

# Notation Index

$\mathbb{N}$	set of natural numbers $\{1, 2, 3, \dots\}$	
$\mathbb{Z}$ ( $\mathbb{Z}_+$ )	set of (nonnegative) integers	
$\mathbb{Q}$ ( $\mathbb{Q}_+$ )	set of (nonnegative) rationals	
$\mathbb{R}$ ( $\mathbb{R}_+$ )	set of (nonnegative) real numbers	
$\subset$	proper subset	
$\subseteq$	subset or equal	
$\dot{\cup}$	disjoint union	
$X \Delta Y$	symmetric difference of sets $X$ and $Y$	
$\ x\ _2$	Euclidean norm of a vector $x$	
$\ x\ _\infty$	infinity-norm of a vector $x$	
$x \bmod y$	the unique number $z$ with $0 \leq z < y$ and $\frac{x-z}{y} \in \mathbb{Z}$	
$x^T, A^T$	transpose of vector $x$ and matrix $A$	
$\lceil x \rceil$	smallest integer greater than or equal to $x$	
$\lfloor x \rfloor$	greatest integer less than or equal to $x$	
$f = O(g)$	$O$ -notation	4
$f = \Theta(g)$	$\Theta$ -notation	4
size( $x$ )	encoding length of $x$ ; length of the binary string $x$	6, 71, 360
$\log x$	logarithm of $x$ with basis 2	6
$V(G)$	vertex set of graph $G$	13
$E(G)$	edge set of graph $G$	13
$G[X]$	subgraph of $G$ induced by $X \subseteq V(G)$	14
$G - v$	subgraph of $G$ induced by $V(G) \setminus \{v\}$	14
$G - e$	graph obtained by deleting edge $e$ from $G$	14
$G + e$	graph obtained by adding edge $e$ to $G$	14
$G + H$	sum of graphs $G$ and $H$	14
$G/X$	the graph resulting from $G$ by contracting the subset $X$ of vertices	14
$E(X, Y)$	set of edges between $X \setminus Y$ and $Y \setminus X$	14
$E^+(X, Y)$	set of directed edges from $X \setminus Y$ to $Y \setminus X$	14
$\delta(X), \delta(v)$	$E(X, V(G) \setminus X), E(\{v\}, V(G) \setminus \{v\})$	14
$\Gamma(X), \Gamma(v)$	set of neighbours of vertex set $X$ , of vertex $v$	14
$\delta^+(X), \delta^+(v)$	set of edges leaving vertex set $X$ , vertex $v$	14
$\delta^-(X), \delta^-(v)$	set of edges entering vertex set $X$ , vertex $v$	14
$2^S$	power set of $S$	15

