A

Basic Notations

In this section we recall some notions from graph theory, linear algebra and complexity of algorithms that are used in this monograph.

Graphs

We briefly give the terminology in graph theory needed in this monography. For standard graph theoretical terms not defined here we refer to [56, 175].

A graph $G(V, E)$ consists of a nonempty finite set V called the *vertex set* and an *edge set* E, where an edge is an unordered pair of distinct vertices; hence we can write $x \in e$ to mean that vertex x is incident with the edge e. For simplicity we write xy (instead of $\{u, v\}$) for an edge with end-vertices x and y.

We use $V(G)$ and $E(G)$ to denote the vertex set and edge set of G, respectively. We have tried to use $n = |V|$ and $m = |E|$ consistently for the respective numbers of vertices and edges ($|S|$ denotes the *cardinality* of a set S). Graphs as we have defined them are also referred to as *simple graphs*, since they do not have multiple edges or loops.

An edge $e = uv$ connects the vertices u and v, and we say that u and v are *adjacent* or u is a *neighbor* of v (and vice versa). We write $v \sim u$ to express more explicitly that v is adjacent to u (and vice versa). In particular we use $\sum_{uv \in E}$ if we sum over all edges of a graph and $\sum_{v \sim u}$ if we sum over all vertices v that are adjacent to some vertex u. The number of neighbors of v is called the *degree* of v and denoted by $d(v)$. If all the vertices of a graph G have the same degree k, then G is *k-regular*, or simply *regular*.

The *complement* G^c of a graph G has the same vertex set as G and two vertices u and v are adjacent in G^c if and only if they are not adjacent in G.

A graph is called *complete* if every pair of vertices are adjacent. We denote the complete graph with n vertices by K_n.

A graph H is a *subgraph* of G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. A subgraph H of G is an *induced subgraph* if two vertices of $V(H)$ are adjacent if and only if they are adjacent in G. If $U \subseteq V(G)$, then $G[U]$ denotes the induced subgraph of G with vertex set U. If U is some set of vertices of G, we write $G - U$ for $G[V \setminus U]$. We write $G - v$ rather than $G - \{v\}$ and say *deletion* of vertex v. For a subset F of $E(G)$ we write $G - F = G(V, E \setminus F)$. Instead of $G - \{e\}$ for an edge e we write $G - e$ and say *deleting* edge e. By G/e we denote the graph obtained from G by *contracting* the edge $e = uv$ into a new vertex v_e which becomes adjacent to all the former neighbors of both u and of v. We delete any multiple edges or loops.

A graph H that can be obtained from G by a series of deletions and contractions of edges and deletions of isolated vertices is called a *minor* of G.

A *clique* is a subgraph that is complete. A set of vertices is *independent* if no two of its elements are adjacent.

A *path* with k vertices from u to v in a graph is a sequence of k distinct vertices starting with u and ending with v such that consecutive vertices are adjacent. We denote a path with k vertices by P_k. If there is a path between any two vertices of a graph G, then G is *connected*, otherwise *disconnected*. A maximal connected induced subgraph of G is called *(connected) component* of G.

A *cycle* is a connected graph where every vertex has exactly two neighbors. A graph containing no cycles is called a *forest*. A connected forest is called a *tree*.

A graph $G(V, E)$ is called *k-partite* if V admits a partition into k classes such that vertices in the same partition class must not be adjacent. Instead of 2-partite one usually says *bipartite*. An k-partite graph in which every two vertices from different partition classes are adjacent is called *complete* and is denoted by $K_{n_1,\dots,n_k}$.

Linear Algebra

We recall the main results of the linear algebra of symmetric matrices over the real numbers. For further details we refer the reader to the relevant literature, e.g. [100]. However, as most of the results are inspired by the close analogy between the continuous Laplace-Beltrami operator on Riemannian manifolds and the graph Laplacian, we will often use a different notion and terminology as we will say *function* (over a subset of $\mathbb{N}$) instead of *vector* and write $f(i)$ (or $x(i)$) instead of x_i for the i-th component of the functions/vectors f and $\mathbf{x}$, respectively.

