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Glossary

This is not a complete list of the symbols. In principle, we try to extract some of them which are used in different sections.

Subsection 2.1.1

$\boldsymbol{g_Y}$**:** the morphism $T \times_S Y \longrightarrow U \times_S Y$ induced by $g : T \longrightarrow U$

$\boldsymbol{p_X}$**:** the projection $X \times_S U \longrightarrow U$, forgetting the X-component

Subsection 2.1.2

$\boldsymbol{\mathcal{H}om(V_1, V_2), N(V_1, V_2)}$**:** For given vector bundles V_i $(i = 1, 2)$, $\mathcal{H}om(V_1, V_2)$ denotes the sheaf of homomorphisms from V_1 to V_2, and $N(V_1, V_2)$ denotes the corresponding vector bundle

$\boldsymbol{F^*}$**,** $\boldsymbol{F^\vee}$**,** $\mathbb{P}(\boldsymbol{F^\vee})$**,** $\mathbb{P}_{\boldsymbol{F}}$**:** Let F be a vector bundle on Y. The complement of the image of the 0-section in F is denoted by F^*, and the dual bundle of F is denoted by $F^\vee$. The projectivization is denoted by $\mathbb{P}(F^\vee)$ or $\mathbb{P}_F$.

Subsection 2.1.3

$\boldsymbol{U}$**-coherent sheaf:** a coherent sheaf over $U \times X$, which is flat over U. We will often omit to denote "U-", if there are no risk of confusion.

$\boldsymbol{U}$**-torsion-free sheaf:** a U-coherent sheaf E such that $E_{|\{u\}\times X}$ is torsion-free for each $u \in U$. We will often omit to denote "U-", if there are no risk of confusion.

$\boldsymbol{E(m)}$**:** $E \otimes p_U^* \mathcal{O}_X(m)$ on $U \times X$ for a given polarization $\mathcal{O}_X(1)$

Subsection 2.1.4

$\boldsymbol{Z_G, Z/G}$: quotient stack of Z with an action of G

Subsection 2.1.5

$\boldsymbol{\mathcal{H}om(C^\bullet, D^\bullet), C^{\bullet\vee}, \mathcal{H}om(C^\bullet, D^\bullet)^\vee}$: complexes induced by $C^\bullet$ and $D^\bullet$

Subsection 2.1.6

$\boldsymbol{C_1(V_1^*, V_2^*)}$, $\boldsymbol{C_2(V_1^*, V_2^*)}$: complexes induced by filtered vector bundles V_i^* $(i = 1, 2)$.

$\boldsymbol{\mathcal{RH}om'_1(E_{1*}, E_{2*}), \mathcal{RH}om'_2(E_{1*}, E_{2*})}$: complexes (objects in the derived category) induced by filtered sheaves E_{i*} $(i = 1, 2)$

Subsection 2.2.1

$\boldsymbol{Y^s(L), Y^{ss}(L)}$: set of (semi)stable points with respect to a G-polarization L

$\boldsymbol{Y^{ss}//G}$: categorical quotient

Subsection 2.2.2

$\boldsymbol{\mu_\lambda(P, L)}$: weight of the action of λ on the fiber of L over $\lim_{t\to 0}\lambda(t)\cdot P$

$\boldsymbol{G_l(V)}$: Grassmann variety of l-dimensional subspaces of a vector space V

$\boldsymbol{G'_l(V)}$: Grassmann variety of l-dimensional quotients of a vector space V

Subsection 2.3.1

$\boldsymbol{L_{\mathcal{X}/\mathcal{Y}}, L_f}$: cotangent complex of $f : \mathcal{X} \longrightarrow \mathcal{Y}$

$\boldsymbol{o(h)}$: obstruction class

$\boldsymbol{\Omega_{\mathcal{X}/\mathcal{Y}}, \Omega_f}$: sheaf of Kahler differentials of $f : \mathcal{X} \longrightarrow \mathcal{Y}$

Subsection 2.3.2

$\boldsymbol{\Theta_{Y/S}}$: the relative tangent bundle of a smooth morphism $Y \longrightarrow S$

$\boldsymbol{Y(W_\bullet)}$: quotient stack $N(W_{-1}, W_0)_{GL(W_{-1})\times \mathrm{GL}(W_0)}$

Subsection 2.3.3

$\mathfrak{k}(E_\bullet, V_{\bullet,\bullet}, \phi)$: cone of $\mathcal{H}om(\mathcal{O}_{U_1\times X}, V_{1\bullet})^\vee \longrightarrow \mathcal{H}om\big(\mathcal{O}_{U_1\times X}, V_{0\bullet}\big)^\vee$

$\mathfrak{r}(E_\bullet, V_{\bullet,\bullet}, \phi)$: induced morphism $\mathfrak{k}(E_\bullet, V_{\bullet,\bullet}, \phi) \longrightarrow L_{U_0\times X/U_1\times X}$

$\mathrm{Ob}(E_\bullet, V_{\bullet,\bullet}, \phi)$: $p_{X*}\big(\mathfrak{k}(E_\bullet, V_{\bullet,\bullet}, \phi) \otimes \omega_X\big)$

$\mathrm{ob}(E_\bullet, V_{\bullet,\bullet}, \phi)$: induced morphism $\mathrm{Ob}(E_\bullet, V_{\bullet,\bullet}, \phi) \longrightarrow L_{U_0/U_1}$

$\mathfrak{k}(\mathcal{V}, \varphi), \mathfrak{r}(\mathcal{V}, \varphi)$: We set $\mathfrak{k}(\mathcal{V}, \varphi) := \mathfrak{k}(E_\bullet, V_{\bullet,\bullet}, \phi)$ and $\mathfrak{r}(\mathcal{V}, \varphi) := \mathfrak{r}(E_\bullet, V_{\bullet,\bullet}, \phi)$ in the derived category.

$\mathrm{Ob}(\mathcal{F}, \varphi), \mathrm{ob}(\mathcal{F}, \varphi)$: We set $\mathrm{Ob}(\mathcal{F}, \varphi) := \mathrm{Ob}(E_\bullet, V_{\bullet,\bullet}, \phi)$ and $\mathrm{ob}(\mathcal{F}, \varphi) := \mathrm{ob}(E_\bullet, V_{\bullet,\bullet}, \phi)$ in the derived category.

Second half of Subsection 2.3.3

$\mathfrak{k}(E_\bullet, \varphi)$, $\mathfrak{r}(E_\bullet, \varphi)$: We set $\mathfrak{k}(E_\bullet, \varphi) := \mathcal{H}om(\mathcal{O}_{U_0\times D}, V_{\bullet|U_0\times D})^\vee$, and let $\mathfrak{r}(E_\bullet, \varphi) : \mathfrak{k}(E_\bullet, \varphi) \longrightarrow L_{U_0\times D/U_1\times D}$ be the induced morphism.

$\mathrm{Ob}(\mathcal{F}, \varphi), \mathrm{ob}(\mathcal{F}, \varphi)$: We set $Rp_{D\,*}\big(\mathfrak{k}(\mathcal{F}, \varphi) \otimes \omega_D\big)$, and let $\mathrm{ob}(\mathcal{F}, \varphi)$ be the induced morphism $\mathrm{Ob}(\mathcal{F}, \varphi) \longrightarrow L_{U_0/U_1}$.

Subsection 2.4.1

$A_*(\mathcal{X})$: Chow group of an algebraic stack $\mathcal{X}$ with rational coefficient

$[\mathcal{X}, \phi]$, $[\mathcal{X}]$: virtual fundamental class associated to an obstruction theory ϕ. If there are no risk of confusion, we use $[\mathcal{X}]$ instead of $[\mathcal{X}, \phi]$.

Subsection 2.4.2

$\mathfrak{g}(f)$, $\mathrm{Ob}(f)$, $\mathrm{ob}(f)$: Let $f : p_U^* F \longrightarrow p_U^* V$ on $U \times X$. Then, $\mathfrak{g}(f) := p_U^* \mathcal{H}om(V, F)$ and $\mathrm{Ob}(f) := Rp_{X*}\big(\mathfrak{g}(f) \otimes \omega_X\big)$. And, $\mathrm{ob}(f)$ denotes the induced morphism $\mathrm{Ob}(f) \longrightarrow L_U$.

Subsection 2.4.3

$\mathfrak{g}(F, f)$, $\mathrm{Ob}(F, f)$, $\mathrm{ob}(F, f)$: For a given $f : F \longrightarrow p_U^* V$, we set $\mathfrak{g}(F, f) := \big(\mathcal{H}om(p_U^* V, F) \longrightarrow \mathcal{H}om(F, F)\big)$ and $\mathrm{Ob}(F, f) := Rp_{X*}\big(\mathfrak{g}(F, f) \otimes \omega_X\big)$. And $\mathrm{ob}(F, f)$ denotes the induced morphism $\mathrm{Ob}(F, f) \longrightarrow L_U$.

Subsection 2.4.4

$\mathfrak{g}(V^*)$, $\mathrm{Ob}(V^*)$, $\mathrm{ob}(V^*)$**:** Let V, F be vector bundles on $\mathcal{D}/S$. Let $\widetilde{g} : \mathcal{D} \times_S T \longrightarrow \mathcal{D}$ be a morphism induced by $g : T \longrightarrow S$. For a filtration

$$V = V^{(1)} \supset V^{(2)} \supset \cdots \supset V^{(l)} \supset V^{(l+1)} \supset \widetilde{g}^* F,$$

we set $\mathfrak{g}(V^*) := C_2(V^*, V^*)^\vee[-1]$ and $\mathrm{Ob}(V^*) := Rp_{X*}\big(\mathfrak{g}(V^*) \otimes \omega_X\big)$. And $\mathrm{ob}(V^*)$ denotes the induced morphism $\mathrm{Ob}(V^*) \longrightarrow L_{T/S}$.

Subsections 2.6.1, 2.6.2

$f_j, y(j), x(i,j)$**:** vectors in $\mathcal{U} = \bigoplus_{i=1}^{N} \mathbb{Q} \cdot e_i$

$y^{(2)}(j), x_1(j), x_2(j)$**:** vectors in $\mathcal{U} = \bigoplus_{i=1}^{N^{(1)}} \mathbb{Q} \cdot e_i^{(1)} \oplus \bigoplus_{i=1}^{N^{(2)}} \mathbb{Q} \cdot e_i^{(2)}$

Subsection 3.1.1

Pic_X**:** Picard variety

$\mathcal{P}oin_X$**:** Poincare bundle

$\det_E$**:** morphism $U \longrightarrow \mathcal{P}oin_X$ induced by the determinant bundle of a U-coherent sheaf E on $U \times X$

$\mathcal{O}r(E)$**:** orientation bundle

Subsection 3.1.2

$F_*(E)$**:** quasi-parabolic structure

$\mathrm{Gr}_i(E)$**:** $F_i(E)/F_{i+1}(E)$

$\mathrm{Cok}_i(E)$**:** $E/F_{i+1}(E)$

Subsection 3.1.3

$[\phi]$**:** reduced L-section

$L = (L_1, L_2)$**:** pair of line bundles

$[\phi] = ([\phi_1], [\phi_2])$**:** pair of reduced L-sections

Subsection 3.1.4

$\boldsymbol{H^{\mathrm{ev}}(X)}$**:** the even part of $H^*(X)$

$\boldsymbol{\mathcal{T}ype}$**:** set of types

$\boldsymbol{\mathcal{T}ype^{\circ}}$**:** set of types whose parabolic part is trivial

$\boldsymbol{\mathcal{T}ype_r}$**:** set of types whose ranks are r

$\boldsymbol{\mathcal{T}ype^{\circ}_r}$**:** $\mathcal{T}ype^{\circ} \cap \mathcal{T}ype_r$

$\mathbf{type}(\boldsymbol{E}, \boldsymbol{F_*})$**:** type of (E, F_*)

$\mathbf{depth}(\boldsymbol{y})$**:** depth of $\boldsymbol{y}$, $\max\{i \mid y_i \neq 0\}$

$\mathbf{rank}(\boldsymbol{y})$**:** the $H^0(X)$-component of y

$\boldsymbol{\chi(y)}$**:** $\int_X \mathrm{Td}(X) \cdot y$

$\boldsymbol{\mathcal{M}(y)}$**:** moduli stack of quasi-parabolic sheaves of type $\boldsymbol{y}$

$\boldsymbol{\mathcal{M}(\widehat{y})}$**:** moduli stack of oriented quasi-parabolic sheaves of type $\boldsymbol{y}$

$\boldsymbol{\mathcal{M}(y, L)}$**:** moduli stack of quasi-parabolic L-Bradlow pairs of type $\boldsymbol{y}$ whose L-sections are non-trivial everywhere

$\boldsymbol{\mathcal{M}(\widehat{y}, L)}$**:** moduli stack of oriented quasi-parabolic L-Bradlow pairs of type $\boldsymbol{y}$ whose L-sections are non-trivial everywhere

$\boldsymbol{\mathcal{M}(y, [L])}$**:** moduli stack of quasi-parabolic reduced L-Bradlow pairs of type $\boldsymbol{y}$

$\boldsymbol{\mathcal{M}(\widehat{y}, [L])}$**:** moduli stack of oriented quasi-parabolic reduced L-Bradlow pairs of type $\boldsymbol{y}$

$\boldsymbol{\mathcal{M}(\widehat{y}, [L])}$**:** moduli stack of oriented quasi-parabolic reduced $\boldsymbol{L}$-Bradlow pairs of type $\boldsymbol{y}$

$\boldsymbol{\mathcal{M}(m, y)}$**:** substack of $\mathcal{M}(\boldsymbol{y})$ determined by the condition O_m

$\boldsymbol{\mathcal{M}(m, \widehat{y})}$**:** substack of $\mathcal{M}(\widehat{\boldsymbol{y}})$ determined by the condition O_m

$\boldsymbol{\mathcal{M}(m, \widehat{y})}$**:** substack of $\mathcal{M}(\widehat{\boldsymbol{y}})$ determined by the condition O_m

$\boldsymbol{\mathcal{M}(m, y, L)}$**:** substack of $\mathcal{M}(\boldsymbol{y}, L)$ determined by the condition O_m

$\boldsymbol{\mathcal{M}(m, \widehat{y}, L)}$ substack of $\mathcal{M}(\widehat{\boldsymbol{y}}, L)$ determined by the condition O_m

$\boldsymbol{\mathcal{M}(m, y, [L])}$**:** substack of $\mathcal{M}(\boldsymbol{y}, [L])$ determined by the condition O_m

$\boldsymbol{\mathcal{M}(m, \widehat{y}, [L])}$**:** substack of $\mathcal{M}(\boldsymbol{y}, [L])$ determined by the condition O_m

$\boldsymbol{\mathcal{M}(m, \widehat{y}, [L])}$**:** substack of $\mathcal{M}(\widehat{\boldsymbol{y}}, [\boldsymbol{L}])$ determined by the condition O_m

$\boldsymbol{\widehat{E}^u}$**:** universal sheaf in the oriented case

$\boldsymbol{E^u}$**:** universal sheaf in the unoriented case

Subsection 3.1.5

$\mathcal{O}_{\mathrm{rel}}(1)$: relative tautological line bundle on $\mathcal{M}(\widehat{\boldsymbol{y}}, [L])$. Pull backs are also denoted by the same notation.

Subsection 3.2.1

H_E: Hilbert polynomial of E

P_E: reduced Hilbert polynomial

$h^0(E)$ dimension of $H^0(X, E)$

Subsection 3.2.2

ϵ_i: We set $\epsilon_i := \alpha_{i+1} - \alpha_i$ for a given system of weights α_*.

H_{E_*}: Hilbert polynomial of a parabolic sheaf E_*

P_{E_*}: reduced Hilbert polynomial of E_*

$\mathrm{par\text{-}deg}(E_*)$: degree of E_*

$\mu(E_*)$: slope of E_*

$h^0(E_*)$: We set $h^0(E_*) := \alpha_1 h^0\big(E(-D)\big) + \sum \epsilon_i h^0\big(F_{i+1}(E)\big)$.

Subsection 3.2.3

$\mathcal{P}^{\mathrm{br}}$: set of polynomials δ with $\mathbb{R}$-coefficients such that (i) $\deg(\delta) \leq \dim X - 1$, (ii) $\delta(t) > 0$ for any sufficiently large t.

δ_{top}: For any $\delta \in \mathcal{P}^{\mathrm{br}}$, the coefficient of t^{d-1} in δ is denoted by δ_{top}, which may be 0.

$H^{\delta}(E_*, \phi)$: δ-Hilbert polynomial of (E_*, ϕ)

$P^{\delta}_{(E_*,\phi)}$: reduced δ-Hilbert polynomial of (E_*, ϕ)

$\mu^{\delta}(E_*, \phi)$: slope of (E_*, ϕ) with δ

$H^{\boldsymbol{\delta}}_{(E_*,\phi)}$: $\boldsymbol{\delta}$-Hilbert polynomial of $(E_*, \boldsymbol{\phi})$

$P^{\boldsymbol{\delta}}_{(E_*,\phi)}$: reduced $\boldsymbol{\delta}$-Hilbert polynomial of $(E_*, \boldsymbol{\phi})$

Subsection 3.2.4

$\boldsymbol{H_y(t), H_{y,i}(t), H_y(t)}$**:** polynomials associated to $\boldsymbol{y}$.