$K_n$	complete graph on $n$ vertices	15
$P_{[x,y]}$	$x$ - $y$ -subpath of $P$	16
$\text{dist}(v, w)$	length of the shortest $x$ - $y$ -path	16
$c(F)$	$\sum_{e \in F} c(e)$ (assuming that $c : E \rightarrow \mathbb{R}$ and $F \subseteq E$ )	16
$K_{n,m}$	complete bipartite graph on $n$ and $m$ vertices	32
$cr(J, l)$	number of times the polygon $J$ crosses line $l$	34, 534
$G^*$	planar dual of $G$	40
$e^*$	an edge of $G^*$ ; the dual of $e$	40
$x^\top y, xy$	scalar product of the vectors $x$ and $y$	49
$x \leq y$	for vectors $x$ and $y$ : inequality holds in each component	49
$\text{rank}(A)$	the rank of matrix $A$	51
$\dim X$	dimension of a nonempty set $X \subseteq \mathbb{R}^n$	51
$I$	identity matrix	53
$e_j$	$j$ -th unit vector	53
$A_J$	submatrix of $A$ consisting of the rows in $J$ only	54
$b_J$	subvector of $b$ consisting of the components with indices in $J$	54
$\mathbb{1}$	vector whose components are all one	57
$A^J$	submatrix of $A$ consisting of the columns in $J$ only	57
$\text{conv}(X)$	convex hull of all vectors in $X$	65
$\det A$	determinant of a matrix $A$	72
$\text{sgn}(\pi)$	signum of permutation $\pi$	73
$E(A, x)$	ellipsoid	81
$B(x, r)$	Euclidean ball with center $x$ and radius $r$	81
$\text{volume}(X)$	volume of the non-empty set $X \subseteq \mathbb{R}^n$	81
$\ A\ $	norm of matrix $A$	82
$X^\circ$	polar set of $X$	93
$P_I$	integer hull of polyhedron $P$	99
$\Xi(A)$	maximum absolute value of the subdeterminants of matrix $A$	101
$P', P^{(i)}$	first and $i$ -th Gomory-Chvátal truncation of $P$	115
$LR(\lambda)$	Lagrangian relaxation	120
$\delta(X_1, \dots, X_p)$	multicut	141
$c_\pi((x, y))$	reduced cost of edge $(x, y)$ with respect to $\pi$	155
$(\bar{G}, \bar{c})$	metric closure of $(G, c)$	157
$\text{ex}_f(v)$	difference between incoming and outgoing flow of $v$	165
$\text{value}(f)$	value of an $s$ - $t$ -flow $f$	165
$\leftrightarrow$		
$\bar{G}$	digraph resulting from $G$ by adding the reverse edges	167
$\overleftarrow{e}$	reverse edge of directed edge $e$	167
$u_f(e)$	residual capacity of edge $e$ with respect to flow $f$	167
$G_f$	residual graph with respect to flow $f$	167
$G_f^L$	level graph of $G_f$	174
$\lambda_{st}$	minimum capacity of a cut separating $s$ and $t$	181
$\lambda(G)$	minimum capacity of a cut in $G$ (edge-connectivity)	188

$\nu(G)$	maximum cardinality of a matching in $G$	228
$\tau(G)$	minimum cardinality of a vertex cover in $G$	228
$T_G(x)$	Tutte matrix of $G$ , depending on vector $x$	230
$q_G(X)$	number of odd connected components in $G - X$	232
$\alpha(G)$	maximum cardinality of a stable set in $G$	250
$\zeta(G)$	minimum cardinality of an edge cover in $G$	250
$r(X)$	rank of $X$ in an independence system	305
$\sigma(X)$	closure of $X$ in an independence system	305
$\mathcal{M}(G)$	cycle matroid of an undirected graph $G$	308
$\rho(X)$	lower rank of $X$ in an independence system	308
$q(E, \mathcal{F})$	rank quotient of an independence system $(E, \mathcal{F})$	308
$C(X, e)$	for $X \in \mathcal{F}$ : the unique circuit in $X \cup \{e\}$ (or $\emptyset$ if $X \cup \{e\} \in \mathcal{F}$ )	313
$(E, \mathcal{F}^*)$	dual of an independence system $(E, \mathcal{F})$	313
$P(f)$	polymatroid for a submodular function $f$	341
$\sqcup$	blank symbol	360
$\{0, 1\}^*$	set of all binary strings	360
$P$	class of polynomially solvable decision problems	368
$NP$	class of decision problems with certificates for yes-instances	368
$\bar{x}$	negation of the literal $x$	371
$coNP$	class of complements of problems in $NP$	382
$OPT(x)$	value of an optimum solution for instance $x$	384
$A(x)$	value of the output of algorithm $A$ for an optimization problem on input $x$	384
$\text{largest}(x)$	largest integer appearing in instance $x$	385
$H(n)$	$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$	394
$\chi(G)$	chromatic number of $G$	409
$\omega(G)$	maximum cardinality of a clique in $G$	409
$\text{Exp}(X)$	expectation of the random variable $X$	416
$\text{Prob}(X)$	probability of an event $X$	416
$\text{SUM}(I)$	sum over all elements in $I$	449
$NF(I)$	output of the NEXT-FIT ALGORITHM for instance $I$	451
$FF(I)$	output of the FIRST-FIT ALGORITHM for instance $I$	451
$FFD(I)$	output of the FIRST-FIT-DECREASING ALGORITHM for instance $I$	453
$G_i^{(a,b)}$	shifted grid	533
$Q(n)$	convex hull of the incidence vectors of the tours in $K_n$	545
$HK(K_n, c)$	Held-Karp bound for the TSP instance $(K_n, c)$	552
$c_F(X), c_F(x)$	facility cost of a solution	565, 585
$c_S(X), c_S(x)$	service cost of a solution	565, 585

