. **539**, 480 (2000)
116. S. Ganguly, G.H. Van Bavel, A. Brown, J. Geophys. Res. **105**, 16,063 (2000)
117. D.L. Hysell, J.L. Chau, J. Geophys. Res. **106**, 16,063 (2001)
118. H. Soltwisch, Rev. Sci. Instrum. **57**, 1939 (1986)
119. A.J.H. Donné, T. Edlington, E. Joffrin, H.R. Koslowski, C. Nieswand, S.E. Segre, P.E. Stott, C. Walker, Rev. Sci. Instrum. **70**, 726 (1999)
120. D.L. Brower, Y. Jiang, W.X. Ding, S.D. Terry, N.E. Lanier, J.K. Anderson, C.B. Forest, D. Holly, Rev. Sci. Instrum. **72**, 1077 (2001)

121. D.L. Brower, Y. Yiang, S.D. Terry, et al., Rev. Sci. Instrum. **74**, 1534 (2003)
122. K. Tanaka, K. Kawahata, A. Ejiri, CHS Group, J. Howard, S. Okajima, Rev. Sci. Instrum. **70**, 730 (1999)
123. M.K. Bird, Space Sci. Rev. **33**, 99 (1982)
124. L.R.O. Storey, Phil. Trans. Roy. Soc. (London) Ser. A **246**, 113 (1953)
125. H. Rishbeth, O.K. Garriot, *Introduction to Ionospheric Physics* (Academic, New York, 1969)
126. E.R. Schmerling, J. Atmospheric Terr. Phys. **12**, 8 (1958)
127. J.E. Titheridge, Radio Sci. **23**, 831 (1988)
128. J.E. Titheridge, World Data Center A for STP **UAG -93** (1985)
129. B.W. Reinisch, *Modern Ionosondes* (EGS Katlenburg-Lindau, 1996)
130. P.E. Argo, M.C. Kelley, J. Geophys. Res. **91**, 5539 (1986)
131. G.C. Hussey, K. Schlegel, C. Haldoupis, J. Geophys. Res. **103**, 6991 (1998)
132. M.L. Parkinson, D.P. Monselesan, P.R. Smith, P.L. Dyson, R.J. Morris, J. Geophys. Res. **102**, 24,075 (1997)
133. R.K. Fisher, R.W. Gould, Phys. Rev. Lett. **22**, 1033 (1969)
134. R.K. Fisher, R.W. Gould, Phys. Fluids **14**, 857 (1971)
135. T. Trottenberg, B. Brede, D. Block, A. Piel, Phys. Plasmas **10**, 4627 (2003)
136. E. Michel, C. Béghin, A. Gonfalone, I.F. Ivanov, Ann. Géophys. **31**, 873 (1975)
137. L.R.O. Storey, J. Thiel, Phys. Fluids **21**, 2325 (1978)
138. L.R.O. Storey, J. Thiel, J. Geophys. Res. **89**, 969 (1984)
139. V. Rohde, A. Piel, H. Thiemann, K.I. Oyama, J. Geophys. Res. **98**, 19,163 (1993)
140. A. Piel, G. Oelerich, Phys. Fluids **27**, 273 (1984)
141. A. Piel, G. Oelerich, Phys. Fluids **28**, 1366 (1985)
142. G. Oelerich-Hill, A. Piel, Phys. Fluids **B1**, 275 (1989)
143. C.D. Child, Phys. Rev. **32**, 498 (1911)
144. I. Langmuir, Phys. Rev. **2**, 450 (1913)
145. H.M. Mott-Smith, I. Langmuir, Phys. Rev. **28**, 727 (1925)
146. M.J. Druyvesteyn, Z. Phys. **64**, 787 (1930)
147. E.O. Johnson, L. Malter, Phys. Rev. **80**, 58 (1950)
148. J. Allen, Physica Scr. **45**, 497 (1992)
149. I.H. Hutchinson, *Principles of Plasma Diagnostics* (Cambridge University Press, Cambridge, 1987)
