Appendix A
Examples of Calabi-Yau 3-manifolds with complex multiplication

Introduction

The previous examples of Calabi-Yau manifolds with CM occur as fibers of a family over a Shimura variety, which has a dense set of complex multiplication fibers. Here we give some examples, which are not necessarily fibers of a non-trivial family with a dense set of complex multiplication fibers.

The first two sections give two different classes of examples by using involutions on $K3$ surfaces. In each of the both Sections we will use a modified version of the construction of Viehweg and Zuo to obtain $K3$ surfaces, which are suitable for the construction of a Borcea-Voisin tower.

In the third section we will prove that a $K3$ surface with a degree 3 automorphism has complex multiplication. By using methods, which has been introduced in Section 9.1 and Section 9.2, we will use this automorphism and the Fermat curve of degree 3 for the construction of a Calabi-Yau 3-manifold with complex multiplication.

A.1 Construction by degree 2 coverings of a ruled surface

We start by finding curves with complex multiplication. The following proposition yields some examples:

Proposition A.1.1. *Let $0 < d_1, d < m$, and ξ_k denote a primitive k-th. root of unity for all $k \in \mathbb{N}$. Then the curve C, which is locally given by*

$$y^m = x^{d_1} \prod_{i=1}^{n-2} (x - \xi_{n-2}^i)^d,$$

is covered by the Fermat curve $\mathbb{F}_{(n-2)m}$ *locally given by*

$$y^{(n-2)m} + x^{(n-2)m} + 1 = 0$$

and has complex multiplication.

Proof. (see Theorem 2.4.4) □

Example A.1.2. By the preceding proposition, the curves locally given by

$$y^4 = x_1^8 + x_0^8, \quad y^4 = x_1(x_1^7 + x_0^7), \quad y^4 = x_1(x_1^6 + x_0^6)x_0$$

have complex multiplication. These curves are degree 4 covers of the projective line and have the genus 9 as one can easily calculate by the Hurwitz formula.

The curves of the preceding example have a natural interpretation as cyclic covers of $\mathbb{P}^1$ of degree 4. One can identify these covers with the set of their 8 branch points in $\mathbb{P}^1$. Thus let $\mathcal{P}_8$ denote the configuration space of 8 different points in $\mathbb{P}^1$. We use a modified version of the construction in [58], Section 5 to construct $K3$ surfaces with complex multiplication by Example $A.1.2$ in a first step. This method is nearly the same method as in Section 8.2.

For our application, it is sufficient to work with $\mathbb{P}^1$-bundles over $\mathbb{P}^1$ resp., with rational ruled surfaces. Let $\pi_n : \mathbb{P}_n \to \mathbb{P}^1$ denote the rational ruled surface given by $\mathbb{P}(\mathcal{O}_{\mathbb{P}^1} \oplus \mathcal{O}_{\mathbb{P}^1}(n))$ and σ denote a non-trivial global section of $\mathcal{O}_{\mathbb{P}^1}(8)$, which has the 8 different zero points represented by a point $q \in \mathcal{P}_8$. The sections E_σ, E_0 and E_∞ of $\mathbb{P}(\mathcal{O} \oplus \mathcal{O}(8))$ are induced by

$$\mathrm{id} \oplus \sigma : \mathcal{O} \to \mathcal{O} \oplus \mathcal{O}(8), \quad \mathrm{id} \oplus 0 : \mathcal{O} \to \mathcal{O} \oplus \mathcal{O}(8)$$
$$\text{and} \quad 0 \oplus \mathrm{id} : \mathcal{O}(8) \to \mathcal{O} \oplus \mathcal{O}(8)$$

resp., by the corresponding surjections onto the cokernels of these embeddings as described in [26], **II**. Proposition 7.12.

Remark A.1.3. The divisors E_σ and E_0 intersect each other transversally over the 8 zero points of σ. Recall that $\mathrm{Pic}(\mathbb{P}_8)$ has a basis given by a fiber and an arbitrary section. Hence by the fact that E_σ and E_0 do not intersect E_∞, one concludes that they are linearly equivalent with self-intersection number 8. Since E_∞ is a section, it intersects each fiber transversally. Thus one has that $E_\infty \sim E_0 - (E_0.E_0)F$, where F denotes a fiber. Therefore one concludes

$$E_\infty.E_\infty = E_\infty.(E_0 - (E_0.E_0)F) = -(E_0.E_0) = -8.$$

Next we establish a morphism $\mu : \mathbb{P}_2 \to \mathbb{P}_8$ over $\mathbb{P}^1$. By [26], **II**. Proposition 7.12., this is the same as to give a surjection $\pi_2^*(\mathcal{O} \oplus \mathcal{O}(8)) \to \mathcal{L}$, where $\mathcal{L}$ is an invertible sheaf on $\mathbb{P}_2$. By the composition

$$\pi_2^*(\mathcal{O}\oplus\mathcal{O}(8)) = \pi_2^*(\mathcal{O})\oplus\pi_2^*\mathcal{O}(8) \hookrightarrow \bigoplus_{i=0}^{4}\pi_2^*\mathcal{O}(2i) = Sym^4(\pi_2^*(\mathcal{O}\oplus\mathcal{O}(8))) \to \mathcal{O}_{\mathbb{P}_2}(4),$$

where the last morphism is induced by the natural surjection $\pi_2^*(\mathcal{O}\oplus\mathcal{O}(2)) \to \mathcal{O}_{\mathbb{P}_2}(1)$ (see [26], **II**. Proposition 7.11), we obtain a morphism μ^* of sheaves. This morphism μ^* is not a surjection onto $\mathcal{O}_{\mathbb{P}_2}(4)$, but onto its image $\mathcal{L} \subset \mathcal{O}_{\mathbb{P}_2}(4)$. Over $\mathbb{A}^1 \subset \mathbb{P}^1$ all rational ruled surfaces are locally given by $\mathrm{Proj}(\mathbb{C}[x])[y_1, y_2]$, where x has the weight 0. Hence we have locally that $\pi_2^*(\mathcal{O} \oplus \mathcal{O}(8)) = \mathcal{O}e_1 \oplus \mathcal{O}e_2$. Over $\mathbb{A}^1$ the morphism μ^* is given by

$$e_1 \to y_1^4, e_2 \to y_2^4$$

such that the sheaf $\mathcal{L} = im(\mu^*) \subset \mathcal{O}_{\mathbb{P}_2}(4)$ is invertible. Thus the morphism $\mu : \mathbb{P}_2 \to \mathbb{P}_8$ corresponding to μ^* is locally given by the ring homomorphism

$$(\mathbb{C}[x])[y_1, y_2] \to (\mathbb{C}[x])[y_1, y_2] \text{ via } y_1 \to y_1^8 \text{ and } y_2 \to y_2^8.$$

