Appendix A

Mapping as Probabilistic State Estimation

In the following, we provide a brief summary of the fundamental mapping techniques developed in the area of probabilistic robotics. For more detailed overviews, we refer you to Thrun et al. (2005), Thrun (2002), and Frese (2006a).

In probabilistic robotics, robot mapping is treated as a state estimation problem: The robot's environment is a dynamic system with a state that is only partially observable by the robot through its perceptions and is affected by the indeterministic results of its actions. Thus, the robot can only estimate the true state of the environment from its history of observations and actions (provided by odometry measurements or motion controls).

The state of the environment is described as a set of random variables that are often continuous and comprise variables for all modeled properties of the external world as well as for the robot's own state. For instance, if the environment is modeled by the positions of five important point landmarks in a two-dimensional coordinate system, this would result in 5×2 continuous random variables (two variables for each landmark, representing its x and y coordinates) plus typically three continuous variables describing the robot's pose. The complete state is then referred to by a single n-dimensional random vector x with one dimension for each random variable. In a dynamic system, state variables may change over time. Typically, discrete time steps are assumed and indices are used to indicate the time step considered, e.g., x_t for the state at time step t, starting with x_0.

The state estimation problem now is the task of computing the conditional joint probability density function $p(x_t|a_{1:t}, o_{1:t})$ over all state variables from a given starting distribution $p(x_0)$ and the sets of observations $o_{1:t} = o_1, ..., o_t$ and actions $a_{1:t} = a_1, ..., a_t$. a_i here is the action that leads from x_{i-1} to x_i, o_i is the observation corresponding to x_i, and we assume that the robot starts in x_0 by performing action a_1. The conditional probability distribution represents the robot's belief about the state of the world.

Generally, it is assumed that a successor state x_t only depends on the previous state x_{t-1} and (although indeterministically) on the action a_t chosen at time point $t-1$. This means the entire history of states passed before x_{t-1} plays no role once x_{t-1} is known. This is known as the *Markov property*. The dynamic system then becomes a (partially observable) Markov chain and $p(x_t|a_{1:t}, o_{1:t}) = p(x_t|x_{t-1}, a_t)$.

However, the Markov property is only an approximation when it comes to robot applications in the real world. One reason for this is that the state model is always only an approximate model of the current state of the world, focusing on the most relevant aspects for the application at hand. Hence, some parameters affecting the result of actions are typically not modeled. For instance, if the state vector does not contain the current status of the robot's battery, the result of the robot's actions does not depend on the current state alone but also on the sequence of actions that lead to it (if the series of actions has completely exhausted the battery, the robot is unlikely to move at all).

A.1 The Recursive Bayes Filter

In probabilistic robotic approaches it is assumed that development of the state of the dynamic system is determined by the laws of probability theory. The fundamental formula underlying these approaches is the *recursive Bayes filter*, which is derived by applying Bayes law to express the *posterior probability distribution* $p(x_t|o_{1:t}, a_{1:t})$:

$$\begin{aligned} p(x_t|o_{1:t}, a_{1:t}) &= \frac{p(o_t|x_t, o_{1:t-1}, a_{1:t})\, p(x_t|o_{1:t-1}, a_{1:t})}{p(o_t|o_{1:t-1}, a_{1:t})} \text{ (Bayes law)} \quad (A.1) \\ &= \eta\, p(o_t|x_t, o_{1:t-1}, a_{1:t})\, p(x_t|o_{1:t-1}, a_{1:t}) \quad (A.2) \end{aligned}$$

where η is the *normalization factor* that ensures that the probabilities integrate to one. η is independent of x_t and can be reformulated by applying the formula of total probability:

$$\begin{aligned} \eta &= \frac{1}{p(o_t|o_{1:t-1}, a_{1:t})} \\ &= \frac{1}{\int p(o_t|x_t, o_{1:t-1}, a_{1:t})\, p(x_t|o_{1:t-1}, a_{1:t})\, dx_t} \text{ (tot. prob.)} \quad (A.3) \end{aligned}$$

Assuming that the Markov property holds as described above, Eq. A.2 can now be reduced by realizing that given the state x_t, the previous history of actions, observations, and states provides no further evidence for the probability distribution of receiving observation o_t:

$$\begin{aligned} p(x_t|o_{1:t}, a_{1:t}) &= \eta\, p(o_t|x_t, o_{1:t-1}, a_{1:t})\, p(x_t|o_{1:t-1}, a_{1:t}) \\ &= \eta\, p(o_t|x_t)\, p(x_t|o_{1:t-1}, a_{1:t}) \text{ (Markov prop.)} \quad (A.4) \end{aligned}$$

$p(o_t|x_t)$ is typically referred to as the *sensor model* as it describes the probability of obtaining a particular observation given the current state of the world and has to be determined for the sensor apparatus of the robot at hand.