In fact we can interpret a vector $\mathbf{x}$ with components indexed by the vertices V of a given Graph $G(V, E)$ as a real-valued function f on V, i.e. $f\colon V \to \mathbb{R}$. Furthermore, the set of all such functions obviously forms a vector space that is isomorphic to $\mathbb{R}^n$ and thus we can denote this vector space in abuse of language simple by $\mathbb{R}^n$. We also have the scalar product for such functions f and g given by $\langle f, g\rangle = \sum_{v \in V} f(x)\, g(x)$ and hence the space of our function forms a Hilbert space.

Let $\mathbf{A} = (A_{ij})$ be a real $n \times n$ matrix. An *eigenvalue* of $\mathbf{A}$ is a number λ satisfying $\mathbf{A}\,\mathbf{x} = \lambda\,\mathbf{x}$ for a nonzero vector $\mathbf{x}$. Any such vector $\mathbf{x}$ is called an *eigenvector* of the matrix $\mathbf{A}$ belonging (affording) to the eigenvalue λ. Due to our convention we will say *eigenfunction* instead of eigenvector. The space of all eigenfunctions of $\mathbf{A}$ belonging to λ together with the null function, is called the *eigenspace* $\mathcal{E}_\lambda$ of λ. The dimension of the eigenspace is called the *geometric multiplicity* of λ. The eigenvalues of $\mathbf{A}$ are the roots of the *characteristic polynomial* $\det(\mathbf{A} - \lambda\mathbf{I})$ of $\mathbf{A}$.

For a symmetric real $n \times n$ matrix $\mathbf{A}$ all eigenvalues are real and the geometric multiplicity is equal to the algebraic multiplicity of λ, i.e., the multiplicity of λ as root of the characteristic polynomial. Furthermore there exists an orthogonal basis of the $\mathbb{R}^n$ consisting of eigenfunctions of $\mathbf{A}$.

The *spectrum* of a matrix is the list of its eigenvalues together with their multiplicities. The *spectral radius* $\rho(\mathbf{A})$ of a matrix is the maximum of the absolute values of its eigenvalues.

The *trace* of a square matrix $\mathbf{A}$ is the sum of the diagonal entries and is denoted by $\mathrm{tr}(\mathbf{A})$. The trace of a square matrix is also equal to the sum of its eigenvalues, i.e., $\mathrm{tr}(\mathbf{A}) = \sum_{i=1}^{n} a_{ii} = \sum_{i=1}^{n} \lambda_i$.

A matrix $\mathbf{B}$ is called a *principal submatrix* of a symmetric matrix $\mathbf{A}$ if it is obtained by removing corresponding rows and columns from $\mathbf{A}$.

Algorithms and Their Complexity

In the analysis of an algorithm first of all we are interested in its *complexity*, which is measured by the number of elementary operations that it requires. The complexity of an algorithm depends on the size of its input. An algorithm is said to be an $\mathcal{O}(g(N))$ *algorithm* of its input size N for some function $g(\cdot)$ if the running time never exceeds $c\,g(N)$ for some positive constant c. An algorithm is a *polynomial algorithm* if $g(N)$ is a polynomial in N.

There are many interesting algorithmic problems concerning graphs for which no polynomial algorithm are known. Many of those problems belong to the class of *NP-complete* problems. For a detailed introduction to the class of NP-complete problems, see [80].

A problem is a *decision problem* if it requires the answer “yes” or “no”. A problem is understood as a family of *instances*. For example, we consider the *Hamilton cycle problem*: given a graph, decide whether or not it has a Hamilton cycle. Every graph provides an instance of this problem.

A decision problem S belongs to the complexity class P if and only if there exists a polynomial algorithm which, given any instance of S, produces answer “yes” or “no” such that the answer of the algorithm on input x is “yes” if and only if x is a “yes” instance for S.

A decision problem belongs to the complexity class *NP* if, for every “yes” instance of the problem, there exists a short “proof”, called a *certificate*, of polynomial size such that, using the certificate, one can verify in polynomial time that the instance is indeed a “yes” instance.

Given a pair of decision problems S and T, we say that S is *polynomially reducible* to T if there is a polynomial algorithm $\mathcal{A}$ that transforms an instance x of S into an instance $\mathcal{A}(x)$ of T such that the second instance has the same answer as the first one. That is, x is a “yes” instance of S if and only if $\mathcal{A}(x)$ is a “yes” instance of T.