$\boldsymbol{H_y^{\alpha_*}}$**:** Hilbert polynomial associated to $(\boldsymbol{y}, \alpha_*)$

$\boldsymbol{P_y^{\alpha_*}}$**:** reduced Hilbert polynomial associated to $(\boldsymbol{y}, \alpha_*)$

$\boldsymbol{\deg(y, \alpha_*)}$**:** degree associated to $(\boldsymbol{y}, \alpha_*)$

$\boldsymbol{\mu(y, \alpha_*)}$**:** slope associated to $(\boldsymbol{y}, \alpha_*)$

$\boldsymbol{\deg(y)}$**:** degree associated to y

$\boldsymbol{\mu(y)}$**:** slope associated to y

$\boldsymbol{H_y^{\alpha_*,\delta}}$**:** δ-Hilbert polynomial associated to $(\boldsymbol{y}, \alpha_*)$

$\boldsymbol{P_y^{\alpha_*,\delta}}$**:** reduced δ-Hilbert polynomial associated to $(\boldsymbol{y}, \alpha_*)$

Subsection 3.3.1

$\boldsymbol{\mathcal{M}^s(y, \alpha_*)}$**:** moduli stack of stable parabolic sheaves of type $\boldsymbol{y}$ with weight α_*

$\boldsymbol{\mathcal{M}^s(\widehat{y}, \alpha_*)}$**:** moduli stack of stable oriented parabolic sheaves of type $\boldsymbol{y}$ with weight α_*

$\boldsymbol{\mathcal{M}^s(y, L, \alpha_*, \delta)}$**:** moduli stack of δ-stable L-Bradlow pairs of type $\boldsymbol{y}$ with weight α_*, whose L-sections are non-trivial everywhere

$\boldsymbol{\mathcal{M}^s(\widehat{y}, L, \alpha_*, \delta)}$**:** moduli stack of δ-stable oriented L-Bradlow pairs of type $\boldsymbol{y}$ with weight α_*, whose L-sections are non-trivial everywhere

$\boldsymbol{\mathcal{M}^s(y, [L], \alpha_*, \delta)}$**:** moduli stack of δ-stable reduced L-Bradlow pairs of type $\boldsymbol{y}$ with weight α_*

$\boldsymbol{\mathcal{M}^s(\widehat{y}, [L], \alpha_*, \delta)}$**:** moduli stack of δ-stable oriented reduced L-Bradlow pairs of type $\boldsymbol{y}$ with weight α_*

$\boldsymbol{\mathcal{M}^s(\widehat{y}, [L], \alpha_*, \delta)}$**:** moduli stack of $\boldsymbol{\delta}$-stable oriented reduced $\boldsymbol{L}$-Bradlow pairs of type $\boldsymbol{y}$ with weight α_*

We replace the superscript “s” with “ss” to denote moduli stacks of semistable objects.

$\boldsymbol{\widetilde{\mathcal{M}}^s_m(\widehat{y}, \alpha_*), \widetilde{\mathcal{M}}^s(\widehat{y}, \alpha_*)}$**:** full flag bundle over $\mathcal{M}^s(\widehat{\boldsymbol{y}}, \alpha_*)$ associated to the vector bundle $p_{X*}\big(\widehat{E}^u(m)\big)$

$\boldsymbol{\widetilde{\mathcal{M}}^s_m(\widehat{y}, [L], \alpha_*, \delta), \widetilde{\mathcal{M}}^s(\widehat{y}, [L], \alpha_*, \delta)}$**:** full flag bundle on $\mathcal{M}^s(\widehat{\boldsymbol{y}}, [L], \alpha_*, \delta)$ associated to $p_{X*}\big(\widehat{E}^u(m)\big)$

$\widetilde{\mathcal{M}}^s_m(\widehat{y},[L],\alpha_*,\delta)$, $\widetilde{\mathcal{M}}^s(\widehat{y},[L],\alpha_*,\delta)$**:** full flag bundle on $\mathcal{M}^s(\widehat{y},[L],\alpha_*,\delta)$ associated to $p_{X*}\big(\widehat{E}^u(m)\big)$

Subsection 3.3.3.

$\widetilde{\mathcal{M}}^{ss}_m\big(y,[L],\alpha_*,(\delta,\ell)\big)$, $\widetilde{\mathcal{M}}^{ss}\big(y,[L],\alpha_*,(\delta,\ell)\big)$**:** moduli stack of (δ,ℓ)-semistable $(E,[\phi],\mathcal{F})$

$\widetilde{\mathcal{M}}^{ss}_m\big(\widehat{y},[L],\alpha_*,(\delta,\ell)\big)$, $\widetilde{\mathcal{M}}^{ss}\big(\widehat{y},[L],\alpha_*,(\delta,\ell)\big)$**:** moduli stack of (δ,ℓ)-semistable $(E,\rho,[\phi],\mathcal{F})$

Subsection 3.4.1

$\mu_{\max}(E)$, $\mu_{\min}(E)$**:** We denote the slope of the first (resp. last) term of the Harder-Narasimhan filtration by $\mu_{\max}(E)$ (resp. $\mu_{\min}(E)$).

Subsection 3.4.2

$\mathcal{SS}(y,L,\alpha_*,\delta^{(0)})$**:** a family of parabolic L-Bradlow pairs with type $\boldsymbol{y}$, weight α_*, and non-vanishing L-section which are δ-μ-semistable for some $\delta\leq\delta^{(0)}$ in $\mathcal{P}^{\mathrm{br}}$

$\mathcal{SS}(y,L,\alpha_*)$**:** $\bigcup_{\delta\in\mathcal{P}^{\mathrm{br}}}\mathcal{SS}\big(\boldsymbol{y},L,\alpha_*,\delta\big)$

$\mathcal{SS}(y,\boldsymbol{L},\alpha_*,\boldsymbol{\delta}^{(0)})$**:** a family of parabolic $\boldsymbol{L}$-Bradlow pairs with a similar property

Subsection 3.4.3

$\widetilde{\mathrm{YOK}}(m,K,y,L,\delta)$**:** See Definition 3.4.10

$\widetilde{\mathcal{YOK}}(N,K,y,L,\delta)$**:** $\bigcup_{m\geq N}\widetilde{\mathrm{YOK}}(m,K,\boldsymbol{y},L,\delta)$

$\mathcal{YOK}(N,K,y,L,\delta)$**:** Yokogawa family

Subsections 3.5.2–3.5.3

$\mathrm{Cr}(y,\alpha_*,L)$**:** set of critical values for $(\boldsymbol{y},\alpha_*,L)$

$\mathrm{Cr}(y,\alpha_*,\boldsymbol{L})$**:** set of $\boldsymbol{\delta}\in(\mathcal{P}^{\mathrm{br}})^2$ such that the 1-stability conditions do not hold for $\big(\boldsymbol{y},\alpha_*,\boldsymbol{L},\boldsymbol{\delta}\big)$

$\mathrm{Cr}(y,\alpha_*,\boldsymbol{L},\delta_1)$**:** set of $\delta_2\in\mathcal{P}^{\mathrm{br}}$ such that the 1-stability conditions do not hold for $\big(\boldsymbol{y},\alpha_*,\boldsymbol{L},(\delta_1,\delta_2)\big)$

Subsection 3.6.1

$\boldsymbol{y(m)}$**:** $y(m) := y \cdot \mathrm{ch}\big(\mathcal{O}(m)\big)$

$\mathbf{det}(\boldsymbol{y})$**:** the $H^2(X)$-component of y

Subsection 3.6.2

$\boldsymbol{V_{m,X}}$**:** vector bundle $V_m \otimes \mathcal{O}_X$

$\boldsymbol{Q(m,y)}$**:** moduli scheme of quotient sheaves of $V_{m,X}$ with type y

$\boldsymbol{\mathcal{E}^u}$**:** universal quotient sheaf

$\boldsymbol{Q^\circ(m,y)}$**:** moduli of quotient sheaves of $V_{m,X}$ with type y satisfying (TFV)-condition

$\boldsymbol{Q(m,\widehat{y})}$**:** moduli of quotient oriented sheaves of $V_{m,X}$ with type y

$\boldsymbol{Q^\circ(m,\widehat{y})}$**:** moduli of quotient oriented sheaves of $V_{m,X}$ with type y satisfying (TFV)-condition

Subsection 3.6.3

$\boldsymbol{Q^\circ(m,y)}$**:** moduli of quotient parabolic sheaves of $V_{m,X}$ with type $\boldsymbol{y}$ satisfying (TFV)-condition

$\boldsymbol{\widehat{Z}_m, Z_m}$**:** See (3.12).

$\boldsymbol{Z_m}$**:** Gieseker space

$\boldsymbol{G_{m,i}}$**:** Grassmann variety of $H_{\boldsymbol{y},i}(m)$-dimensional quotients of V_m

Subsections 3.6.4, 3.6.5, 3.6.6

$\boldsymbol{Q^\circ(m,y,L)}$**:** moduli of quotient quasi-parabolic L-Bradlow pairs of $V_{m,X}$ with type $\boldsymbol{y}$ satisfying (TFV)-condition

$\boldsymbol{Q^\circ(m,y,[L])}$**:** moduli of quotient quasi-parabolic reduced L-Bradlow pairs of $V_{m,X}$ with type $\boldsymbol{y}$ satisfying (TFV)-condition

$\boldsymbol{Q^\circ(m,y,[L])}$**:** moduli of quotient quasi-parabolic reduced $\boldsymbol{L}$-Bradlow pairs

Subsection 3.6.7

$\boldsymbol{Q^\circ(m,\widehat{y},[L])}$**:** moduli of quotient quasi-parabolic oriented reduced L-Bradlow pairs of $V_{m,X}$ with type $\boldsymbol{y}$ satisfying (TFV)-condition.
We use the symbols $Q^\circ(m,\widehat{\boldsymbol{y}})$, $Q^\circ(m,\widehat{\boldsymbol{y}},L)$, $Q^\circ(m,\widehat{\boldsymbol{y}},[\boldsymbol{L}])$, etc., in similar meanings.

Subsection 4.1.1

$\boldsymbol{\mathcal{A}_m(y)}$**:** $\mathcal{A}_m(\boldsymbol{y}) := Z_m \times \prod_i G_{m,i}$

$\boldsymbol{\mathcal{A}_m(y,[L])}$**:** $\mathcal{A}_m(\boldsymbol{y},[L]) := \mathcal{A}_m(\boldsymbol{y}) \times \mathbb{P}_m$

$\boldsymbol{\mathcal{A}_m(y,[L])}$**:** $\mathcal{A}_m(\boldsymbol{y},[\boldsymbol{L}]) := \mathcal{A}_m(\boldsymbol{y}) \times \mathbb{P}_m^{(1)} \times \mathbb{P}_m^{(2)}$

$\boldsymbol{\mathcal{L}_y(A,B_*), \mathcal{L}_{y,L}(A,B_*,C), \mathcal{L}_{y,L}(A,B_*,C_1,C_2)}$**:** We set

$$\mathcal{L}_{\boldsymbol{y}}(A,B_*) := \mathcal{O}_{Z_m}(A) \otimes \bigotimes_{i=1}^{l} \mathcal{O}_{G_{m,i}}(B_i), \quad \text{on } \mathcal{A}_m(\boldsymbol{y})$$

$$\mathcal{L}_{\boldsymbol{y},L}(A,B_*,C) := \mathcal{L}_{\boldsymbol{y}}(A,B_*) \otimes \mathcal{O}_{\mathbb{P}_m}(C) \quad \text{on } \mathcal{A}_m(\boldsymbol{y},L),$$

$$\mathcal{L}_{\boldsymbol{y},\boldsymbol{L}}(A,B_*,C_1,C_2) := \mathcal{L}_{\boldsymbol{y}}(A,B_*) \otimes \mathcal{O}_{\mathbb{P}_m^{(1)}}(C_1) \otimes \mathcal{O}_{\mathbb{P}_m^{(2)}}(C_2) \quad \text{on } \mathcal{A}_m(\boldsymbol{y},\boldsymbol{L}).$$

$\boldsymbol{\mathcal{A}_m^{ss}(y,A,B_*), \mathcal{A}_m^{ss}(y,L,A,B_*,C), \mathcal{A}_m^{ss}(y,L,A,B_*,C_1,C_2)}$**:**
Let $\mathcal{A}_m^{ss}(\boldsymbol{y},A,B_*)$ denote the set of semistable points of $\mathcal{A}_m(\boldsymbol{y})$ with respect to $\mathcal{L}_{\boldsymbol{y}}(A,B_*)$.
We use the symbols $\mathcal{A}_m^{ss}(\boldsymbol{y},L,A,B_*,C)$ and $\mathcal{A}_m^{ss}(\boldsymbol{y},\boldsymbol{L},A,B_*,C_1,C_2)$ in similar ways.

$\boldsymbol{Q^{ss}(m,y,\alpha_*), Q^s(m,y,\alpha_*)}$**:** maximal open subset of $Q^\circ(m,\boldsymbol{y})$ which consists of the points $(q,\mathcal{E},F_*)$ such that the parabolic sheaf $(\mathcal{E}(-m),F_*,\alpha_*)$ is (semi)stable

We use the following symbols in similar ways:

$$Q^{ss}(m,\boldsymbol{y},[L],\alpha_*,\delta), \quad Q^s(m,\boldsymbol{y},[L],\alpha_*,\delta),$$
$$Q^{ss}(m,\boldsymbol{y},[\boldsymbol{L}],\alpha_*,\boldsymbol{\delta}), \quad Q^s(m,\boldsymbol{y},[\boldsymbol{L}],\alpha_*,\boldsymbol{\delta}).$$

Subsection 4.1.2

$\boldsymbol{V/W_*}$**:** $\big(V_m/W_i \,|\, i=1,\ldots,l\big)$

$\boldsymbol{\Psi\big(q,E_*,\phi,W_*,[\widetilde{\phi}]\big)}$**:** See (4.1).

Subsection 4.2.1

$\boldsymbol{V,Q,\mathcal{L},\mathcal{A}}$**:** We set

$$V := V_m, \quad Q := Q^{ss}(m,\boldsymbol{y},[L],\alpha_*,\delta)$$

$$\mathcal{L} := \mathcal{L}_{\boldsymbol{y},L}\big(P_{\boldsymbol{y}}^{\delta,\alpha_*}(m),\epsilon_*,\delta(m)\big), \quad \mathcal{A} := \mathcal{A}_m(\boldsymbol{y},[L])$$

$\boldsymbol{\mathcal{L}_\gamma}$**:** Take a sufficiently large k such that $\mathcal{L}^{\otimes k}$ is an actual line bundle, and we set $\mathcal{L}_\gamma := \mathcal{L}^{\otimes k} \otimes \mathcal{O}_{\mathbb{P}_m}(\gamma)$. (Note that k is not the ground field.)

$\mathbf{Flag}(\boldsymbol{V}, \underline{\boldsymbol{N}})$**:** full flag variety of V

$\widetilde{\boldsymbol{Q}}, \widetilde{\boldsymbol{\mathcal{A}}}$**:** We set $\widetilde{Q} := Q \times \mathrm{Flag}(V, \underline{N})$ and $\widetilde{\mathcal{A}} := \mathcal{A} \times \mathrm{Flag}(V, \underline{N})$.

$\widetilde{\boldsymbol{\Psi}}_{\boldsymbol{m}}$**:** induced morphism $\widetilde{Q} \longrightarrow \widetilde{\mathcal{A}}$

$\widetilde{\boldsymbol{\mathcal{L}}}(\boldsymbol{\gamma}, \boldsymbol{n}_*)$**:** $\mathcal{L}_\gamma \otimes \bigotimes_{i=1}^N \rho_i^* \mathcal{O}_{G_i(V)}(n_i)$.

$\widetilde{\boldsymbol{\mathcal{A}}}^{\boldsymbol{ss}}(\boldsymbol{\gamma}, \boldsymbol{n}_*), \widetilde{\boldsymbol{\mathcal{A}}}^{\boldsymbol{s}}(\boldsymbol{\gamma}, \boldsymbol{n}_*)$**:** set of (semi)stable points with respect to $\widetilde{\mathcal{L}}(\gamma, n_*)$

Subsections 4.2.2, 4.2.3

$\boldsymbol{Q}_{\pm}^{\boldsymbol{ss}}$**:** $Q^{ss}\big(m, \boldsymbol{y}, [L], \alpha_*, \delta_\pm\big)$

$\widetilde{\boldsymbol{Q}}^{\boldsymbol{ss}}(\boldsymbol{\delta}, \boldsymbol{\ell})$**:** maximal subset of $\widetilde{Q}$, which consists of the points $\big(q, E_*, [\phi], \mathcal{F}\big)$ such that $(E_*, [\phi], \mathcal{F})$ is (δ, ℓ)-semistable

Subsection 4.3.1

$\widetilde{\boldsymbol{\mathcal{L}}}_{\boldsymbol{1}}, \widetilde{\boldsymbol{\mathcal{L}}}_{\boldsymbol{2}}$**:** Let k' be a number such that $k' \cdot (\gamma_1 - \gamma_2) = 1$, and we set

$$\widetilde{\mathcal{L}}_1 := \widetilde{\mathcal{L}}(\gamma_1, n_*)^{\otimes k'}, \quad \widetilde{\mathcal{L}}_2 := \widetilde{\mathcal{L}}(\gamma_2, n_*)^{\otimes k'}.$$

$\widetilde{\boldsymbol{\mathcal{B}}}, \widetilde{\boldsymbol{\mathcal{B}}}_{\boldsymbol{1}}, \widetilde{\boldsymbol{\mathcal{B}}}_{\boldsymbol{2}}$**:** Let $\pi_1 : \widetilde{\mathcal{A}} \longrightarrow \mathbb{P}_m$ denote the projection, and we set

$$\widetilde{\mathcal{B}} := \mathbb{P}\big(\pi_1^* \mathcal{O}_{\mathbb{P}_m}(0) \oplus \pi_1^* \mathcal{O}_{\mathbb{P}_m}(1)\big).$$

That is a $\mathbb{P}^1$-bundle over $\widetilde{\mathcal{A}}$. We set $\widetilde{\mathcal{B}}_1 := \mathbb{P}\big(\pi_1^* \mathcal{O}_{\mathbb{P}_m}(0)\big)$ and $\widetilde{\mathcal{B}}_2 := \mathbb{P}\big(\pi_1^* \mathcal{O}_{\mathbb{P}_m}(1)\big)$.