Construction A.1.4. One has a commutative diagram

$$\begin{array}{ccccc}
\mathcal{Y}' & \xrightarrow{\tau'} & \mathbb{P}_2' & \xrightarrow{\mu'} & \mathbb{P}^1 \times \mathbb{P}^1 \\
\uparrow{\scriptstyle\delta} & & \uparrow{\scriptstyle\delta_2} & & \uparrow{\scriptstyle\delta_8} \\
\hat{\mathcal{Y}} & \xrightarrow{\hat{\tau}} & \hat{\mathbb{P}}_2 & \xrightarrow{\hat{\mu}} & \hat{\mathbb{P}}_8 \\
\downarrow{\scriptstyle\rho} & & \downarrow{\scriptstyle\rho_2} & & \downarrow{\scriptstyle\rho_8} \\
\mathcal{Y} & \xrightarrow[\sqrt[2]{\frac{\mu^* E_\sigma}{3\cdot(\mu^* E_0)_{red}}}]{\tau} & \mathbb{P}_2 & \xrightarrow[\sqrt[4]{\frac{E_\infty + 8\cdot F}{E_0}}]{\mu} & \mathbb{P}_8 \\
\downarrow{\scriptstyle\pi} & & \downarrow{\scriptstyle\pi_2} & & \downarrow{\scriptstyle\pi_8} \\
\mathbb{P}^1 & \xrightarrow{\mathrm{id}} & \mathbb{P}^1 & \xrightarrow{\mathrm{id}} & \mathbb{P}^1
\end{array}$$

of morphisms between normal varieties with:

(a) δ, δ_2, δ_8, ρ, ρ_2 and ρ_8 are birational.
(b) π is a family of curves, π_2 and π_8 are $\mathbb{P}^1$-bundles.

Proof. One must only explain δ_8 and ρ_8. Recall that E_σ is a section of $\mathbb{P}(\mathcal{O} \oplus \mathcal{O}(8))$, which intersects E_0 transversally in exactly 8 points. The morphism ρ_8 is the blowing up of the 8 intersection points of $E_0 \cap E_\sigma$. The preimage of the 8 points given by $q \in \mathcal{P}_8$ with respect to $\pi_8 \circ \rho_8$ consists of the exceptional divisor $\hat{D}_1$ and the proper transform $\tilde{D}_2$ of the preimage of these 8 points with respect to ρ_8 given by 8 rational curves with self-intersection number -1. The morphism δ_8 is obtained by blowing down $\tilde{D}_2$. □

Remark A.1.5. The section σ has the zero divisor given by some $q \in \mathcal{P}_8$. Hence one obtains $\mu^*(E_\sigma) \cong C$, where $C \to \mathbb{P}^1$ is a cyclic cover of degree 4 as in Example A.1.2 ramified over the 8 points given by σ. The surface $\mathcal{Y}$ is a cyclic degree 2 cover of $\mathbb{P}_2$ ramified over C. Thus it has an involution. It is given by the invertible sheaf

$$\mathcal{L} = \omega_{\mathbb{P}_2}^{-1}$$

and the divisor

$$B = \mu^*(E_\sigma), \quad \text{where} \quad \mathcal{O}(B) \cong \mathcal{L}^2,$$

with the notation of [6] **I**. Section 17. Thus [6] **I**. Lemma 17.1 implies that $\mathcal{Y}$ is a $K3$ surface.

By Lemma 10.4.1, there is only one elliptic curve with a cyclic degree 4 cover onto $\mathbb{P}^1$. Let $\mathbb{E}$ denote this curve, which is locally given by

$$y^4 = x(x-1)^2.$$

One can easily see that $\mathbb{E}$ has the j invariant 1728. Thus $\mathbb{E}$ has complex multiplication.

We fix some notation. Let $n \in \mathbb{N}$, let ξ be a fixed primitive n-th. root of unity and let C_1 and C_2 be curves locally given by

$$y^n = f_1(x) \quad \text{and} \quad y^n = f_2(x),$$

where $f_1, f_2 \in \mathbb{C}[x]$. By $(x,y) \to (x, \xi y)$, one can define an automorphism γ_i on C_i for $i = 1,2$. The surface $C_1 \times C_2/\langle(1,1)\rangle$ is the quotient of $C_1 \times C_2$ by $\langle(\gamma_1, \gamma_2)\rangle$.

Proposition A.1.6. *The surface $\mathcal{Y}$ is birationally equivalent to $C \times \mathbb{E}/\langle(1,1)\rangle$.*[1]

Proof. Let $\tilde{E}_\bullet$ denote the proper transform of the section $E_\bullet$ with respect to ρ_8. Then $\hat{\mu}$ is the Kummer covering given by

$$\sqrt[4]{\frac{\tilde{E}_\infty + 8 \cdot F}{\tilde{E}_0 + \hat{D}_1}},$$

where $\hat{D}_1$ denotes the exceptional divisor of ρ_8. Thus the morphism μ' is the Kummer covering

$$\sqrt[4]{\frac{(\delta_8)_*\tilde{E}_\infty + 8 \cdot (\delta_8)_* F}{(\delta_8)_*\tilde{E}_0 + (\delta_8)_*\hat{D}_1}} = \sqrt[4]{\frac{\mathbb{P}^1 \times \{\infty\} + 8 \cdot (P \times \mathbb{P}^1)}{\mathbb{P}^1 \times \{0\} + \Delta \times \mathbb{P}^1}},$$

[1] Similarly to [58], Construction 5.2, we show that $\mathcal{Y}'$ is birationally equivalent to $C \times \mathbb{E}/\langle(1,1)\rangle$.

where Δ is the divisor of the 8 different points in $\mathbb{P}^1$ given by $q \in \mathcal{P}_8$ and $P \in \mathbb{P}^1$ is the point with the fiber F. Since $E_0 + E_\sigma$ is a normal crossing divisor, $\tilde{E}_\sigma$ neither meets $\tilde{E}_0$ nor $\tilde{D}_2$, where $\tilde{D}_2$ is the proper transform of $\pi_8^*(\Delta)$. Therefore $(\delta_8)_* \tilde{E}_\sigma$ neither meets

$$(\delta_8)_* \tilde{E}_0 = \mathbb{P}^1 \times \{0\} \quad \text{nor} \quad (\delta_8)_* \tilde{E}_\infty = \mathbb{P}^1 \times \{\infty\}.$$

Hence one can choose coordinates in $\mathbb{P}^1$ such that $(\delta_8)_* \tilde{E}_\sigma = \mathbb{P}^1 \times \{1\}$.

