Appendix A
Kronecker Product of Matrices

This appendix is a brief description of the Kronecker product of matrices and its properties. For a detailed treatment the reader is referred to [43], [54], [83].

Definition A.1. Let $\mathbf{A} = \{a_{ij}\}$ and $\mathbf{B} = \{b_{kl}\}$ be matrices, $\mathbf{A} \in \mathbb{R}^{n \times n}$ and $\mathbf{B} \in \mathbb{R}^{m \times m}$. Then the Kronecker product of $\mathbf{A}$ and $\mathbf{B}$, denoted by $\mathbf{A} \otimes \mathbf{B}$, is the block matrix

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & \cdots & a_{1n}\mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{n1}\mathbf{B} & \cdots & a_{nn}\mathbf{B} \end{bmatrix} \in \mathbb{R}^{nm \times nm}. \tag{A.1}$$

The Kronecker product is also known as the direct product or the tensor product.

1. The Kronecker product "$\otimes$" is a bilinear operator. If k is a scalar, and $\mathbf{A}$, $\mathbf{B}$ and C are square matrices, such that $\mathbf{B}$ and C are of the same order, then

$$\mathbf{A} \otimes (\mathbf{B} + \mathbf{C}) = \mathbf{A} \otimes \mathbf{B} + \mathbf{A} \otimes \mathbf{C}, \tag{A.2a}$$

$$(\mathbf{B} + \mathbf{C}) \otimes \mathbf{A} = \mathbf{B} \otimes \mathbf{A} + \mathbf{C} \otimes \mathbf{A}, \tag{A.2b}$$

$$k\,(\mathbf{A} \otimes \mathbf{B}) = (k\mathbf{A}) \otimes \mathbf{B} = \mathbf{A} \otimes (k\mathbf{B})\,. \tag{A.2c}$$

2. If $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ are square matrices such that the products $\mathbf{AC}$ and $\mathbf{BD}$ exist, then $(\mathbf{A} \otimes \mathbf{B})\,(\mathbf{C} \otimes \mathbf{D})$ exists and

$$(\mathbf{A} \otimes \mathbf{B})\,(\mathbf{C} \otimes \mathbf{D}) = \mathbf{AC} \otimes \mathbf{BD}. \tag{A.3}$$

3. If $\mathbf{A}$ and $\mathbf{B}$ are invertible matrices, then

$$(\mathbf{A} \otimes \mathbf{B})^{-1} = \mathbf{A}^{-1} \otimes \mathbf{B}^{-1}. \tag{A.4}$$

4. If $\mathbf{A}$ and $\mathbf{B}$ are square matrices, then for the transpose $(\mathbf{A}^T)$ we have

$$(\mathbf{A} \otimes \mathbf{B})^T = \mathbf{A}^T \otimes \mathbf{B}^T. \tag{A.5}$$

5. Let $\mathbf{A}$ and $\mathbf{B}$ be square matrices of orders n and m, respectively. If $\{\lambda_i \mid i = 1, \ldots, n\}$ are eigenvalues of $\mathbf{A}$ and $\{\mu_j \mid j = 1, \ldots, m\}$ are eigenvalues of $\mathbf{B}$, then $\{\lambda_i \mu_j \mid i = 1, \ldots, n,\ j = 1, \ldots, m\}$ are eigenvalues of $\mathbf{A} \otimes \mathbf{B}$. Also,

$$\det(\mathbf{A} \otimes \mathbf{B}) = (\det \mathbf{A})^m (\det \mathbf{B})^n, \tag{A.6a}$$

$$\operatorname{rank}(\mathbf{A} \otimes \mathbf{B}) = \operatorname{rank} \mathbf{A} \ \operatorname{rank} \mathbf{B}, \tag{A.6b}$$

$$\operatorname{trace}(\mathbf{A} \otimes \mathbf{B}) = \operatorname{trace} \mathbf{A} \ \operatorname{trace} \mathbf{B}. \tag{A.6c}$$

Appendix B
Generators and Fundamental Matrices for P1-TS Systems

This appendix contains the relationship between the ordered set containing vertices of the hypercuboid $D^n = [-\alpha_1, \beta_1] \times \ldots \times [-\alpha_n, \beta_n]$, the generators and fundamental matrices for the P1-TS systems with $n = 1, 2, 3, 4$ inputs $z_1, \ldots, z_4$. They are helpful for the fuzzy rules transformation into the crisp function and vice-versa.

B.1 Formulas for $n = 1$

B.1.1 Vertices of the Interval $D^1 = [-\alpha_1, \beta_1]$

$\boldsymbol{\gamma}_1 = -\alpha_1$, $\boldsymbol{\gamma}_2 = \beta_1$.

B.1.2 Generator

$$\mathbf{g}_1(z_1) = \begin{bmatrix} 1 \\ z_1 \end{bmatrix}. \tag{B.1}$$

B.1.3 Fundamental Matrix and Its Inverse

- General case

$$\boldsymbol{\Omega}_1 = \begin{bmatrix} 1 & 1 \\ -\alpha_1 & \beta_1 \end{bmatrix}, \qquad \boldsymbol{\Omega}_1^{-1} = \frac{1}{V_1} \begin{bmatrix} \beta_1 & -1 \\ \alpha_1 & 1 \end{bmatrix}, \tag{B.2}$$

for $V_1 = \alpha_1 + \beta_1 > 0$.

- Unity interval $D^1 = [0, 1]$

$$\boldsymbol{\Omega}_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \qquad \boldsymbol{\Omega}_1^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}. \tag{B.3}$$

- Interval symmetrical around zero $D^1 = [-\alpha_1, \alpha_1]$

$$\boldsymbol{\Omega}_1 = \begin{bmatrix} 1 & 1 \\ -\alpha_1 & \alpha_1 \end{bmatrix}, \qquad \boldsymbol{\Omega}_1^{-1} = \frac{1}{2\alpha_1}\begin{bmatrix} \alpha_1 & -1 \\ \alpha_1 & 1 \end{bmatrix}. \tag{B.4}$$

B.2 Formulas for $n = 2$

B.2.1 Vertices of the Rectangle $D^2 = [-\alpha_1, \beta_1] \times [-\alpha_2, \beta_2]$,

$\gamma_1 = (-\alpha_1, -\alpha_2)$, $\gamma_2 = (\beta_1, -\alpha_2)$, $\gamma_3 = (-\alpha_1, \beta_2)$, $\gamma_4 = (\beta_1, \beta_2)$.

B.2.2 Generator

$$\mathbf{g}_2(z_1, z_2) = [1, z_1, z_2, z_1 z_2]^T. \tag{B.5}$$

B.2.3 Fundamental Matrix and Its Inverse

- General case

$$\boldsymbol{\Omega}_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\alpha_1 & \beta_1 & -\alpha_1 & \beta_1 \\ -\alpha_2 & -\alpha_2 & \beta_2 & \beta_2 \\ \alpha_1\alpha_2 & -\alpha_2\beta_1 & -\alpha_1\beta_2 & \beta_1\beta_2 \end{bmatrix}, \quad \boldsymbol{\Omega}_2^{-1} = \frac{1}{V_2}\begin{bmatrix} \beta_1\beta_2 & -\beta_2 & -\beta_1 & 1 \\ \alpha_1\beta_2 & \beta_2 & -\alpha_1 & -1 \\ \alpha_2\beta_1 & -\alpha_2 & \beta_1 & -1 \\ \alpha_1\alpha_2 & \alpha_2 & \alpha_1 & 1 \end{bmatrix}, \tag{B.6}$$

where $V_2 = (\alpha_1 + \beta_1)(\alpha_2 + \beta_2) > 0$.

- Unity square $D^2 = [0, 1]^2$

$$\boldsymbol{\Omega}_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \boldsymbol{\Omega}_2^{-1} = \begin{bmatrix} 1 & -1 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{B.7}$$

- Rectangle symmetrical around zero $D^2 = [-\alpha_1, \alpha_1] \times [-\alpha_2, \alpha_2]$

$$\boldsymbol{\Omega}_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\alpha_1 & \alpha_1 & -\alpha_1 & \alpha_1 \\ -\alpha_2 & -\alpha_2 & \alpha_2 & \alpha_2 \\ \alpha_1\alpha_2 & -\alpha_1\alpha_2 & -\alpha_1\alpha_2 & \alpha_1\alpha_2 \end{bmatrix}, \; \boldsymbol{\Omega}_2^{-1} = \frac{1}{4\alpha_1\alpha_2}\begin{bmatrix} \alpha_1\alpha_2 & -\alpha_2 & -\alpha_1 & 1 \\ \alpha_1\alpha_2 & \alpha_2 & -\alpha_1 & -1 \\ \alpha_1\alpha_2 & -\alpha_2 & \alpha_1 & -1 \\ \alpha_1\alpha_2 & \alpha_2 & \alpha_1 & 1 \end{bmatrix}. \tag{B.8}$$

B.3 Formulas for $n = 3$

B.3.1 Vertices of the Cuboid $D^3 = [-\alpha_1, \beta_1] \times [-\alpha_2, \beta_2] \times [-\alpha_3, \beta_3]$

$\boldsymbol{\gamma}_1 = (-\alpha_1, -\alpha_2, -\alpha_3)$, $\boldsymbol{\gamma}_2 = (\beta_1, -\alpha_2, -\alpha_3)$, $\boldsymbol{\gamma}_3 = (-\alpha_1, \beta_2, -\alpha_3)$,
$\boldsymbol{\gamma}_4 = (\beta_1, \beta_2, -\alpha_3)$, $\boldsymbol{\gamma}_5 = (-\alpha_1, -\alpha_2, \beta_3)$, $\boldsymbol{\gamma}_6 = (\beta_1, -\alpha_2, \beta_3)$,
$\boldsymbol{\gamma}_7 = (-\alpha_1, \beta_2, \beta_3)$, $\boldsymbol{\gamma}_8 = (\beta_1, \beta_2, \beta_3)$.