$\boldsymbol{\mathcal{O}}_{\widetilde{\boldsymbol{\mathcal{B}}}}(\mathbf{1}), \widetilde{\boldsymbol{\mathcal{B}}}^{\boldsymbol{ss}}, \widetilde{\boldsymbol{\mathcal{B}}}^{\boldsymbol{s}}$**:** We set $\mathcal{O}_{\widetilde{\mathcal{B}}}(1) := \mathcal{O}_{\mathbb{P}}(1) \otimes \widetilde{\mathcal{L}}_1$. Let $\widetilde{\mathcal{B}}^{ss}$ (resp. $\widetilde{\mathcal{B}}^{s}$) denote the set of the semistable (resp. stable) points with respect to $\mathcal{O}_{\widetilde{\mathcal{B}}}(1)$.

$\mathbf{TH}, \mathbf{TH}_{\mathbf{1}}, \mathbf{TH}_{\mathbf{2}}, \mathbf{TH}^*, \mathbf{TH}^{\boldsymbol{ss}}$**:** We put $\mathrm{TH} := \widetilde{\mathcal{B}} \times_{\widetilde{\mathcal{A}}} \widetilde{Q}$, $\mathrm{TH}_i := \widetilde{\mathcal{B}}_i \times_{\widetilde{\mathcal{A}}} \widetilde{Q}$ and $\mathrm{TH}^* := \mathrm{TH} - \big(\mathrm{TH}_1 \cup \mathrm{TH}_2\big)$. We also put $\mathrm{TH}^{ss} := \widetilde{\mathcal{B}}^{ss} \times_{\widetilde{\mathcal{A}}} \widetilde{Q}$.

Subsection 4.4.1

$\boldsymbol{\rho}, \overline{\boldsymbol{\rho}}$**:** naturally defined torus actions. (Please do not confuse with orientations.)

$\mathfrak{I} = (\boldsymbol{y}_{\boldsymbol{1}}, \boldsymbol{y}_{\boldsymbol{2}}, \boldsymbol{I}_{\boldsymbol{1}}, \boldsymbol{I}_{\boldsymbol{2}})$ decomposition data. See Definition 4.4.2.

$\mathbf{Dec}(\boldsymbol{m}, \boldsymbol{y}, \boldsymbol{\alpha}_*, \boldsymbol{\delta})$**:** set of decomposition data for $(m, \boldsymbol{y}, \alpha_*, \delta)$

Subsection 4.4.2

$\boldsymbol{Q^{(1)}, Q^{(2)}}$**:** $Q^{(1)} := Q^{ss}\big(m, \boldsymbol{y}_1, [L], \alpha_*, \delta\big)$ and $Q^{(2)} := Q^{ss}(m, \boldsymbol{y}_2, \alpha_*)$

$\mathbf{Flag}(\boldsymbol{V^{(i)}}, \boldsymbol{I_i}), \mathbf{Flag}^{(i)}$**:** For $i = 1, 2$, we set

$$\mathrm{Flag}^{(i)} = \mathrm{Flag}(V^{(i)}, I_i) := \left\{ \mathcal{F}_*^{(i)} \;\middle|\; \begin{array}{l} \text{filtration indexed by } \underline{N}, \\ \dim \mathrm{Gr}_j^{\mathcal{F}^{(i)}} = 1 \ (j \in I_i), \text{or } = 0 \ (j \notin I_i) \end{array} \right\}.$$

$\boldsymbol{\widetilde{Q}^{(i)}, \widetilde{Q}_{\mathrm{split}}(\mathfrak{I})}$**:** We set $\widetilde{Q}^{(i)} := Q^{(1)} \times \mathrm{Flag}^{(i)}$ and $\widetilde{Q}_{\mathrm{split}}(\mathfrak{I}) := \widetilde{Q}^{(1)} \times \widetilde{Q}^{(2)}$.

$\mathbf{TH}_{\mathbf{split}}(\mathfrak{I}), \mathbf{TH}^*_{\mathbf{split}}(\mathfrak{I}), \mathbf{TH}_{\boldsymbol{i},\mathbf{split}}(\mathfrak{I})$**:** We set

$$\mathrm{TH}_{\mathrm{split}}(\mathfrak{I}) := \mathrm{TH} \times_{\widetilde{Q}} \widetilde{Q}_{\mathrm{split}}(\mathfrak{I}), \quad \mathrm{TH}^*_{\mathrm{split}}(\mathfrak{I}) := \mathrm{TH}^* \times_{\widetilde{Q}} \widetilde{Q}_{\mathrm{split}}(\mathfrak{I})$$

$$\mathrm{TH}_{i,\mathrm{split}}(\mathfrak{I}) := \mathrm{TH}_i \times_{\widetilde{Q}} \widetilde{Q}_{\mathrm{split}}(\mathfrak{I}).$$

Subsection 4.5.1

$\boldsymbol{\widehat{Q}, \widehat{\widetilde{Q}}}, \widehat{\mathbf{TH}}, \widehat{\mathbf{TH}}^*, \widehat{\mathbf{TH}}^{\boldsymbol{ss}}$**:** We set

$$\widehat{Q} := Q^{ss}(\widehat{\boldsymbol{y}}, [L], \alpha_*, \delta), \quad \widehat{\widetilde{Q}} := \widehat{Q} \times \mathrm{Flag}(V, \underline{N}),$$

$$\widehat{\mathrm{TH}} := \mathrm{TH} \times_{\widetilde{Q}} \widehat{\widetilde{Q}}, \quad \widehat{\mathrm{TH}}^* := \mathrm{TH}^* \times_{\mathrm{TH}} \widehat{\mathrm{TH}}, \quad \widehat{\mathrm{TH}}^{ss} = \mathrm{TH}^{ss} \times_{\mathrm{TH}} \widehat{\mathrm{TH}}.$$

$\boldsymbol{\widehat{M}}$**:** We set $\widehat{M} := \mathrm{TH}^{ss} / \mathrm{GL}(V)$, that is the enhanced master space in the oriented case.

$(\boldsymbol{\widehat{E}^{\widehat{M}}, F_*^{\widehat{M}}, [\phi^{\widehat{M}}], \rho^{\widehat{M}}})$**:** universal object induced on $\widehat{M} \times X$

$\boldsymbol{\mathcal{F}^{\widehat{M}}}$**:** full flag bundles on $\widehat{M}$ associated to $p_{X*}\big(\widehat{E}^{\widehat{M}}(m)\big)$

$\boldsymbol{\overline{\rho}}$**:** torus action on $\widehat{M}$

Subsection 4.5.2

$\widehat{\mathbf{TH}}_{\boldsymbol{i}}^{\boldsymbol{ss}}, \boldsymbol{\widehat{M}_i}$**:** We set $\widehat{\mathrm{TH}}_i^{ss} := \mathrm{TH}_i \times_{\mathrm{TH}} \widehat{\mathrm{TH}}^{ss}$ and $\widehat{M}_i := \widehat{\mathrm{TH}}_i^{ss} / \mathrm{GL}(V)$.

Subsection 4.5.3

$\boldsymbol{\widehat{M}^*}$**:** $\widehat{M} - \big(\widehat{M}_1 \cup \widehat{M}_2\big)$

$\boldsymbol{\widetilde{Q}^{(1)}(\delta, k)}$**:** maximal open subset of $\widetilde{Q}^{(1)}$ determined by the (δ, k)-semistability

$\widetilde{\boldsymbol{Q}}_{+}^{(\mathbf{2})}$**:** maximal open subset of $\widetilde{Q}^{(2)}$ determined by the condition that the underlying reduced $\mathcal{O}(-m)$-Bradlow pair is ϵ-stable for any sufficiently small $\epsilon > 0$.

$\widehat{\widetilde{\boldsymbol{Q}}}_{\mathbf{split}}(\mathfrak{I})$**:** fiber product of $\widetilde{Q}^{(1)}(\delta,k) \times \widetilde{Q}_{+}^{(2)}$ and $\widehat{\widetilde{Q}}$ over $\widetilde{Q}$

$\widehat{\mathbf{TH}}_{\mathbf{split}}^{\boldsymbol{ss}}(\mathfrak{I})$**:** $\widehat{\mathrm{TH}}^{*} \times_{\widehat{\widetilde{Q}}} \widehat{\widetilde{Q}}_{\mathrm{split}}(\mathfrak{I})$

$\widehat{\boldsymbol{M}}^{\boldsymbol{G_m}}(\mathfrak{I})$**:** quotient stack of $\widehat{\mathrm{TH}}_{\mathrm{split}}^{ss}(\mathfrak{I})$ by a natural $\mathrm{GL}(V^{(1)}) \times \mathrm{GL}(V^{(2)})$-action

$\boldsymbol{\varphi}_{\mathfrak{I}}$**:** naturally defined morphism $\widehat{M}^{G_m}(\mathfrak{I}) \longrightarrow \widehat{M}$

Subsection 4.5.5

$\widehat{\widetilde{\boldsymbol{\mathcal{B}}}}^{\boldsymbol{ss}}$**:** $\widehat{Z}_m \times_{Z_m} \widetilde{\mathcal{B}}^{ss}$

$\mathfrak{B}'$**:** quotient stack of $\widehat{\widetilde{\mathcal{B}}}^{ss}$ by a natural $\mathrm{GL}(V_m)$-action

$\mathfrak{B}$**:** G_m-equivariant open neighbourhood of $\widehat{M}$ in $\mathfrak{B}'$

Subsection 4.5.6

$\widehat{\widetilde{\boldsymbol{\mathcal{B}}}}_{\boldsymbol{i}}^{\boldsymbol{ss}}, \mathfrak{B}_{\boldsymbol{i}}$**:** We set $\widehat{\widetilde{\mathcal{B}}}_{i}^{ss} := \widetilde{\mathcal{B}}_i \times_{\widetilde{\mathcal{B}}} \widehat{\widetilde{\mathcal{B}}}^{ss}$. The quotient stack $\widehat{\widetilde{\mathcal{B}}}_{i}^{ss} / \mathrm{GL}(V)$ is denoted by $\mathfrak{B}_i$.

$\mathfrak{Q}$**:** decomposition type for $\widetilde{\mathcal{A}}$

$\widehat{\widetilde{\boldsymbol{\mathcal{A}}}}$**:** $\widetilde{\mathcal{A}} \times_{Z_m} \widehat{Z}_m$

$\mathfrak{C}_{\mathbf{1}}(\mathfrak{Q})$**:** locally closed regular subvariety of $\widehat{\widetilde{\mathcal{A}}}$ associated to $\mathfrak{Q}$

$\mathfrak{C}_{\mathbf{2}}(\mathfrak{Q}), \mathfrak{C}_{\mathbf{3}}(\mathfrak{Q})$**:** We set

$$\mathfrak{C}_2(\mathfrak{Q}) := \widetilde{\mathcal{B}}^{*} \times_{\widetilde{\mathcal{A}}} \mathfrak{C}_1(\mathfrak{Q}), \quad \mathfrak{C}_3(\mathfrak{Q}) := \big(\mathfrak{C}_2(\mathfrak{Q}) / \mathrm{GL}(V)\big) \cap \mathfrak{B}.$$

Subsection 4.5.7

$\mathfrak{B}_{\mathbf{0}}^{\boldsymbol{G_m}}$**:** $\bigsqcup_{\mathfrak{I} \in \mathcal{S}(m,\boldsymbol{y})} \mathfrak{C}_3\big(\mathfrak{Q}(\mathfrak{I})\big)$

$\boldsymbol{\psi}_{\mathbf{1}}$**:** naturally defined morphism $\bigsqcup_{\mathfrak{I} \in \mathcal{S}(m,\boldsymbol{y})} \widehat{M}^{G_m}(\mathfrak{I}) \longrightarrow \widehat{M} \times_{\mathfrak{B}} \mathfrak{B}_0^{G_m}$

Subsection 4.6.1

$\widetilde{\boldsymbol{\mathcal{M}}}^{\boldsymbol{ss}}\big(\widehat{\boldsymbol{y}}_{\mathbf{1}}, [\boldsymbol{L}], \boldsymbol{\alpha}_{*}, (\boldsymbol{\delta}, \boldsymbol{k})\big)$**:** moduli of (δ,k)-semistable $(E_{*}^{(1)}, [\phi^{(1)}], \rho^{(1)}, \mathcal{F}^{(1)})$ with type $\boldsymbol{y}_1$

$\boldsymbol{(\widehat{E}^{u}_{1*}, \phi^{u}_{1}, \rho^{u}_{1}, \mathcal{F}^{u}_{1})}$**:** universal object

$\boldsymbol{\widetilde{\mathcal{M}}^{ss}(y_1, L, \alpha_*, (\delta, k))}$**:** moduli of (δ, k)-stable $(E^{(1)}_*, \phi^{(1)}, \mathcal{F}^{(1)})$ with type $\boldsymbol{y}_1$

$\boldsymbol{(E^{u}_{1*}, \phi^{u}_{1}, \mathcal{F}^{u}_{1})}$**:** universal object

$\boldsymbol{\widetilde{\mathcal{M}}^{ss}(\widehat{y}_2, \alpha_*, +)}$**:** moduli of $(E^{(2)}_*, \rho^{(2)}, \mathcal{F}^{(2)})$ such that the associated reduced $\mathcal{O}_X(-m)$-Bradlow pair $\big(E^{(2)}_*, \mathcal{F}^{(2)}_{\min}\big)$ is ϵ-stable for any sufficiently small $\epsilon > 0$

$\boldsymbol{(\widehat{E}^{u}_{2}, \rho^{u}_{2}, \mathcal{F}^{u}_{2})}$**:** universal object

algebraic stack $\boldsymbol{\mathcal{S}}$ and line bundle $\boldsymbol{\mathcal{O}_{\mathrm{rel}}(1/r_2)}$: See Proposition 4.6.1.

Subsection 4.7.1

$\boldsymbol{\mathcal{L}_i}$**:** Take k' such that $k' \cdot (\gamma_2 - \gamma_1) = 1$, and we put $\mathcal{L}_i := \mathcal{L}_{\gamma_i}^{k'}$.

$\boldsymbol{\mathcal{B}, \mathcal{B}_i}$**:** We set $\mathcal{B} := \mathbb{P}\big(\mathcal{O}_{\mathbb{P}_m}(0) \oplus \mathcal{O}_{\mathbb{P}_m}(1)\big)$, which is a $\mathbb{P}^1$-bundle over $\mathcal{A}$. We put $\mathcal{B}_1 = \mathbb{P}\big(\mathcal{O}_{\mathbb{P}_m}(0)\big)$ and $\mathcal{B}_2 = \mathbb{P}\big(\mathcal{O}_{\mathbb{P}_m}(1)\big)$, which are naturally regarded as closed subschemes of $\mathcal{B}$.

$\boldsymbol{\mathcal{O}_{\mathcal{B}}(1), \mathcal{B}^{ss}}$**:** We put $\mathcal{O}_{\mathcal{B}}(1) := \mathcal{O}_{\mathbb{P}}(1) \otimes \mathcal{L}_1$, where $\mathcal{O}_{\mathbb{P}}(1)$ is the tautological line bundle. Let $\mathcal{B}^{ss}$ denote the set of the semistable points of $\mathcal{B}$ with respect to $\mathcal{O}_{\mathcal{B}}(1)$.

$\boldsymbol{\mathrm{TH}^{ss}, \widehat{\mathrm{TH}}^{ss}, \widehat{\mathrm{TH}}_i^{ss}}$**:** We set $\mathrm{TH}^{ss} := Q \times_{\mathcal{A}} \mathcal{B}^{ss}$ and $\widehat{\mathrm{TH}}^{ss} := \mathrm{TH}^{ss} \times_Q \widehat{Q}$. We also put $\widehat{\mathrm{TH}}_i^{ss} := \mathcal{B}_i \times_{\mathcal{A}} \widehat{\mathrm{TH}}^{ss}$.

$\boldsymbol{\widehat{M}, \widehat{M}_i, \widehat{M}^*}$**:** We put $\widehat{M} := \widehat{\mathrm{TH}}^{ss} / \mathrm{GL}(V)$, $\widehat{M}_i := \widehat{\mathrm{TH}}_i^{ss} / \mathrm{GL}(V)$ and $\widehat{M}^* := \widehat{M} - \big(\widehat{M}_1 \cup \widehat{M}_2\big)$.