## Author Index

- Aardal, K., 355, 565–567, 575, 595, 597, 598
- Aarts, E., 545, 559, 561
- Ackermann, W., 130, 133
- Adleman, L.M., 383, 391
- Agrawal, A., 502, 523
- Agrawal, M., 383, 391
- Aho, A.V., 8, 12, 391
- Ahuja, R.K., 147, 154, 163, 176, 194, 195, 224
- Ajtai, M., 426, 434
- Albrecht, C., 162, 163
- Alizadeh, F., 403, 434
- Alspach, B., 335
- Alt, H., 229, 254
- Anderson, I., 234, 253, 254
- Anstee, R.P., 299, 303
- Aoshima, K., 32, 46
- Appel, K., 411, 434
- Applegate, D., 543, 553, 556, 559, 560
- Archer, A., 577, 596
- Arkin, E.M., 222, 224
- Armstrong, R.D., 218, 224
- Arora, S., 422, 423, 434, 493, 523, 532, 535, 537–539, 560, 565, 596
- Arya, V., 580, 581, 583, 597
- Asano, T., 420, 434
- Ausiello, G., 391, 422, 434
- Avi-Itzak, B., 148
- Avis, D., 56, 68
- Bachem, A., 283, 302, 356
- Baker, B.S., 454, 464
- Balakrishnan, V.K., 147
- Balinski, M.L., 281, 282, 563, 566, 597
- Ball, M.O., 148, 254, 276, 281, 282, 302, 523, 559
- Bansal, N., 462, 464
- Bar-Hillel, Y., 12
- Bar-Yehuda, R., 398, 399, 431, 434
- Becker, A., 431, 434
- Becker, M., 483, 488
- Becvar, J., 448
- Bellare, M., 420, 422, 434, 435
- Bellman, R.E., 152, 154, 156–158, 163, 196, 208, 225, 443, 448
- Benders, J.F., 597
- Berge, C., 20, 46, 230, 234, 241, 242, 244, 251, 254, 280, 281, 409, 435
- Berman, L., 495, 496, 525
- Berman, P., 430, 435, 496, 499, 523, 532, 560
- Bern, M., 492, 523
- Bertsimas, D.J., 68, 125, 491, 514, 523, 524
- Bienstock, D., 303, 476, 488, 489, 597, 598
- Birkhoff, G., 259, 280, 282
- Bixby, R.E., 334, 355, 559, 560
- Björner, A., 355
- Bland, R.G., 55, 56, 68, 80, 97
- Blum, M., 440, 441, 448
- Blum, N., 254
- Bock, F.C., 136, 148
- Boesch, F., 487, 488
- Bollobás, B., 46
- Bondy, J.A., 46
- Borchers, A., 498, 523
- Borgwardt, K.-H., 56, 68
- Borradaile, G., 191, 195, 493, 523
- Borůvka, O., 127, 148, 149
- Bovet, D.P., 391
- Boyd, E.A., 119, 125