B.3.2 Generator

$$\mathbf{g}_3(z_1, z_2, z_3) = [1, z_1, z_2, z_1 z_2, z_3, z_1 z_3, z_2 z_3, z_1 z_2 z_3]^T . \tag{B.9}$$

B.3.3 Fundamental Matrix and Its Inverse

- General case

$$\boldsymbol{\Omega}_3 = \begin{bmatrix} 1 & -\alpha_1 & -\alpha_2 & \alpha_1\alpha_2 & -\alpha_3 & \alpha_1\alpha_3 & \alpha_2\alpha_3 & -\alpha_1\alpha_2\alpha_3 \\ 1 & \beta_1 & -\alpha_2 & -\alpha_2\beta_1 & -\alpha_3 & -\beta_1\alpha_3 & \alpha_2\alpha_3 & \alpha_2\beta_1\alpha_3 \\ 1 & -\alpha_1 & \beta_2 & -\alpha_1\beta_2 & -\alpha_3 & \alpha_1\alpha_3 & -\alpha_3\beta_2 & \alpha_1\alpha_3\beta_2 \\ 1 & \beta_1 & \beta_2 & \beta_1\beta_2 & -\alpha_3 & -\beta_1\alpha_3 & -\alpha_3\beta_2 & -\beta_1\alpha_3\beta_2 \\ 1 & -\alpha_1 & -\alpha_2 & \alpha_1\alpha_2 & \beta_3 & -\alpha_1\beta_3 & -\alpha_2\beta_3 & \alpha_1\alpha_2\beta_3 \\ 1 & \beta_1 & -\alpha_2 & -\alpha_2\beta_1 & \beta_3 & \beta_1\beta_3 & -\alpha_2\beta_3 & -\alpha_2\beta_1\beta_3 \\ 1 & -\alpha_1 & \beta_2 & -\alpha_1\beta_2 & \beta_3 & -\alpha_1\beta_3 & \beta_2\beta_3 & -\alpha_1\beta_2\beta_3 \\ 1 & \beta_1 & \beta_2 & \beta_1\beta_2 & \beta_3 & \beta_1\beta_3 & \beta_2\beta_3 & \beta_1\beta_2\beta_3 \end{bmatrix}^T , \tag{B.10}$$

$$\boldsymbol{\Omega}_3^{-1} = \frac{1}{V_3} \begin{bmatrix} \beta_1\beta_2\beta_3 & -\beta_2\beta_3 & -\beta_1\beta_3 & \beta_3 & -\beta_1\beta_2 & \beta_2 & \beta_1 & -1 \\ \alpha_1\beta_2\beta_3 & \beta_2\beta_3 & -\alpha_1\beta_3 & -\beta_3 & -\alpha_1\beta_2 & -\beta_2 & \alpha_1 & 1 \\ \alpha_2\beta_1\beta_3 & -\alpha_2\beta_3 & \beta_1\beta_3 & -\beta_3 & -\alpha_2\beta_1 & \alpha_2 & -\beta_1 & 1 \\ \alpha_1\alpha_2\beta_3 & \alpha_2\beta_3 & \alpha_1\beta_3 & \beta_3 & -\alpha_1\alpha_2 & -\alpha_2 & -\alpha_1 & -1 \\ \beta_1\alpha_3\beta_2 & -\alpha_3\beta_2 & -\beta_1\alpha_3 & \alpha_3 & \beta_1\beta_2 & -\beta_2 & -\beta_1 & 1 \\ \alpha_1\alpha_3\beta_2 & \alpha_3\beta_2 & -\alpha_1\alpha_3 & -\alpha_3 & \alpha_1\beta_2 & \beta_2 & -\alpha_1 & -1 \\ \alpha_2\beta_1\alpha_3 & -\alpha_2\alpha_3 & \beta_1\alpha_3 & -\alpha_3 & \alpha_2\beta_1 & -\alpha_2 & \beta_1 & -1 \\ \alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & \alpha_1\alpha_3 & \alpha_3 & \alpha_1\alpha_2 & \alpha_2 & \alpha_1 & 1 \end{bmatrix} , \tag{B.11}$$

where $V_3 = (\alpha_1 + \beta_1)(\alpha_2 + \beta_2)(\alpha_3 + \beta_3) > 0$.

- Unity cube $D^3 = [0,1]^3$

$$\mathbf{\Omega}_3 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \ \mathbf{\Omega}_3^{-1} = \begin{bmatrix} 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{B.12}$$

- Cuboid symmetrical around zero $D^3 = [-\alpha_1, \alpha_1] \times [-\alpha_2, \alpha_2] \times [-\alpha_3, \alpha_3]$

$$\mathbf{\Omega}_3 = \begin{bmatrix} 1 & -\alpha_1 & -\alpha_2 & \alpha_1\alpha_2 & -\alpha_3 & \alpha_1\alpha_3 & \alpha_2\alpha_3 & -\alpha_1\alpha_2\alpha_3 \\ 1 & \alpha_1 & -\alpha_2 & -\alpha_1\alpha_2 & -\alpha_3 & -\alpha_1\alpha_3 & \alpha_2\alpha_3 & \alpha_1\alpha_2\alpha_3 \\ 1 & -\alpha_1 & \alpha_2 & -\alpha_1\alpha_2 & -\alpha_3 & \alpha_1\alpha_3 & -\alpha_2\alpha_3 & \alpha_1\alpha_2\alpha_3 \\ 1 & \alpha_1 & \alpha_2 & \alpha_1\alpha_2 & -\alpha_3 & -\alpha_1\alpha_3 & -\alpha_2\alpha_3 & -\alpha_1\alpha_2\alpha_3 \\ 1 & -\alpha_1 & -\alpha_2 & \alpha_1\alpha_2 & \alpha_3 & -\alpha_1\alpha_3 & -\alpha_2\alpha_3 & \alpha_1\alpha_2\alpha_3 \\ 1 & \alpha_1 & -\alpha_2 & -\alpha_1\alpha_2 & \alpha_3 & \alpha_1\alpha_3 & -\alpha_2\alpha_3 & -\alpha_1\alpha_2\alpha_3 \\ 1 & -\alpha_1 & \alpha_2 & -\alpha_1\alpha_2 & \alpha_3 & -\alpha_1\alpha_3 & \alpha_2\alpha_3 & -\alpha_1\alpha_2\alpha_3 \\ 1 & \alpha_1 & \alpha_2 & \alpha_1\alpha_2 & \alpha_3 & \alpha_1\alpha_3 & \alpha_2\alpha_3 & \alpha_1\alpha_2\alpha_3 \end{bmatrix}^T, \tag{B.13}$$

$$\mathbf{\Omega}_3^{-1} = \frac{1}{v_3} \begin{bmatrix} \alpha_1\alpha_2\alpha_3 & -\alpha_2\alpha_3 & -\alpha_1\alpha_3 & \alpha_3 & -\alpha_1\alpha_2 & \alpha_2 & \alpha_1 & -1 \\ \alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & -\alpha_1\alpha_3 & -\alpha_3 & -\alpha_1\alpha_2 & -\alpha_2 & \alpha_1 & 1 \\ \alpha_1\alpha_2\alpha_3 & -\alpha_2\alpha_3 & \alpha_1\alpha_3 & -\alpha_3 & -\alpha_1\alpha_2 & \alpha_2 & -\alpha_1 & 1 \\ \alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & \alpha_1\alpha_3 & \alpha_3 & -\alpha_1\alpha_2 & -\alpha_2 & -\alpha_1 & -1 \\ \alpha_1\alpha_2\alpha_3 & -\alpha_2\alpha_3 & -\alpha_1\alpha_3 & \alpha_3 & \alpha_1\alpha_2 & -\alpha_2 & -\alpha_1 & 1 \\ \alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & -\alpha_1\alpha_3 & -\alpha_3 & \alpha_1\alpha_2 & \alpha_2 & -\alpha_1 & -1 \\ \alpha_1\alpha_2\alpha_3 & -\alpha_2\alpha_3 & \alpha_1\alpha_3 & -\alpha_3 & \alpha_1\alpha_2 & -\alpha_2 & \alpha_1 & -1 \\ \alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & \alpha_1\alpha_3 & \alpha_3 & \alpha_1\alpha_2 & \alpha_2 & \alpha_1 & 1 \end{bmatrix}, \tag{B.14}$$

where $v_3 = 8\alpha_1\alpha_2\alpha_3$.