150. F.F. Chen, Plasma Phys. **7**, 47 (1965)
151. N. Hershkowitz, in *Plasma Diagnostics*, ed. by O. Auciello, D. Flamm (Academic, San Diego CA, 1989)
152. J.R. Anderson, K.D. Goodfellow, J.E. Polk, V.K. Rawlin, J.S. Sovey, IEEE Aerospace Conf. Proc. **4**, 99 (2000)
153. J.R. Brophy, Rev. Sci. Instrum. **73**, 1071 (2002)
154. I. Langmuir, Phys. Rev. **33**, 954 (1929)
155. M.A. Raadu, Phys. Rep. **178**, 25 (1989)
156. H. Schamel, Phys. Rep. **140**, 161 (1986)
157. N. Hershkowitz, G.L. Payne, C. Chan, J.R. deKock, Plasma Phys. **23**, 903 (1981)
158. N. Nershkowitz, Space Sci. Rev. **41**, 351 (1985)
159. L. Block, Astrophys. Space Sci. **55**, 59 (1978)
160. P. Coakley, N. Hershkowitz, Phys. Fluids **22**, 1171 (1979)
161. J. Dawson, Phys. Fluids **4**, 869 (1961)
162. P.C. Clemmow, J.P. Dougherty, *Electrodynamics of Particles and Plasmas* (Addison-Wesley, Reading MA, 1969)
163. O. Buneman, Phys. Rev. Lett. **1**, 8 (1958)
164. J.R. Pierce, J. Appl. Phys. **15**, 721 (1944)
165. J.R. Pierce, J. Appl. Phys. **19**, 231 (1948)
166. J.R. Pierce, J. Appl. Phys. **21**, 1063 (1950)
167. J. Frey, C.K. Birdsall, J. Appl. Phys. **36**, 2962 (1965)

168. J.R. Cary, D.S. Lemons, J. Appl. Phys. **53**, 3303 (1982)
169. I.N. Kolyshkin, V. I.Kuznetsov, A.Y. Ender, Sov. Phys. Tech. Phys. **29**, 882 (1984a)
170. S. Kuhn, Phys. Fluids **27**, 1821 (1984)
171. S. Kuhn, Phys. Fluids **27**, 1834 (1984)
172. S. Kuhn, M. Hörhager, J. Appl. Phys. **60**, 1952 (1986)
173. S. Kuhn, Contrib. Plasma Phys. **34**, 495 (1994)
174. A.Y. Ender, S. Kuhn, V.I. Kuznetsov, Phys. Plasmas **13**, 113506 (2006)
175. B.B. Godfrey, Phys. Fluids **30**, 1553 (1987)
176. W.S. Lawson, Phys. Fluids B **1**, 1483 (1989)
177. W.S. Lawson, Phys. Fluids B **1**, 1493 (1989)
178. H. Schamel, V. Maslov, Phys. Rev. Lett. **70**, 1105 (1993)
179. H. Kolinsky, H. Schamel, Phys. Plasmas **1**, 2359 (1994)
180. T.L. Crystal, S. Kuhn, Phys. Fluids **28**, 2116 (1985)
181. H. Matsumoto, H. Yokoyama, D. Summers, Phys. Plasmas **3**, 177 (1996)
182. A.E. Hramov, A.A. Koronovskii, I.S. Rempen, Chaos **16**, 013123 (2006)
183. N. Krahnstöver, T. Klinger, F. Greiner, A. Piel, Phys. Lett. A **239**, 103 (1998)
184. H. Friedel, R. Grauer, K.H. Spatschek, Phys. Plasmas **5**, 3187 (1998)
185. A.A. Koronovskii, I.S. Rempen, A.E. Khramov, Tech. Phys. Lett. **29**, 998 (2003)
186. M. Kočan, S. Kuhn, D. Tskhakaya (jr.), Contrib. Plasma Phys. **46**, 322 (2006)
187. C.K. Birdsall, W.B. Bridges, J. Appl. Phys. **32**, 2611 (1961)
188. C.K. Birdsall, W.B. Bridges, *Electron Dynamics in Diode Regions* (Academic, New York, 1966)