A decision problem is *NP-hard* if all problems in NP can be polynomially reduced this problem. If the problem is NP-hard and also belongs to NP then it is *NP-complete*. Polynomial transformations are transitive. Hence, in order to prove that a problem W is NP-hard, it is sufficient to prove that there is some NP-complete problem which is polynomial reducible to W.

B

Eigenfunctions Used in Figures

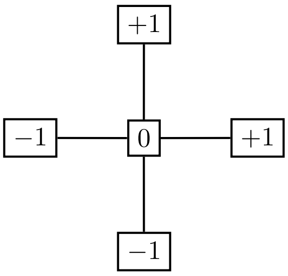

Fig. 3.1. Laplacian $\mathbf{L}$, $\lambda_2 = 1$ (multiplicity $r = 3$).

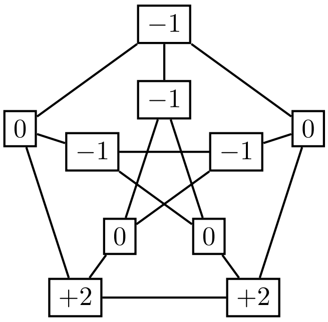

Fig. 3.2. Laplacian $\mathbf{L}$, $\lambda_2 = 2$ (multiplicity $r = 5$).

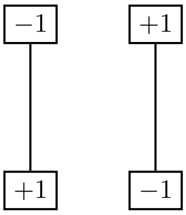

Fig. 3.3. Laplacian $\mathbf{L}$, $\lambda_3 = 2$ (multiplicity $r = 2$).

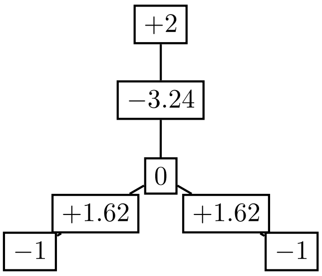

Fig. 3.4. Laplacian $\mathbf{L}$, $\lambda_5 = (3 + \sqrt{5})/2$ (multiplicity $r = 2$; numbers rounded to 3 digits).

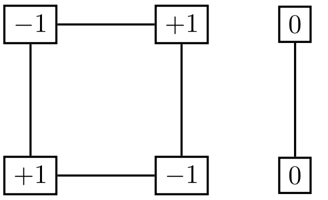

Fig. 3.5. Laplacian $\mathbf{L}$, $\lambda_6 = 4$ (simple).

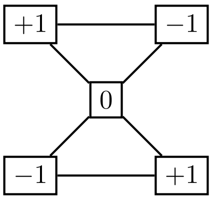

Fig. 3.7. Laplacian $\mathbf{L}$, $\lambda_3 = 2$ (multiplicity $r = 2$).

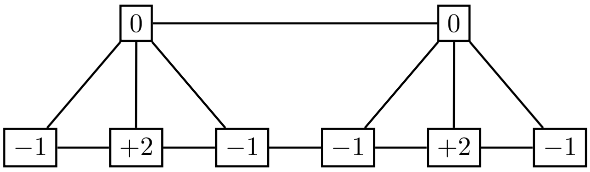

Fig. 3.6. Laplacian $\mathbf{L}$, $\lambda_5 = 4$ (multiplicity $r = 2$).

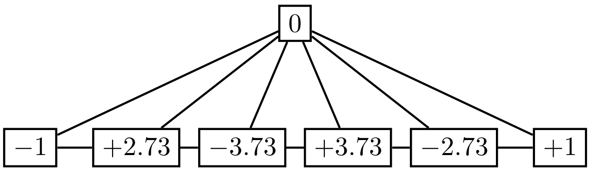

Fig. 3.8. Laplacian $\mathbf{L}$, $\lambda_6 = 3 + \sqrt{3}$ (simple; numbers rounded to 3 digits).

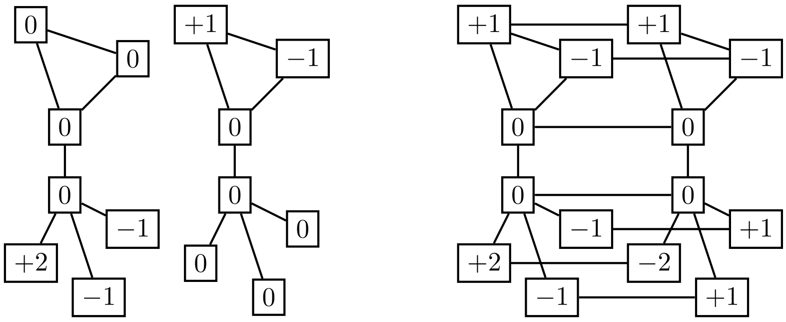

Fig. 4.3. Laplacian $\mathbf{L}$, $\lambda_3(G) = 1$ (multiplicity $r = 2$), $\lambda_5(G) = 3$ (simple), and $\lambda_7(G \square K_2) = 3$ (multiplicity $r = 3$).