B.4 Formulas for $n = 4$

B.4.1 Vertices of the Hypercuboid $D^4 = [-\alpha_1, \beta_1] \times \ldots \times [-\alpha_4, \beta_4]$

$\gamma_1 = (-\alpha_1, -\alpha_2, -\alpha_3, -\alpha_4)$, $\gamma_2 = (\beta_1, -\alpha_2, -\alpha_3, -\alpha_4)$,

$\boldsymbol{\gamma}_3 = (-\alpha_1, \beta_2, -\alpha_3, -\alpha_4)$, $\boldsymbol{\gamma}_4 = (\beta_1, \beta_2, -\alpha_3, -\alpha_4)$,
$\boldsymbol{\gamma}_5 = (-\alpha_1, -\alpha_2, \beta_3, -\alpha_4)$, $\boldsymbol{\gamma}_6 = (\beta_1, -\alpha_2, \beta_3, -\alpha_4)$,
$\boldsymbol{\gamma}_7 = (-\alpha_1, \beta_2, \beta_3, -\alpha_4)$, $\boldsymbol{\gamma}_8 = (\beta_1, \beta_2, \beta_3, -\alpha_4)$,
$\boldsymbol{\gamma}_9 = (-\alpha_1, -\alpha_2, -\alpha_3, \beta_4)$, $\boldsymbol{\gamma}_{10} = (\beta_1, -\alpha_2, -\alpha_3, \beta_4)$,
$\boldsymbol{\gamma}_{11} = (-\alpha_1, \beta_2, -\alpha_3, \beta_4)$, $\boldsymbol{\gamma}_{12} = (\beta_1, \beta_2, -\alpha_3, \beta_4)$,
$\boldsymbol{\gamma}_{13} = (-\alpha_1, -\alpha_2, \beta_3, \beta_4)$, $\boldsymbol{\gamma}_{14} = (\beta_1, -\alpha_2, \beta_3, \beta_4)$,
$\boldsymbol{\gamma}_{15} = (-\alpha_1, \beta_2, \beta_3, \beta_4)$, $\boldsymbol{\gamma}_{16} = (\beta_1, \beta_2, \beta_3, \beta_4)$.

B.4.2 Generator

$$\mathbf{g}_4(z_1, z_2, z_3, z_4) = \begin{array}{l}[1, z_1, z_2, z_1z_2, z_3, z_1z_3, z_2z_3, z_1z_2z_3, z_4, z_1z_4, \\ z_2z_4, z_1z_2z_4, z_3z_4, z_1z_3z_4, z_2z_3z_4, z_1z_2z_3z_4].\end{array} \tag{B.15}$$

B.4.3 Fundamental Matrix and Its Inverse

- General case

$$\boldsymbol{\Omega}_4 = [\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}], \tag{B.16}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\alpha_1 & \beta_1 & -\alpha_1 & \beta_1 \\ -\alpha_2 & -\alpha_2 & \beta_2 & \beta_2 \\ \alpha_1\alpha_2 & -\alpha_2\beta_1 & -\alpha_1\beta_2 & \beta_1\beta_2 \\ -\alpha_3 & -\alpha_3 & -\alpha_3 & -\alpha_3 \\ \alpha_1\alpha_3 & -\beta_1\alpha_3 & \alpha_1\alpha_3 & -\beta_1\alpha_3 \\ \alpha_2\alpha_3 & \alpha_2\alpha_3 & -\alpha_3\beta_2 & -\alpha_3\beta_2 \\ -\alpha_1\alpha_2\alpha_3 & \alpha_2\beta_1\alpha_3 & \alpha_1\alpha_3\beta_2 & -\beta_1\alpha_3\beta_2 \\ -\alpha_4 & -\alpha_4 & -\alpha_4 & -\alpha_4 \\ \alpha_1\alpha_4 & -\beta_1\alpha_4 & \alpha_1\alpha_4 & -\beta_1\alpha_4 \\ \alpha_2\alpha_4 & \alpha_2\alpha_4 & -\beta_2\alpha_4 & -\beta_2\alpha_4 \\ -\alpha_1\alpha_2\alpha_4 & \alpha_2\beta_1\alpha_4 & \alpha_1\beta_2\alpha_4 & -\beta_1\beta_2\alpha_4 \\ \alpha_3\alpha_4 & \alpha_3\alpha_4 & \alpha_3\alpha_4 & \alpha_3\alpha_4 \\ -\alpha_1\alpha_3\alpha_4 & \beta_1\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 & \beta_1\alpha_3\alpha_4 \\ -\alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & \alpha_3\beta_2\alpha_4 & \alpha_3\beta_2\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_2\beta_1\alpha_3\alpha_4 & -\alpha_1\alpha_3\beta_2\alpha_4 & \beta_1\alpha_3\beta_2\alpha_4 \end{bmatrix}, \tag{B.17}$$

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\alpha_1 & \beta_1 & -\alpha_1 & \beta_1 \\ -\alpha_2 & -\alpha_2 & \beta_2 & \beta_2 \\ \alpha_1\alpha_2 & -\alpha_2\beta_1 & -\alpha_1\beta_2 & \beta_1\beta_2 \\ \beta_3 & \beta_3 & \beta_3 & \beta_3 \\ -\alpha_1\beta_3 & \beta_1\beta_3 & -\alpha_1\beta_3 & \beta_1\beta_3 \\ -\alpha_2\beta_3 & -\alpha_2\beta_3 & \beta_2\beta_3 & \beta_2\beta_3 \\ \alpha_1\alpha_2\beta_3 & -\alpha_2\beta_1\beta_3 & -\alpha_1\beta_2\beta_3 & \beta_1\beta_2\beta_3 \\ -\alpha_4 & -\alpha_4 & -\alpha_4 & -\alpha_4 \\ \alpha_1\alpha_4 & -\beta_1\alpha_4 & \alpha_1\alpha_4 & -\beta_1\alpha_4 \\ \alpha_2\alpha_4 & \alpha_2\alpha_4 & -\beta_2\alpha_4 & -\beta_2\alpha_4 \\ -\alpha_1\alpha_2\alpha_4 & \alpha_2\beta_1\alpha_4 & \alpha_1\beta_2\alpha_4 & -\beta_1\beta_2\alpha_4 \\ -\alpha_4\beta_3 & -\alpha_4\beta_3 & -\alpha_4\beta_3 & -\alpha_4\beta_3 \\ \alpha_1\alpha_4\beta_3 & -\beta_1\alpha_4\beta_3 & \alpha_1\alpha_4\beta_3 & -\beta_1\alpha_4\beta_3 \\ \alpha_2\alpha_4\beta_3 & \alpha_2\alpha_4\beta_3 & -\beta_2\alpha_4\beta_3 & -\beta_2\alpha_4\beta_3 \\ -\alpha_1\alpha_2\alpha_4\beta_3 & \alpha_2\beta_1\alpha_4\beta_3 & \alpha_1\beta_2\alpha_4\beta_3 & -\beta_1\beta_2\alpha_4\beta_3 \end{bmatrix}, \tag{B.18}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\alpha_1 & \beta_1 & -\alpha_1 & \beta_1 \\ -\alpha_2 & -\alpha_2 & \beta_2 & \beta_2 \\ \alpha_1\alpha_2 & -\alpha_2\beta_1 & -\alpha_1\beta_2 & \beta_1\beta_2 \\ -\alpha_3 & -\alpha_3 & -\alpha_3 & -\alpha_3 \\ \alpha_1\alpha_3 & -\beta_1\alpha_3 & \alpha_1\alpha_3 & -\beta_1\alpha_3 \\ \alpha_2\alpha_3 & \alpha_2\alpha_3 & -\alpha_3\beta_2 & -\alpha_3\beta_2 \\ -\alpha_1\alpha_2\alpha_3 & \alpha_2\beta_1\alpha_3 & \alpha_1\alpha_3\beta_2 & -\beta_1\alpha_3\beta_2 \\ \beta_4 & \beta_4 & \beta_4 & \beta_4 \\ -\alpha_1\beta_4 & \beta_1\beta_4 & -\alpha_1\beta_4 & \beta_1\beta_4 \\ -\alpha_2\beta_4 & -\alpha_2\beta_4 & \beta_2\beta_4 & \beta_2\beta_4 \\ \alpha_1\alpha_2\beta_4 & -\alpha_2\beta_1\beta_4 & -\alpha_1\beta_2\beta_4 & \beta_1\beta_2\beta_4 \\ -\alpha_3\beta_4 & -\alpha_3\beta_4 & -\alpha_3\beta_4 & -\alpha_3\beta_4 \\ \alpha_1\alpha_3\beta_4 & -\beta_1\alpha_3\beta_4 & \alpha_1\alpha_3\beta_4 & -\beta_1\alpha_3\beta_4 \\ \alpha_2\alpha_3\beta_4 & \alpha_2\alpha_3\beta_4 & -\alpha_3\beta_2\beta_4 & -\alpha_3\beta_2\beta_4 \\ -\alpha_1\alpha_2\alpha_3\beta_4 & \alpha_2\beta_1\alpha_3\beta_4 & \alpha_1\alpha_3\beta_2\beta_4 & -\beta_1\alpha_3\beta_2\beta_4 \end{bmatrix}, \tag{B.19}$$