189. D.T. Farley, B.B. Balsley, R.F. Woodman, J.P. McClure, J. Geophys. Res. **75**, 7199 (1970)
190. S.L. Ossakow, J. Atmos. Terr. Phys. **43**, 437 (1981)
191. M.C. Kelley, et al., J. Geophys. Res. **91**, 5487 (1986)
192. R.F. Pfaff, H.A. Sobral, M.A. Abdu, W.E. Swartz, J.W. LaBelle, M.F. Larsen, R.A. Goldberg, F.J. Schmidlin, Geophys. Res. Lett. **24**, 1663 (1997)
193. C.T. Steigies, D. Block, M. Hirt, A. Piel, J. Geophys. Res. **106**, 12765 (2001)
194. L.D. Landau, J. Phys. USSR **10**, 25 (1946)
195. J.H. Malmberg, C.B. Wharton, Phys. Rev. Lett. **17**, 175 (1966)
196. H. Weitzner, Phys. Fluids **6**, 1123 (1963)
197. J. Lacina, Plasma Phys. **14**, 605 (1972)
198. J. Weiland, Eur. J. Phys. **2**, 171 (1981)
199. J. Lacina, Plasma Phys. Control. Fusion **36**, 601 (1884)
200. R.W.B. Best, Physica Scr. **59**, 55 (1999)
201. D.D. Ryutov, Plasma Phys. Control. Fusion **41**, A1 (1999)
202. W.E. Drummond, Phys. Plasmas **11**, 552 (2004)
203. R. Bilato, M. Brambilla, Comm. Nonlin. Sci. Num. Sim. **13**, 18 (2008)
204. I. Alexeff, O. Ishihara, IEEE Trans. Plasma Sci. **6**, 212 (1978)
205. D. Sagan, Am. J. Phys. **62**, 450 (1994)
206. G. Brodin, Am. J. Phys. **65**, 66 (1997)
207. D. Anderson, R. Fedele, M. Lisak, Am. J. Phys. **69**, 1262 (2001)
208. R.W. Gould, T.M. O'Neill, J.H. Malmberg, Phys. Rev. Lett. **19**, 219 (1967)
209. J.H. Malmberg, C.B. Wharton, R.W. Gould, T.M. O'Neill, Phys. Fluids **11**, 1147 (1968)
210. A.Y. Wong, D.R. Baker, Phys. Rev. **188**, 326 (1969)
211. D.R. Nicholson, *Introduction to Plasma Theory* (Wiley, New York, 1983)
212. B.R. Ripin, R.E. Pechacek, Phys. Rev. Lett. **24**, 1330 (1970)
213. D.R. Baker, N.R. Ahern, A.Y. Wong, Phys. Rev. Lett. **20**, 318 (1968)
214. R.W. Hockney, J.W. Eastwood, *Computer Simulation Using Particles* (Adam Hilger, Bristol, 1991)
215. C.K. Birdsall, B. Langdon, *Plasma Physics via Computer Simulation* (Adam Hilger, Bristol, 1991)
216. J.P. Verboncoeur, M.V. Alves, V. Vahedi, C.K. Birdsall, J. Comp. Phys. **104**, 321 (1992)

217. D.J. Sullivan, IEEE Trans. Nucl. Sci. **26**, 4274 (1979)
218. D.J. Sullivan, IEEE Trans. Nucl. Sci. **30**, 3426 (1983)
219. H.A. Davis, R.R. Bartsch, T.J.T. Kwan, E.G. Sherwood, R.M. Stringfield, Phys. Rev. Lett. **59**, 288 (1987)
220. S. Burkhart, J. Appl. Phys. **62**, 75 (1987)
221. W. Jiang, M. Kristiansen, Phys. Plasmas **8**, 3781 (2001)
222. L. Spitzer, *Diffuse Matter in Space* (Wiley, New-York, 1968)
223. K.G. Spears, R.P. Kampf, T.J. Robinson, J. Phys. Chem. **92**, 5297 (1988)
224. A. Bouchoule, A. Plain, L. Boufendi, J.P. Blondeau, C. Laure, J. Appl. Phys. **70**, 1991 (1991)