In the next two steps, Eq. A.4 is turned into a recursive version: $p(x_t|o_{1:t}, a_{1:t})$ is computed from $p(x_{t-1}|o_{1:t-1}, a_{1:t-1})$. First, the law of total probability is applied to the rightmost term:

$$\begin{aligned} p(x_t|o_{1:t}, a_{1:t}) &= \eta\, p(o_t|x_t)\, p(x_t|o_{1:t-1}, a_{1:t}) \\ &= \eta\, p(o_t|x_t) \int p(x_t|x_{t-1}, o_{1:t-1}, a_{1:t})\, p(x_{t-1}|o_{1:t-1}, a_{1:t})\, dx_{t-1} \\ &\quad \text{(tot. prob.)} \end{aligned} \tag{A.5}$$

Then, a_t is removed from $p(x_{t-1}|...)$ under the conditional independence assumption that controls are not selected based on current state:

$$\begin{aligned} p(x_t|o_{1:t}, a_{1:t}) &= \eta\, p(o_t|x_t) \int p(x_t|x_{t-1}, o_{1:t-1}, a_{1:t})\, p(x_{t-1}|o_{1:t-1}, a_{1:t})\, dx_{t-1} \\ &= \eta\, p(o_t|x_t) \int p(x_t|x_{t-1}, o_{1:t-1}, a_{1:t})\, p(x_{t-1}|o_{1:t-1}, a_{1:t-1})\, dx_{t-1} \\ &\quad \text{(cond. ind.)} \end{aligned} \tag{A.6}$$

In a final step, the Markov property is used again to simplify the leftmost term of the integral by considering that given the current state and the action taken in this state, past actions and observations bear no further evidence on the successor state and thus can be dropped from the conditional probability:

$$\begin{aligned} p(x_t|o_{1:t}, a_{1:t}) &= \eta\, p(o_t|x_t) \int p(x_t|x_{t-1}, o_{1:t-1}, a_{1:t})\, p(x_{t-1}|o_{1:t-1}, a_{1:t-1})\, dx_{t-1} \\ &= \eta\, p(o_t|x_t) \int p(x_t|x_{t-1}, a_t)\, p(x_{t-1}|o_{1:t-1}, a_{1:t-1})\, dx_{t-1} \\ &\quad \text{(Markov prop.)} \end{aligned} \tag{A.7}$$

The term $p(x_t|x_{t-1}, a_t)$ is the so-called *motion model* that similarly to the sensor model has to be determined for the robot. Given these two models, the recursive Bayes filter as provided in Eq. A.7 describes how the probability distribution for time step t can be determined from the probability distribution for time step $t-1$.

However, in most realistic cases exact computation and representation of the probability distribution is not feasible. Exceptions are discrete state spaces, where the integral is replaced by a sum, and situations in which the probabilities follow a particular parametric distribution and can be computed by a closed form version of Eq. A.7 (see Sect. A.2 about parametric filters). In all other cases, approximations have to be employed which can broadly be classified into parametric filters (Sect. A.2) and particle filters (Sect. A.3).

In general, incorporating a new action by computing the result of the integral in Eq. A.7 is referred to as the *prediction step*, while incorporating a new observation by multiplication of the sensor model with the result of the prediction step (followed by normalization) is called the *correction step*.

A.2 Parametric Filters

Parametric filters employ parametric probability density functions in order to represent $p(x_t|o_{1:t}, a_{1:t})$. As a consequence, their application is limited to scenarios in which $p(x_t|o_{1:t}, a_{1:t})$ is either guaranteed to always follow this particular parametric distribution or at least to be reasonably approximated by this kind of parametric distribution.

The main approaches in this class are the *Kalman filter* (Kalman, 1960) and the *information filter* (Mutambara, 1998), which both employ multivariate normal distributions. We will only describe the Kalman and its extension, the *extended Kalman filter*, here.

A.2.1 Kalman Filter

The Kalman filter was developed as a filtering method for systems which fulfill the following conditions:

1. the state space is continuous (no discrete state variables),
2. the Markov assumption underlying the recursive Bayes filter holds,
3. the motion model is linear Gaussian (linear in its arguments plus Gaussian noise),
4. the sensor model is linear Gaussian,
5. the initial state probability $p(x_0)$ is Gaussian.

Under these conditions the posterior of the Bayes filter always stays normally distributed and the integral and multiplications of Eq. A.7 can be computed in closed form.

In the Kalman filter, the normal probability distributions are represented by their means and covariance matrices. The linear Gaussian assumption for the motion model means that the system develops according to the linear equation:

$$x_t = A_t x_{t-1} + B_t a_t + v_t \tag{A.8}$$

where A_t and B_t are non-random matrices ensuring the linearity of the state transition and v_t is an additive random noise vector that is normally distributed with 0 mean and covariance matrix Q_t.