$$\mathbf{D} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\alpha_1 & \beta_1 & -\alpha_1 & \beta_1 \\ -\alpha_2 & -\alpha_2 & \beta_2 & \beta_2 \\ \alpha_1\alpha_2 & -\alpha_2\beta_1 & -\alpha_1\beta_2 & \beta_1\beta_2 \\ \beta_3 & \beta_3 & \beta_3 & \beta_3 \\ -\alpha_1\beta_3 & \beta_1\beta_3 & -\alpha_1\beta_3 & \beta_1\beta_3 \\ -\alpha_2\beta_3 & -\alpha_2\beta_3 & \beta_2\beta_3 & \beta_2\beta_3 \\ \alpha_1\alpha_2\beta_3 & -\alpha_2\beta_1\beta_3 & -\alpha_1\beta_2\beta_3 & \beta_1\beta_2\beta_3 \\ \beta_4 & \beta_4 & \beta_4 & \beta_4 \\ -\alpha_1\beta_4 & \beta_1\beta_4 & -\alpha_1\beta_4 & \beta_1\beta_4 \\ -\alpha_2\beta_4 & -\alpha_2\beta_4 & \beta_2\beta_4 & \beta_2\beta_4 \\ \alpha_1\alpha_2\beta_4 & -\alpha_2\beta_1\beta_4 & -\alpha_1\beta_2\beta_4 & \beta_1\beta_2\beta_4 \\ \beta_3\beta_4 & \beta_3\beta_4 & \beta_3\beta_4 & \beta_3\beta_4 \\ -\alpha_1\beta_3\beta_4 & \beta_1\beta_3\beta_4 & -\alpha_1\beta_3\beta_4 & \beta_1\beta_3\beta_4 \\ -\alpha_2\beta_3\beta_4 & -\alpha_2\beta_3\beta_4 & \beta_2\beta_3\beta_4 & \beta_2\beta_3\beta_4 \\ \alpha_1\alpha_2\beta_3\beta_4 & -\alpha_2\beta_1\beta_3\beta_4 & -\alpha_1\beta_2\beta_3\beta_4 & \beta_1\beta_2\beta_3\beta_4 \end{bmatrix}. \tag{B.20}$$

$$\mathbf{\Omega}_4^{-1} = \frac{1}{V_4}\left[\mathbf{P}, \mathbf{Q}\right], \tag{B.21}$$

where $V_4 = (\alpha_1 + \beta_1)(\alpha_2 + \beta_2)(\alpha_3 + \beta_3)(\alpha_4 + \beta_4) > 0$ and

$$\mathbf{P} = \begin{bmatrix} \beta_1\beta_2\beta_3\beta_4 & -\beta_2\beta_3\beta_4 & -\beta_1\beta_3\beta_4 & \beta_3\beta_4 & -\beta_1\beta_2\beta_4 & \beta_2\beta_4 & \beta_1\beta_4 & -\beta_4 \\ \alpha_1\beta_2\beta_3\beta_4 & \beta_2\beta_3\beta_4 & -\alpha_1\beta_3\beta_4 & -\beta_3\beta_4 & -\alpha_1\beta_2\beta_4 & -\beta_2\beta_4 & \alpha_1\beta_4 & \beta_4 \\ \alpha_2\beta_1\beta_3\beta_4 & -\alpha_2\beta_3\beta_4 & \beta_1\beta_3\beta_4 & -\beta_3\beta_4 & -\alpha_2\beta_1\beta_4 & \alpha_2\beta_4 & -\beta_1\beta_4 & \beta_4 \\ \alpha_1\alpha_2\beta_3\beta_4 & \alpha_2\beta_3\beta_4 & \alpha_1\beta_3\beta_4 & \beta_3\beta_4 & -\alpha_1\alpha_2\beta_4 & -\alpha_2\beta_4 & -\alpha_1\beta_4 & -\beta_4 \\ \beta_1\alpha_3\beta_2\beta_4 & -\alpha_3\beta_2\beta_4 & -\beta_1\alpha_3\beta_4 & \alpha_3\beta_4 & \beta_1\beta_2\beta_4 & -\beta_2\beta_4 & -\beta_1\beta_4 & \beta_4 \\ \alpha_1\alpha_3\beta_2\beta_4 & \alpha_3\beta_2\beta_4 & -\alpha_1\alpha_3\beta_4 & -\alpha_3\beta_4 & \alpha_1\beta_2\beta_4 & \beta_2\beta_4 & -\alpha_1\beta_4 & -\beta_4 \\ \alpha_2\beta_1\alpha_3\beta_4 & -\alpha_2\alpha_3\beta_4 & \beta_1\alpha_3\beta_4 & -\alpha_3\beta_4 & \alpha_2\beta_1\beta_4 & -\alpha_2\beta_4 & \beta_1\beta_4 & -\beta_4 \\ \alpha_1\alpha_2\alpha_3\beta_4 & \alpha_2\alpha_3\beta_4 & \alpha_1\alpha_3\beta_4 & \alpha_3\beta_4 & \alpha_1\alpha_2\beta_4 & \alpha_2\beta_4 & \alpha_1\beta_4 & \beta_4 \\ \beta_1\beta_2\alpha_4\beta_3 & -\beta_2\alpha_4\beta_3 & -\beta_1\alpha_4\beta_3 & \alpha_4\beta_3 & -\beta_1\beta_2\alpha_4 & \beta_2\alpha_4 & \beta_1\alpha_4 & -\alpha_4 \\ \alpha_1\beta_2\alpha_4\beta_3 & \beta_2\alpha_4\beta_3 & -\alpha_1\alpha_4\beta_3 & -\alpha_4\beta_3 & -\alpha_1\beta_2\alpha_4 & -\beta_2\alpha_4 & \alpha_1\alpha_4 & \alpha_4 \\ \alpha_2\beta_1\alpha_4\beta_3 & -\alpha_2\alpha_4\beta_3 & \beta_1\alpha_4\beta_3 & -\alpha_4\beta_3 & -\alpha_2\beta_1\alpha_4 & \alpha_2\alpha_4 & -\beta_1\alpha_4 & \alpha_4 \\ \alpha_1\alpha_2\alpha_4\beta_3 & \alpha_2\alpha_4\beta_3 & \alpha_1\alpha_4\beta_3 & \alpha_4\beta_3 & -\alpha_1\alpha_2\alpha_4 & -\alpha_2\alpha_4 & -\alpha_1\alpha_4 & -\alpha_4 \\ \beta_1\alpha_3\beta_2\alpha_4 & -\alpha_3\beta_2\alpha_4 & -\beta_1\alpha_3\alpha_4 & \alpha_3\alpha_4 & \beta_1\beta_2\alpha_4 & -\beta_2\alpha_4 & -\beta_1\alpha_4 & \alpha_4 \\ \alpha_1\alpha_3\beta_2\alpha_4 & \alpha_3\beta_2\alpha_4 & -\alpha_1\alpha_3\alpha_4 & -\alpha_3\alpha_4 & \alpha_1\beta_2\alpha_4 & \beta_2\alpha_4 & -\alpha_1\alpha_4 & -\alpha_4 \\ \alpha_2\beta_1\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & \beta_1\alpha_3\alpha_4 & -\alpha_3\alpha_4 & \alpha_2\beta_1\alpha_4 & -\alpha_2\alpha_4 & \beta_1\alpha_4 & -\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 & \alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_4 & \alpha_2\alpha_4 & \alpha_1\alpha_4 & \alpha_4 \end{bmatrix}, \tag{B.22}$$