225. G.S. Selwyn, J. Singh, R.S. Bennett, J. Vac. Sci. Technol. A **7**, 2758 (1989)
226. P.K. Shukla, Phys. Plasmas **8**, 1791 (2001)
227. A. Piel, A. Melzer, Plasma Phys. Control. Fusion **44**, R1 (2002)
228. V.N. Tsytovich, G.E. Morfill, H. Thomas, Plasma Phys. Rep. **28**, 623 (2002)
229. G.E. Morfill, V.N. Tsytovich, H. Thomas, Plasma Phys. Rep. **29**, 1 (2003)
230. H. Thomas, G.E. Morfill, V.N. Tsytovich, Plasma Phys. Rep. **29**, 895 (2003)
231. V.N. Tsytovich, G.E. Morfill, H. Thomas, Plasma Phys. Rep. **30**, 816 (2004)
232. S. Vladimirov, K. Ostrikov, Phys. Rep. **393**, 175 (2004)
233. O. Ishihara, J. Phys. D: Appl. Phys. **40**, R121 (2007)
234. M. Bonitz, P. Ludwig, H. Baumgartner, C. Henning, A. Filinov, D. Block, O. Arp, A. Piel, S. Käding, Y. Ivanov, A. Melzer, H. Fehske, V. Filinov, Phys. Plasmas **15**, 055704 (2008)
235. A. Piel, O. Arp, D. Block, I. Pilch, T. Trottenberg, S. Käding, A. Melzer, H. Baumgartner, C. Henning, M. Bonitz, Plasma Phys. Control. Fusion **50**, 124003 (2008)
236. P.K. Shukla, B. Eliasson, Rev. Mod. Phys. **81**, 25 (2009)
237. A. Bouchoule, *Dusty Plasmas: Physics, Chemistry, and Technological Impacts in Plasma Processing* (Wiley, New York, 1999)
238. F. Verheest, *Waves in Dusty Space Plasmas* (Kluwer, Dordrecht, 2000)
239. P.K. Shukla, M.M. Mamun, *Introduction to Dusty Plasma Physics* (IOP Bristol, 2002)
240. S.V. Vladimirov, K. Ostrikov, A. Samarian, *Physics and Applications of Complex Plasmas* (World Scientific, Singapore, 2005)
241. V. Tsytovich, G.E. Morfill, S.V. Vladimirov, H.M. Thomas, *Elementary Physics of Complex Plasmas* (Springer, Berlin, 2007)
242. O. Arp, D. Block, M. Klindworth, A. Piel, Phys. Plasmas **12**, 122102 (2005)
243. E.C. Whipple, Rep. Prog. Phys. **44**, 1197 (1981)
244. E.J. Sternglass, Westinghouse Res. Lab. **Sci. Pap. 1772** (1954)
245. B.T. Draine, E.E. Salpeter, Ap. J. **231**, 77 (1979)
246. M.K. Wallis, M.H.A. Hassan, Astronom. Astrophys. **121**, 10 (1983)
247. N. Meyer-Vernet, Astronom. Astrophys. **105**, 98 (1982)
248. T.N. Woods, P.C. Chamberlin, J.W. Harder, R.A. Hock, M. Snow, F.G. Eparvier, J. Fontenla, W.E. McClintock, E.C. Richard, Geophys. Res. Lett. **36**, L01101 (2009)
249. D.A. Verner, G.F. Ferland, K.T. Korista, D.G. Yakovlev, Ap. J. **465**, 487 (1996)
250. M.M. Abbas, D. Cantosic, P.D. Craven, R.B. Hoover, L.A. Hoover, L.A. Taylor, J.F. Spann, A. LeClair, E.A. West, ESA **SP-643**, 165 (2007)
251. D.R. Criswell, in *Photon and Particle Interactions with Surfaces in Space*, ed. by R.J.L. Grard (Springer, Heidelberg, 1973), pp. 545–556