The linear Gaussian sensor model assumption means that observations depend on the state according to the equation:

$$o_t = C_t x_t + w_t \tag{A.9}$$

where C_t is a non-random matrix and w_t is an additive noise vector that is normally distributed with 0 mean and covariance matrix R_t.

The closed form formulas for updating the mean and covariance matrix based on the recursive Bayes filter are then given by

$$
\begin{aligned}
\overline{\mu_t} &= A_t\mu_{t-1} + B_t a_t && \text{(A.10)}\\
\overline{\Sigma_t} &= A_t\Sigma_{t-1}A_t^T + Q_t && \text{(A.11)}\\
K_t &= \overline{\Sigma_t}C_t^T(C_t\overline{\Sigma_t}C_t^T + R_t)^{-1} && \text{(A.12)}\\
\mu_t &= \overline{\mu_t} + K_t(o_t - C_t\overline{\mu_t}) && \text{(A.13)}\\
\Sigma_t &= (I - K_tC_t)\overline{\Sigma_t} && \text{(A.14)}
\end{aligned}
$$

I here stands for the identity matrix, and the auxiliary quantity K_t is called *Kalman gain*. The difference between observation and expected observation $(o_t - C_t\overline{\mu_t})$ in Eq. A.13 is referred to as the *innovation*.

In the context of robot localization and mapping, the most limiting factor of the Kalman filter is that it does not provide a good approximation when multiple distinct hypotheses exist (for instance, being in one particular room or another in the case of localization), meaning that the real probability distribution is multimodal. In contrast, the normal distribution employed in the Kalman filter is unimodal. When using the Kalman filter to approximate a multimodal distribution, the mean will most likely lie somewhere between the modes of the real distribution.

During mapping, the mean vector and covariance matrix grow whenever an observed feature is classified as not yet contained in the map during the data association step. The Kalman filter approach generally relies on a correct data association as otherwise the filter tends to diverge rather quickly. The size of the covariance matrix Σ_t grows quadratically with the number of features contained in the map. If the dimensionality of the observation vector is small compared to the dimensionality n of the state space, the time complexity of the Kalman filter is dominated by the $O(n^2)$ costs of the matrix multiplications.

A.2.2 Extended Kalman Filter

The requirements of linear state transition and linear sensor model are rather restrictive and rarely hold in practice. Therefore, different extensions have been conceived for the Kalman filter, in which the linearity assumption is weakened. In the *extended Kalman filter* a normally distributed approximation of the posterior is computed by using a linear approximation of the motion model and the sensor model when computing the covariance matrices in the prediction and correction steps. The linearizations used are the first-order Taylor expansions developed around μ_t and $\overline{\mu_t}$, respectively.

As we are now considering potentially nonlinear functions, the motion model and sensor model of Eqs. A.8 and A.9 are replaced with

$$
\begin{aligned}
x_t &= f(a_t, x_{t-1}) + v_t && \text{(A.15)}\\
o_t &= g(x_t) + w_t && \text{(A.16)}
\end{aligned}
$$

This results in the following update equations for the extended Kalman filter:

$$\begin{aligned}\overline{\mu_t} &= f(a_t, \mu_{t-1}) && \text{(A.17)}\\ \overline{\Sigma_t} &= F_t \Sigma_{t-1} F_t^T + Q_t && \text{(A.18)}\\ K_t &= \overline{\Sigma_t} G_t^T (G_t \overline{\Sigma_t} G_t^T + R_t)^{-1} && \text{(A.19)}\\ \mu_t &= \overline{\mu_t} + K_t(o_t - g(\overline{\mu_t})) && \text{(A.20)}\\ \Sigma_t &= (I - K_t G_t)\overline{\Sigma_t} && \text{(A.21)}\end{aligned}$$

where f and g have been replaced by the Jacobians $F_t = \frac{\partial f}{\partial x}\Big|_{\mu, a_t}$ and $G_t = \frac{\partial g}{\partial x}\Big|_{\overline{\mu_t}}$ in Eqs. A.18, A.19, and A.21 because of the linearization.

The time complexity of the extended Kalman filter is the same as for the basic Kalman filter.

A.3 Nonparametric Filters

In contrast to parametric filters, nonparametric filters can approximate arbitrary probability distributions. They typically represent the probability distributions by a finite number of samples over the state space. Two main groups can be distinguished: *Histogram filters* partition the state space into regions and store a single probability value for each region. In a *particle filter* the samples are randomly drawn from the probability distribution they represent. We will focus on particle filters here.

A.3.1 Particle Filter

In the particle filter approach, a set of n samples (or *particles*) $\mathbf{x}_{k,t}, 0 \leq k < n$ is maintained at each time step. The probability is represented by the density of the particles. More precisely, the likelihood for a particular state x_t to be part of the sample set at time step t is supposed to be proportional to $p(x_t | a_{1:t}, o_{1:t})$.