$$
\mathbf{Q} = \begin{bmatrix}
-\beta_1\beta_2\beta_4 & \beta_2\beta_4 & \beta_1\beta_4 & -\beta_4 & -\beta_1\beta_2\beta_3 & \beta_2\beta_3 & \beta_1\beta_3 & -\beta_3 \\
-\alpha_1\beta_2\beta_4 & -\beta_2\beta_4 & \alpha_1\beta_4 & \beta_4 & -\alpha_1\beta_2\beta_3 & -\beta_2\beta_3 & \alpha_1\beta_3 & \beta_3 \\
-\alpha_2\beta_1\beta_4 & \alpha_2\beta_4 & -\beta_1\beta_4 & \beta_4 & -\alpha_2\beta_1\beta_3 & \alpha_2\beta_3 & -\beta_1\beta_3 & \beta_3 \\
-\alpha_1\alpha_2\beta_4 & -\alpha_2\beta_4 & -\alpha_1\beta_4 & -\beta_4 & -\alpha_1\alpha_2\beta_3 & -\alpha_2\beta_3 & -\alpha_1\beta_3 & -\beta_3 \\
\beta_1\beta_2\beta_4 & -\beta_2\beta_4 & -\beta_1\beta_4 & \beta_4 & -\beta_1\alpha_3\beta_2 & \alpha_3\beta_2 & \beta_1\alpha_3 & -\alpha_3 \\
\alpha_1\beta_2\beta_4 & \beta_2\beta_4 & -\alpha_1\beta_4 & -\beta_4 & -\alpha_1\alpha_3\beta_2 & -\alpha_3\beta_2 & \alpha_1\alpha_3 & \alpha_3 \\
\alpha_2\beta_1\beta_4 & -\alpha_2\beta_4 & \beta_1\beta_4 & -\beta_4 & -\alpha_2\beta_1\alpha_3 & \alpha_2\alpha_3 & -\beta_1\alpha_3 & \alpha_3 \\
\alpha_1\alpha_2\beta_4 & \alpha_2\beta_4 & \alpha_1\beta_4 & \beta_4 & -\alpha_1\alpha_2\alpha_3 & -\alpha_2\alpha_3 & -\alpha_1\alpha_3 & -\alpha_3 \\
-\beta_1\beta_2\alpha_4 & \beta_2\alpha_4 & \beta_1\alpha_4 & -\alpha_4 & \beta_1\beta_2\beta_3 & -\beta_2\beta_3 & -\beta_1\beta_3 & \beta_3 \\
-\alpha_1\beta_2\alpha_4 & -\beta_2\alpha_4 & \alpha_1\alpha_4 & \alpha_4 & \alpha_1\beta_2\beta_3 & \beta_2\beta_3 & -\alpha_1\beta_3 & -\beta_3 \\
-\alpha_2\beta_1\alpha_4 & \alpha_2\alpha_4 & -\beta_1\alpha_4 & \alpha_4 & \alpha_2\beta_1\beta_3 & -\alpha_2\beta_3 & \beta_1\beta_3 & -\beta_3 \\
-\alpha_1\alpha_2\alpha_4 & -\alpha_2\alpha_4 & -\alpha_1\alpha_4 & -\alpha_4 & \alpha_1\alpha_2\beta_3 & \alpha_2\beta_3 & \alpha_1\beta_3 & \beta_3 \\
\beta_1\beta_2\alpha_4 & -\beta_2\alpha_4 & -\beta_1\alpha_4 & \alpha_4 & \beta_1\alpha_3\beta_2 & -\alpha_3\beta_2 & -\beta_1\alpha_3 & \alpha_3 \\
\alpha_1\beta_2\alpha_4 & \beta_2\alpha_4 & -\alpha_1\alpha_4 & -\alpha_4 & \alpha_1\alpha_3\beta_2 & \alpha_3\beta_2 & -\alpha_1\alpha_3 & -\alpha_3 \\
\alpha_2\beta_1\alpha_4 & -\alpha_2\alpha_4 & \beta_1\alpha_4 & -\alpha_4 & \alpha_2\beta_1\alpha_3 & -\alpha_2\alpha_3 & \beta_1\alpha_3 & -\alpha_3 \\
\alpha_1\alpha_2\alpha_4 & \alpha_2\alpha_4 & \alpha_1\alpha_4 & \alpha_4 & \alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & \alpha_1\alpha_3 & \alpha_3
\end{bmatrix}. \tag{B.23}
$$

- Unity hypercube $D^4 = [0,1]^4$

$$
\mathbf{\Omega}_4 = \begin{bmatrix}
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \\
0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1 \\
0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1 \\
0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1 \\
0&0&0&0&1&1&1&1&0&0&0&0&1&1&1&1 \\
0&0&0&0&0&1&0&1&0&0&0&0&0&1&0&1 \\
0&0&0&0&0&0&1&1&0&0&0&0&0&0&1&1 \\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1 \\
0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1 \\
0&0&0&0&0&0&0&0&0&1&0&1&0&1&0&1 \\
0&0&0&0&0&0&0&0&0&0&1&1&0&0&1&1 \\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1 \\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{bmatrix}, \tag{B.24}
$$

$$
\mathbf{\Omega}_4^{-1} = \begin{bmatrix}
1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 \\
0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & 1 & 0 & -1 \\
0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} . \tag{B.25}
$$

- Hypercube symmetrical around zero $D^4 = [-\alpha_1, \alpha_1] \times \ldots \times [-\alpha_4, \alpha_4]$

$$
\mathbf{\Omega}_4 = [\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}] , \tag{B.26}
$$

where

$$
\mathbf{A} = \begin{bmatrix}
1 & 1 & 1 & 1 \\
-\alpha_1 & \alpha_1 & -\alpha_1 & \alpha_1 \\
-\alpha_2 & -\alpha_2 & \alpha_2 & \alpha_2 \\
\alpha_1\alpha_2 & -\alpha_1\alpha_2 & -\alpha_1\alpha_2 & \alpha_1\alpha_2 \\
-\alpha_3 & -\alpha_3 & -\alpha_3 & -\alpha_3 \\
\alpha_1\alpha_3 & -\alpha_1\alpha_3 & \alpha_1\alpha_3 & -\alpha_1\alpha_3 \\
\alpha_2\alpha_3 & \alpha_2\alpha_3 & -\alpha_2\alpha_3 & -\alpha_2\alpha_3 \\
-\alpha_1\alpha_2\alpha_3 & \alpha_1\alpha_2\alpha_3 & \alpha_1\alpha_2\alpha_3 & -\alpha_1\alpha_2\alpha_3 \\
-\alpha_4 & -\alpha_4 & -\alpha_4 & -\alpha_4 \\
\alpha_1\alpha_4 & -\alpha_1\alpha_4 & \alpha_1\alpha_4 & -\alpha_1\alpha_4 \\
\alpha_2\alpha_4 & \alpha_2\alpha_4 & -\alpha_2\alpha_4 & -\alpha_2\alpha_4 \\
-\alpha_1\alpha_2\alpha_4 & \alpha_1\alpha_2\alpha_4 & \alpha_1\alpha_2\alpha_4 & -\alpha_1\alpha_2\alpha_4 \\
\alpha_3\alpha_4 & \alpha_3\alpha_4 & \alpha_3\alpha_4 & \alpha_3\alpha_4 \\
-\alpha_1\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 \\
-\alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 \\
\alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_3\alpha_4
\end{bmatrix} , \tag{B.27}
$$