By the definition of τ, we obtain that $\hat{\tau}$ is given by

$$\sqrt[2]{\frac{\rho_2^* \mu^*(E_\sigma)}{\rho_2^* \mu^*(E_0)}} = \sqrt[2]{\frac{\hat{\mu}^*(\tilde{E}_\sigma)}{\hat{\mu}^*(\tilde{E}_0)}},$$

and τ' is given by

$$\sqrt[2]{\frac{\mu'^*(\mathbb{P}^1 \times \{1\})}{\mu'^*(\mathbb{P}^1 \times \{0\})}}.$$

By the fact that the last function is the root of the pullback of a function on $\mathbb{P}^1 \times \mathbb{P}^1$ with respect to μ', it is possible to reverse the order of the field extensions corresponding to τ' and μ' such that the resulting varieties obtained by Kummer coverings are birationally equivalent. Hence we have the composition of $\beta : \mathbb{P}^1 \times \mathbb{P}^1 \to \mathbb{P}^1 \times \mathbb{P}^1$ given by

$$\sqrt[2]{\frac{\mathbb{P}^1 \times \{1\}}{\mathbb{P}^1 \times \{0\}}}$$

with

$$\sqrt[4]{\frac{\beta^*(\mathbb{P}^1 \times \{\infty\}) + 8 \cdot (P \times \mathbb{P}^1)}{\beta^*(\mathbb{P}^1 \times \{0\}) + (\Delta \times \mathbb{P}^1)}},$$

which yields the covering variety isomorphic to $\mathbb{E} \times C/\langle(1,1)\rangle$. □

As in Section 8.2 we conclude:

Corollary A.1.7. *If the curve C has complex multiplication, the $K3$-surface $\mathcal{Y}$ has complex multiplication, too.*

By the the preceding corollary, our Example $A.1.2$ yields 3 different $K3$ surfaces with complex multiplication locally given by

$$y_2^2 + y_1^4 + x_1^8 + x_0^8, \quad y_2^2 + y_1^4 + x_1(x_1^7 + x_0^7), \quad y_2^2 + y_1^4 + x_1(x_1^6 + x_0^6)x_0.$$

Proposition A.1.8. *For $i = 1, 2$ assume that C_i is a Calabi-Yau i-manifold with complex multiplication endowed with the involution ι_i such that ι_i acts by -1 on $\Gamma(\omega_{C_i})$. By blowing up the singular locus of $C_1 \times C_2/\langle(\iota_1, \iota_2)\rangle$, one obtains a Calabi-Yau 3-manifold with complex multiplication.*

Proof. It is well-known that an involution on a Calabi-Yau 2-manifold resp., a $K3$ surface, which acts by -1 on $\Gamma(\omega)$, has a smooth divisor of fixed points or it has not any fixed point. Thus the proof follows from the same methods as in Section 7.2. □

Now we need some elliptic curves with complex multiplication:

Example A.1.9. Elliptic curves with CM has been well studied by number theorists. Some examples of elliptic curves with complex multiplication are given by the following list:

equation	j invariant
$y_1^2 x_0 = x_1^3 - x_0^3$	0
$y_1^2 x_0 = x_1(x_1 - x_0)(x_1 - 2x_0)$	1728
$y^2 x_0 = x_1(x_1 - x_0)(x_1 - (1+\sqrt{2})^2 x_0)$	8000
$y^2 x_0 = x_1(x_1 - x_0)(x_1 - \frac{1}{4}(3+i\sqrt{7})^2 x_0)$	−3375
$y^2 x_0 = x_1^3 - 15x_1x_0^2 + 22x_0^3$	54000
$y^2 x_0 = x^3 - 595x_1x_0^2 + 5586x_0^3$	16581375

Note that the equations allow an explicit definition of an involution on these elliptic curves. (see Section 7.4)

A.1.10. By combining our 3 examples of $K3$ surfaces and the 6 elliptic curves and using Proposition A.1.8, we have 18 examples of Calabi-Yau 3-manifolds with complex multiplication. By [60], one has equations to determine the Hodge numbers of these examples. Let C_2 be a $K3$ surface satisfying the assumptions of Proposition A.1.8, let N be the number of curves in the ramification locus of the quotient map $C_2 \to C_2/\iota_2$ and let N' be given by

$$N' = g_1 + \ldots + g_N,$$

where g_i denotes the genus of the i-th. curve in the ramification locus. Then one has for the Calabi-Yau 3-manifold, which results by Proposition A.1.8:

$$h^{1,1} = 11 + 5N - N'$$

$$h^{2,1} = 11 + 5N' - N$$

In our case the ramification locus of $C_2 \to C_2/\iota_2$ is given by one genus 9 curve. Thus in our case the Hodge numbers are given by

$$h^{1,1} = 7 \text{ and } h^{2,1} = 55.$$

A.2 Construction by degree 2 coverings of $\mathbb{P}^2$

Example A.2.1. By Proposition A.1.1, the projective curves given by

$$y^6 = x_1^6 + x_0^6, \quad y^6 = x_1(x_1^5 + x_0^5), \quad y^6 = x_1(x_1^4 + x_0^4)x_0$$

have complex multiplication. These curves have the genus 10 as one can easily calculate by the Hurwitz formula.

Let $\mathcal{P}_6$ denote the configuration space of 6 different points in $\mathbb{P}^1$. Again we use a modified version of the construction in [58], Section 5. Let σ denote a non-trivial global section of $\mathcal{O}_{\mathbb{P}^1}(6)$, which has the 6 different zero points represented by a point $q \in \mathcal{P}_6$.