252. J.E. McCoy, D.R. Criswell, *Proceedings of 5th Lunar Science Conference* pp. 2991–3005 (1974)
253. C.F. Bohren, D.R. Huffman, *Absorption and Scattering of Light by Small Particles* (Wiley-VCH, Weinheim, 1998)
254. T.J. Stubbs, R.R. Vondrak, W.M. Farrell, Adv. Space Res. **37**, 59 (2006)
255. C. Cui, J. Goree, IEEE Trans. Plasma Sci. **22**, 151 (1994)
256. C. Hollenstein, Plasma Phys. Control. Fusion **42**, R93 (2000)
257. M. Horanyi, C.K. Goertz, Ap. J. **361**, 155 (1990)
258. O. Havnes, Adv. Space Res. **4**, 75 (1984)
259. O. Havnes, G.E. Morfill, C.K. Goertz, J. Geophys. Res. **89**, 10,999 (1984)

260. M.S. Barnes, J.H. Keller, J.C. Forster, J.A. O'Neill, D.K. Coultas, Phys. Rev. Lett. **68**, 313 (1992)
261. E.B. Tomme, D.A. Law, B.M. Annaratone, J.E. Allen, Phys. Rev. Lett. **85**, 2518 (2000)
262. P.S. Epstein, Phys. Rev. **23**, 710 (1924)
263. A. Melzer, T. Trottenberg, A. Piel, Phys. Lett. A **191**, 301 (1994)
264. T. Trottenberg, A. Melzer, A. Piel, Plasma Sources Sci. Technol. **4**, 450 (1995)
265. A. Homann, A. Melzer, A. Piel, Phys. Rev. E **59**, 3835 (1999)
266. T. Nitter, Plasma Sources Sci. Technol. **5**, 93 (1996)
267. A.V. Ivlev, U. Konopka, G.E. Morfill, Phys. Rev. E **62**, 2739 (2000)
268. S. Nunomura, T. Misawa, N. Ohno, S. Takamura, Phys. Rev. Lett. **83**, 1970 (1999)
269. C. Arnas, M. Mikikian, F. Doveil, Phys. Rev. E **60**, 7420 (1999)
270. G.M. Jellum, D.B. Graves, Appl. Phys. Lett. **57**, 2077 (1990)
271. L. Talbot, R.K. Cheng, R.W. Schefer, D.R. Willis, J. Fluid Mech. **101**, 37 (1980)
272. O. Havnes, T. Nitter, V. Tsytovich, G.E. Morfill, T. Hartquist, Plasma Sources Sci. Technol. **3**, 448 (1994)
273. I.H. Hutchinson, Plasma Phys. Control. Fusion **47**, 71 (2005)
274. S.A. Khrapak, A.V. Ivlev, G.E. Morfill, H.M. Thomas, Phys. Rev. E **66**, 046414 (2002)
275. S.A. Khrapak, A. Ivlev, G.E. Morfill, S.K. Zhdanov, Phys. Rev. Lett. **90**, 225002 (2003)
276. S.A. Khrapak, A.V. Ivlev, G.E. Morfill, Phys. Rev. E **70**, 056405 (2004)
277. I.H. Hutchinson, Plasma Phys. Control. Fusion **48**, 185 (2006)
278. C. Zafiu, A. Melzer, A. Piel, Phys. Plasmas **9**, 4794 (2002)
279. C. Zafiu, A. Melzer, A. Piel, Phys. Plasmas **10**, 4582 (2003)
280. M. Hirt, D. Block, A. Piel, Phys. Plasmas **11**, 5690 (2004)
281. G.E. Morfill, H.M. Thomas, U. Konopka, H. Rothermel, M. Zuzic, A. Ivlev, J. Goree, Phys. Rev. Lett. **83**, 1598 (1999)
282. M. Klindworth, A. Piel, A. Melzer, U. Konopka, H. Rothermel, K. Tarantik, G. Morfill, Phys. Rev. Lett. **93**, 195002 (2004)
283. G. Praburam, J. Goree, Phys. Plasmas **3**, 1212 (1996)