$\boldsymbol{\mathfrak{I}}$**:** decomposition type

$\boldsymbol{S(y, \alpha_*, \delta)}$**:** set of decomposition types for $(\boldsymbol{y}, \alpha_*, \delta)$

$\boldsymbol{\widehat{M}^{G_m}(\mathfrak{I})}$**:** moduli of tuples $\big((E^{(1)}_*, \phi), E^{(2)}_*, \rho\big)$, where (i) $(E^{(1)}_*, \phi)$ is δ-stable L-Bradlow pair of type $\boldsymbol{y}_1$, (ii) $E^{(2)}_*$ is stable of type $\boldsymbol{y}_2$, (iii) ρ is an orientation of $E^{(1)} \oplus E^{(2)}$

$\boldsymbol{(\widehat{E}^{\widehat{M}}, [\phi^{\widehat{M}}], \rho^{\widehat{M}})}$**:** universal object on $\widehat{M} \times X$

$\boldsymbol{\mathcal{S}, \mathcal{O}_{\mathrm{rel}}(1/r_2)}$**:** See Proposition 4.7.4 for the case in which a 2-stability condition is satisfied.

Subsection 4.7.2

$\boldsymbol{\mathcal{A}, \mathcal{L}, \mathcal{L}_\gamma}$**:** We set $\mathcal{A} := \mathcal{A}_m(\boldsymbol{y}, [\boldsymbol{L}])$, $\mathcal{L} := \mathcal{L}_{\boldsymbol{y},\boldsymbol{L}}\big(P_{\boldsymbol{y}}^{\alpha_*,\boldsymbol{\delta}}(m), \epsilon_*, \boldsymbol{\delta}(m)\big)$ and $\mathcal{L}_\gamma := \mathcal{L} \otimes \mathcal{O}_{\mathbb{P}_m^{(1)}(1)}(\gamma)$

$\mathcal{A}^{ss}(\mathcal{L}_\gamma)$: open subset of semistable points with respect to $\mathcal{L}_\gamma$

$Q, Q^{ss}_{\pm}$: We set $Q := Q^{ss}(m, \boldsymbol{y}, [\boldsymbol{L}], \alpha_*, \boldsymbol{\delta})$ and $Q^{ss}_{\pm} := Q^{ss}(m, \boldsymbol{y}, [\boldsymbol{L}], \alpha_*, \boldsymbol{\delta}_{\pm})$.

$\widehat{Q}$: $Q(\widehat{\boldsymbol{y}}, [\boldsymbol{L}], \alpha_*, \boldsymbol{\delta})$

$\mathcal{L}_i$: line bundle $\mathcal{L}_{\gamma_i}^{\otimes k'}$

$\mathcal{B}, \mathcal{B}_i$: We put $\mathcal{B} := \mathbb{P}\big(\mathcal{O}_{\mathbb{P}^{(1)}_m}(0) \oplus \mathcal{O}_{\mathbb{P}^{(1)}_m}(1)\big)$ which is a $\mathbb{P}^1$-bundle over $\mathcal{A}$. We put $\mathcal{B}_1 = \mathbb{P}\big(\mathcal{O}_{\mathbb{P}^{(1)}_m}(0)\big)$ and $\mathcal{B}_2 = \mathbb{P}\big(\mathcal{O}_{\mathbb{P}^{(1)}_m}(1)\big)$, which are naturally regarded as the closed subschemes of $\mathcal{B}$.

$\mathcal{O}_{\mathcal{B}}(1), \mathcal{B}^{ss}$: We put $\mathcal{O}_{\mathcal{B}}(1) := \mathcal{O}_{\mathbb{P}}(1) \otimes \mathcal{L}_1$, where $\mathcal{O}_{\mathbb{P}}(1)$ is the tautological line bundle. Let $\mathcal{B}^{ss}$ denote the set of the semistable points of $\mathcal{B}$ with respect to $\mathcal{O}_{\mathcal{B}}(1)$.

$\mathrm{TH}^{ss}, \widehat{\mathrm{TH}}^{ss}, \widehat{\mathrm{TH}}_i^{ss}$: We put $\mathrm{TH}^{ss} := Q \times_{\mathcal{A}} \mathcal{B}^{ss}$, $\widehat{\mathrm{TH}}^{ss} := \mathrm{TH}^{ss} \times_Q \widehat{Q}$, and $\widehat{\mathrm{TH}}_i^{ss} := \mathcal{B}_i \times_{\mathcal{A}} \widehat{\mathrm{TH}}^{ss}$.

$\widehat{M}, \widehat{M}_i, \widehat{M}^*$: We define $\widehat{M} := \widehat{\mathrm{TH}}^{ss} / \mathrm{GL}(V)$, $\widehat{M}_i := \widehat{\mathrm{TH}}_i^{ss} / \mathrm{GL}(V)$ and $\widehat{M}^* := \widehat{M} - \big(\widehat{M}_1 \cup \widehat{M}_2\big)$.

$\mathfrak{I}$: decomposition type

$S(\boldsymbol{y}, \boldsymbol{\alpha}_*, \boldsymbol{\delta})$: set of decomposition types

$\widehat{M}^{G_m}(\mathfrak{I})$: moduli stack of tuples $\big(E^{(1)}_*, \phi_1, E^{(2)}_*, [\phi_2], \rho\big)$ such that (i) $(E^{(1)}_*, \phi_1)$ is a δ_1-semistable L_1-Bradlow pair of type $\boldsymbol{y}_1$, (ii) $(E^{(2)}_*, [\phi_2])$ is a δ_2-semistable reduced L_2-Bradlow pair of type $\boldsymbol{y}_2$, (iii) ρ is an orientation of $E^{(1)} \oplus E^{(2)}$

$(\widehat{\boldsymbol{E}}^{\widehat{M}}_*, [\phi_1^{\widehat{M}}], [\phi_2^{\widehat{M}}], \rho)$ universal object on $\widehat{M} \times X$

$\mathcal{O}^{(i)}_{\mathrm{rel}}(1)$: the line bundles on $\mathcal{M}(m, \widehat{\boldsymbol{y}}, [\boldsymbol{L}])$ which are the pull back of the tautological line bundles on $\mathcal{M}(m, \widehat{\boldsymbol{y}}, [L_i])$ via the natural morphism $\mathcal{M}(m, \widehat{\boldsymbol{y}}, [\boldsymbol{L}]) \longrightarrow \mathcal{M}(m, \widehat{\boldsymbol{y}}, [L_i])$

$\mathcal{S}, \mathcal{O}_{i,\mathrm{rel}}(1), \mathcal{O}_{i,\mathrm{rel}}(1/r_2)$: See Proposition 4.7.9 for the case of oriented reduced $\boldsymbol{L}$-Bradlow pairs.

Subsection 5.1.1

$\mathfrak{g}(V_\bullet), \mathrm{Ob}(V_\bullet), \mathrm{ob}(V_\bullet)$: For $V_\bullet$ on $U \times X$, we set $\mathfrak{g}(V_\bullet) := \mathcal{H}om(V_\bullet, V_\bullet)^\vee[-1]$ and $\mathrm{Ob}(V_\bullet) := Rp_{X*}\big(\mathfrak{g}(V_\bullet) \otimes \omega_X\big)$. Let $\mathrm{ob}(V_\bullet)$ denote the naturally defined morphism $\mathrm{Ob}(V_\bullet) \longrightarrow L_U$.

$W_\bullet$, $\mathrm{GL}(W_\bullet)$, $W_{i\,X}$: Let W_i $(i = -1, 0)$ be vector spaces with $\dim W_i = \operatorname{rank} V_i$. We set $\mathrm{GL}(W_\bullet) := \mathrm{GL}(W_{-1}) \times \mathrm{GL}(W_0)$ and $W_{i\,X} := W_i \otimes \mathcal{O}_X$.

$\boldsymbol{Y(W_\bullet), \Phi(V_\bullet)}$**:** Let $Y(W_\bullet)$ denote the quotient stack of $N(W_{-1\,X}, W_{0\,X})$ by a natural action of $\mathrm{GL}(W_\bullet)$. The classifying map $U \times X \longrightarrow Y(W_\bullet)$ is denoted by $\Phi(V_\bullet)$.

$\boldsymbol{\Phi(E), \mathrm{Ob}(E), \mathrm{ob}(E)}$**:** If E is a vector bundle of rank R, the classifying map $U \times X \longrightarrow X_{\mathrm{GL}(R)}$ is denoted by $\Phi(E)$. We set $\mathrm{Ob}(E) := Rp_{X*}\big(\mathcal{H}om(E,E) \otimes \omega_X\big)$. The naturally induced morphism $\mathrm{Ob}(E) \longrightarrow L_U$ is denoted by $\mathrm{ob}(E)$.

Subsection 5.1.2

$\boldsymbol{\mathfrak{w}}$**:** naturally induced morphism $Y(W_\bullet) \longrightarrow X_{G_m}$

$\boldsymbol{\Phi(\det(E))}$**:** classifying map $U \times X \longrightarrow X_{G_m}$ of $\det(E)$

$\boldsymbol{i : \mathcal{O}[-1] \longrightarrow \mathfrak{g}(V_\bullet), \mathrm{tr} : \mathfrak{g}(V_\bullet) \longrightarrow \mathcal{O}[-1]}$**:** naturally defined maps

$\boldsymbol{\mathfrak{g}^\circ(V_\bullet)}$**:** trace-free part $\mathrm{Ker}(\mathrm{tr})$

$\boldsymbol{\mathfrak{g}^d(V_\bullet)}$**:** diagonal part $\mathrm{Im}(i)$

$\boldsymbol{\mathrm{Ob}^\circ(V_\bullet), \mathrm{Ob}^d(V_\bullet)}$**:** We set $\mathrm{Ob}^\circ(V_\bullet) := Rp_{X*}\big(\mathfrak{g}^\circ(V_\bullet) \otimes \omega_X\big)$ and $\mathrm{Ob}^d(V_\bullet) := Rp_{X*}\big(\mathfrak{g}^d(V_\bullet) \otimes \omega_X\big)$.

Subsection 5.1.3

$\boldsymbol{A(W_\bullet), B(W_\bullet)}$**:** We set $A(W_\bullet) := X_{\mathrm{GL}(W_0)}$ and $B(W_\bullet) := \mathrm{Spec}(k)_{\mathrm{GL}(W_0)}$.

$\boldsymbol{\Gamma, \Psi(V_\bullet)}$**:** The natural morphism $Y(W_\bullet) \longrightarrow A(W_\bullet)$ is denoted by Γ. We set $\Psi(V_\bullet) := \Gamma \circ \Phi(V_\bullet)$.

$\boldsymbol{\mathfrak{h}(V_\bullet)}$**:** $\mathcal{H}om(V_0, V_\bullet)^\vee[-1]$

$\boldsymbol{\mathrm{Ob}^G(V_\bullet)}$**:** $Rp_{X*}\big(\mathfrak{h}(V_\bullet) \otimes \omega_X\big)$

Subsection 5.1.4

$\boldsymbol{H}$**:** Hilbert polynomial H_y associated to a type y

$\boldsymbol{\mathcal{E}^u}$**:** universal sheaf on $\mathcal{M}(m,y) \times X$

$\boldsymbol{\mathcal{V}_\bullet}$**:** canonical resolution of $\mathcal{E}^u(m)$

$\boldsymbol{\mathcal{V}'}$**:** $p_{X*}\big(\mathcal{V}_0\big)$

$\boldsymbol{\mathrm{Ob}(m,y), \mathrm{ob}(m,y)}$**:** $\mathrm{Ob}(m,y) := \mathrm{Ob}(\mathcal{V}_\bullet)$ and $\mathrm{ob}(m,y) := \mathrm{ob}(\mathcal{V}_\bullet)$

Subsection 5.1.5

$\mathfrak{g}(\mathcal{P}oin)$**:** $\Phi(\mathcal{P}oin)^* L_{X_{G_m}/X}$

$\mathbf{Ob}(\mathcal{P}oin)$**:** $Rp_{X*}\big(\mathfrak{g}(\mathcal{P}oin) \otimes \omega_X\big)$

Subsection 5.2.1

$\mathbf{Ob}_{\mathbf{rel}}(V_\bullet, \rho)$**:** cone of the natural morphism $\mathrm{Ob}^d(V_\bullet) \longrightarrow \det_E^* L_{\mathrm{Pic}}$

$\mathbf{ob}_{\mathbf{rel}}(V_\bullet, \rho)$**:** naturally defined morphism $\mathrm{Ob}_{\mathrm{rel}}(V_\bullet, \rho) \longrightarrow L_{U_1/U}$

Subsection 5.2.2

$\mathbf{Ob}_{\mathbf{rel}}(V_\bullet, \rho^u)$**,** $\mathbf{ob}_{\mathbf{rel}}(V_\bullet, \rho^u)$**:** Let π denote the projection $\mathcal{O}r(E)^* \longrightarrow U$. A complex $\mathrm{Ob}_{\mathrm{rel}}(V_\bullet, \rho^u)$ is induced by the universal orientation ρ^u of $\pi^* E$ with a morphism $\mathrm{ob}_{\mathrm{rel}}(V_\bullet, \rho^u) : \mathrm{Ob}_{\mathrm{rel}}(V_\bullet, \rho^u) \longrightarrow L_{\mathcal{O}r(E)^*/U}$.

Subsection 5.3.1

$P_\bullet \simeq L$**:** a resolution by locally free sheaves

$Y_0(W_\bullet, P_\bullet)$**:** quotient stack of $N(P_{-1}, W_{0\,X})$ by a natural action of $\mathrm{GL}(W_\bullet)$

$Y_1(W_\bullet, P_\bullet)$**:** quotient stack of X by a natural action of $\mathrm{GL}(W_\bullet)$

$Y_2(W_\bullet, P_\bullet)$**:** quotient stack of

$$N(W_{-1\,X}, W_{0\,X}) \times_X N(P_0, W_{0\,X}) \times_X N(P_{-1}, W_{-1\,X})$$

by a natural action of $\mathrm{GL}(W_\bullet)$

$\mathfrak{g}_{\mathbf{rel}}(V_\bullet, \widetilde{\phi}_\bullet)$**:** $\mathcal{H}om\big(p_{U_2}^* P_\bullet, F_{2,X}^* V_\bullet\big)^\vee$

$\gamma(\widetilde{\phi}_\bullet)$**:** naturally defined morphism $\mathfrak{g}_{\mathrm{rel}}(V_\bullet, \widetilde{\phi})[-1] \longrightarrow \mathfrak{g}(V_\bullet)$

$\mathfrak{g}(V_\bullet, \widetilde{\phi}_\bullet)$**:** cone of $\gamma(\widetilde{\phi}_\bullet)$

$\Phi(V_\bullet, \widetilde{\phi}_\bullet)$**:** induced morphism $U_2 \times X \longrightarrow Y(W_\bullet, P_\bullet)$

$\Phi_i(V_\bullet, \widetilde{\phi}_\bullet)$**:** induced morphism $U_2 \times X \longrightarrow Y_i(W_\bullet, P_\bullet)$

$\mathbf{Ob}_{\mathbf{rel}}\big(V_\bullet, \widetilde{\phi}_\bullet\big)$**:** $Rp_{X*}\big(\mathfrak{g}_{\mathrm{rel}}(V_\bullet, \widetilde{\phi}_\bullet) \otimes \omega_X\big)$

Subsection 5.3.3

$\boldsymbol{A(W_\bullet, P_\bullet)}$: quotient stack of $N(\mathcal{O}_X, W_{0\,X})$ by a natural $\mathrm{GL}(W_0)$-action

$\boldsymbol{B(W_\bullet, P_\bullet)}$: quotient stack of W_0 by a natural $\mathrm{GL}(W_0)$-action. We have a natural isomorphism $J_L : B(W_\bullet, P_\bullet) \simeq A(W_\bullet, P_\bullet)$.

$\boldsymbol{\Gamma_L}$: natural morphism $Y(W_\bullet, P_\bullet) \longrightarrow A(W_\bullet, P_\bullet)$

$\boldsymbol{\Psi(V_\bullet, \widetilde{\phi}_\bullet)}$: composite $\Gamma_L \circ \Phi(V_\bullet, \widetilde{\phi}_\bullet) : U_2 \times X \longrightarrow A(W_\bullet, P_\bullet)$

$\boldsymbol{\Xi(V_\bullet, \overline{\phi})}$: induced map $U_2 \longrightarrow B(W_\bullet, P_\bullet)$

$\boldsymbol{\mathfrak{h}_{\mathrm{rel}}(V_\bullet, \widetilde{\phi}_\bullet)}$: $\mathcal{H}om\big(p_{U_2}^* \mathcal{O}_X, F_{2X}^* V_\bullet\big)^\vee$

$\boldsymbol{\gamma(\overline{\phi})}$: naturally induced morphism $\mathfrak{h}_{\mathrm{rel}}(V_\bullet, \widetilde{\phi}_\bullet)[-1] \longrightarrow \mathfrak{h}(V_\bullet)$

$\boldsymbol{\mathfrak{h}(V_\bullet, \widetilde{\phi}_\bullet)}$: cone of $\gamma(\overline{\phi})$

$\boldsymbol{\mathrm{Ob}^G_{\mathrm{rel}}(V_\bullet, \widetilde{\phi}_\bullet)}$: $Rp_{X*}\big(\mathfrak{h}_{\mathrm{rel}}(V_\bullet, \widetilde{\phi}_\bullet) \otimes \omega_X\big)$

Subsection 5.3.5

$\boldsymbol{\mathcal{Z}(V_\bullet, L)}$: quotient stack of $N(p_U^* L, V_0)$ by a natural action of $N(p_U^* L, V_{-1})$

$\boldsymbol{\mathfrak{g}_{\mathrm{rel}}(V_\bullet, \phi)}$: $\mathcal{H}om(L, V_\bullet)^\vee$

$\boldsymbol{\mathrm{Ob}_{\mathrm{rel}}(V_\bullet, \phi), \mathrm{ob}_{\mathrm{rel}}(V_\bullet, \phi)}$: We set $\mathrm{Ob}_{\mathrm{rel}}(V_\bullet, \phi) := Rp_{X*}\big(\mathfrak{g}_{\mathrm{rel}}(V_\bullet, \phi) \otimes \omega_X\big)$. A naturally induced morphism $\mathrm{Ob}_{\mathrm{rel}}(V_\bullet, \phi) \longrightarrow L_{U_2/U}$ is denoted by $\mathrm{ob}_{\mathrm{rel}}(V_\bullet, \phi)$.