Here the sections E_σ, E_0 and E_∞ of $\mathbb{P}(\mathcal{O} \oplus \mathcal{O}(6))$ are induced by

$$\mathrm{id} \oplus \sigma : \mathcal{O} \to \mathcal{O} \oplus \mathcal{O}(6), \quad \mathrm{id} \oplus 0 : \mathcal{O} \to \mathcal{O} \oplus \mathcal{O}(6)$$
$$\text{and} \quad 0 \oplus \mathrm{id} : \mathcal{O}(6) \to \mathcal{O} \oplus \mathcal{O}(6)$$

resp., by the corresponding surjections onto the cokernels of these embeddings as described in [26], **II**. Proposition 7.12.

One concludes similarly to the preceding section that

$$E_\infty.E_\infty = E_\infty.(E_0 - (E_0.E_0)F) = -(E_0.E_0) = -6.$$

By the composition

$$\pi_1^*(\mathcal{O} \oplus \mathcal{O}(6)) = \pi_1^*(\mathcal{O}) \oplus \pi_1^*\mathcal{O}(6) \hookrightarrow \bigoplus_{i=0}^{6} \pi_1^*\mathcal{O}(i) = Sym^6(\pi_1^*(\mathcal{O} \oplus \mathcal{O}(6))) \to \mathcal{O}_{\mathbb{P}_1}(6),$$

where the last morphism is induced by the natural surjection $\pi_2^*(\mathcal{O} \oplus \mathcal{O}(1)) \to \mathcal{O}_{\mathbb{P}_1}(1)$ (see [26], **II**. Proposition 7.11), we obtain a morphism μ^* of sheaves as in the preceding section. The morphism $\mu : \mathbb{P}_1 \to \mathbb{P}_6$ corresponding to μ^* is locally given by the ring homomorphism

$$(\mathbb{C}[x])[y_1, y_2] \to (\mathbb{C}[x])[y_1, y_2] \quad \text{via} \quad y_1 \to y_1^6 \quad \text{and} \quad y_2 \to y_2^6.$$

Construction A.2.2. One has a commutative diagram

$$\begin{array}{ccccc}
\mathcal{Y}' & \xrightarrow{\tau'} & \mathbb{P}_1' & \xrightarrow{\mu'} & \mathbb{P}^1 \times \mathbb{P}^1 \\
\uparrow{\scriptstyle \delta} & & \uparrow{\scriptstyle \delta_1} & & \uparrow{\scriptstyle \delta_6} \\
\hat{\mathcal{Y}} & \xrightarrow{\hat{\tau}} & \hat{\mathbb{P}}_1 & \xrightarrow{\hat{\mu}} & \hat{\mathbb{P}}_6 \\
\downarrow{\scriptstyle \rho} & & \downarrow{\scriptstyle \rho_1} & & \downarrow{\scriptstyle \rho_6} \\
\mathcal{Y} & \xrightarrow[\sqrt[2]{\frac{\mu^* E_\sigma}{3\cdot(\mu^* E_0)_{red}}}]{\tau} & \mathbb{P}_1 & \xrightarrow[\sqrt[6]{\frac{E_\infty + 6\cdot F}{E_0}}]{\mu} & \mathbb{P}_6 \\
\downarrow{\scriptstyle \pi} & & \downarrow{\scriptstyle \pi_1} & & \downarrow{\scriptstyle \pi_6} \\
\mathbb{P}^1 & \xrightarrow{\mathrm{id}} & \mathbb{P}^1 & \xrightarrow{\mathrm{id}} & \mathbb{P}^1
\end{array}$$

of morphisms between normal varieties with:

(a) δ, δ_1, δ_1, ρ, ρ_1 and ρ_6 are birational.
(b) π is a family of curves, π_1 and π_6 are $\mathbb{P}^1$-bundles.

Proof. One must only explain δ_6 and ρ_6. These morphisms are given by blowing up morphisms similar to the preceding section. □

Remark A.2.3. The section σ has the zero divisor given by some $q \in \mathcal{P}_6$. Hence one obtains $\mu^*(E_\sigma) \cong C$, where $C \to \mathbb{P}^1$ is a cyclic cover of degree 6 as in Example A.2.1 ramified over the 6 points given by σ. The surface $\mathcal{Y}$ is a cyclic degree 2 cover of $\mathbb{P}_1$ ramified over C. Thus it is birationally equivalent to the $K3$ surface given the degree 2 cover of $\mathbb{P}^2$ ramified over C.

Let C' denote the projective smooth curve locally given by

$$y^6 = x(x-1).$$

By Proposition A.1.1, it has complex multiplication.

Proposition A.2.4. *The surface $\mathcal{Y}$ is birationally equivalent to $C \times C'/\langle(1,1)\rangle$.*

Proof. Let $\tilde{E}_\bullet$ denote the proper transform of the section $E_\bullet$ with respect to ρ_6. Then $\hat{\mu}$ is the Kummer covering given by

$$\sqrt[6]{\frac{\tilde{E}_\infty + 6 \cdot F}{\tilde{E}_0 + \hat{D}_1}},$$

where $\hat{D}_1$ denotes the exceptional divisor of ρ_6. Thus the morphism μ' is the Kummer covering

$$\sqrt[6]{\frac{(\delta_6)_*\tilde{E}_\infty + 6 \cdot (\delta_6)_*F}{(\delta_6)_*\tilde{E}_0 + (\delta_6)_*\hat{D}_1}} = \sqrt[6]{\frac{\mathbb{P}^1 \times \{\infty\} + 6 \cdot (P \times \mathbb{P}^1)}{\mathbb{P}^1 \times \{0\} + \Delta \times \mathbb{P}^1}},$$

where Δ is the divisor of the 6 different points in $\mathbb{P}^1$ given by $q \in \mathcal{P}_6$ and $P \in \mathbb{P}^1$ is the point with the fiber F. Since $E_0 + E_\sigma$ is a normal crossing divisor, $\tilde{E}_\sigma$ neither meets $\tilde{E}_0$ nor $\tilde{D}_2$, where $\tilde{D}_2$ is the proper transform of $\pi_6^*(\Delta)$. Therefore $(\delta_6)_*\tilde{E}_\sigma$ neither meets

$$(\delta_6)_*\tilde{E}_0 = \mathbb{P}^1 \times \{0\} \text{ nor } (\delta_6)_*\tilde{E}_\infty = \mathbb{P}^1 \times \{\infty\}.$$

Hence one can choose coordinates in $\mathbb{P}^1$ such that $(\delta_6)_*\tilde{E}_\sigma = \mathbb{P}^1 \times \{1\}$.