284. E. Thomas, K. Avinash, R.L. Merlino, Phys. Plasmas **11**, 1770 (2004)
285. M. Lampe, G. Joyce, G. Ganguli, V. Gavroshchaka, Phys. Plasmas **7**, 3851 (2000)
286. U. Konopka, G.E. Morfill, L. Ratke, Phys. Rev. Lett. **84**, 891 (2000)
287. A. Melzer, Phys. Rev. E **67**, 016411 (2003)
288. V.M. Bedanov, F. Peeters, Phys. Rev. B **49**, 2667 (1994)
289. M. Kong, B. Partoens, F.M. Peeters, New J. Phys. **5**, 32.1 (2003)
290. V.A. Schweigert, F. Peeters, Phys. Rev. B **51**, 7700 (1995)
291. M. Klindworth, A. Melzer, A. Piel, V.A. Schweigert, Phys. Rev. B **61**, 8404 (2000)
292. F. Diedrich, E. Peik, J.M. Chen, W. Quint, H. Walther, Phys. Rev. Lett. **59**, 2931 (1987)
293. D.J. Wineland, J.C. Bergquist, W.M. Itano, J.J. Bollinger, C.H. Manney, Phys. Rev. Lett. **59**, 2935 (1987)
294. S.L. Gilbert, J.J. Bollinger, D.J. Wineland, Phys. Rev. Lett. **60**, 2022 (1988)
295. M. Drewsen, C. Brodersen, L. Hornekaer, J. Hangst, J. Schiffer, Phys. Rev. Lett. **81**, 2878 (1998)
296. W. Luck, M. Klier, H. Wesslau, Ber. Bunsenges. Phys. Chem. **50**, 485 (1963)
297. P.A. Hiltner, I.M. Krieger, J. Phys. Chem. **73**, 2386 (1969)
298. A. Kose, M. Ozaki, K. Takano, Y. Kobayashi, S. Hachiso, J. Coll. Interface Sci. **44**, 330 (1973)
299. R. Williams, R.S. Crandall, Phys. Lett. A **48**, 225 (1974)
300. H. Ikezi, Phys. Fluids **29**, 1764 (1986)
301. A. Melzer, V. Schweigert, I. Schweigert, A. Homann, S. Peters, A. Piel, Phys. Rev. E **54**, R46 (1996)
302. K. Takahashi, T. Oishi, K. Shimomai, Y. Hayashi, S. Nishino, Phys. Rev. E **58**, 7805 (1998)
303. A.V. Zobnin, A.P. Nefedov, V.A. Sinel'shchikov, O.A. Sinkevich, A.D. Usachev, V.S. Filippov, V.E. Fortov, Plasma Phys. Rep. **26**, 415 (2000)

304. M. Zuzic, A.V. Ivlev, J. Goree, G.E. Morfill, H.M. Thomas, H. Rothermel, U. Konopka, R. Sütterlin, D.D. Goldbeck, Phys. Rev. Lett. **85**, 4064 (2000)
305. M. Nambu, S.V. Vladimirov, P.K. Shukla, Phys. Lett. A **203**, 40 (1995)
306. F. Melandsø, J. Goree, Phys. Rev. E **52**, 5312 (1995)
307. V.A. Schweigert, I.V. Schweigert, A. Melzer, A. Homann, A. Piel, Phys. Rev. E **54**, 4155 (1996)
308. O. Ishihara, S.V. Vladimirov, Phys. Rev. E **57**, 3392 (1998)
309. M. Lampe, G. Joyce, G. Ganguli, IEEE Trans. Plasma Sci. **33**, 57 (2005)
310. F. Melandsø, Phys. Rev. E **55**, 7495 (1997)
311. G. Lapenta, Phys. Plasmas **6**, 1442 (1999)
312. D. Winske, W. Daughton, D.S. Lemons, M.S. Murillo, Phys. Plasmas **7**, 2320 (2000)
313. G. Lapenta, Phys. Rev. E **66**, 026409 (2002)
314. A. Melzer, V. Schweigert, A. Piel, Phys. Rev. Lett. **83**, 3194 (1999)