Subsection 5.4.1

$\boldsymbol{g_M}$: Let M be a line bundle on U_3. Let $\Phi_M : U_3 \longrightarrow \mathrm{Spec}(k)_{G_m}$ denote the classifying map for M. A morphism $g : U_3 \longrightarrow T$ and Φ_M induce the morphism $U_3 \longrightarrow T \times \mathrm{Spec}(k)_{G_m}$, which is denoted by g_M.

$\boldsymbol{Y(W_\bullet, [P_\bullet])}$: quotient stack $Y(W_\bullet, P_\bullet)$ by G_m

$\boldsymbol{Y_i(W_\bullet, [P_\bullet])\ (i = 0, 1, 2)}$: quotient stack $Y_i(W_\bullet, P_\bullet)$ by G_m

$\boldsymbol{\mathfrak{g}'_{\mathrm{rel}}(V_\bullet, [\widetilde{\phi}_\bullet])}$: $\mathcal{H}om\big(p_{U_3}^* P_\bullet \otimes p_X^* M,\ F_{3X}^* V_\bullet\big)^\vee$

$\boldsymbol{\Phi\big(V_\bullet, [\widetilde{\phi}_\bullet]\big)}$: induced morphism $U_3 \times X \longrightarrow Y(W_\bullet, [P_\bullet])$

$\boldsymbol{(U \times X)_{G_m}, Y(W_\bullet)_{G_m}, X_{G_m}}$: quotient stacks by the trivial G_m-actions

$\boldsymbol{\gamma([\widetilde{\phi}_\bullet])}$: naturally induced morphism $\mathfrak{g}'_{\mathrm{rel}}(V_\bullet, [\widetilde{\phi}_\bullet])[-1] \longrightarrow \mathfrak{g}(V_\bullet)$

$\boldsymbol{\mathfrak{g}'(V_\bullet, [\widetilde{\phi}_\bullet])}$: cone of $\gamma([\widetilde{\phi}_\bullet])$

$\boldsymbol{\mathrm{Ob}'_{\mathrm{rel}}\big(V_\bullet, [\widetilde{\phi}_\bullet]\big)}$: $Rp_{X*}\big(\mathfrak{g}'_{\mathrm{rel}}(V_\bullet, [\widetilde{\phi}_\bullet]) \otimes \omega_X\big)$

$\mathbf{Ob_{rel}}(\boldsymbol{V_\bullet},[\widetilde{\phi}_\bullet]), \mathbf{ob_{rel}}(\boldsymbol{V_\bullet},[\widetilde{\phi}_\bullet])$**:** We set

$$\mathrm{Ob}_{\mathrm{rel}}(V_\bullet,[\widetilde{\phi}_\bullet]) := \mathrm{Cone}\Big(\mathrm{Ob}'_{\mathrm{rel}}(V_\bullet,[\widetilde{\phi}_\bullet]) \longrightarrow F^*_{3,M}L_{U_{G_m}/U}[1]\Big)[-1].$$

The induced morphism $\mathrm{Ob}_{\mathrm{rel}}(V_\bullet,[\widetilde{\phi}_\bullet]) \longrightarrow L_{U_3/U}$ is denoted by $\mathrm{ob}_{\mathrm{rel}}(V_\bullet,[\widetilde{\phi}_\bullet])$.

Subsection 5.4.3

$\boldsymbol{A(W_\bullet,[P_\bullet])}$**:** quotient stack of $A(W_\bullet,P_\bullet)$ by a natural G_m-action

$\boldsymbol{B(W_\bullet,[P_\bullet])}$**:** quotient stack of $B(W_\bullet,P_\bullet)$ by a natural G_m-action. We have a natural isomorphism $J_{[L]}: B(W_\bullet,[P_\bullet])\times X \simeq A(W_\bullet,[P_\bullet])$.

$\mathfrak{h}'_{\mathbf{rel}}(\boldsymbol{V_\bullet},[\widetilde{\phi}_\bullet])$**:** $\mathcal{H}om\big(p^*_{U_3}\mathcal{O}_X\otimes p^*_X M,\, F^*_{3X}V_\bullet\big)^\vee$

$\mathbf{Ob'}^{\boldsymbol{G}}_{\mathbf{rel}}(\boldsymbol{V_\bullet},[\widetilde{\phi}])$**:** $Rp_{X*}\big(\mathfrak{h}'_{\mathrm{rel}}(V_\bullet,[\widetilde{\phi}_\bullet])\otimes\omega_X\big)$

$\mathbf{Ob}^{\boldsymbol{G}}_{\mathbf{rel}}(\boldsymbol{V_\bullet},[\widetilde{\phi}])$**:** cone of the induced morphism

$$\mathrm{Ob}'^{G}_{\mathrm{rel}}(V_\bullet,[\widetilde{\phi}_\bullet]) \longrightarrow F^*_{3,M}L_{U_{G_m}/U}[1]$$

$\boldsymbol{\Gamma_{[L]}}$**:** natural morphism $Y(W_\bullet,[P_\bullet]) \longrightarrow A(W_\bullet,[P_\bullet])$

$\boldsymbol{\Psi(V_\bullet,[\widetilde{\phi}_\bullet])}$ composite $\Gamma_{[L]}\circ\Phi(V_\bullet,[\widetilde{\phi}_\bullet]) : U_3\times X \longrightarrow A(W_\bullet,[P_\bullet])$.

$\boldsymbol{\Xi(V_\bullet,[\overline{\phi}])}$**:** induced morphism $U_3 \longrightarrow B(W_\bullet,[P_\bullet])$

Subsection 5.5.1

$\boldsymbol{V^*_D}$**:** filtered vector bundle $V_D^{(1)}\supset V_D^{(2)}\supset\cdots\supset V_D^{(l+1)}$ on $U_4\times D$, induced by a parabolic structure of $F^*_{4X}E$

vector spaces $\boldsymbol{W^{(h)}}$**:** We set $W^{(1)} := W_0$ and $W^{(l+1)} := W_{-1}$. We take vector spaces $W^{(h)}$ $(h=2,\ldots,l)$ such that $\dim W^{(h)} = \mathrm{rank}\, V_D^{(h)}$.

$\boldsymbol{W_{i\,D}, W_D^{(h)}}$**:** We set $W_{i\,D} := W_i\otimes\mathcal{O}_D$ and $W_D^{(h)} := W^{(h)}\otimes\mathcal{O}_D$.

$\boldsymbol{Y_D(W_\bullet)}$**:** quotient stack of $N(W_{-1\,D},W_{0,D})$ by a natural action of $\mathrm{GL}(W_\bullet)$

$\boldsymbol{Y_D(W_\bullet,W^*)}$**:** quotient stack of $\prod_{h=1}^{l} N(W_D^{(h+1)},W_D^{(h)})$ by a natural action of $\prod_{h=1}^{l+1}\mathrm{GL}(W^{(h)})$

$\boldsymbol{\Phi_D(V_\bullet,F_*)}$**:** induced morphism $U_4\times D\longrightarrow Y_D(W_\bullet,W^*)$

$\boldsymbol{\Phi(V_{\bullet|D})}$**:** induced morphism $U_4\times D\longrightarrow Y_D(W_\bullet)$

$\boldsymbol{\mathfrak{g}_D(V_\bullet,F_*)}$**:** $C_1(V^*_D,V^*_D)^\vee[-1]$

$\mathfrak{g}_{\mathbf{rel}}(\boldsymbol{V_\bullet}, \boldsymbol{F_*})$**:** $C_2(V_D^*, V_D^*)^\vee[-1]$

$\mathfrak{g}(\boldsymbol{V_{\bullet|D}})$**:** $\mathcal{H}om(V_{\bullet|D}, V_{\bullet|D})^\vee[-1]$

$\mathbf{Ob_{rel}}(\boldsymbol{V_\bullet}, \boldsymbol{F_*}), \mathbf{ob_{rel}}(\boldsymbol{V_\bullet}, \boldsymbol{F_*})$**:** We set $\mathrm{Ob}_{\mathrm{rel}}(V_\bullet, F_*) := Rp_{D*}\big(\mathfrak{g}_{\mathrm{rel}}(V_\bullet, F_*) \otimes \omega_D\big)$. An induced morphism $\mathrm{Ob}_{\mathrm{rel}}(V_\bullet, F_*) \longrightarrow L_{U_4/U}$ is denoted by $\mathrm{ob}_{\mathrm{rel}}(V_\bullet, F_*)$.

$\mathbf{Ob}(\boldsymbol{V_{\bullet\,|\,D}}), \mathbf{ob}(\boldsymbol{V_{\bullet|D}})$**:** We set $\mathrm{Ob}(V_{\bullet\,|\,D}) := Rp_{D*}\big(\mathfrak{g}(V_{\bullet|D}) \otimes \omega_D\big)$. The induced morphism $\mathrm{Ob}(V_{\bullet\,|\,D}) \longrightarrow F_4^* L_U$ is denoted by $\mathrm{ob}(V_{\bullet\,|\,D})$.

Subsection 5.6.1

$\boldsymbol{\mathcal{V}_\bullet}$**:** canonical resolution of $\mathcal{E}^u(m)$ on $\mathcal{M}(m, y) \times X$

$\boldsymbol{P_\bullet}$**:** canonical resolution of $L(m)$ for a sufficiently large m

$\mathbf{Ob_{rel}}(\boldsymbol{m}, \boldsymbol{y}), \mathbf{ob_{rel}}(\boldsymbol{m}, \boldsymbol{y})$**:** From $\mathcal{V}_\bullet$, we obtain

$$\mathrm{ob}_{\mathrm{rel}}(m, \boldsymbol{y}) : \mathrm{Ob}_{\mathrm{rel}}(m, \boldsymbol{y}) \longrightarrow L_{\mathcal{M}(m,\boldsymbol{y})/\mathcal{M}(m,y)}.$$

$\mathbf{Ob_{rel}}(\boldsymbol{m}, \boldsymbol{y}, \boldsymbol{L}), \mathbf{ob_{rel}}(\boldsymbol{m}, \boldsymbol{y}, \boldsymbol{L})$**:** From $\mathcal{V}_\bullet$ and $P_\bullet$, we obtain

$$\mathrm{ob}_{\mathrm{rel}}(m, y, L) : \mathrm{Ob}_{\mathrm{rel}}(m, y, L) \longrightarrow L_{\mathcal{M}(m,y,L)/\mathcal{M}(m,y)}.$$

$\mathbf{Ob_{rel}}(\boldsymbol{m}, \boldsymbol{y}, [\boldsymbol{L}]), \mathbf{ob_{rel}}(\boldsymbol{m}, \boldsymbol{y}, [\boldsymbol{L}])$**:** From $\mathcal{V}_\bullet$ and $P_\bullet$, we obtain

$$\mathrm{ob}_{\mathrm{rel}}(m, y, [L]) : \mathrm{Ob}_{\mathrm{rel}}(m, y, [L]) \longrightarrow L_{\mathcal{M}(m,y,[L])/\mathcal{M}(m,y)}.$$

$\mathbf{Ob_{rel}}(\boldsymbol{m}, \widehat{\boldsymbol{y}}), \mathbf{ob_{rel}}(\boldsymbol{m}, \widehat{\boldsymbol{y}})$**:** From $\mathcal{V}_\bullet$, we obtain

$$\mathrm{ob}_{\mathrm{rel}}(m, \widehat{y}) : \mathrm{Ob}_{\mathrm{rel}}(m, \widehat{y}) \longrightarrow L_{\mathcal{M}(m,\widehat{y})/\mathcal{M}(m,y)}.$$

Subsection 5.6.2

$\mathbf{Ob}(\boldsymbol{m}, \widehat{\boldsymbol{y}}, [\boldsymbol{L}]), \mathbf{ob}(\boldsymbol{m}, \widehat{\boldsymbol{y}}, [\boldsymbol{L}])$**:** We obtain a morphism

$$\mathrm{ob}(m, \widehat{\boldsymbol{y}}, [L]) : \mathrm{Ob}(m, \widehat{\boldsymbol{y}}, [L]) \longrightarrow L_{\mathcal{M}(m,\widehat{\boldsymbol{y}},[L])}$$

from $\mathrm{ob}(m, y)$, $\mathrm{ob}_{\mathrm{rel}}(m, \boldsymbol{y})$, $\mathrm{ob}_{\mathrm{rel}}(m, [L])$ and $\mathrm{ob}_{\mathrm{rel}}(m, \widehat{y})$. Similarly, we obtain

$$\mathrm{ob}(m, \boldsymbol{y}, L) : \mathrm{Ob}(m, \boldsymbol{y}, L) \longrightarrow L_{\mathcal{M}(m,\boldsymbol{y},L)/k}$$

$$\mathrm{ob}(m, \widehat{\boldsymbol{y}}) : \mathrm{Ob}(m, \widehat{\boldsymbol{y}}) \longrightarrow L_{\mathcal{M}(m,\widehat{\boldsymbol{y}})/k}$$

$$\mathrm{ob}(m, \widehat{\boldsymbol{y}}, [\boldsymbol{L}]) : \mathrm{Ob}(m, \widehat{\boldsymbol{y}}, [\boldsymbol{L}]) \longrightarrow L_{\mathcal{M}(m,\widehat{\boldsymbol{y}},[\boldsymbol{L}])/k}$$

Subsection 5.7.1

$\mathbb{P}_{\boldsymbol{m}}$**:** $\mathbb{P}(V_m^\vee)$

$\boldsymbol{Z_1}$**:** $\mathbb{P}\big(\mathcal{O}_{\mathbb{P}_m}(0)\oplus\mathcal{O}_{\mathbb{P}_m}(1)\big)$ that is a $\mathbb{P}^1$-bundle over $\mathbb{P}_m$

$\boldsymbol{Z_2}$**:** $Z_1\times \mathrm{Flag}(V_m,\underline{N})$

$\widetilde{\boldsymbol{\mathcal{Q}}}$**:** quotient stack of Z_2 by a natural action of $\mathrm{GL}(V_m)$

$\boldsymbol{W_0}$**:** We often discuss under the setting $W_0=V_m$.

$\boldsymbol{B^*(W_\bullet,[P_\bullet])}$**:** $(\mathbb{P}_m)_{\mathrm{GL}(V_m)}$

$\boldsymbol{\Psi_1}$**:** $\Xi(\mathcal{V}_\bullet,[\overline{\phi}])$

$\mathbf{Ob}^{\boldsymbol{G}}(\boldsymbol{\mathcal{V}_\bullet},[\widetilde{\boldsymbol{\phi}}_\bullet])$**:** cone of $\mathrm{Ob}^G_{\mathrm{rel}}(\mathcal{V}_\bullet,[\widetilde{\phi}_\bullet])[-1]\longrightarrow \mathrm{Ob}^G(\mathcal{V}_\bullet)$

$\boldsymbol{\mathcal{N}}$**:** $\mathcal{M}(m,\widehat{\boldsymbol{y}},[L])\times_{B^*(W_\bullet,[P_\bullet])}\widetilde{\mathcal{Q}}$

$\boldsymbol{\Psi_2}$**:** induced morphism $\mathcal{M}\longrightarrow\widetilde{\mathcal{Q}}$

$\boldsymbol{\mathfrak{p}}$**:** projection $\widehat{M}\longrightarrow\mathcal{M}(m,\widehat{\boldsymbol{y}},[L])$

$\mathbf{Ob}(\widehat{\boldsymbol{M}})$**:** cone of $\Psi_2^*L_{\widetilde{\mathcal{Q}}/B^*(W_\bullet,[P_\bullet])}[-1]\longrightarrow\mathfrak{p}^*\,\mathrm{Ob}(m,\boldsymbol{y},[L])$

$\mathbf{ob}(\widehat{\boldsymbol{M}})$**:** induced morphism $\mathrm{Ob}(\widehat{M})\longrightarrow L_{\widehat{M}}$

Subsection 5.7.2

$\widetilde{\boldsymbol{\mathcal{Q}}}^*$**:** quotient stack of $V_m^*\times\mathrm{Flag}(V_m,\underline{N})$ by a natural $\mathrm{GL}(V_m)$-action

$\boldsymbol{B^*(W_\bullet,P_\bullet)}$**:** quotient stack of V_m^* by a natural $\mathrm{GL}(V_m)$-action

$\mathbf{Ob}^{\boldsymbol{G}}(\boldsymbol{\mathcal{V}_\bullet},\widetilde{\boldsymbol{\phi}}_\bullet)$**:** cone of $\mathrm{Ob}^G_{\mathrm{rel}}(\mathcal{V}_\bullet,\widetilde{\phi}_\bullet)[-1]\longrightarrow\mathrm{Ob}^G(\mathcal{V}_\bullet)$

$\boldsymbol{\Psi_4}$**:** naturally induced morphism $\widehat{M}^*\longrightarrow\widetilde{\mathcal{Q}}^*$

$\overline{\boldsymbol{F}}$**:** $\mathrm{Flag}(V_m,\underline{N})_{\mathrm{GL}(V_m)}$. We have a naturally defined morphism $\Gamma_1:\widetilde{\mathcal{Q}}^*\longrightarrow\overline{F}$.