By the definition of τ, we obtain that $\hat{\tau}$ is given by

$$\sqrt[2]{\frac{\rho_1^*\mu^*(E_\sigma)}{\rho_1^*\mu^*(E_0)}} = \sqrt[2]{\frac{\hat{\mu}^*(\tilde{E}_\sigma)}{\hat{\mu}^*(\tilde{E}_0)}},$$

and τ' is given by

$$\sqrt[2]{\frac{\mu'^*(\mathbb{P}^1 \times \{1\})}{\mu'^*(\mathbb{P}^1 \times \{0\})}}.$$

By the fact that the last function is the root of the pullback of a function on $\mathbb{P}^1 \times \mathbb{P}^1$ with respect to μ', it is possible to reverse the order of the field extensions corresponding to τ' and μ' such that the resulting varieties obtained by Kummer coverings are birationally equivalent. Hence we have the composition of $\beta : \mathbb{P}^1 \times \mathbb{P}^1 \to \mathbb{P}^1 \times \mathbb{P}^1$ given by

$$\sqrt[2]{\frac{\mathbb{P}^1 \times \{1\}}{\mathbb{P}^1 \times \{0\}}}$$

with

$$\sqrt[6]{\frac{\beta^*(\mathbb{P}^1 \times \{\infty\}) + 6 \cdot (P \times \mathbb{P}^1)}{\beta^*(\mathbb{P}^1 \times \{0\}) + (\Delta \times \mathbb{P}^1)}},$$

which yields the covering variety isomorphic to $C' \times C/\langle(1,1)\rangle$. □

Hence $\mathcal{Y}$ is birationally equivalent to $C' \times C/\langle(1,1)\rangle$. As in Section 8.2 we conclude:

Corollary A.2.5. *If the curve C has complex multiplication, the $K3$-surface $\mathcal{Y}$ has complex multiplication, too.*

A.2.6. By the preceding corollary, our Example $A.2.1$ yields 3 different $K3$ surfaces with complex multiplication as degree 2 covers of $\mathbb{P}^2$, which are locally given by

$$y_2^2 + y_1^6 = x_1^6 + x_0^6, \quad y_2^2 + y_1^6 = x_1(x_1^5 + x_0^5), \quad y_2^2 + y_1^6 = x_1(x_1^4 + x_0^4)x_0.$$

By an elliptic curve with complex multiplication, these $K3$ surfaces yield Calabi-Yau 3-manifolds with complex multiplication. We obtain 18 Calabi-Yau 3-manifolds with complex multiplication by using Example $A.1.9$. By the same methods as in $A.1.10$, one calculates easily that the resulting Calabi-Yau 3-manifolds have the Hodge numbers

$$h^{1,1} = 6 \text{ and } h^{2,1} = 60.$$

A.3 Construction by a degree 3 quotient

Consider the $K3$ surface

$$S = V((y_2^3 - y_1^3)y_1 + (x_1^3 - x_0^3)x_0) \subset \mathbb{P}^3.$$

By using the partial derivatives of the defining equation, one can easily verify that S is smooth. First we prove that this surface has complex multiplication. In a second step we consider an automorphism of degree 3 on this surface, which allows the construction of a Calabi-Yau 3-manifold with complex multiplication.

Proposition A.3.1. *The $K3$ surface S has complex multiplication.*

Proof. Consider the isomorphic curves

$$C_1 = V(z_1^4 - (y_2^3 - y_1^3)y_1) \subset \mathbb{P}^2,$$

$$C_2 = V(z_2^4 - (x_1^3 - x_0^3)x_0) \subset \mathbb{P}^2.$$

Since the elliptic curve with j invariant 0 given by

$$V(y^2 x_0 + x_1^3 + x_0^3) \subset \mathbb{P}^2$$

has complex multiplication, one concludes as in Remark 7.4.2 that C_1 and C_2 have complex multiplication, too. The $K3$ surface S is birationally equivalent to

$$T = C_1 \times C_2 / \langle (1,1) \rangle.$$

This follows from the rational map $C_1 \times C_2 \to S$ given by

$$((z_1 : y_2 : y_1), (z_2 : x_1 : x_0)) \to (\frac{z_2}{z_1} y_2 : \frac{z_2}{z_1} y_1 : x_1 : x_0).$$ [2]

There exists a suitable sequence of blowing ups turning $C_1 \times C_2$ into $\widetilde{C_1 \times C_2}$ such that

$$\tilde{T} = \widetilde{C_1 \times C_2} / \langle (1,1) \rangle$$

is smooth. Since we only blow up points, $\widetilde{C_1 \times C_2}$ has CM, too (see Corollary 7.1.6). Thus the quotient has CM. Since $\tilde{T}$ is birationally equivalent to S, there exists a sequence of blowing ups of smooth points and blowing downs to smooth points, which turns $\tilde{T}$ into S. By Corollary 7.1.6, the fact that $\tilde{T}$ has CM implies that S has CM. □

A.3.2. Let ξ denote $e^{\frac{2\pi i}{3}}$. The $K3$ surface S has an automorphism γ of degree 3 given by

$$(y_2 : y_1 : x_1 : x_0) \to (\xi y_2 : y_1 : \xi x_1 : x_0).$$

On $\{x_0 = 1\}$ we have the 4 fixed points given by

$$(0 : \sqrt[4]{-1} : 0 : 1).$$

[2] In [9], Section 5 one finds a similar rational map.

Now assume $x_0 = 0$. By the equation of S, this yields

$$(y_2^3 - y_1^3)y_1 = 0.$$

Thus in addition the line given by $y_1 = x_0 = 0$ is fixed.