The basic particle filter algorithm works as follows:

1. For every sample $\mathbf{x}_{k,t-1}$ from the previous sample set, a sample $\overline{\mathbf{x}}_{k,t}$ is randomly determined according to $p(x_t | a_t, \mathbf{x}_{k,t-1})$ (prediction step).

2. For every new sample $\overline{\mathbf{x}}_{k,t}$, an importance factor $w_{k,t}$ is computed which is the probability of observing o_t given $\overline{\mathbf{x}}_{k,t}$ (correction step):

$$w_{k,t} = p(o_t | \overline{\mathbf{x}}_{k,t}) \quad \text{(A.22)}$$

3. Particles $\mathbf{x}_{k,t}$ are randomly chosen from the set $\overline{\mathbf{x}}_{k,t}$ determined in step 1 according to the importance weights determined in step 2 (resampling step). The result is a new set of particles representing the posterior for time step t.

Due to its stochastic nature, the particle filter has a variance which depends on the number of particles used: the higher the number of particles, the lower the variance. Strictly speaking, the particle filter only converges to the correct posterior for an infinite number of particles. A related problem is the so-called *particle depletion problem*, the problem that no particle may reside near the correct state because the number of particles is too small to cover all regions with a high probability (the probability distribution has too many modes). Overall, the "right" number of particles depends on the dimensionality of the state space and the complexity of the represented probability distribution. For many applications, a rather small number of particles has been demonstrated to be sufficient.

The variance problem caused by losing diversity of particles can be alleviated by reducing the frequency of resampling. In this case, the important factors are also stored and, when no resampling is done, are updated by the following operation:

$$w_{k,t} = p(o_t|\mathbf{x}_{k,t})\; w_{k,t-1} \tag{A.23}$$

In a resampling step, the importance factors then have to be reinitialized to 1. The decision on when to perform resampling can, for instance, be based on the variance of the importance factors: If the variance is high, resampling is advisable; otherwise not.

A.3.2 Rao-Blackwellized Particle Filter and FastSLAM

One important particle filter approach is the so-called Rao-Blackwellized particle filter (Doucet et al., 2000). The general idea is to marginalize out substructures from the posterior distribution. In the context of robot mapping, the key realization is that if the state vector x_t is split into two parts, the robot's pose s_t and the rest of the state variables simply referred to as the map m, the posterior for the complete history of poses $s_{0:t}$ (the robot's *trajectory*), and m can be factorized in the following way:

$$p(s_{0:t}, m|o_{1:t}, a_{1:t}) \quad = \quad p(s_{0:t}|o_{1:t}, a_{1-t})\; p(m|s_{0:t}, o_{1:t}, a_{1:t}) \tag{A.24}$$

The result is a product of two probability distributions: The right term describes the problem of estimating the map for known poses and can typically be computed exactly. The left part is a probability distribution that estimates the trajectory of the robot from the history of observations and actions and will be computed approximately using a particle filter approach. Accordingly, Rao-Blackwellized particle filters developed for robot mapping work in the following way: The particles represent the distribution over all trajectories, which means each particle represents a particular trajectory. In addition, each particle maintains its own map posterior $p(m|s_{0:t}, o_{1:t}, a_{1:t})$ based on its trajectory.

Rao-Blackwellized particle filters for robot mapping have been first described for a simple discrete state space by Murphy (2000) and then been realized on real robots for feature-based representations by Montemerlo et al. (2002) and later for grid maps (Hähnel et al., 2003a) under the term FastSLAM.

A.3.2.1 Feature-Based FastSLAM

Montemerlo et al. applied the idea of Rao-Blackwellized particle filters to feature-based representations (Montemerlo, 2003; Montemerlo et al., 2002). In this case, the map m consists of the positions m_i of l landmarks. Given the sequence of poses $s_{0:t}$, the position of each feature is conditionally independent of the positions of the other features. Thus, each feature location can be marginalized out individually and Eq. A.24 is replaced with:

$$p(s_{0:t}, m|o_{1:t}, a_{1:t}) = p(s_{0:t}|o_{1:t}, a_{1:t}) \prod_{i=1}^{l} p(m_i|s_{0:t}, o_{1_t}, a_{1_t}) \tag{A.25}$$

Each particle of the particle filter used for $p(s_{0:t}|o_{1:t}, a_{1:t})$ then maintains a set of extended Kalman filters, one for estimating the position of each landmark. The landmark estimators are organized in a binary tree so that the resampling step can be performed in logarithmic time, resulting in an overall time complexity of $O(n \log l)$ for one update, where n is the number of particles employed. In addition, the prediction step and the correction step only require the robot's current pose and not the complete trajectory; therefore each particle only needs to maintain the current pose. Data association can be performed for each particle individually or by sampling over different data associations. An improvement that incorporates o_t in the prediction step is described in Montemerlo et al. (2003) and the resulting algorithm is called Fast-SLAM 2.0.