$\mathbf{Ob}(\widehat{\boldsymbol{M}}^*)$**:** cone of $\Psi_4^*L_{\widetilde{\mathcal{Q}}/B^*(W_\bullet,P_\bullet)}[-1]\longrightarrow\mathrm{Ob}(m,\widehat{\boldsymbol{y}},L)$. It is isomorphic to the cone of $\Psi_4^*\Gamma_1^*L_{\overline{F}/B(W_\bullet)}[-1]\longrightarrow\mathfrak{p}_2^*\,\mathrm{Ob}(m,\widehat{\boldsymbol{y}},L)$

$\mathbf{ob}(\widehat{\boldsymbol{M}}^*)$**:** naturally induced morphism $\mathrm{Ob}(\widehat{M}^*)\longrightarrow L_{\widehat{M}^*}$

Subsection 5.7.3

$\widetilde{\boldsymbol{\mathcal{M}}}(\boldsymbol{m},\widehat{\boldsymbol{y}},[\boldsymbol{L}])$**:** full flag bundle associated to $p_{X*}\widehat{E}^u(m)$ over $\mathcal{M}(m,\widehat{\boldsymbol{y}},[L])$. We have a naturally defined morphisms

$$\Psi_{11}:\widetilde{\mathcal{M}}(m,\widehat{\boldsymbol{y}},[L])\longrightarrow\overline{F},\quad \mathfrak{p}_1:\widetilde{\mathcal{M}}(m,\widehat{\boldsymbol{y}},[L])\longrightarrow\mathcal{M}(m,\widehat{\boldsymbol{y}},[L])$$

$\widetilde{\mathbf{Ob}}(\boldsymbol{m}, \widehat{\boldsymbol{y}}, [\boldsymbol{L}])$**:** cone of $\Psi_{11}^{*} L_{\overline{F}/B(W_{\bullet})}[-1] \longrightarrow \mathfrak{p}_1^{*} \operatorname{Ob}(m, \widehat{\boldsymbol{y}}, [L])$

$\widetilde{\mathbf{ob}}(\boldsymbol{m}, \widehat{\boldsymbol{y}}, [\boldsymbol{L}])$**:** induced morphism $\widetilde{\operatorname{Ob}}(m, \widehat{\boldsymbol{y}}, [L]) \longrightarrow L_{\widetilde{\mathcal{M}}(m, \widehat{\boldsymbol{y}}, [L])}$

$\mathcal{Q}_1$**:** quotient stack of $\mathbb{P}_m \times \operatorname{Flag}(V, \underline{N})$ by a natural $\operatorname{GL}(V_m)$-action. We have a naturally defined morphism

$$\Psi_{13} : \widetilde{\mathcal{M}}(m, \widehat{\boldsymbol{y}}, [L]) \longrightarrow \mathcal{Q}_1.$$

$\widetilde{\mathbf{Ob}}_{\mathbf{2}}(\boldsymbol{m}, \widehat{\boldsymbol{y}}, [\boldsymbol{L}])$**:** cone of $\Psi_{13}^{*} L_{\mathcal{Q}_1/B^{*}(W,[P])}[-1] \longrightarrow \mathfrak{p}_1^{*} \operatorname{Ob}(m, \widehat{\boldsymbol{y}}, [L])$

$\widetilde{\mathbf{ob}}_{\mathbf{2}}(\boldsymbol{m}, \widehat{\boldsymbol{y}}, [\boldsymbol{L}])$**:** naturally induced morphism $\widetilde{\operatorname{Ob}}_2(m, \widehat{\boldsymbol{y}}, [L]) \longrightarrow L_{\widetilde{\mathcal{M}}(m, \widehat{\boldsymbol{y}}, [L])}$

Subsection 5.7.4

$\widetilde{\boldsymbol{\mathcal{M}}}(\boldsymbol{m}, \boldsymbol{y}, \boldsymbol{L})$**:** full flag bundle associated to $p_{X*}E^{u}(m)$ over $\mathcal{M}(m, \boldsymbol{y}, L)$. We have naturally defined morphisms

$$\Psi_{11} : \widetilde{\mathcal{M}}(m, \boldsymbol{y}, L) \longrightarrow \overline{F}, \quad \mathfrak{p}_1 : \widetilde{\mathcal{M}}(m, \boldsymbol{y}, L) \longrightarrow \mathcal{M}(m, \boldsymbol{y}, L)$$

$\widetilde{\mathbf{Ob}}(\boldsymbol{m}, \boldsymbol{y}, \boldsymbol{L})$**:** cone of $\Psi_{11}^{*} L_{\overline{F}/B(W_{\bullet})}[-1] \longrightarrow \mathfrak{p}_1^{*} \operatorname{Ob}(m, \boldsymbol{y}, L)$

$\widetilde{\mathbf{ob}}(\boldsymbol{m}, \boldsymbol{y}, \boldsymbol{L})$**:** induced morphism $\widetilde{\operatorname{Ob}}(m, \boldsymbol{y}, L) \longrightarrow L_{\widetilde{\mathcal{M}}(m, \boldsymbol{y}, L)}$

Subsection 5.7.5

$\widetilde{\boldsymbol{\mathcal{M}}}(\boldsymbol{m}, \widehat{\boldsymbol{y}})$**:** full flag bundle associated to $p_{X*}\widehat{E}^{u}(m)$ over $\mathcal{M}(m, \widehat{\boldsymbol{y}})$. We have naturally defined morphisms

$$\mathfrak{p}_1 : \widetilde{\mathcal{M}}(m, \widehat{\boldsymbol{y}}) \longrightarrow \mathcal{M}(m, \widehat{\boldsymbol{y}}), \quad \Psi_{11} : \widetilde{\mathcal{M}}(m, \widehat{\boldsymbol{y}}) \longrightarrow \overline{F}$$

$\widetilde{\mathbf{Ob}}(\boldsymbol{m}, \widehat{\boldsymbol{y}})$**:** cone of $\Psi_{11}^{*} L_{\overline{F}/B(W_{\bullet})}[-1] \longrightarrow \mathfrak{p}_1^{*} \operatorname{Ob}(m, \widehat{\boldsymbol{y}})$

$\widetilde{\mathbf{ob}}(\boldsymbol{m}, \widehat{\boldsymbol{y}})$**:** naturally induced morphism $\widetilde{\operatorname{Ob}}(m, \widehat{\boldsymbol{y}}) \longrightarrow L_{\widetilde{\mathcal{M}}(m, \widehat{\boldsymbol{y}})}$

Subsection 5.7.6

$\mathcal{Q}$**:** quotient stack of $\mathbb{P}\big(\mathcal{O}_{\mathbb{P}_m}(0) \oplus \mathcal{O}_{\mathbb{P}_m}(1)\big)$ by a natural $\operatorname{GL}(V_m)$-action. We have a natural morphism $\Psi_1 : \widehat{M} \longrightarrow \mathcal{Q}$.

$\mathbf{Ob}(\widehat{\boldsymbol{M}})$**:** cone of $\Psi_1^{*} L_{\mathcal{Q}/B^{*}(W,[P])}[-1] \longrightarrow \mathfrak{p}^{*} \operatorname{Ob}(m, \widehat{\boldsymbol{y}}, [L])$

$\mathbf{ob}(\widehat{\boldsymbol{M}})$**:** naturally induced morphism $\operatorname{Ob}(\widehat{M}) \longrightarrow L_{\widehat{M}}$

$\mathbf{Ob}(\widehat{\boldsymbol{M}}^{*})$**,** $\mathbf{ob}(\widehat{\boldsymbol{M}}^{*})$**:** We set $\mathrm{Ob}(\widehat{M}^{*}) := \mathrm{ob}(m, \widehat{\boldsymbol{y}}, L)$. The naturally induced morphism $\mathrm{Ob}(\widehat{M}^{*}) \longrightarrow L_{\widehat{M}^{*}}$ is denoted by $\mathrm{ob}(\widehat{M}^{*})$.

Subsection 5.7.7

$\mathbf{Ob}(\widehat{\boldsymbol{M}}), \mathbf{ob}(\widehat{\boldsymbol{M}})$**:** Let $\mathrm{Ob}(\widehat{M})$ be the cone of

$$\Psi_1^{*} L_{\mathcal{Q}/B^{*}(W,[P])}[-1] \longrightarrow \mathrm{Ob}(m, \widehat{\boldsymbol{y}}, [\boldsymbol{L}]).$$

The naturally induced morphism $\mathrm{Ob}(\widehat{M}) \longrightarrow L_{\widehat{M}}$ is denoted by $\mathrm{ob}(\widehat{M})$.

$\mathbf{Ob}(\widehat{\boldsymbol{M}}^{*})$**,** $\mathbf{ob}(\widehat{\boldsymbol{M}}^{*})$**:** We set $\mathrm{Ob}(\widehat{M}^{*}) := \mathrm{Ob}(m, \widehat{\boldsymbol{y}}, L_1, [L_2])$. The naturally induced morphism $\mathrm{Ob}(\widehat{M}^{*}) \longrightarrow L_{\widehat{M}^{*}}$ is denoted by $\mathrm{ob}(\widehat{M}^{*})$.

Subsection 5.8.1

$\widetilde{\boldsymbol{\mathcal{M}}}_{\mathbf{split}}$**:** $\widetilde{\mathcal{M}}^{ss}(\boldsymbol{y}_1, L, \alpha_{*}, (\delta, k_0)) \times \widetilde{\mathcal{M}}^{ss}(\widehat{\boldsymbol{y}}_2, \alpha_{*}, +)$

$\mathbf{Ob}(\widehat{\boldsymbol{M}}^{\boldsymbol{G_m}}(\mathfrak{J}))$**,** $\mathbf{ob}(\widehat{\boldsymbol{M}}^{\boldsymbol{G_m}}(\mathfrak{J}))$**:** obstruction theory of $\widehat{M}^{G_m}(\mathfrak{J})$. See Proposition 5.8.1.

$\widetilde{\mathbf{Ob}}(\widehat{\boldsymbol{M}}^{\boldsymbol{G_m}}(\mathfrak{J})), \widetilde{\mathbf{ob}}(\widehat{\boldsymbol{M}}^{\boldsymbol{G_m}}(\mathfrak{J}))$**:** obstruction theory of $\widehat{M}^{G_m}(\mathfrak{J}) \times A^1$ over A^1. See Proposition 5.8.1.

$\widetilde{\mathbf{Ob}}_{\boldsymbol{a}}(\widehat{\boldsymbol{M}}^{\boldsymbol{G_m}}(\mathfrak{J}))$**,** $\widetilde{\mathbf{ob}}_{\boldsymbol{a}}(\widehat{\boldsymbol{M}}^{\boldsymbol{G_m}}(\mathfrak{J}))$**:** obstruction theory of $\widehat{M}^{G_m}(\mathfrak{J})$. See Proposition 5.8.1.

Subsection 5.8.2

$\boldsymbol{\mathcal{M}_i}$ $\boldsymbol{(i = 1, 2, 3)}$ We set $\mathcal{M}_1 := \mathcal{M}(m, \boldsymbol{y}_1, L)$, $\mathcal{M}_2 := \mathcal{M}(m, \boldsymbol{y}_2)$ and $\mathcal{M}_3 := \mathcal{M}_1 \times \mathcal{M}_2$. They are equipped with the obstruction theories $\mathrm{Ob}(\mathcal{M}_i)$.

$\widehat{\boldsymbol{\mathcal{M}}}_{\mathbf{3}}$**:** moduli of $(E_1, F_{1\,*}, \phi, E_2, F_{2\,*}, \rho)$ such that (i) $(E_1, F_{1\,*}, \phi) \in \mathcal{M}_1$, (ii) $(E_2, F_{2\,*}) \in \mathcal{M}_2$, (iii) ρ is an orientation of $E_1 \oplus E_2$.

$\boldsymbol{\mathcal{E}_i^u}$**,** $\boldsymbol{\mathcal{V}_{\bullet}^{(i)}}$**:** Let $\mathcal{E}_i^u$ be universal sheaves on $\mathcal{M}_i \times X$ $(i = 1, 2)$. Let $\mathcal{V}^{(i)}$ be the canonical resolution of $\mathcal{E}_i^u$.

$\mathbf{Ob}_{\mathbf{rel}}(\widehat{\boldsymbol{\mathcal{M}}}_{\mathbf{3}}/\boldsymbol{\mathcal{M}}_{\mathbf{3}})$**,** $\mathbf{Ob}(\widehat{\boldsymbol{\mathcal{M}}}_{\mathbf{3}})$**,** $\mathbf{ob}(\widehat{\boldsymbol{\mathcal{M}}}_{\mathbf{3}})$**:** complex and morphisms induced by the universal orientation.

Subsection 5.8.3

$\boldsymbol{\mathcal{M}_0, \widehat{\mathcal{M}}_0}$**:** We set $\mathcal{M}_0 := \mathcal{M}(m, \boldsymbol{y}, L)$ and $\widehat{\mathcal{M}}_0 := \mathcal{M}(m, \widehat{\boldsymbol{y}}, L)$. We have a naturally defined morphism $\mathfrak{F} : \mathcal{M}_3 \longrightarrow \mathcal{M}_0$.

$\boldsymbol{\mathcal{V}_D^{(i)(j)}}$**:** $\mathrm{Ker}\big(\mathcal{V}^{(i)}_{|D} \longrightarrow \mathrm{Cok}^{(i)}_{j-1}\big)$.

$\boldsymbol{W_b^{(a)}}$**:** We take decompositions $W_0 = W_0^{(1)} \oplus W_0^{(2)}$ and $W_{-1} = W_{-1}^{(1)} \oplus W_{-1}^{(2)}$, where $\dim W_0^{(i)} = H_{\boldsymbol{y}_i}(m)$ and $\dim W_{-1}^{(i)} = H_{\boldsymbol{y}_i}(m) - \mathrm{rank}(\boldsymbol{y}_i)$.

$\boldsymbol{Y(W_\bullet^{(1)}, W_\bullet^{(2)})}$**:** $Y(W_\bullet^{(1)}) \times Y(W_\bullet^{(2)})$.

$\boldsymbol{Y(W_\bullet^{(1)}, W_\bullet^{(2)}, P_\bullet)}$**:** $Y(W_\bullet^{(1)}, P_\bullet) \times Y(W_\bullet^{(2)})$.

$\boldsymbol{W^{(i)(j)}}$**:** vector spaces such that $\dim W^{(i)(j)} = \mathrm{rank}\, \mathcal{V}_D^{(i)(j)}$

$\boldsymbol{Y_D(W_\bullet, W^{(1)*}, W^{(2)})}$**:** $Y_D(W_\bullet^{(1)}, W^{(1)*}) \times Y_D(W_\bullet^{(2)}, W^{(2)*})$

$\boldsymbol{Y_D(W_\bullet^{(1)}, W_\bullet^{(2)})}$**:** $Y_D(W_\bullet^{(1)}) \times Y_D(W_\bullet^{(2)})$

$\boldsymbol{\mathrm{Ob}_D(\mathcal{V}, F_*^u)}$**:** $Rp_{D\,*}\big(\mathfrak{g}_D(\mathcal{V}_\bullet, F_*^u) \otimes \omega_D\big)$

$\boldsymbol{\mathrm{Ob}_D(\mathcal{V}^{(i)}, F_*^{u(i)})}$**:** $Rp_{D\,*}\big(\mathfrak{g}_D(\mathcal{V}_\bullet^{(i)}, F_*^{u(i)}) \otimes \omega_D\big)$

Subsection 5.8.4

$\boldsymbol{B(W_\bullet^{(1)}, W_\bullet^{(2)})}$**:** $\mathrm{Spec}(k)_{\mathrm{GL}(W_0)\times \mathrm{GL}(W_{-1})}$

$\boldsymbol{\Phi(\mathcal{V}_0'^{(1)}, \mathcal{V}_0'^{(2)})}$**:** induced morphism $\mathcal{M}_3 \longrightarrow B(W_\bullet^{(1)}, W_\bullet^{(2)})$

Subsection 5.8.5

$\boldsymbol{\overline{\mathrm{Ob}}(m, y_2)}$**:** $\mathrm{Ob}^\circ(m, \boldsymbol{y}_2) \oplus \tau_{\leq -1}\, \mathrm{Ob}^d(m, \boldsymbol{y}_2)$

$\boldsymbol{\mathrm{Ob}_1(\widehat{\mathcal{M}}_3)}$**:** cone of the morphism of $\tau_{\leq -1}\,\mathrm{Ob}^d\big(\mathcal{V}_\bullet^{(1)} \oplus \mathcal{V}_\bullet^{(2)}\big)$ to $\mathrm{Ob}(m, \boldsymbol{y}_1, L) \oplus \overline{\mathrm{Ob}}(m, \boldsymbol{y}_2)$

$\boldsymbol{\widetilde{\mathrm{Ob}}_a(\widehat{\mathcal{M}}_3), \widetilde{\mathrm{Ob}}(\widehat{\mathcal{M}}_3)}$**:** Let $\widetilde{\mathrm{Ob}}_a(\widehat{\mathcal{M}}_3)$ be the cone of φ_a. The family version is $\widetilde{\mathrm{Ob}}(\widehat{\mathcal{M}}_3)$.

$\boldsymbol{\widehat{\mathcal{M}}_2, \mathcal{S}', F_1, G_1'}$**:** See Lemma 5.8.6.