Proposition A.3.3. *The automorphism γ acts via pullback by ξ^2 on $\Gamma(\omega_S)$.*

Proof. By the multiplication of i with z_1 and z_2, one defines an action of the group of the 4-th. roots of unity on the curves C_1 and C_2 given by

$$V(z_1^4 - (y_2^3 - y_1^3)y_1) \subset \mathbb{P}^2 \text{ and } C_2 = V(z_2^4 - (x_1^3 - x_0^3)x_0) \subset \mathbb{P}^2.$$

The -1 eigenspace in $\Gamma(\omega_{C_1})$ and $\Gamma(\omega_{C_2})$ with respect to the action of i comes from the cohomology of the elliptic curve E_0 given by

$$y^2 x_0 = x_1^3 - x_0^3$$

(see Section 4.2). Note that the action of $\langle(1,1)\rangle$ on $\omega_{C_1 \times C_2}$ fixes exactly the tensor product of the -1 eigenspaces in $\Gamma(\omega_{C_1})$ and $\Gamma(\omega_{C_2})$. Thus one concludes that $\Gamma(\omega_S)$ is given by the tensor product of the -1 eigenspaces in $\Gamma(\omega_{C_1})$ and $\Gamma(\omega_{C_2})$.

The automorphism $\gamma_{\mathbb{F}_3} : E_0 \to E_0$ given by $x_1 \to \xi x_1$ is the generator of the Galois group of the degree 3 cover, which allows an identification of E_0 with the Fermat curve $\mathbb{F}_3$ of degree 3. It acts via pullback by ξ on $\Gamma(\omega_{\mathbb{F}_3})$. Thus the corresponding automorphisms $\varphi_{C_1} : C_1 \to C_1$ and $\varphi_{C_2} : C_2 \to C_2$ act by ξ on the -1 eigenspace with respect to $H^0(\omega_{C_1})$ and $H^0(\omega_{C_2})$. Note that $(\varphi_{C_1}, \varphi_{C_2})$ yields an automorphism of $C_1 \times C_2/\langle(1,1)\rangle$. By the birational map to S, this automorphism corresponds to γ and one verifies easily that γ acts via pullback by ξ^2 on $\Gamma(\omega_S)$. □

A.3.4. Consider the blowing up $\tilde{\mathbb{P}}^3$ of $\mathbb{P}^3$ with respect to $\{y_2 = x_1 = 0\}$. Let $\tilde{S}$ denote the proper transform of the blowing up of S with respect to the latter blowing up, which has the exceptional divisor E consisting of four -1 curves over the 4 points given by $(0 : \sqrt[4]{-1} : 0 : 1)$. Consider the projection

$$p : S\backslash\{y_2 = x_1 = 0\} \hookrightarrow \mathbb{P}^3\backslash\{y_2 = x_1 = 0\} \to \mathbb{P}^1 \text{ given by } (y_2 : y_1 : x_1 : x_0) \to (y_2 : x_1).$$

Over $\{x_0 = 1\}$ one has an embedding of an open subset of $\tilde{\mathbb{P}}^3$ into $\mathbb{P}^1 \times \mathbb{A}^3$, which yields an open embedding e of an open subset U of $\tilde{S}$ into $\mathbb{P}^1 \times \mathbb{A}^3$. Note that $\mathbb{P}^1 \times \mathbb{A}^3$ is endowed with a natural projection $pr_1 : \mathbb{P}^1 \times \mathbb{A}^3 \to \mathbb{P}^1$. Over $U \setminus \{y_2 = x_1 = 0\}$ one has

$$p = pr_1 \circ e.$$

Thus by gluing, p extends to a morphism $\tilde{S} \to \mathbb{P}^1$, which is a family of projective curves of degree 4. This family has a section $D = \{y_1 = x_0 = 0\}$.

One checks easily the singular loci of the fibers do not meet D (since $y_2 \neq 0$ or $x_1 \neq 0$). By $\tilde{S} \times \mathbb{F}_3 \to \mathbb{P}^1$, we have a family of surfaces. Let denote the generator of the Galois group of $\mathbb{F}_3 \to \mathbb{P}^1$, which acts via pullback by ξ on $\omega_{\mathbb{F}_3}$. The quotient map onto $\tilde{S} \times \mathbb{F}_3/\langle(\gamma, \gamma_{\mathbb{F}_3})\rangle$ yields three quotient singularities of type $A_{3,2}$ with the notation of [6], **III**. Section 5. As in Section 9.2 described one must blow up the three corresponding sections obtained from D and in a second step one blows up the fixed locus of the exceptional divisor over D. Now we blow down the image of the proper transform of the exceptional divisor over D and obtain the orbifold X_1. Note that the exceptional divisor E of the blowing up $\tilde{S} \to S$ and the 3 points on $\mathbb{F}_3$ fixed by $\gamma_{\mathbb{F}_3}$ yield a singular locus consisting of 12 curves.

On $S \times \mathbb{F}_3$ we blow up the 12 points given by the product of $\{(0 : \sqrt[4]{-1} : 0 : 1)\}$ with the three points fixed by $\gamma_{\mathbb{F}_3}$. Since $(\gamma, \gamma_{\mathbb{F}_3})$ acts by ξ on all local parameters of each of these points, the exceptional divisor over these points is contained in the ramification locus of the quotient map onto

$$X_2 = \widetilde{S \times \mathbb{F}_3}/\langle(\gamma, \gamma_{\mathbb{F}_3})\rangle.$$

X_2 is a orbifold with three $A_{3,2}$ singularities obtained from D. By gluing the complements of the singular loci of X_1 and X_2, one obtains a Calabi-Yau 3-manifold X. By the same arguments as in Section 9.2, the Calabi-Yau manifold X has obviously complex multiplication.

Thus the Calabi-Yau manifold X is obtained by the method of S. Cynk and K. Hulek [13], which we have written down in Proposition 10.4.3.

A.3.5. For the computation of the Hodge numbers we use the same methods as in Section 10.3. During Section 10.3 these methods are explained in-depth. The automorphism γ of S acts on $\tilde{S}$, too. The quotient map φ onto $M = \tilde{S}/\gamma$ is ramified over E and $D = \{y_1 = x_0 = 0\}$. Since D is a rational curve on a $K3$ surface, the adjunction formula implies that $D.D = -2$. By the Hurwitz formula, one has

$$\varphi^* K_M \sim -2D - E.$$

Since

$$3 \cdot K_M^2 = (\varphi^* K_M)^2,$$

one concludes that

$$c_1(M)^2 = K_M^2 = -4.$$

Thus the Noether formula

$$\chi(\mathcal{O}_M) = \frac{1}{12}(c_1(M)^2 + c_2(M)) \quad \text{and} \quad c_2(M) - 2 = b_2(M)$$

tell us that $b_2(M) = 14$. Since we have blown up 4 points, one obtains $h_0^{1,1}(S) = 10$. Thus