315. A. Melzer, V.A. Schweigert, A. Piel, Physica Scripta **61**, 494 (2000)
316. G.A. Hebner, M.E. Riley, B.M. Marder, Phys. Rev. E **68**, 046401 (2003)
317. G.A. Hebner, M.E. Riley, Phys. Rev. E **69**, 026405 (2004)
318. H. Totsuji, C. Totsuji, T. Ogawa, K. Tsuruta, Phys. Rev. E **71**, 045401 (2005)
319. C. Henning, H. Baumgartner, A. Piel, P. Ludwig, V. Golubnichiy, M. Bonitz, D. Block, Phys. Rev. E **74**, 056403 (2006)
320. C. Henning, P. Ludwig, A. Filinov, A. Piel, M. Bonitz, Phys. Rev. E **76**, 036404 (2007)
321. H. Baumgartner, H. Kählert, V. Golobnychiy, C. Henning, S. Käding, A. Melzer, M. Bonitz, Contrib. Plasma Phys. **47**, 281 (2007)
322. D. Block, M. Kroll, O. Arp, A. Piel, S. Käding, A. Melzer, C. Henning, H. Baumgartner, P. Ludwig, M. Bonitz, Plasma Phys. Control. Fusion **49**, B109 (2007)
323. A. Piel, O. Arp, M. Klindworth, A. Melzer, Phys. Rev. E **77**, 026407 (2008)
324. M. Bonitz, D. Block, O. Arp, V. Golubnychiy, H. Baumgartner, P. Ludwig, A. Piel, A. Filinov, Phys. Rev. Lett. **96**, 075001 (2006)
325. R.W. Hasse, V.V. Avilov, Phys. Rev. A **44**, 4506 (1991)
326. A. Homann, A. Melzer, S. Peters, R. Madani, A. Piel, Phys. Rev. E **56**, 7138 (1997)
327. F.M. Peeters, X. Wu, Phys. Rev. A **35**, 3109 (1987)
328. A. Homann, A. Melzer, S. Peters, R. Madani, A. Piel, Phys. Lett. A **242**, 173 (1998)
329. X. Wang, A. Bhattacharjee, S. Hu, Phys. Rev. Lett. **86**, 2569 (2001)
330. S. Nunomura, J. Goree, S. Hu, X. Wang, A. Bhattacharjee, Phys. Rev. E **65**, 066402 (2002)
331. S. Nunomura, D. Samsonov, J. Goree, Phys. Rev. Lett. **84**, 5141 (2000)
332. A. Piel, V. Nosenko, J. Goree, Phys. Rev. Lett. **89**, 085004 (2002)
333. D. Samsonov, J. Goree, Z. Ma, A. Bhattacharjee, H.M. Thomas, G.E. Morfill, Phys. Rev. Lett. **83**, 3649 (1999)
334. A. Melzer, S. Nunomura, D. Samsonov, J. Goree, Phys. Rev. E **62**, 4162 (2000)
335. V. Nosenko, J. Goree, Z.W. Ma, D.H.E. Dubin, A. Piel, Phys. Rev. E **68**, 056409 (2003)
336. D.H.E. Dubin, Phys. Plasmas **7**, 3895 (2000)
337. O. Havnes, T. Aslaksen, T.W. Hartquist, F. Li, F. Melandsø, G.E. Morfill, T. Nitter, J. Geophys. Res. **100**, 1731 (1995)
338. O. Havnes, F. Li, T.W. Hartquist, T. Aslaksen, A. Brattli, Planet. Space Sci. **49**, 223 (2001)
339. Y. Feng, J. Goree, B. Liu, Rev. Sci. Instrum. **78**, 053704 (2007)
340. S. Nunomura, J. Goree, S. Hu, X. Wang, A. Bhattacharjee, K. Avinash, Phys. Rev. Lett. **89**, 035001 (2002)
341. N.N. Rao, P.K. Shukla, M.Y. Yu, Planet. Space Sci. **38**, 543 (1990)
342. A. Barkan, R.L. Merlino, N. D'Angelo, Phys. Plasmas **2**, 3563 (1995)