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\alpha_1 & \alpha_1 & -\alpha_1 & \alpha_1 \\ -\alpha_2 & -\alpha_2 & \alpha_2 & \alpha_2 \\ \alpha_1\alpha_2 & -\alpha_1\alpha_2 & -\alpha_1\alpha_2 & \alpha_1\alpha_2 \\ \alpha_3 & \alpha_3 & \alpha_3 & \alpha_3 \\ -\alpha_1\alpha_3 & \alpha_1\alpha_3 & -\alpha_1\alpha_3 & \alpha_1\alpha_3 \\ -\alpha_2\alpha_3 & -\alpha_2\alpha_3 & \alpha_2\alpha_3 & \alpha_2\alpha_3 \\ \alpha_1\alpha_2\alpha_3 & -\alpha_1\alpha_2\alpha_3 & -\alpha_1\alpha_2\alpha_3 & \alpha_1\alpha_2\alpha_3 \\ -\alpha_4 & -\alpha_4 & -\alpha_4 & -\alpha_4 \\ \alpha_1\alpha_4 & -\alpha_1\alpha_4 & \alpha_1\alpha_4 & -\alpha_1\alpha_4 \\ \alpha_2\alpha_4 & \alpha_2\alpha_4 & -\alpha_2\alpha_4 & -\alpha_2\alpha_4 \\ -\alpha_1\alpha_2\alpha_4 & \alpha_1\alpha_2\alpha_4 & \alpha_1\alpha_2\alpha_4 & -\alpha_1\alpha_2\alpha_4 \\ -\alpha_3\alpha_4 & -\alpha_3\alpha_4 & -\alpha_3\alpha_4 & -\alpha_3\alpha_4 \\ \alpha_1\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 \\ \alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 \\ -\alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_3\alpha_4 \end{bmatrix}, \tag{B.28}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\alpha_1 & \alpha_1 & -\alpha_1 & \alpha_1 \\ -\alpha_2 & -\alpha_2 & \alpha_2 & \alpha_2 \\ \alpha_1\alpha_2 & -\alpha_1\alpha_2 & -\alpha_1\alpha_2 & \alpha_1\alpha_2 \\ -\alpha_3 & -\alpha_3 & -\alpha_3 & -\alpha_3 \\ \alpha_1\alpha_3 & -\alpha_1\alpha_3 & \alpha_1\alpha_3 & -\alpha_1\alpha_3 \\ \alpha_2\alpha_3 & \alpha_2\alpha_3 & -\alpha_2\alpha_3 & -\alpha_2\alpha_3 \\ -\alpha_1\alpha_2\alpha_3 & \alpha_1\alpha_2\alpha_3 & \alpha_1\alpha_2\alpha_3 & -\alpha_1\alpha_2\alpha_3 \\ \alpha_4 & \alpha_4 & \alpha_4 & \alpha_4 \\ -\alpha_1\alpha_4 & \alpha_1\alpha_4 & -\alpha_1\alpha_4 & \alpha_1\alpha_4 \\ -\alpha_2\alpha_4 & -\alpha_2\alpha_4 & \alpha_2\alpha_4 & \alpha_2\alpha_4 \\ \alpha_1\alpha_2\alpha_4 & -\alpha_1\alpha_2\alpha_4 & -\alpha_1\alpha_2\alpha_4 & \alpha_1\alpha_2\alpha_4 \\ -\alpha_3\alpha_4 & -\alpha_3\alpha_4 & -\alpha_3\alpha_4 & -\alpha_3\alpha_4 \\ \alpha_1\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 \\ \alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 \\ -\alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_3\alpha_4 \end{bmatrix}, \tag{B.29}$$

$$\mathbf{D} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -\alpha_1 & \alpha_1 & -\alpha_1 & \alpha_1 \\ -\alpha_2 & -\alpha_2 & \alpha_2 & \alpha_2 \\ \alpha_1\alpha_2 & -\alpha_1\alpha_2 & -\alpha_1\alpha_2 & \alpha_1\alpha_2 \\ \alpha_3 & \alpha_3 & \alpha_3 & \alpha_3 \\ -\alpha_1\alpha_3 & \alpha_1\alpha_3 & -\alpha_1\alpha_3 & \alpha_1\alpha_3 \\ -\alpha_2\alpha_3 & -\alpha_2\alpha_3 & \alpha_2\alpha_3 & \alpha_2\alpha_3 \\ \alpha_1\alpha_2\alpha_3 & -\alpha_1\alpha_2\alpha_3 & -\alpha_1\alpha_2\alpha_3 & \alpha_1\alpha_2\alpha_3 \\ \alpha_4 & \alpha_4 & \alpha_4 & \alpha_4 \\ -\alpha_1\alpha_4 & \alpha_1\alpha_4 & -\alpha_1\alpha_4 & \alpha_1\alpha_4 \\ -\alpha_2\alpha_4 & -\alpha_2\alpha_4 & \alpha_2\alpha_4 & \alpha_2\alpha_4 \\ \alpha_1\alpha_2\alpha_4 & -\alpha_1\alpha_2\alpha_4 & -\alpha_1\alpha_2\alpha_4 & \alpha_1\alpha_2\alpha_4 \\ \alpha_3\alpha_4 & \alpha_3\alpha_4 & \alpha_3\alpha_4 & \alpha_3\alpha_4 \\ -\alpha_1\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 \\ -\alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_3\alpha_4 \end{bmatrix}. \tag{B.30}$$

$$\mathbf{\Omega}_4^{-1} = \frac{1}{16\alpha_1\alpha_2\alpha_3\alpha_4}\left[\mathbf{P}, \mathbf{Q}\right], \tag{B.31}$$

where

$$\mathbf{P} = \begin{bmatrix} \alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 & \alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_4 & \alpha_2\alpha_4 & \alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 & -\alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_4 & -\alpha_2\alpha_4 & \alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 & -\alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_4 & \alpha_2\alpha_4 & -\alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 & \alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_4 & -\alpha_2\alpha_4 & -\alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 & \alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_4 & -\alpha_2\alpha_4 & -\alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 & -\alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_4 & \alpha_2\alpha_4 & -\alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 & -\alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_4 & -\alpha_2\alpha_4 & \alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 & \alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_4 & \alpha_2\alpha_4 & \alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 & \alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_4 & \alpha_2\alpha_4 & \alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 & -\alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_4 & -\alpha_2\alpha_4 & \alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 & -\alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_4 & \alpha_2\alpha_4 & -\alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 & \alpha_3\alpha_4 & -\alpha_1\alpha_2\alpha_4 & -\alpha_2\alpha_4 & -\alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 & \alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_4 & -\alpha_2\alpha_4 & -\alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 & -\alpha_1\alpha_3\alpha_4 & -\alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_4 & \alpha_2\alpha_4 & -\alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & -\alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 & -\alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_4 & -\alpha_2\alpha_4 & \alpha_1\alpha_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4 & \alpha_2\alpha_3\alpha_4 & \alpha_1\alpha_3\alpha_4 & \alpha_3\alpha_4 & \alpha_1\alpha_2\alpha_4 & \alpha_2\alpha_4 & \alpha_1\alpha_4 \end{bmatrix}, \tag{B.32}$$

$$\mathbf{Q} = \begin{bmatrix}
-\alpha_4 & -\alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & \alpha_1\alpha_3 & -\alpha_3 & \alpha_1\alpha_2 & -\alpha_2 & -\alpha_1 & 1 \\
\alpha_4 & -\alpha_1\alpha_2\alpha_3 & -\alpha_2\alpha_3 & \alpha_1\alpha_3 & \alpha_3 & \alpha_1\alpha_2 & \alpha_2 & -\alpha_1 & -1 \\
\alpha_4 & -\alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & -\alpha_1\alpha_3 & \alpha_3 & \alpha_1\alpha_2 & -\alpha_2 & \alpha_1 & -1 \\
-\alpha_4 & -\alpha_1\alpha_2\alpha_3 & -\alpha_2\alpha_3 & -\alpha_1\alpha_3 & -\alpha_3 & \alpha_1\alpha_2 & \alpha_2 & \alpha_1 & 1 \\
\alpha_4 & -\alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & \alpha_1\alpha_3 & -\alpha_3 & -\alpha_1\alpha_2 & \alpha_2 & \alpha_1 & -1 \\
-\alpha_4 & -\alpha_1\alpha_2\alpha_3 & -\alpha_2\alpha_3 & \alpha_1\alpha_3 & \alpha_3 & -\alpha_1\alpha_2 & -\alpha_2 & \alpha_1 & 1 \\
-\alpha_4 & -\alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & -\alpha_1\alpha_3 & \alpha_3 & -\alpha_1\alpha_2 & \alpha_2 & -\alpha_1 & 1 \\
\alpha_4 & -\alpha_1\alpha_2\alpha_3 & -\alpha_2\alpha_3 & -\alpha_1\alpha_3 & -\alpha_3 & -\alpha_1\alpha_2 & -\alpha_2 & -\alpha_1 & -1 \\
-\alpha_4 & \alpha_1\alpha_2\alpha_3 & -\alpha_2\alpha_3 & -\alpha_1\alpha_3 & \alpha_3 & -\alpha_1\alpha_2 & \alpha_2 & \alpha_1 & -1 \\
\alpha_4 & \alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & -\alpha_1\alpha_3 & -\alpha_3 & -\alpha_1\alpha_2 & -\alpha_2 & \alpha_1 & 1 \\
\alpha_4 & \alpha_1\alpha_2\alpha_3 & -\alpha_2\alpha_3 & \alpha_1\alpha_3 & -\alpha_3 & -\alpha_1\alpha_2 & \alpha_2 & -\alpha_1 & 1 \\
-\alpha_4 & \alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & \alpha_1\alpha_3 & \alpha_3 & -\alpha_1\alpha_2 & -\alpha_2 & -\alpha_1 & -1 \\
\alpha_4 & \alpha_1\alpha_2\alpha_3 & -\alpha_2\alpha_3 & -\alpha_1\alpha_3 & \alpha_3 & \alpha_1\alpha_2 & -\alpha_2 & -\alpha_1 & 1 \\
-\alpha_4 & \alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & -\alpha_1\alpha_3 & -\alpha_3 & \alpha_1\alpha_2 & \alpha_2 & -\alpha_1 & -1 \\
-\alpha_4 & \alpha_1\alpha_2\alpha_3 & -\alpha_2\alpha_3 & \alpha_1\alpha_3 & -\alpha_3 & \alpha_1\alpha_2 & -\alpha_2 & \alpha_1 & -1 \\
\alpha_4 & \alpha_1\alpha_2\alpha_3 & \alpha_2\alpha_3 & \alpha_1\alpha_3 & \alpha_3 & \alpha_1\alpha_2 & \alpha_2 & \alpha_1 & 1
\end{bmatrix}. \quad \text{(B.33)}$$

For $n \geq 5$ it is preferred to generate formulas recurrently using symbolic computations on a computer.