$$h_1^{1,1}(S) = h_2^{1,1}(S) = 5.$$

By the fact that one has an exceptional divisor consisting of 12 copies of $\mathbb{P}^2$ and 6 rational ruled surfaces and $b_1(S) = 0$, one obtains as in Section 10.3:

$$h^{1,1}(X) = h_0^{0,0}(\mathbb{F}_3) \cdot h_0^{1,1}(S) + h_0^{0,0}(S) \cdot h_0^{1,1}(\mathbb{F}_3) + 18 = 10 + 1 + 18 = 29$$

$$h^{2,1}(X) = h_1^{1,0}(\mathbb{F}_3) \cdot h_2^{1,1}(S) = 5$$

References

1. Abramenko, P.: Lineare Algebraische Gruppen: Eine elementare Einführung. Univ. Bielefeld, Sonderforschungsbereich **343** (1994) *preprint*.
2. André, Y.: Mumford-Tate groups of mixed Hodge structures and the theorem of the fixed part. Comp. Math. **82** (1992) 1-24.
3. André, Y.: Distribution des points CM sur les sous-variétés des variétés de modules de variétés abéliennes. (1997) *preprint*.
4. Artin, M.: Some numerical criteria for contractibility of curves on algebraic surfaces. Am. J. Math. **84** (1962) 485-496.
5. Baily, W. L., Borel, A.: Compactification of arithmetic quotients of bounded symmetric domains. Ann. of Math. **84** (1966) 442-528.
6. Barth, W., Hulek, K., Peters, C., Van de Ven, A.: Compact Complex Surfaces. Second Enlarged Edition, Ergebnisse der Mathematik und ihrer Grenzgebiete **3**, Vol. **4**, Springer-Verlag (2004) Berlin Heidelberg New York Tokyo.
7. Birkenhake, C., Lange, H.: Complex abelian varieties. Grundlehren der mathematischen Wissenschaften **302**, Springer-Verlag (1980) Berlin Heidelberg New York.
8. Borcea, C.: Calabi-Yau Threefolds and Complex Multiplication. In: Essays on Mirror Manifolds, Internat. Press, Hong Kong (1992) 489-502.
9. Borcea, C.: $K3$ surfaces with Involution and Mirror Pairs of Calabi-Yau Manifolds. In: Mirror Symmetry **II**, Studies in Advanced Mathematics **1**, AMS/Internat. Press, Providence, RI (1997) 717-743.
10. Borel, A.: Linear Algebraic Groups. Second Enlarged Edition, Graduate Texts in Mathematics **126**, Springer-Verlag (1991) New York Berlin Heidelberg.
11. Coleman, R.: Torsion points on curves. In: Galois representations and arithmetic algebraic geometry, Advanced Studies in Pure Mathematics **12**, Amsterdam (1987) 235-247.
12. Cox, D., Katz, S.: Mirror Symmetry and Algebraic Geometry. Mathematical Surveys and Monographs **68**, AMS (1999) USA.
13. Cynk, S., Hulek, K.: Construction and examples of higher-dimensional modular Calabi-Yau manifolds. Canad. Math. Bull. **50**, (2007) 486-503.
14. Deligne, P., Mumford, D.: The irreducibility of the space of curves of given genus. IHES **36** (1969) 75-110.
15. Deligne, P.: Travaux de Shimura. In: Seminaire Bourbaki, **389** (1970/71), Lecture Notes in Mathematics **244**, Springer-Verlag, Berlin (1971) 123-165.
16. Deligne, P.: Variétés de Shimura: interprétation modulaire, et techniques de construction de modèles canoniques. In: Automorphic forms, representations and L-functions (Proc. Sympos. Pure Math., Oregon State Univ., Corvalis, Ore., 1977), Part 2, Proc. Sympos. Pure Math., XXXIII, AMS, Providence, R.I. (1979) 247-289.

17. Deligne, P.: Hodge cycles on Abelian varieties. In: Hodge Cycles, Motives, and Shimura Varieties, Lecture Notes in Mathematics **900**, Springer-Verlag, Berlin Heidelberg New York (1982) 9-100.
18. Deligne, P., Mostow, G. D.: Monodromy of hypergeometric functions and non-lattice integral monodromy. IHES **63** (1986) 5-89.
19. Doran, B.: Hurwitz Spaces and Moduli Spaces as Ball Quotients via Pull-Back. (2004) arXiv math.AG/0404363v1.
20. Esnault, H., Viehweg, E.: Lectures on Vanishing Theorems. DMV-Seminar **20**, Birkhäuser (1992) Basel Boston Berlin.
21. Fulton, W., Harris, J.: Representation Theory: A First Course. Graduate Texts in Mathematics, Readings in Mathematics **129**, Springer-Verlag (1991) Berlin Heidelberg New York.
22. Gross, B.: On the Periods of Abelian Integrals and a Formula of Chowla and Selberg. Invent. Math. **45** (1978) 193-211.
23. Gukov, S., Vafa, C.: Rational Conformal Field Theories and Complex Multiplication. Communications in Mathematical Physics **246** (2004) 181-210.
24. Hansen, V. L.: Braids and Coverings. London Mathematical Society Student Texts **18**, Cambridge University Press (1990) Cambridge.
25. Harris, J., Morrison, I.: Moduli of Curves. Graduate Texts in Mathematics **187**, Springer-Verlag (1998) Berlin Heidelberg New-York.
26. Hartshorne, R.: Algebraic Geometry. Graduate Texts in Mathematics **52**, Springer-Verlag (1977) Berlin Heidelberg New York.
27. Helgason, S.: Differential Geometry, Lie Groups, and Symmetric Spaces. Graduate Studies in Mathematics **34**, AMS (2001) Providence, RI.
28. Herrlich, F., Schmithüsen, G.: An extraordinary origami curve. (2005) arXiv math.AG/0509195v1.
29. de Jong, J., Noot, R.: Jacobians with complex multiplication. In: Arithmetic Algebraic Geometry (Texel, 1989), Progress in Math. **89**, Birkhäuser, Boston (1991) 177-192.
30. Jost, J.: Compact Riemann Surfaces. Second Edition, An Introduction to Contemporary Mathematics, Springer-Verlag (2002) Berlin Heidelberg New York.
31. Kobayashi, S., Nomizu, K.: Foundations of Differential Geometry. **I**. and **II**, Interscience Publishers, a division of John Wiley & Sons (1963) New York London.
32. Koblitz, N., Rohrlich, D.: Simple factors in the Jacobian of a Fermat curve. Canadian J. Math. **20** (1978) 1183-1205.
33. Kowalsky, H.-J., Michler, G.: Lineare Algebra. Walter de Gruyter (1995) Berlin New York.
34. Krazer, A., Wirtinger, W.: Abelsche Funktionen und allgemeine Thetafunktionen. Enzykl. Math. Wiss. **II**. B. **7** (1920) 604-873.
35. Lang, S.: Complex Multiplication. Grundlehren der mathematischen Wissenschaften **255**, Springer-Verlag (1983) Berlin Heidelberg New York.
36. Looijenga, E.: Uniformization by Lauricella functions - an overview of the theory of Deligne-Mostow. (2005) arXiv math.CV/0507534v1.
37. Lynker, M., Periwal, V., Schimmrigk, R.: Black Hole Attractor Varieties and Complex Multiplication. In: Proceedings of the Field's Institute Workshop "Arithmetic, Geometry and Physics around Calabi-Yau Varieties and Mirror Symmetry" 2001, Fields Inst. Commun. **38**, AMS, Providence, RI (2003).
38. Lynker, M., Schimmrigk, R., Stewart, S.: Complex Multiplication of Exactly Solvable Calabi-Yau Varieties. (2003) arXiv hep-th/0312319v1.
39. Milne, J. S.: Jacobian Varieties. In: Arithmetic Geometry, Springer-Verlag, New York Berlin Heidelberg (1986) 167-212.
40. Milne, J. S.: Introduction to Shimura Varieties. (2004) *preprint*.
41. Möller, M.: Shimura- and Teichmüller curves. (2005) arXiv math.AG/0501333v1.
42. Möller, M., Viehweg, E., Zuo K.: Stability of Hodge bundles and a numerical characterization of Shimura varieties. (2007) arXiv math.AG:/0706.3462v1.