A.3.2.2 Grid-Based FastSLAM

In the approach developed in Hähnel et al. (2003a) (see also Stachniss et al. (2005)) the maps associated with each particle are grid maps which are updated using a standard occupancy grid mapping approach to compute $p(m|s_{0:t}, o_{1:t}, a_{1:t}) = p(m|s_{0:t}, o_{1:t})$. The particle filter is only updated after several steps and is based on odometry estimates derived from scan matching and a learned parametric motion model. This results in a significant performance gain, allowing us to map large environments or use fewer particles.

A similar improvement to the prediction step as developed in the FastSLAM 2.0 algorithm is described for the grid-based variant in Grisetti et al. (2007a). More improvements are discussed in Grisetti et al. (2007b).

Appendix B

Qualitative Spatial Reasoning

Research on qualitative spatial reasoning (QSR) started at the end of the 1980s inspired by earlier work on temporal reasoning (Allen, 1983; van Beek, 1992; Freksa, 1992a). One goal of this research field is the development of efficient reasoning formalisms about sets of qualitative spatial relations. The most important reasoning problem is the *satisfiability* problem of deciding whether a set of spatial statements is consistent. The results of this research typically come in the form of so-called constraint calculi for expressing and reasoning about a particular aspect of space like mereotopology (Egenhofer, 1989; Randell et al., 1992; Renz & Nebel, 1999), direction and orientation (Dylla & Moratz, 2005; Frank, 1991; Freksa, 1992b; Ligozat, 1993, 1998; Moratz et al., 2000, 2005; Renz & Mitra, 2004; Schlieder, 1995), and position (Moratz et al., 2003). The expressiveness is limited to conjunctive relational statements, but as a result a better efficiency is achieved compared to more general reasoning approaches such as full geometric reasoning. Overviews on QSR research can be found in Cohn & Hazarika (2001) and Ligozat & Renz (2004).

B.1 Qualitative Constraint Calculi

An *n-ary qualitative constraint calculus* is concerned with n-ary relations over a potentially infinite *domain* $\mathcal{D}$ of (spatial) objects. The most common forms are binary calculi ($n = 2$) and ternary constraint calculi ($n = 3$). Typically the relations are derived from a *jointly exhaustive and pairwise disjoint* (JEPD) set of so-called *base relations* $\mathcal{B}$.

Definition B.1 (JEPD set of n-ary relations)**.** *A set of n-ary relations $\mathcal{B} \subseteq 2^{\mathcal{D}^n}$ over a domain $\mathcal{D}$ is called jointly exhaustive and pairwise disjoint (JEPD) if the relations of $\mathcal{B}$ cover $\mathcal{D}^n$ ($\bigcup_{R \in \mathcal{B}} R = \mathcal{D}^n$) and no two relations overlap ($\forall R, S \in \mathcal{B} : R \cap S = \emptyset$).*

The complete set of relations considered in a calculus is the set of *general relations* $\mathcal{R}_{\mathcal{B}}$ derived from the base relations $\mathcal{B}$ as follows.

Definition B.2 (Set of general relations of a set of JEPD relations)**.** *Given a JEPD set of n-ary relations $\mathcal{B}$, the set of general relations $\mathcal{R}_\mathcal{B}$ of $\mathcal{B}$ is the set of all unions of relations from $\mathcal{B}$:*

$$\mathcal{R}_\mathcal{B} = \left\{ R \subseteq \mathcal{D}^n \mid \exists X \subseteq \mathcal{B} : R = \bigcup_{x \in X} x \right\} \tag{B.1}$$

In the following, we will simply write $\{B_1, B_2, ..., B_m\}$ for the general relation which is the union of the base relations $B_1, B_2, ..., B_m \in \mathcal{B}$. $\emptyset$ will be used for the empty relation and U for the universal relation ($U = \bigcup_{B \in \mathcal{B}} B = \mathcal{D}^n$).

A qualitative constraint calculus employs operations defined over its set of general relations for elementary reasoning. The calculus has to be closed under these operations, meaning that the result is always again a relation from $\mathcal{R}_\mathcal{B}$. The operations can be grouped into three classes:

1. set-theoretic operations,
2. unary operations that permute the order of objects in the relation tuples, and
3. composition operations that combine information from two (or more) relations.

The set-theoretic operations are the standard *complement* ($\bar{\ }$), *union* ($\cup$), and *intersection* ($\cap$) operations applied to the relations of $\mathcal{R}_\mathcal{B}$.