Appendix C
Proofs of Theorems, Remarks and Algorithms

C.1 Proof of Remark 3.2

Proof. First we prove (3.2).

(1) From (2.43), (A.5) and (A.3) we have

$$\begin{aligned}\mathbf{\Omega}_{k+1}\mathbf{\Omega}_{k+1}^T &= \left(\begin{bmatrix} 1 & 1 \\ -\alpha_{k+1} & \beta_{k+1}\end{bmatrix} \otimes \mathbf{\Omega}_k\right)\left(\begin{bmatrix} 1 & 1 \\ -\alpha_{k+1} & \beta_{k+1}\end{bmatrix} \otimes \mathbf{\Omega}_k\right)^T \\ &= \left(\begin{bmatrix} 1 & 1 \\ -\alpha_{k+1} & \beta_{k+1}\end{bmatrix} \otimes \mathbf{\Omega}_k\right)\left(\begin{bmatrix} 1 & 1 \\ -\alpha_{k+1} & \beta_{k+1}\end{bmatrix}^T \otimes \mathbf{\Omega}_k^T\right) \\ &= \begin{bmatrix} 2 & \beta_{k+1}-\alpha_{k+1} \\ \beta_{k+1}-\alpha_{k+1} & \alpha_{k+1}^2+\beta_{k+1}^2\end{bmatrix} \otimes \mathbf{\Omega}_k\mathbf{\Omega}_k^T,\end{aligned} \tag{C.1}$$

for $k = 0, 1, 2, \ldots, n-1$. This ends the proof of the first part of Remark 3.2.

(2) Now we prove the orthogonality condition: $\beta_k = \alpha_k$ for $k = 1, \ldots, n$. According to the equation (C.1) we see that $\prod_{k=1}^{n}(\beta_k - \alpha_k)$ is the element in the first row and the last column of the matrix $\mathbf{\Omega}_{k+1}\mathbf{\Omega}_{k+1}^T$, $(k = 0, 1, 2, \ldots, n-1)$. By using recurrence we conclude that the necessary condition under which the rows of the matrix $\mathbf{\Omega} = \mathbf{\Omega}_n$ are orthogonal is

$$\prod_{i=1}^{k}(\beta_i - \alpha_i) = 0$$

for $k = 1, 2, \ldots, n$, where n is the number of system inputs.

Now we prove the sufficient condition. In this case $\beta_k = \alpha_k$ holds for $k = 1, \ldots, n$. According to (C.1)

$$\mathbf{\Omega}_{k+1}\mathbf{\Omega}_{k+1}^T = 2\begin{bmatrix}1 & 0\\ 0 & \alpha_{k+1}^2\end{bmatrix} \otimes \mathbf{\Omega}_k\mathbf{\Omega}_k^T, \qquad k = 0, 1, 2, \ldots, n-1.$$

holds. Using the above recurrency we obtain that $\beta_k = \alpha_k$ is a sufficient condition for orthogonality of $\mathbf{\Omega}$. This ends the proof of the second part of Remark 3.2. □

C.2 Proof of Remark 3.3

Proof. Let us take the following notation

$$\mathbf{\Lambda}_1 = \begin{bmatrix}1 & 0\\ 0 & \alpha_1\end{bmatrix} \otimes 1 = \begin{bmatrix}1 & 0\\ 0 & \alpha_1\end{bmatrix},$$

$$\mathbf{\Lambda}_2 = \begin{bmatrix}1 & 0\\ 0 & \alpha_2\end{bmatrix} \otimes \mathbf{\Lambda}_1 = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \alpha_1 & 0 & 0\\ 0 & 0 & \alpha_2 & 0\\ 0 & 0 & 0 & \alpha_1\alpha_2\end{bmatrix},$$

$$\mathbf{\Lambda}_3 = \begin{bmatrix}1 & 0\\ 0 & \alpha_3\end{bmatrix} \otimes \mathbf{\Lambda}_2 = \begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \alpha_1\alpha_2 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \alpha_3 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \alpha_1\alpha_3 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_2\alpha_3 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_1\alpha_2\alpha_3\end{bmatrix},$$

and so forth. This means that the recurrence (3.4) holds. By induction we obtain

$$\mathbf{\Omega}_1\mathbf{\Omega}_1^T = 2^1\mathbf{\Lambda}_1^2,$$

$$\mathbf{\Omega}_2\mathbf{\Omega}_2^T = 2^2\mathbf{\Lambda}_2^2,$$

$$\vdots$$

$$\mathbf{\Omega}_k\mathbf{\Omega}_k^T = 2^k\mathbf{\Lambda}_k^2 \qquad \text{for} \qquad k = 1, \ldots, n.$$

Thus, we can neglect the subscripts, i.e. $\mathbf{\Omega}_n = \mathbf{\Omega}$ and $\mathbf{\Lambda}_n = \mathbf{\Lambda}$ and simply write

$$\mathbf{\Omega}\mathbf{\Omega}^T = 2^n\mathbf{\Lambda}^2.$$

Taking into account popular features of matrix calculus such, as $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$, $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ and $c\mathbf{A} = \mathbf{A}c$ for $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{m\times m}$ and $c \in \mathbb{R}$, after simple transformations we obtain

$$\mathbf{\Lambda}^{-1}\mathbf{\Omega}\mathbf{\Omega}^T\left(\mathbf{\Lambda}^{-1}\right)^T = 2^n\mathbf{I}.$$

Taking into account symmetry of $\mathbf{\Lambda}$ we obtain

$$\left(2^{-\frac{n}{2}}\mathbf{\Lambda}^{-1}\mathbf{\Omega}\right)\left(2^{-\frac{n}{2}}\mathbf{\Lambda}^{-1}\mathbf{\Omega}\right)^T = \mathbf{I}.$$

This means that

$$\left(2^{-\frac{n}{2}}\mathbf{\Lambda}^{-1}\mathbf{\Omega}\right)^{-1} = \left(2^{-\frac{n}{2}}\mathbf{\Lambda}^{-1}\mathbf{\Omega}\right)^T,$$

or equivalently

$$\mathbf{\Omega}^{-1}\mathbf{\Lambda}2^{\frac{n}{2}} = \mathbf{\Omega}^T\mathbf{\Lambda}^{-1}2^{-\frac{n}{2}},$$

and finally

$$\mathbf{\Omega}^{-1} = 2^{-n}\mathbf{\Omega}^T\mathbf{\Lambda}^{-2}.$$

This ends the proof of Remark 3.3. □

C.3 Proof of Corollary 5.27

Proof. First we prove (5.176). Let us define the generator by

$$\mathbf{g} = \left[1,\ z_1,\ z_2,\ \ldots\ ,\ z_{n-1},\ z_n,\ z_1z_2,\ z_1z_3,\ \ldots\ ,\ \prod_{i=1}^{n} z_i\right]^T,$$

and the corresponding fundamental matrix by $\mathbf{\Omega} = \left[\mathbf{g}\left(\boldsymbol{\gamma}_1\right) \ldots \mathbf{g}\left(\boldsymbol{\gamma}_{2^n}\right)\right]$. The linear mapping f in (5.175) is a special case of the function f_0 given by (2.26), where the vector $\boldsymbol{\theta}$ is of the form: $\boldsymbol{\theta} = [0, r_1, r_2, \ldots, r_n, 0, \ldots, 0]^T$. After filling the matrix $\mathbf{\Omega}$ for the above generator with the vectors $\boldsymbol{\gamma}_v \in \Gamma^n$, from (2.30) we obtain

$$\mathbf{q} = \begin{bmatrix} 1 & -\alpha_1 & \cdots & -\alpha_n & * & \cdots & * \\ 1 & -\alpha_1 & \cdots & \beta_n & * & \cdots & * \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 1 & \beta_1 & \ldots & \beta_n & * & \cdots & * \end{bmatrix} \boldsymbol{\theta},$$

where $\boldsymbol{\theta} = [0, r_1, r_2, \ldots, r_n, 0, \ldots, 0]^T$ and the symbols "$*$" are nonzero elements depending on α_i and β_j. Thus,

$$\mathbf{q} = \begin{bmatrix} 0 & \boldsymbol{\gamma}_1^T & 0 & \cdots & 0 \\ 0 & \boldsymbol{\gamma}_2^T & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \boldsymbol{\gamma}_{2^n}^T & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \mathbf{r} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \mathbf{L}^T\mathbf{r}.$$

This ends the proof of (5.176).