Subsection 5.8.6

$\boldsymbol{F_i, \overline{F}_i}$**:** F_i denotes the full flag of $V_m^{(i)}$, and $\overline{F}_i$ denotes the quotient of F_i by a natural $\mathrm{GL}(V_m^{(i)})$-action.

$\mathbf{Ob}(\widehat{M}^{G_m}(\mathfrak{I}))$**:** We have naturally defined morphisms:

$$g : \widehat{M}^{G_m}(\mathfrak{I}) \longrightarrow \overline{F}_1 \times \overline{F}_2, \quad \pi_1 : \widehat{M}^{G_m}(\mathfrak{I}) \longrightarrow \widehat{\mathcal{M}}_3$$

Then, $\mathrm{Ob}(\widehat{M}^{G_m}(\mathfrak{I}))$ denotes the cone of the morphism

$$g^* L_{\overline{F}_1 \times \overline{F}_2 / B(W_\bullet^{(1)}, W_\bullet^{(2)})}[-1] \longrightarrow \pi_1^* \mathrm{Ob}(\widehat{\mathcal{M}}_3).$$

$\mathbf{Flag}(V_m^{(i)}, I_i)$**:** full flag of $V^{(i)}$ indexed by I_i.

Subsections 5.8.7–5.8.8

$\mathcal{M}_{\mathrm{split}}, \mathbf{ob}(\widehat{M}^{G_m}), \mathbf{Ob}(\widehat{M}^{G_m}), \widetilde{\mathbf{ob}}(\widehat{M}^{G_m}), \widetilde{\mathbf{Ob}}(\widehat{M}^{G_m})$**:**
See Subsection 5.8.7 for the case in which a 2-stability condition is satisfied, and Subsection 5.8.8 for the case of oriented reduced $\boldsymbol{L}$-Bradlow pairs.

Subsection 5.9.1

$\iota_i^* \mathbf{Ob}(\widehat{M})^{\mathbf{inv}}, \mathbf{ob_1}(\widehat{M}_i)$**:** G_m-invariant of $\iota_i^* \mathrm{Ob}(\widehat{M})$, where $\iota_i : \widehat{M}_i \longrightarrow \widehat{M}$. We have the induced morphism $\mathrm{ob}_1(\widehat{M}_i) : \iota_i^* \mathrm{Ob}(\widehat{M})^{\mathrm{inv}} \longrightarrow L_{\widehat{M}_i}$.

$\iota_i^* \mathbf{Ob}(\widehat{M})^{\mathbf{mov}}$**:** moving part of $\iota_i^* \mathrm{Ob}(\widehat{M})$

$\varphi_{\mathfrak{I}}^* \mathbf{Ob}(\widehat{M})^{\mathbf{inv}}, \mathbf{ob_1}(\widehat{M}^{G_m}(\mathfrak{I}))$**:** G_m-invariant part of $\varphi_{\mathfrak{I}}^* \mathrm{Ob}(\widehat{M})$, where $\varphi_{\mathfrak{I}} : \widehat{M}^{G_m}(\mathfrak{I}) \longrightarrow \widehat{M}$. The induced morphism $\varphi_{\mathfrak{I}}^* \mathrm{Ob}(\widehat{M})^{\mathrm{inv}} \longrightarrow L_{\widehat{M}^{G_m}(\mathfrak{I})}$ is denoted by $\mathrm{ob}_1(\widehat{M}^{G_m}(\mathfrak{I}))$.

$\varphi_{\mathfrak{I}}^* \mathbf{Ob}(\widehat{M})^{\mathbf{mov}}$**:** moving part of $\varphi_{\mathfrak{I}}^* \mathrm{Ob}(\widehat{M})$

$\mathfrak{N}(\widehat{M}_i)$**:** normal bundle of $\widehat{M}_i$ in $\widehat{M}$, isomorphic to $\mathcal{O}_{\mathrm{rel}}\big((-1)^{i-1}\big)$. The weight of the induced G_m-action is $(-1)^i$.

$\mathfrak{N}(\widehat{M}^{G_m}(\mathfrak{I}))$**:** virtual normal bundle of $\widehat{M}^{G_m}(\mathfrak{I})$ in $\widehat{M}$

$\mathfrak{N}(E_i^{\widehat{M}}, E_j^{\widehat{M}})$**:** $-\sum_{l=0,1,2}(-1)^l R^l p_{X*} \mathcal{RH}om\big(E_i^{\widehat{M}}, E_j^{\widehat{M}}\big)$

$\mathfrak{N}(L, E_2^{\widehat{M}})$**:** $\sum_{l=0,1,2}(-1)^l R^l p_{X\,*} \mathcal{H}om\big(L, E_2^{\widehat{M}}\big)$

$\mathfrak{N}_D(E_{i\,*}^{\widehat{M}}, E_{j\,*}^{\widehat{M}})$**:** $-\sum_{l=0,1}(-1)^l R^l p_{D\,*} \mathcal{RH}om_2'\big(E_{i|D\,*}^{\widehat{M}}, E_{j|D\,*}^{\widehat{M}}\big)$

N_0**:** normal bundle of $\widehat{M}^{G_m}(\mathfrak{I}) \subset \widehat{M} \times_{\widehat{\mathcal{M}}_0} \widehat{\mathcal{M}}_3$

$\mathfrak{N}(\widehat{E}_i^u, \widehat{E}_j^u)$**:** $-\sum_{l=0,1,2}(-1)^l R^l p_{X*} \mathcal{RH}om(\widehat{E}_i^u, \widehat{E}_j^u)$

$\mathfrak{N}(L, \widehat{E}^u_2)$: $\sum_{l=0,1,2}(-1)^l R^l p_{X\,*}\mathcal{H}om(L, \widehat{E}^u_2)$

$\mathfrak{N}_D(\widehat{E}^u_{i\,*}, \widehat{E}^u_{j\,*})$: $-\sum_{l=0,1}(-1)^l R^l p_{D\,*}\mathcal{RH}om'_2\big(\widehat{E}^u_{i|D\,*}, \widehat{E}^u_{j|D\,*}\big)$

Subsections 6.1.1, 6.1.2

$[\mathcal{M}]$, $[\mathcal{M}]^{\mathrm{vir}}$: virtual fundamental class of the moduli stacks $\mathcal{M}$

$\int_{\mathcal{M}} \Phi$: evaluation of a cohomology class Φ via the virtual fundamental class $[\mathcal{M}]$

$[\widehat{M}^{G_m}(\mathfrak{I})]$: virtual fundamental class of $\widehat{M}^{G_m}(\mathfrak{I})$

Subsection 6.3.1

$M(c, L)$: moduli of L-abelian pairs (E, ϕ) such that $c_1(E) = c$

$(\mathcal{L}^u, \phi^u)$: universal object on $M(c, L) \times X$

$M(\widehat{c}, [L])$: moduli space of oriented reduced L-abelian pairs $(E, [\phi], \rho)$ such that $c_1(E) = c$

$(\widehat{\mathcal{L}}^u, [\phi^u], \rho^u)$: universal object on $M(\widehat{c}, [L]) \times X$

$\mathcal{L}$: line bundle such that $c_1(\mathcal{L}) = c \in H^2(X, \mathbb{Z})$. If $H^1(X, \mathcal{O}_X) = 0$, it is determined uniquely up to isomorphisms.

$d(c, L)$: $\dim H^0(X, L^{-1} \otimes \mathcal{L}) - 1 = \dim M(c, L)$

$\chi(L^{-1} \otimes \mathcal{L})$: $\sum_{l=0,1,2}(-1)^l \dim H^l(X, L^{-1} \otimes \mathcal{L})$

$O(c, L)$, $[M(c, L)]_0$: If $H^1(X, \mathcal{O}_X) = 0$ and $p_g > 0$, $M(c, L)$ is smooth. The actual dimension is denoted by $O(c, L)$. The naive fundamental class is denoted by $[M(c, L)]_0$. We have $[M(c, L)] = \mathrm{Eu}\big(O(c, L)\big) \cap [M(c, L)]_0$.

p_g, K_X: We set $p_g := \dim H^2(X, \mathcal{O}_X)$. Let K_X denote the canonical line bundle of X.

$\mathrm{SW}(c, L)$, $\mathrm{SW}(c)$: Assume $H^1(X, \mathcal{O}_X) = 0$ and $p_g > 0$. If $[M(c, L)] \neq 0$, the expected dimension of $M(c, L)$ is 0. Hence, we can regard $[M(c, L)]$ as a number, which is denoted by $\mathrm{SW}(c, L)$. In the case $L = \mathcal{O}_X$, it is also denoted by $\mathrm{SW}(c)$.

$\mathrm{Ob}(\widehat{c})$, $[M(\widehat{c}, [L])]_0$: Assume $H^1(X, \mathcal{O}_X) = 0$ and $H^2(X, L^{-1} \otimes \mathcal{L}) = 0$. Then, $M(\widehat{c}, [L])$ is smooth and equipped with a perfect obstruction theory. The obstruction bundle is denoted by $\mathrm{Ob}(\widehat{c})$. The naive fundamental class is denoted by $[M(\widehat{c}, [L])]_0$.

Subsection 6.3.2

$\widetilde{\mathrm{SW}}(a)$: Seiberg-Witten invariant associated to a Spin^c-structure ξ with det $(\xi) = a$

Subsection 6.3.3

$\boldsymbol{X^{[y]}}$**:** parabolic Hilbert scheme of ideals with type $\boldsymbol{y}$

$\mathbf{SGL}(\boldsymbol{W_\bullet})$**:** $\{(g_{-1}, g_0) \in \mathrm{GL}(W_\bullet), \mid \det(g_{-1})\det(g_0) = 1\}$

$\overline{\boldsymbol{Y}}(\boldsymbol{W_\bullet})$**:** quotient stack $N(W_{-1\,X}, W_{0\,X})_{\mathrm{SGL}(W_\bullet)}$

$\overline{\boldsymbol{Y}}_{\boldsymbol{D}}(\boldsymbol{W_\bullet})$**:** quotient stack $N(W_{-1\,D}, W_{0\,D})_{\mathrm{SGL}(W_\bullet)}$

$\overline{\boldsymbol{Y}}_{\boldsymbol{D}}(\boldsymbol{W_\bullet}, \boldsymbol{W^*})$**:** quotient stack of $\prod_{i=1}^{l} N\big(W^{(i+1)}, W^{(i)}\big)$ by a natural action of $\prod_{i=2}^{l} \mathrm{GL}(W^{(i)}) \times \mathrm{SGL}(W_\bullet)$

$\mathbf{Ob}^{\circ}_{\boldsymbol{D}}(\boldsymbol{y})$**:** $Rp_{D\,*}\big(\mathfrak{g}^{\circ}_{D}(V_\bullet, F_*) \otimes \omega_D\big)$

$\mathbf{Ob}^{\circ}_{\boldsymbol{D}}(\boldsymbol{y})$**:** $Rp_{D\,*}\big(\mathfrak{g}^{\circ}(V_{\bullet|D}) \otimes \omega_D\big)$

$\mathbf{Ob}(\boldsymbol{y})$**:** cone of $\mathrm{Ob}^{\circ}_{D}(y) \longrightarrow \mathrm{Ob}^{\circ}(y) \oplus \mathrm{Ob}^{\circ}_{D}(\boldsymbol{y})$

$\mathbf{ob}(\boldsymbol{y})$**:** naturally induced morphism $\mathrm{Ob}(\boldsymbol{y}) \longrightarrow L_{X^{[\boldsymbol{y}]}}$

Subsection 6.3.4

$\boldsymbol{y}(-\boldsymbol{c})$**:** For $\boldsymbol{y} \in \mathcal{T}ype$, we set $\boldsymbol{y}(-c) := \boldsymbol{y} \cdot \exp(-c)$, where c is the $H^2(X)$-component of y.

$\boldsymbol{\mathcal{Z}}(\boldsymbol{y}(-\boldsymbol{c}))$**:** universal 0-scheme of $X^{[\boldsymbol{y}(-c)]} \times X$

$\boldsymbol{\mathcal{K}}$**:** $\mathcal{L}^{u}_{c} \otimes L^{-1} \otimes \mathcal{O}_{\mathcal{Z}(\boldsymbol{y}(-c))}$ on $M(c, L) \times X^{\boldsymbol{y}(-c)} \times X$

$\boldsymbol{\mathfrak{V}}$**:** $p_{X*}\mathcal{K}$

$\widetilde{\boldsymbol{\mathcal{K}}}$**,** $\widetilde{\boldsymbol{\mathfrak{V}}}$**:** Assume $H^1(X, \mathcal{O}_X) = 0$. Take $\mathcal{L}$ with $c_1(\mathcal{L}) = c$ and we set $\widetilde{\mathcal{K}} := \mathcal{O}_{\mathcal{Z}(\boldsymbol{y}(-c))} \otimes \mathcal{L} \otimes L^{-1}$ on $X^{[\boldsymbol{y}(-c)]} \times X$, and $\widetilde{\mathfrak{V}} := p_{X*}\mathcal{K}$ on $X^{[\boldsymbol{y}(-c)]}$.

Subsection 6.3.5

$\boldsymbol{\phi'}$**:** L-section of $\det(E)$ induced by ϕ of E

$\mathbf{det}_{\boldsymbol{E},\boldsymbol{\phi}}$**:** morphism $U_2 \longrightarrow M(c, L)$ induced by $(\det(E), \phi')$

$\mathbf{Ob}(\boldsymbol{V_\bullet}, \widetilde{\boldsymbol{\phi}}_{\bullet})$**:** $Rp_{X*}\big(\mathfrak{g}(V_\bullet, \widetilde{\phi}_\bullet) \otimes \omega_X\big)$

$\mathbf{Ob}(\boldsymbol{M}(\boldsymbol{c}, \boldsymbol{L}))$**:** obstruction theory of $M(c, L)$

$\boldsymbol{\mathfrak{g}}_{\mathbf{rel}}$**,** $\boldsymbol{\mathfrak{g}}(\mathbf{det}(\boldsymbol{V_\bullet}), \boldsymbol{\phi'})$**:** We set $\mathfrak{g}_{\mathrm{rel}} := \mathcal{H}om\big(P_\bullet, \det(V_\bullet)\big)^{\vee}$. Let $\mathfrak{g}(\det(V_\bullet, \phi'))$ denote the cone of the morphism $\mathfrak{g}_{\mathrm{rel}}[-1] \longrightarrow \mathcal{O}[-1]$.

$\widetilde{\boldsymbol{Y}}_{\boldsymbol{i}}(\boldsymbol{W_\bullet}, \boldsymbol{P_\bullet})$**:** Let $\widetilde{Y}_0(W_\bullet, P_\bullet)$ be a quotient stack of

$$N(W_{-1\,X}, W_{0\,X}) \times N(P_{-1}, W_{0\,X})$$

by a natural $\mathrm{GL}(W_\bullet)$-action. We set $\widetilde{Y}_1(W_\bullet, P_\bullet) = Y(W_\bullet)$ and $\widetilde{Y}_2(W_\bullet, P_\bullet) := Y_2(W_\bullet, P_\bullet)$.

$\boldsymbol{\widetilde{Y}(W_\bullet)}$**:** Fiber product of $\widetilde{Y}_i(W_\bullet, P_\bullet)$ $(i = 1, 2)$ over $\widetilde{Y}_0(W_\bullet, P_\bullet)$ is denoted by $\widetilde{Y}(W_\bullet, P_\bullet)$.

$\boldsymbol{Z_i(W_\bullet, P_\bullet)}$**:** We set $Z_0(W_\bullet, P_\bullet) := N\big(P_{-1}, \det(W_{\bullet,X})\big)_{G_m}$, $Z_1(W_\bullet, P_\bullet) := X_{G_m}$ and $Z_2(W_\bullet, P_\bullet) := N\big(P_0, \det(W_{\bullet,X})\big)_{G_m}$.

$\boldsymbol{Z(W_\bullet, P_\bullet)}$**:** Fiber product of $Z_1(W_\bullet, P_\bullet)$ $(i = 1, 2)$ over $Z_0(W_\bullet, P_\bullet)$ is denoted by $Z(W_\bullet, P_\bullet)$.

$\mathbf{Ob}(\boldsymbol{M(c, [L])})$ obstruction theory of $M(c, [L])$

$\mathbf{Ob}(\boldsymbol{V_\bullet, [\widetilde{\phi}_\bullet]})$**:** cone of $\mathrm{Ob}_{\mathrm{rel}}(V_\bullet, [\widetilde{\phi}_\bullet])[-1] \longrightarrow \mathrm{Ob}(V_\bullet)$

$\mathbf{Ob}(\boldsymbol{V_\bullet, \rho, [\widetilde{\phi}_\bullet]})$**:** cone of $\mathrm{Ob}_{\mathrm{rel}}(V_\bullet, \rho)[-1] \longrightarrow \mathrm{Ob}(V_\bullet, [\widetilde{\phi}_\bullet])$

$\mathbf{Ob}(\boldsymbol{M(\widehat{c}, [L])})$**:** obstruction theory of $M(\widehat{c}, [L])$

Subsection 6.3.6

$\boldsymbol{I(E)}$**:** $\det(E)^{-1} \otimes E$

$\boldsymbol{\Xi(E)}$**:** morphism $U \longrightarrow X^{[\boldsymbol{y}(-c)]}$ induced by $I(E)$ with the induced parabolic structure.

$\mathbf{Ob}(\boldsymbol{V_\bullet, F_*})$**:** cone of $\mathrm{Ob}_{\mathrm{rel}}(V_\bullet, F_*)[-1] \longrightarrow \mathrm{Ob}(V_\bullet)$.