43. Moonen, B.: Linearity properties of Shimura varieties. Part **I**, J. Algebraic Geom. **7** (1998) 539-567.
44. Mostow, G.: Generalized Picard lattices arising from half-integral conditions. IHES **63** (1986) 91-106.
45. Mostow, G.: On discontinuous action of monodromy groups on the complex n-ball. J. AMS. **1** (1988) 555-586.
46. Mumford, D.: Families of Abelian Varieties. Proc. Sympos. Pure Math. **9**, AMS (1966) 347-351.
47. Mumford, D.: A note of Shimura's paper "Discontinuous Groups and Abelian Varieties". Math. Ann. **181** (1969) 345-351.
48. Mumford, D.: Abelian Varieties. Oxford University Press (1970) Glasgow New York Tokyo.
49. Narasimhan, R.: Introduction to the Theory of Analytic Spaces. Lecture Notes in Mathematics **25**, Springer-Verlag (1966) Berlin Heidelberg New York.
50. Newlander, A., Nirenberg, L.: Complex analytic coordinates in almost complex manifolds. Ann. of Math. **65** (1957) 391-404.
51. Nikulin, V. V.: Discrete Reflection Groups in Lobachevsky Spaces and Algebraic Surfaces. In: Proc. Intern. Cong. Math., Berkeley (1986) 654-669.
52. Oort, F.: Canonical liftings and dense sets of CM points. In: Arithmetic Geometry. Proc Cortona Symp., Symposia Math. **XXXVII**, Cambridge University Press, Cambridge (1997) 228-234.
53. Platonov, V., Rapinchuk, A.: Algebraic Groups and Number Theory. Academic Press (1994) New York London Tokyo.
54. Satake, I.: Algebraic structures of symmetric domains. Kanô Memorial Lectures **4**, Iwanami Shoten, Princeton University Press (1980) USA.
55. Silverman, J.: Advanced Topics in the Arithmetic of Elliptic Curves. Graduate Texts in Mathematics **151**, Springer-Verlag (1994) Berlin Heidelberg New York.
56. Viehweg, E., Zuo K.: Families over curves with a strictly maximal Higgs field. Asian J. Math. **7** (2003) 575-598.
57. Viehweg, E., Zuo, K.: Discreteness of minimal models of Kodaira dimension zero and subvarieties of moduli stacks. In: Surveys in differential geometry **VIII**, Internat. Press, Somerville, MA (2003) 337-356.
58. Viehweg, E., Zuo K.: Complex multiplication, Griffiths-Yukawa couplings, and rigidity for families of hypersurfaces. J. Alg. Geom. **14** (2005) 481-528.
59. Viehweg, E., Zuo K.: Arakelov inequalities and the uniformization of certain rigid Shimura varieties. J. Diff. Geom. **77** (2007) 291-352.
60. Voisin, C.: Miroirs et involutions sur les surfaces $K3$. Journées de géométrie algébrique d´Orsay, Astérisque **218** (1993) 273-323.
61. Voisin, C.: Théorie de Hodge et géométrie algébrique complexe. Cours spécialisés **10**, SMF (2002) France.
62. Völklein, H.: Groups as Galois Groups: An introduction. Cambridge Studies in Advanced Mathematics **53**, Cambridge University Press (1996) Cambridge.
63. Zarhin, Y.: Very simple 2-adic representations and hyperelliptic jacobians. Moscow Math. J. **2** (2002) 403-431.
64. Zhang, Y. Mumford-Tate Groups, families of Calabi-Yau varieties and analogue André-Oort problems **I**. (2005) *preprint*.

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Edited by J.-M. Morel, F. Takens, B. Teissier, P.K. Maini

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Addresses:

Professor J.-M. Morel, CMLA,
École Normale Supérieure de Cachan,
61 Avenue du Président Wilson, 94235 Cachan Cedex, France
E-mail: Jean-Michel.Morel@cmla.ens-cachan.fr

Professor F. Takens, Mathematisch Instituut,
Rijksuniversiteit Groningen, Postbus 800,
9700 AV Groningen, The Netherlands
E-mail: F.Takens@rug.nl

Professor B. Teissier, Institut Mathématique de Jussieu,
UMR 7586 du CNRS, Équipe "Géométrie et Dynamique",
175 rue du Chevaleret,
75013 Paris, France
E-mail: teissier@math.jussieu.fr

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