On the other hand, according to (2.47) we have $S = f$ if and only if the consequents of the rules are $q_v = f(\boldsymbol{\gamma}_v) = \mathbf{r}^T \boldsymbol{\gamma}_v$ for every $v = 1, \ldots, 2^n$. Thus,

$$q_v = q_{(i_1, i_2, \ldots, i_n)} = \mathbf{r}^T \boldsymbol{\gamma}_v \ ,$$

where $v \leftrightarrow (i_1, \ldots, i_n)$ as in (2.16). Now, if we take into account (2.23), the result in (5.177) is clear. Both (5.177) and (5.176) for the consequents of the rules are the necessary and sufficient conditions which guarantee linearity of the P1-TS system. This ends the proof of Corollary 5.27. □

C.4 Proof of RLS Algorithm from Section 6.4

Proof. Without loss of generality we assume a new simplified notation in which the index j will be neglected, since all computations should be performed for all inputs. For simplicity we will take a notation as shown in Table C.1

Table C.1 Simplified notation for the proof of the algorithm from Section 6.4

Old notation	Number of equation	Simplified notation
$\mathbf{w}(t_k)$	(6.11)	$\mathbf{w}_k$
$d_j(t_k, t_{k+1})$	(6.7)	d_k
$\epsilon_j(t_k, t_{k+1})$	(6.14)	ϵ_k

The gradient of (6.32) with respect to $\mathbf{q}$ must be zero vector

$$\nabla_{\mathbf{q}} E_j(\lambda) = 2 \sum_{k=1}^{K} \lambda^{K-k} \left(\mathbf{w}_k^T \mathbf{q} - d_k \right) \mathbf{w}_k = \mathbf{0}. \tag{C.2}$$

The vector of the consequents $\mathbf{q}$ that satisfies the equation (C.2), we will denote by $\mathbf{q}_K$, since it is computed for the given K data pairs from the available set (6.31). The normal equations are as follows

$$\underbrace{\sum_{k=1}^{K} \lambda^{K-k} d_k \mathbf{w}_k}_{\mathbf{r}_K} = \underbrace{\sum_{k=1}^{K} \lambda^{K-k} \left(\mathbf{w}_k \mathbf{w}_k^T \right)}_{\mathbf{R}_K} \cdot \mathbf{q}_K \ , \tag{C.3}$$

or equivalently

$$\mathbf{q}_K = \mathbf{R}_K^{-1} \mathbf{r}_K \ , \tag{C.4}$$

where

$$\mathbf{r}_K = \lambda \sum_{k=1}^{K-1} \lambda^{K-1-k} d_k \mathbf{w}_k + d_k \mathbf{w}_k = \lambda \mathbf{r}_{K-1} + d_K \mathbf{w}_K \ , \tag{C.5}$$

$$\mathbf{R}_K = \lambda \sum_{k=1}^{K-1} \lambda^{K-1-k} \mathbf{w}_k \mathbf{w}_k^T + \mathbf{w}_k \mathbf{w}_k^T = \lambda \mathbf{R}_{K-1} + \mathbf{w}_K \mathbf{w}_K^T \ . \tag{C.6}$$

Now we consider the Sherman-Morrison-Woodbury matrix identity [161]

$$\left(\mathbf{A} + \mathbf{X}_1 \mathbf{B} \mathbf{X}_2^T\right)^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{X}_1 \left(\mathbf{B}^{-1} + \mathbf{X}_2^T \mathbf{A}^{-1} \mathbf{X}_1\right)^{-1} \mathbf{X}_2^T \mathbf{A}^{-1}, \tag{C.7}$$

which holds for matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{X}_1$ and $\mathbf{X}_2$, where the first two are non-singular. From (C.6)-(C.7) by $\mathbf{A} = \lambda \mathbf{R}_{K-1}$, $\mathbf{X}_1 = \mathbf{X}_2 = \mathbf{w}_K$, $\mathbf{B} = 1$ we obtain

$$\mathbf{R}_K^{-1} = \frac{1}{\lambda}\mathbf{R}_{K-1}^{-1} - \underbrace{\frac{1}{\lambda}\mathbf{R}_{K-1}^{-1}\mathbf{w}_K \left(1 + \mathbf{w}_K^T \frac{1}{\lambda}\mathbf{R}_{K-1}^{-1}\mathbf{w}_K\right)^{-1}}_{\mathbf{h}_K} \mathbf{w}_K^T \frac{1}{\lambda}\mathbf{R}_{K-1}^{-1} \ .$$

By assuming $\mathbf{R}_K^{-1} = \mathbf{P}_K$ (the inverse correlation matrix), the last equation can be written as

$$\mathbf{P}_K = \frac{1}{\lambda}\left(\mathbf{P}_{K-1} - \mathbf{h}_K \mathbf{w}_K^T \mathbf{P}_{K-1}\right), \tag{C.8}$$

where

$$\mathbf{h}_K = \frac{\mathbf{P}_{K-1}\mathbf{w}_K}{\lambda + \mathbf{w}_K^T \mathbf{P}_{K-1}\mathbf{w}_K} \ , \tag{C.9}$$

and $\mathbf{h}_K$ is called the Kalman gain vector. From (C.4) and (C.5) we have

$$\mathbf{q}_K = \mathbf{P}_K \mathbf{r}_K = \lambda \mathbf{P}_K \mathbf{r}_{K-1} + d_K \mathbf{P}_K \mathbf{w}_K \ . \tag{C.10}$$

From (C.9) we obtain

$$\mathbf{P}_{K-1}\mathbf{w}_K = \lambda \mathbf{h}_K + \mathbf{h}_K \mathbf{w}_K^T \mathbf{P}_{K-1}\mathbf{w}_K \ .$$

After multiplying (C.8) by $\mathbf{w}_K$ we get

$$\mathbf{P}_K \mathbf{w}_K = \frac{1}{\lambda}\left(\mathbf{P}_{K-1}\mathbf{w}_K - \mathbf{h}_K \mathbf{w}_K^T \mathbf{P}_{K-1}\mathbf{w}_K\right).$$

Thus,

$$\mathbf{P}_K \mathbf{w}_K = \frac{1}{\lambda}\left(\lambda \mathbf{h}_K + \mathbf{h}_K \mathbf{w}_K^T \mathbf{P}_{K-1}\mathbf{w}_K - \mathbf{h}_K \mathbf{w}_K^T \mathbf{P}_{K-1}\mathbf{w}_K\right) = \mathbf{h}_K. \tag{C.11}$$

From (C.8) we have $\lambda \mathbf{P}_K = \mathbf{P}_{K-1} - \mathbf{h}_K \mathbf{w}_K^T \mathbf{P}_{K-1}$. Let us multiply this equation by $\mathbf{r}_{K-1}$ and add $d_K \mathbf{P}_K \mathbf{w}_K = d_K \mathbf{h}_K$. We obtain

$$\lambda \mathbf{P}_K \mathbf{r}_{K-1} + d_K \mathbf{h}_K = \mathbf{P}_{K-1} \mathbf{r}_{K-1} - \mathbf{h}_K \mathbf{w}_K^T \mathbf{P}_{K-1} \mathbf{r}_{K-1} + d_K \mathbf{h}_K . \quad \text{(C.12)}$$

Taking into account (C.10), (C.12) and (C.11), we obtain the consequents vector

$$\mathbf{q}_K = \lambda \mathbf{P}_K \mathbf{r}_{K-1} + d_K \mathbf{P}_K \mathbf{w}_K = \mathbf{P}_{K-1} \mathbf{r}_{K-1} - \mathbf{h}_K \mathbf{w}_K^T \mathbf{P}_{K-1} \mathbf{r}_{K-1} + d_K \mathbf{h}_K . \quad \text{(C.13)}$$

According to (C.4) the equation $\mathbf{P}_{K-1} \mathbf{r}_{K-1} = \mathbf{q}_{K-1}$ holds. Finally, from (C.13) we get

$$\mathbf{q}_K = \mathbf{q}_{K-1} - \mathbf{h}_K \mathbf{w}_K^T \mathbf{q}_{K-1} + d_K \mathbf{h}_K \ . \quad \text{(C.14)}$$

In the RLS algorithm we take k instead of K because of the data inflow. Taking into account the notation from Table C.1 we conclude that:

- the vector (C.9) is the same as the Kalman gain vector in (6.34),
- the equation (C.14) is equivalent to two equations: (6.33) and (6.35).

This completes the proof of RLS algorithm from Section 6.4. □

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