$\mathbf{Ob}(\boldsymbol{X^{[y(-c)]}})$**:** obstruction theory of $X^{[\boldsymbol{y}(-c)]}$, that is the same as $L_{X^{[\boldsymbol{y}(-c)]}} = \Omega_{X^{[\boldsymbol{y}(-c)]}}$

$\mathbf{Ob}^\circ(\boldsymbol{V_\bullet, F_*})$**:** trace-free part of $\mathrm{Ob}(V_\bullet, F_*)$

Subsection 6.3.7

$\mathbf{Ob}(\boldsymbol{V_\bullet, F_*, \widetilde{\phi}_\bullet})$**:** cone of $\mathrm{Ob}(V_\bullet) \longrightarrow \mathrm{Ob}(V_\bullet, F_*) \oplus \mathrm{Ob}(V_\bullet, \widetilde{\phi}_\bullet)$

$\mathbf{Ob}(\boldsymbol{V_\bullet, F_*, [\widetilde{\phi}_\bullet], \rho})$**:** cone of $\mathrm{Ob}(V_\bullet) \longrightarrow \mathrm{Ob}(V_\bullet, [\widetilde{\phi}_\bullet], \rho) \oplus \mathrm{Ob}(V_\bullet, F_*)$

Subsection 7.1.1

$\mathbf{Map}_{\boldsymbol{f}}\big(\mathbb{Z}^{\mathbf{2}}_{\geq \mathbf{0}}, \boldsymbol{H^*(X)}\big)$, $\mathbf{Map}_{\boldsymbol{f}}\big(\mathbb{Z}^{\mathbf{3}}_{\geq \mathbf{0}}, \boldsymbol{H^*(D)}\big)$**:** Let $\mathrm{Map}_f\big(\mathbb{Z}^2_{\geq 0}, H^*(X)\big)$ denote the set of maps $\varphi : \mathbb{Z}^2_{\geq 0} \longrightarrow H^*(X)$ such that $\big\{(n_1, n_2) \,\big|\, \varphi(n_1, n_2) \neq 0\big\}$ is finite. We use $\mathrm{Map}_f\big(\mathbb{Z}^2_{\geq 0}, H^*(X)\big)$ in a similar meaning.

$\boldsymbol{\mathcal{R}'_l}$**:** $\mathrm{Sym}\Big(\mathrm{Map}_f\big(\mathbb{Z}^{2l}_{\geq 0},\, H^*(X)\big)\Big) \otimes \mathrm{Sym}\Big(\mathrm{Map}_f\big(\mathbb{Z}^{3l}_{\geq 0},\, H^*(D)\big)\Big)$

$\boldsymbol{\mathcal{R}_l, \mathcal{R}}$**:** We set $\mathcal{R}_l := H^*(\mathrm{Pic}) \otimes \mathcal{R}'_l$ and $\mathcal{R} := \mathcal{R}_1$.

$\mathfrak{q}_l$**:** homomorphism of algebras $\mathcal{R}_l \longrightarrow \mathcal{R}^{\otimes l}$

$\mathfrak{r}_l$**:** homomorphism of algebras $\mathcal{R}_l \longrightarrow \mathcal{R}$

Subsection 7.1.2

$\boldsymbol{\mathcal{R}(E_*), \mathcal{R}(E)}$**:** When we are given $\boldsymbol{E}_* = (E_{1*}, \ldots, E_{l*})$, we set $\mathcal{R}(\boldsymbol{E}_*) := \mathcal{R}_l$. It is also denoted by $\mathcal{R}(\boldsymbol{E})$.

$\boldsymbol{\mathcal{R}(E_*), \mathcal{R}(E)}$**:** When we are given E_*, we set $\mathcal{R}(E_*) := \mathcal{R}$. It is also denoted by $\mathcal{R}(E)$.

$\boldsymbol{P(E \cdot e^t)}$**:** image of $P \in \mathcal{R}(E)$ via the homomorphism $\mathcal{R}(E) \longrightarrow \mathcal{R}(E)[t]$ given in Subsection 7.1.2.

Subsection 7.1.3

$\boldsymbol{A^*(\mathcal{Y})}$**:** bivariant theory $A^*(\mathcal{Y} \to \mathcal{Y})$

$\boldsymbol{\mathcal{R}(E_*, \mathcal{Y})}$**:** $\mathcal{R}(\boldsymbol{E}_*) \otimes A^*(\mathcal{Y})$

$\mathbf{deg}\big(\boldsymbol{P(E_*) \cap F([\mathcal{Z}])}\big)$**:** evaluation of $P(\boldsymbol{E}_*) \cdot F \in \mathcal{R}(\boldsymbol{E}_*, \mathcal{Y})$ over $[\mathcal{Z}]$

$\boldsymbol{\int_{\mathcal{M}^{ss}(\widehat{y}, \alpha_*)} P(\widehat{E}^u)}$**:** If the 1-stability condition holds for $(\boldsymbol{y}, \alpha_*)$, we define

$$\int_{\mathcal{M}^{ss}(\widehat{\boldsymbol{y}}, \alpha_*)} P(\widehat{E}^u) := \deg\Big(P(\widehat{E}^u) \cap [\mathcal{M}^{ss}(\widehat{\boldsymbol{y}}, \alpha_*)]\Big).$$

$\boldsymbol{\int_{\mathcal{M}^{s}(\widehat{y}, [L], \alpha_*, \delta)} P(\widehat{E}^u) \cdot \omega^k}$**:** If the 1-stability condition holds for $(\boldsymbol{y}, L, \alpha_*, \delta)$, we set

$$\int_{\mathcal{M}^{s}(\widehat{\boldsymbol{y}}, [L], \alpha_*, \delta)} P(\widehat{E}^u) \cdot \omega^k := \deg\Big(P(\widehat{E}^u) \cdot \omega^k \cap \big[\mathcal{M}^s(\widehat{\boldsymbol{y}}, [L], \alpha_*, \delta)\big]\Big)$$

Subsection 7.1.4

$\boldsymbol{T}$**:** l-dimensional torus $(G_m)^l$

$\boldsymbol{R(T)}$**:** T-equivariant bivariant theory of a point

$\boldsymbol{e^{w \cdot t_i}}$**:** trivial line bundle with the T-action induced by the action of i-th G_m with weight w

$\boldsymbol{A_*^T(\mathcal{Y})}$**:** T-equivariant Chow group of $\mathcal{Y}$

$\boldsymbol{H_*^T(\mathcal{Y})}$**:** T-equivariant homology group of $\mathcal{Y}$

$\boldsymbol{\deg^T\big(P(E_*)\cap[\mathcal{Z}]\big)\in\mathcal{R}(T)}$**:** evaluation of $P(E_*)$ over $[\mathcal{Z}]\in A^T_*(\mathcal{Y})$

$\boldsymbol{\mathcal{R}_T(E_*,\mathcal{Y})}$**:** $\mathcal{R}(E_*)\otimes A^*_T(\mathcal{Y})$

$\boldsymbol{\int_{\widehat{M}}\Phi(\widehat{E}^{\widehat{M}}_*)}$**:** $\deg^{G_m}\big(\Phi(\widehat{E}^{\widehat{M}}_*)\cap[\widehat{M}]\big)$

Subsection 7.1.5

$\boldsymbol{\mathcal{R}_{\mathrm{CH}}}$**:** $\mathrm{Sym}\Big(\mathrm{Map}_f\big(\mathbb{Z}^2_{\geq 0},A^*(X)\big)\Big)\otimes\mathrm{Sym}\Big(\mathrm{Map}_f\big(\mathbb{Z}^3_{\geq 0},A^*(D)\big)\Big)\otimes A^*(\mathrm{Pic})$

$\boldsymbol{\mathcal{R}_{\mathrm{CH}}(E_*)}$**:** When we are given E_*, we set $\mathcal{R}_{\mathrm{CH}}(E_*):=\mathcal{R}_{\mathrm{CH}}$.

$\boldsymbol{\mathcal{R}_{l,\mathrm{CH}},\mathcal{R}_{\mathrm{CH}}(E_*)}$**:** Similar

$\boldsymbol{\mathfrak{Q}}$**:** natural homomorphism $\mathcal{R}_{\mathrm{CH}}(\boldsymbol{E}_*)\longrightarrow A^*(\mathcal{Y})$

Subsection 7.1.6

$\boldsymbol{R[\![t^{-1},t]}$**:** algebra of power series $\sum a_j\cdot t^j$ such that $\big\{j>0|a_j\neq 0\big\}$ are finite.

$\boldsymbol{\mathfrak{R}(t)}$**:** $\mathbb{Q}[\![t^{-1},t]$

$\boldsymbol{\mathfrak{R}(t_1,\ldots,t_k)}$**:** $\mathfrak{R}(t_1,\ldots,t_k):=\mathfrak{R}(t_2,\ldots,t_k)[\![t_1^{-1},t_1]$

$\boldsymbol{\mathrm{Eu}(\mathcal{F}_a)}$**:** equivariant Euler class of $\mathcal{F}_a$

Subsection 7.1.7

$\boldsymbol{e^{\omega}}$**:** a line bundle $\mathcal{L}$ such that $c_1(\mathcal{L})=\omega$.

$\boldsymbol{P(E\otimes e^{\omega})}$**:** image of $P(E)\in\mathcal{R}(E)$ via the naturally defined morphism $\mathcal{R}(E)\longrightarrow\mathcal{R}(E\otimes e^{\omega},\mathcal{Y})$.

Subsection 7.2.1

$\boldsymbol{S(y,\alpha_*,\delta)}$**:** set of $(\boldsymbol{y}_1,\boldsymbol{y}_2)\in(\mathcal{T}ype)^2$ such that (i) $\boldsymbol{y}_1+\boldsymbol{y}_2=\boldsymbol{y}$, (ii) $P^{\alpha_*,\delta}_{\boldsymbol{y}_1}=P^{\alpha_*}_{\boldsymbol{y}_2}=P^{\alpha_*,\delta}_{\boldsymbol{y}}$

$\boldsymbol{\mathcal{M}(y_1,\widehat{y}_2,L,\alpha_*,\delta)}$**:** $\mathcal{M}^{ss}(\boldsymbol{y}_1,L,\alpha_*,\delta)\times\mathcal{M}^{ss}(\widehat{\boldsymbol{y}}_2,\alpha_*)$

$\boldsymbol{E^u_1}$**:** sheaf on $\mathcal{M}(\boldsymbol{y}_1,\widehat{\boldsymbol{y}}_2,L,\alpha_*,\delta)\times X$ obtained as the pull back of the universal sheaf $\mathcal{M}^{ss}(\boldsymbol{y}_1,L,\alpha_*,\delta)\times X$

$\boldsymbol{\widehat{E}^u_2}$**:** sheaf on $\mathcal{M}(\boldsymbol{y}_1,\widehat{\boldsymbol{y}}_2,L,\alpha_*,\delta)\times X$ obtained as the pull back of the universal sheaf $\mathcal{M}^{ss}(\widehat{\boldsymbol{y}}_2,\alpha_*)\times X$

$\boldsymbol{\omega_1}$**:** $c_1\big(\mathcal{O}r(E^u_1)\big)/\,\mathrm{rank}\,\boldsymbol{y}_1$

$\boldsymbol{e^{w\cdot t}}$**:** trivial line bundle with the G_m-action of weight w

Subsection 7.2.2

$\boldsymbol{\Theta}_{\mathbf{rel}}$**:** relative tangent bundle of $\mathcal{M}^s(\widehat{\boldsymbol{y}},[L],\alpha_*,\delta) \longrightarrow \mathcal{M}(\widehat{\boldsymbol{y}})$, if the 1-vanishing condition is satisfied for $(\widehat{\boldsymbol{y}},[L],\alpha_*,\delta)$

$\boldsymbol{N_L(y)}$**:** $\int_X \mathrm{Td}(X)\cdot y\cdot \mathrm{ch}(L^{-1})$

$\boldsymbol{Q(\widehat{E}_1\cdot e^{-s/r_1},\widehat{E}_2\cdot e^{s/r_2})}$**:** equivariant Euler class of the virtual vector bundle (7.16)

Subsection 7.2.3

$\boldsymbol{\Theta}^{(\boldsymbol{i})}_{\mathbf{rel}}$**:** Let $\Theta^{(1)}_{\mathrm{rel}}$ be the relative tangent bundle of the smooth morphism

$$\mathcal{M}^s(\widehat{\boldsymbol{y}},[\boldsymbol{L}],\alpha_*,\boldsymbol{\delta}_{\pm}) \longrightarrow \mathcal{M}(\widehat{\boldsymbol{y}},[L_2]).$$

We use the symbol $\Theta^{(2)}_{\mathrm{rel}}$ in a similar meaning.

$\boldsymbol{\mathcal{O}^{(i)}_{\mathbf{rel}}(1)}$**:** pull back of the tautological line bundle on $\mathcal{M}(\widehat{\boldsymbol{y}},[L_i])$ via the morphism $\mathcal{M}^{ss}(\widehat{\boldsymbol{y}},[\boldsymbol{L}],\alpha_*,\delta) \longrightarrow \mathcal{M}(\widehat{\boldsymbol{y}},[L_i])$.

$\boldsymbol{S(y,\alpha_*,\delta)}$**:** set of $(\boldsymbol{y}_1,\boldsymbol{y}_2)\in(\mathcal{T}ype)^2$ such that

$$P^{\alpha_*}_{\boldsymbol{y}_1}=P^{\alpha_*}_{\boldsymbol{y}_2},\quad \delta_1/\operatorname{rank}\boldsymbol{y}_1=\delta_2/\operatorname{rank}\boldsymbol{y}_2$$

$\boldsymbol{\mathcal{M}(\widehat{y}_1,\widehat{y}_2,[L],\alpha_*,\delta)}$**:** $\mathcal{M}^s(\widehat{\boldsymbol{y}}_1,[L_1],\alpha_*,\delta_1)\times\mathcal{M}^s(\widehat{\boldsymbol{y}}_2,[L_2],\alpha_*,\delta_2)$

$\boldsymbol{\mathcal{O}_{i,\mathbf{rel}}(1)}$**:** Let $\mathcal{O}_{2,\mathrm{rel}}(1)$ denote the tautological line bundle on $\mathcal{M}(\widehat{\boldsymbol{y}}_2,[L_2])$. The pull back is also denoted by the same symbol. We use the symbol $\mathcal{O}_{1,\mathrm{rel}}(1)$ in a similar meaning.

$\boldsymbol{\omega_i, e^{w\cdot\omega_i}}$**:** We set $\omega_i:=c_1(\mathcal{O}_{i,\mathrm{rel}}(1))$ and $e^{w\cdot\omega_i}:=\mathcal{O}_{i,\mathrm{rel}}(w)$.

Subsection 7.3.1

$\int_{\boldsymbol{\mathcal{M}^{ss}(\widehat{y},\alpha_*)}}$: a linear map $\mathcal{R}\longrightarrow\mathbb{Q}$. See Definition 7.3.2.

Subsection 7.4.1

$\boldsymbol{y,a,b,n}$**:** For $y\in\mathcal{T}ype$, we have the decomposition $y=\operatorname{rank}(y)+a+b$, where $a\in H^2(X)$ and $b\in H^4(X)$. The number $n=a^2/2-n$ corresponds to the second Chern class.

$\boldsymbol{NS(X)}$**:** subgroup of $H^2(X,\mathbb{Z})$ generated by algebraic 1-cycles on X

$\boldsymbol{X^{[l]}}$**:** Hilbert scheme of l-points

Subsection 7.4.2

$\boldsymbol{\mathcal{M}_H(\widehat{y})}$**:** moduli stack of torsion-free sheaves of type y which are semistable with respect to H

$\boldsymbol{\mathcal{C}}$**:** ample cone in $NS(X) \otimes \mathbb{R}$

$\boldsymbol{\xi}$**:** element of $NS(X)$

$\boldsymbol{W^{\xi}}$**:** wall determined by ξ

$\boldsymbol{C_{\pm}, H_{\pm}}$**:** For a given ξ, let $C_{\pm}$ be chambers which are divided by the wall W^{ξ}. Let $H_{\pm} \in C_{\pm}$. We assume $H_{-} \cdot \xi < 0 < H_{+} \cdot \xi$.

$\boldsymbol{\mathcal{M}(\widehat{y}_0, \widehat{y}_1)}$**:** $\mathcal{M}(\widehat{y}_0) \times \mathcal{M}(\widehat{y}_1)$

$\boldsymbol{\widehat{E}_i}$**:** pull back of the universal sheaf on $\mathcal{M}(\widehat{y}_i) \times X$

$\boldsymbol{\mathcal{M}^{ss}(\widehat{y}, \alpha)}$**:** moduli stack of torsion-free sheaves with trivial quasi-parabolic structure and a weight α

$\boldsymbol{S(y, \xi)}$**:** set of $(y_0, y_1) \in (\mathcal{T}ype)^2$ such that (i) $y_0 + y_1 = y$, (ii) $a_0 - a_1 = m\xi$ for some $m > 0$

$\boldsymbol{\mathcal{S}}$**:** family of μ-semistable torsion-free sheaves of type y

$\boldsymbol{\overline{\mathcal{S}}}$**:** family of torsion-free sheaves E' of rank one such that (i) $\mu(E') = \mu(y)$, (ii) there is a member E of $\mathcal{S}$, such that E' is a saturated subsheaf of E.

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Edited by J.-M. Morel, F. Takens, B. Teissier, P.K. Maini

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