Definition B.3 (Complement, union, intersection)**.** *Given relations $R, S \in \mathcal{R}_\mathcal{B}$, the operations of complement, union, and intersection are defined as follows:*

$$\bar{R} = U \setminus R = \{ x \mid x \in U \wedge x \notin R \} \quad \textit{(complement)} \tag{B.2}$$
$$R \cup S = \{ x \mid x \in R \vee x \in S \} \quad \textit{(union)} \tag{B.3}$$
$$R \cap S = \{ x \mid x \in R \wedge x \in S \} \quad \textit{(intersection)} \tag{B.4}$$

A set of general relations is trivially closed under these three operations. The results of the operations can directly be computed by applying the set-theoretic operations to the set notation introduced above (e.g., $\{B_1, B_4, B_7\} \cap \{B_4, B_7, B_8\} = \{B_4, B_7\}$).

For general n-ary calculi two operations are required to construct all permutations of the ordering of objects in the relation tuples: The *converse* operation ($^\smile$) exchanges the order of the last and second-to-last objects in the tuples, while the *rotation* operation ($^\frown$) rotates the relation tuples to the right.

Definition B.4 (Converse, rotation)**.** *Given a relation $R \in \mathcal{R}_\mathcal{B}$, the operations of converse and rotation are defined as follows:*

$$\begin{aligned} R^\smile &= \{ (d_1, ..., d_{n-2}, d_n, d_{n-1}) \in \mathcal{D}^n \mid (d_1, ..., d_{n-2}, d_{n-1}, d_n) \in R \} && \textit{(converse)} \\ R^\frown &= \{ (d_n, d_1, ..., d_{n-1}) \in \mathcal{D}^n \mid (d_1, ..., d_{n-1}, d_n) \in R \} && \textit{(rotation)} \end{aligned} \tag{B.5}$$

Converse and rotation can be used to generate all permutations of the tuples and thus allow for a change of perspective: From knowing that relation R holds between objects A, B, C ($(A, B, C) \in R$), it follows that the relation holding between B, A, C is $(R^{\smile})^{\frown}$ (the rotation of the converse of R). For binary calculi, converse and rotation do exactly the same thing, namely swap the elements of all relation pairs, and thus typically only converse is specified. Given that our set of general relations is closed under converse and rotation, these two operations can be specified by providing the results in table form. In principle, it is sufficient to provide the result of applying them to the set of base relations. The result for general relations can then be computed by taking the union of the results for the contained base relations.

Finally, the *composition* ($\circ$) operation takes two relations and combines them into a new relation. For better readability, we define the composition for binary and ternary calculi individually instead of giving a general definition.[1]

Definition B.5 (Binary composition)**.** *Given two binary relations $R, S \in \mathcal{R}_{\mathcal{B}}$, their composition is defined as*

$$R \circ S = \{\ (A, C) \in \mathcal{D}^2 \mid \exists B \in \mathcal{D}\ :\ (A, B) \in R \wedge (B, C) \in S\ \} \tag{B.6}$$

Definition B.6 (Ternary composition)**.** *Given two ternary relations $R, S \in \mathcal{R}_{\mathcal{B}}$, their composition is defined as*

$$R \circ S = \{\ (A, B, D) \in \mathcal{D}^3 \mid \exists C \in \mathcal{D}\ :\ (A, B, C) \in R \wedge (B, C, D) \in S\ \} \tag{B.7}$$

As for the converse and rotation operations, the composition operation of a calculus is typically specified as a table, either for only the base relations (an $m \times m$ matrix for m base relations) or for the complete set of general relations (a $2^m \times 2^m$ matrix for m base relations). In the first case, the composition of two general relations is computed by taking the union of the component-wise compositions.

We now have all the parts in place to give a definition of an n-ary qualitative constraint calculus.

Definition B.7 (n-ary qualitative constraint calculus)**.** *An n-ary qualitative constraint calculus is an 8-tuple $(\mathcal{D}, \mathcal{B}, \bar{\ }, \cup, \cap, ^{\smile}, ^{\frown}, \circ)$ where*

- *the domain $\mathcal{D}$ is a potentially infinite set of objects,*
- *the set of base relations $\mathcal{B} \subseteq 2^{\mathcal{D}^n}$ is a finite, nonempty JEPD set of n-ary relations over $\mathcal{D}$,*
- *$\bar{\ }, \cup, \cap, ^{\smile}, ^{\frown}, \circ$ are the operations over the set of general relations $\mathcal{R}_{\mathcal{B}}$ of $\mathcal{B}$ as defined above, and*
- *$\mathcal{R}_{\mathcal{B}}$ is closed under $\bar{\ }, \cup, \cap, ^{\smile}, ^{\frown}, \circ$.*

[1] Ternary composition has also been defined as a ternary operator in Condotta et al. (2006).