C

List of Symbols

$\mathbb{R}$	set of real numbers
$\mathbb{R}^n$	real vector space of dimension n
$\mathbb{S}^{n-1}$	unit sphere in $\mathbb{R}^n$
$\|S\|$	cardinality of set S
$S^\perp$	orthogonal complement of set $S \subset \mathbb{R}$
$\langle \mathbf{x}, \mathbf{y} \rangle$	scalar product of vectors $\mathbf{x}$ and $\mathbf{y}$
$G(V, E)$	graph with vertex set V and edge set E
$V(G)$	vertex set of G
n	number of vertices ($n = \|V\|$)
V°, ∂V	set of interior and boundary vertices of graph G with boundary
$E(G)$	edge set of graph G
m	number of vertices ($m = \|E\|$)
E°, ∂E	set of interior and boundary edges of graph G with boundary
$d(v)$	degree of vertex v
uv	edge with incident vertices u and v
$y \sim x$	y is adjacent to x (and vice versa)
$\mathcal{G}$	geometric realization of graph G
$\mathrm{dist}(u, v)$	geodesic distance between $u, v \in \mathcal{G}$
$h(v)$	height of a vertex v in rooted tree
$\mathrm{Br}(w, v)$	branch of a tree spanned by edge wv
$B(x, r)$	ball in $\mathcal{G}$ with center x a radius r
$\mathrm{Aut}(G)$	automorphism group of graph G
$G[U]$	induced subgraph of G with vertex set U
G^c	complement of graph G
$G - e$	deletion of edge e
$G - v$	deletion of vertex v
G/e	contraction of edge e
$G\{W\}$	reduced graph
$G \Box H$	Cartesian product of two graphs G and H
$G + H$	disjoint union of two graphs G and H
$G * H$	join of two graphs G and H

K_n	complete graph with n vertices
$K_{n_1,\dots,n_k}$	complete k-partite graph
K_2^d	d-dimensional hypercube
P_k	path with k vertices
$\boldsymbol{\nabla}$	incidence matrix of a graph of dimension $\lvert E\rvert \times \lvert V\rvert$
$\mathbf{A}(G)$	adjacency matrix of graph G
$\mathbf{D}(G)$	degree matrix of graph G
$\mathbf{L}(G)$	Laplacian of graph G
$\mathbf{L}_w(G)$	Laplacian of a weighted graph G with weights w
$\mathbf{L}^\circ(G)$	Dirichlet matrix of graph G with boundary
$\mathbf{M}(G)$	generalized Laplacian of graph G
$\mathbf{A}$, $\mathbf{L}$, $\mathbf{M}$	respective shorthand for adjacency matrix, graph Laplacian, and generalized graph Laplacian, when graph is clear from context
λ_i	i-th eigenvalue of (generalized) graph Laplacian $\mathbf{L}$ (or $\mathbf{M}$)
$\mathcal{E}_\lambda$	eigenspace of eigenvalue λ
$\lambda^\circ(G)$	lowest Dirichlet eigenvalue of graph G
$\mathfrak{S}(f)$	number of strong nodal domains of function f
$\mathfrak{W}(f)$	number of weak nodal domains of function f
$\mathbf{A} \otimes \mathbf{B}$	Kronecker product of two matrices $\mathbf{A}$ and $\mathbf{B}$
$\mathbf{I}$	identity matrix
$\mathbf{J}$	matrix of all ones
$\rho(\mathbf{A})$	spectral radius of matrix $\mathbf{A}$
$\varphi_I(v)$	Walsh function
$\mathcal{O}(\cdot)$	Landau symbol
$\mathcal{F}_0$	set of all functions f on vertex set $V = V^\circ \cup \partial V$ with $f\vert_{\partial V} = 0$

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