343. C. Thompson, A. Barkan, N. D'Angelo, R.L. Merlino, Phys. Plasmas **4**, 2331 (1997)
344. T. Trottenberg, D. Block, A. Piel, Phys. Plasmas **13**, 042105 (2006)
345. S. Ratynskaia, S. Khrapak, A. Zobnin, M.H. Thoma, M. Kretschmer, A. Usachev, V. Yaroshenko, R.A. Quinn, G.E. Morfill, O. Petrov, V. Fortov, Phys. Rev. Lett. **93**, 085001 (2004)

346. A. Piel, M. Klindworth, O. Arp, A. Melzer, M. Wolter, Phys. Rev. Lett. **97**, 205009 (2006)
347. M. Schwabe, M. Rubin-Zuzic, S. Zhdanov, H.M. Thomas, G.E. Morfill, Phys. Rev. Lett. **99**, 095002 (2007)
348. E. Thomas, Jr., Phys. Plasmas **13**, 042107 (2006)
349. I. Pilch, A. Piel, T. Trottenberg, M.E. Koepke, Phys. Plasmas **14**, 123704 (2007)
350. E. Thomas, Jr., R. Fisher, R.L. Merlino, Phys. Plasmas **14**, 123701 (2007)
351. A. von Engel, *Ionized Gases* (Clarendon, Oxford, 1965)
352. W.B. Nottingham, in *Handbuch der Physik*, vol. XXI, ed. by S. Flügge (Springer, Berlin, 1956)
353. J.W. McGowan, P.K. John, *Gaseous Electronics* (North-Holland, Amsterdam, 1974), pp. 9–16
354. L. Malter, E.O. Johnson, W.M. Webster, RCA Rev. **XII**, 415 (1951)
355. R. Timm, A. Piel, Contrib. Plasma Phys. **32**, 599 (1992)
356. V.A. Godyak, N. Sternberg, Phys. Rev. A **42**, 2299 (1990)
357. P. Belenguer, J.P. Boeuf, Phys. Rev. A **41**, 4447 (1990)
358. M.A. Liebermann, V.A. Godyak, IEEE Trans. Plasma Sci. **26**, 955 (1998)
359. V.A. Godyak, A.S. Khanneh, IEEE Trans. Plasma Sci. **41**, 112 (1986)
360. R. Flohr, A. Piel, Contrib. Plasma Phys. **33**, 153 (1993)
361. V.A. Godyak, R.B. Piejak, B.M. Alexandrovich, Plasma Sources Sci. Technol. **1**, 36 (1992)
362. A. Melzer, R. Flohr, A. Piel, Plasma Sources Sci. Technol. **4**, 424 (1995)
363. K. Köhler, J.W. Coburn, D.E. Horne, E. Kay, J. Appl. Phys. **57**, 59 (1985)
364. J.W. Coburn, E. Kay, J. Appl. Phys. **43**, 4965 (1972)
365. J.W. Coburn, H.F. Winters, J. Appl. Phys. **50**, 3189 (1979)
366. J. Hopwood, Plasma Sources Sci. Technol. **1**, 109 (1992)
367. J.H. Keller, J.C. Forster, M.S. Barnes, J. Vac. Sci. Technol. A **11**, 2487 (1993)
368. J.H. Keller, Plasma Sources Sci. Technol. **5**, 166 (1996)
369. V.I. Kolobov, D.J. Economou, Plasma Sources Sci. Technol. **6**, R1 (1997)
370. M.A. Liebermann, A.J. Lichtenberg, *Principles of Plasma Discharges and Material Processing* (Wiley, New York, 1994)
371. B. Chapman, *Glow Discharge Processes* (Wiley, New York, 1980)
372. J.R. Roth, *Industrial Plasma Engineering, Vol. I: Principles* (IOP, Bristol, 1995)
373. R. Hippler, H. Kersten, M. Schmidt, K.H. Schoenbach, *Low Temperature Plasmas*, 2nd edn. (Wiley-VCH, Weinheim, 2008)

Name Index

Subject Index