One simple example of a binary calculus used in this work is the cardinal direction calculus by Ligozat (1998) (cf. Fig. 6.4). It consists of nine base relations for relating points in the plane ($\mathcal{D} = \mathbb{R}^2$) corresponding to the eight cardinal directions $n, ne, w, sw, s, se, e, ne$ and the equal relation eq. The relations are typically written as $A\ ne\ B$ instead of $(A, B) \in ne$, and the meaning is that object A is to the northeast of B.

Tables for the converse and composition of the cardinal direction calculus state, for instance, that the converse of se is nw (if A is to the southeast of B, B is to the northwest of A) and that the composition of n and sw is $\{nw, w, sw\}$ (if A is to the north of B and B is to the southwest of C, it follows that A is either to the northwest, to the west, or to the southwest of C).

B.2 Weak vs. Strong Operations

In our definition of a qualitative constraint calculus, we demand that the set of general relations be closed under converse, rotation, and composition (as well as under the three set-theoretic operations). However, for many sets of JEPD base relations, the corresponding general relations are not closed at least under some of these operations (typically under composition). Often it can be shown that no finite set of relations exists that contains the base relations and is closed under all operations.

In this case, some kind of approximation has to be used in order for us to still remain with a finite set of relations. This is done by weakening the definition of the problematic operations by taking the smallest relation from $\mathcal{R}_{\mathcal{B}}$ that contains the result of the strong operation. For instance, a *weak composition* (in the binary case) would be defined as the union of base relations that have a nonempty intersection with the result of the strong composition:

$$R \circ_w S = \{ B \in \mathcal{B} \mid B \cap (R \circ S) \neq \emptyset \} \tag{B.8}$$

The consequences of having to resort to weak operations for constraint-based reasoning as described in the next section are still the topic of ongoing research. A discussion of this matter can be found in Renz & Ligozat (2005).

B.3 Constraint Networks and Consistency

For the following considerations, we restrict ourselves to binary constraint calculi. All techniques can be straightforwardly transferred to calculi of a higher arity.

A qualitative constraint calculus provides a formal language restricted to stating conjunctive relational facts holding between object constants. This can for instance be used to describe an observed spatial configuration of objects. Uncertainty can be expressed by referring to general relations that are unions of multiple base relations, where U represents complete ignorance.

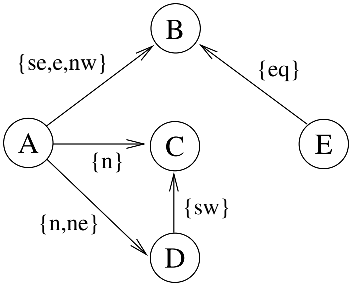

Figure B.1: A constraint network for the cardinal direction calculus

Such a set of relation statements over a finite set of objects can be depicted as a *constraint network*, as shown in Fig. B.1. The objects are denoted by the nodes and each directed edge is labeled by a constraint which is a relation from $\mathcal{R}_\mathcal{B}$ of the employed calculus. If the edge connecting object A with object B is annotated with relation R, this means that the pair of values from $\mathcal{D}$ (e.g., coordinates in the plane) that can be assigned to A and B is restricted to being one contained in R. If no edge connects two nodes, this corresponds to an edge labeled with the universal relation U, which is usually omitted.

A constraint network defines a particular kind of constraint satisfaction problem (CSP). We will call it a spatial CSP here.

Definition B.8 (Spatial constraint satisfaction problem)**.** *A spatial constraint problem is a triple* $(\mathcal{QCC}, \mathcal{V}, \mathcal{C})$ *consisting of a finite set* $\mathcal{V}$ *of variables* $V_1, ..., V_m$ *over the domain of a given qualitative constraint calculus* $\mathcal{QCC}$ *and a set* $\mathcal{C}$ *of binary constraints* $C_{ij} \in \mathcal{R}_\mathcal{B}$ *(*$1 \leq i < j \leq m$*) holding between* V_i *and* V_j *where* $\mathcal{R}_\mathcal{B}$ *is the set of general relations of* $\mathcal{QCC}$*.*

A spatial CSP has a solution if one can assign values from the spatial domain to the variables so that all constraints are satisfied.

Definition B.9 (Assignment)**.** *An assignment is a total function assign* : $\mathcal{V} \rightarrow \mathcal{D}$ *that maps each variable from a spatial CSP to a value from the domain.*

Definition B.10 (Solution)**.** *A solution of a spatial CSP is an assignment assign which satisfies all the constraints:*

$$\forall V_i, V_j \in \mathcal{V}, 1 \leq i < j \leq m : (\mathit{assign}(V_i), \mathit{assign}(V_j)) \in C_{ij} \qquad \text{(B.9)}$$

One fundamental reasoning problem of QSR is the *satisfiability* or *consistency problem*, concerned with whether a spatial CSP has a solution or not.

Definition B.11 (Satisfiable, consistent)**.** *A spatial CSP is said to be satisfiable or consistent if it has at least one solution.*

In contrast to many other kinds of CSPs, the domain in our case is infinite. Therefore, special techniques for checking consistency had to be developed which are based on the operations of the calculus. Before we turn to these techniques, we need to define two more concepts.

Definition B.12 (Scenario, atomic)**.** *A spatial CSP (or constraint network) is called a scenario or atomic if all constraints are base relations.*

Definition B.13 (Refinement)**.** *A refinement of a spatial CSP P with variable set $\mathcal{V}$ and constraints C_{ij} is another spatial CSP P' over the same set of variables with $C'_{ij} \subseteq C_{ij}$ for all $1 \leq i < j \leq |\mathcal{V}|$.*

Thus, refinements can be constructed by removing base relations from the constraints of a CSP. If a constraint network contains constraints that are not base relations (meaning it is not a scenario), we are often interested in finding a scenario that is a refinement of this network. If we can show this scenario to be consistent, we know that the original network is consistent as well. Figure B.2 shows in (a) a non-atomic network and in (b) and (c) the two only scenarios which are refinements of this network.

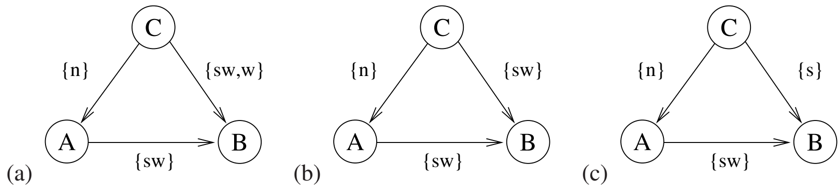

Figure B.2: A non-atomic constraint network (a) with possible scenarios (b) and (c)

B.4 Checking Consistency

The techniques developed for relational constraint problems are based on weaker forms of consistency called *local consistencies* which can be enforced based on the operations of the calculus and which are under particular conditions sufficient to decide consistency. If this is the case, enforcing this local consistency without one constraint becoming the empty relation proves consistency of the original network.

One important form of local consistency is *path consistency*, which (in binary CSPs) means that for every triple of variables each consistent evaluation of the first two variables can be extended to the third variable in such a way that all constraints are satisfied. To enforce path consistency, a syntactic procedure called the *algebraic closure algorithm* is used (Mackworth, 1977; Montanari, 1974). The algorithm runs

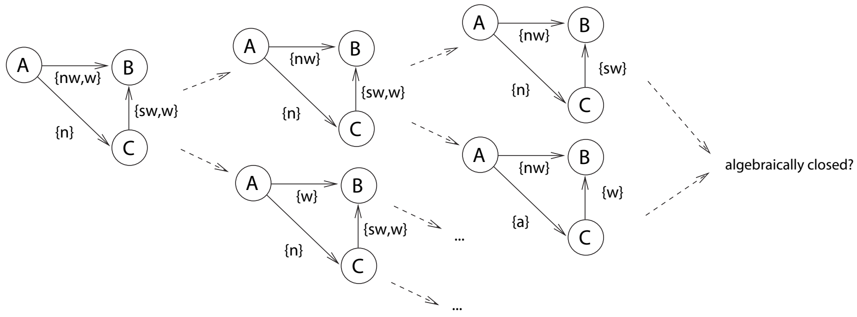

Figure B.3: Backtracking search through the refinements until an algebraically closed scenario is found

in $O(n^3)$ time and in essence performs the following operation on triples of variables V_i, V_j, V_k until a fix point has been reached:

$$C_{ik} = C_{ik} \cap (C_{ij} \circ C_{jk}) \tag{B.10}$$

If one constraint becomes the empty relation in the process, the original network was inconsistent. It has to be noted that for some algorithms this procedure does not actually enforce path consistency but only approximates it. Moreover, for some calculi, algebraic closure is not sufficient to decide overall consistency in general but it is sufficient if the network is a scenario. In this case, a backtracking search can be used in which all possible scenarios of the original network are generated by recursively splitting constraints as indicated in Fig. B.3. Each generated scenario is checked using the algebraic closure algorithm. If one is shown to be consistent, the original network is consistent as well. If no consistent scenario exists, inconsistency of the original network has been shown. Unfortunately, this approach has an exponential worst-case time complexity.

For some calculi larger so-called *maximal tractable subsets* of relations including the base relations are known for which algebraic closure decides consistency. These subsets can be used to increase performance of the backtrack algorithm by only refining until all constraints are relations from the subset (Ladkin & Reinefeld, 1992). Even when algebraic closure cannot be employed as a decision procedure for consistency, it can still be used as an incomplete method that may not discover all inconsistencies.

The standard constraint-based reasoning techniques described here have been realized for the most common spatial calculi in the SparQ toolbox (Wallgrün et al., 2006, 2007). This toolbox is used to implement the approach described in Chap. 6 of this book.

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