- Boyd, S.C., 550, 560  
 Brègman, L.M., 253, 254  
 Brooks, R.L., 408, 435  
 Bruns, W., 122, 125  
 Budach, L., 255  
 Burkard, R.E., 529, 560  
 Busacker, R.G., 207, 224  
 Byrka, J., 575, 597
- Camion, P., 43, 44, 46  
 Caprara, A., 299, 303, 462, 464  
 Carathéodory, C., 68, 69, 122, 125, 126  
 Carr, R., 550, 560  
 Cayley, A., 127, 144, 148  
 Chalasani, P., 557, 560  
 Chan, T.M., 157, 163  
 Chandra, B., 540, 560  
 Chang, R.C., 556, 561  
 Chao, K.-M., 148  
 Charikar, M., 573, 578, 585, 597  
 Chazelle, B., 133, 148  
 Chen, B., 587, 596, 598  
 Chen, J., 417, 435  
 Cheng, X., 522, 524  
 Cheriton, D., 133, 148  
 Cheriyan, J., 180, 195, 520, 523  
 Cherkassky, B.V., 156, 163, 174, 195  
 Cheung, K.K.H., 194, 195  
 Chlebík, M., 430, 435  
 Chlebíková, J., 430, 435  
 Cholesky, A.-L., 96, 401, 403  
 Choukhmane, E., 495, 523  
 Christofides, N., 522, 529, 543, 553, 557, 560  
 Chu, Y., 136, 148  
 Chudak, F.A., 476, 488, 567, 597  
 Chudnovsky, M., 409, 435  
 Church, A., 362  
 Chvátal, V., 56, 68, 108, 115, 119, 125, 282, 303, 394, 409, 435, 549, 559, 560  
 Clementi, A.E.F., 430, 435, 496, 523  
 Cobham, A., 6, 12  
 Coffman, E.G., 464  
 Collatz, L., 336  
 Cook, S.A., 371, 374, 375, 391  
 Cook, W., 102, 104, 109, 119, 122, 125, 194, 224, 275, 282, 302, 334, 434, 487, 543, 559, 560, 598
- Cormen, T.H., 12, 147, 163, 194  
 Cornuéjols, G., 435, 596  
 Correa, J.R., 464  
 Cramer, G., 73, 102, 106, 110, 113  
 Crescenzi, P., 391, 434  
 Crestin, J.P., 43  
 Crowder, H., 559, 560  
 Cunningham, W.H., 125, 194, 195, 218, 224, 275, 276, 279, 282, 302, 327, 334, 335, 355, 550, 559, 560
- Dahlhaus, E., 194, 195  
 Dantzig, G.B., 54, 55, 69, 111, 115, 125, 126, 169, 195, 218, 224, 439, 443, 448, 545, 560  
 Deĭneko, V.G., 529, 560  
 Delaunay, B., 145, 149  
 Demers, A., 465  
 Derigs, U., 276, 281, 282  
 Dessouky, M.I., 190, 196  
 Deza, M.M., 126, 432, 435  
 di Battista, G., 597  
 Dial, R.B., 161, 163  
 Diestel, R., 46, 140, 148  
 Dijkstra, E.W., 26, 130, 148, 153, 154, 157, 161, 163, 209, 214, 223, 472, 474, 495  
 Dilworth, R.P., 251, 255  
 Dinic, E.A., 174, 175, 191, 195  
 Dinur, I., 397, 422, 435  
 Dirac, G.A., 43, 46  
 Dixon, B., 133, 148  
 Doig, A.G., 554, 561  
 Dreyfus, S.E., 163, 493–495, 520, 523  
 Du, D.-Z., 496–498, 522–525  
 Dulmage, A.L., 251, 256
- Edmonds, J., 6, 12, 23, 46, 79, 95, 97, 107, 108, 125, 136, 137, 139, 140, 142, 145, 147–149, 155, 163, 172–174, 192, 195, 208–210, 224, 227, 229, 232, 241, 243, 246–250, 253–255, 257, 259–262, 265, 275, 277, 282, 286, 287, 291, 294, 299–301, 303, 320, 322–324, 326, 328–330, 333–335, 341–344, 354–356, 382, 391, 560  
 Egerváry, E., 280, 282  
 Egoryčev, G.P., 252, 255

- Eisemann, K., 455, 464  
 Eisenbrand, F., 119, 123, 125  
 Eleutério, V., 476, 488  
 Elias, P., 169, 195  
 Englert, M., 540, 560  
 Erdős, P., 254, 255, 302, 433, 435  
 Erickson, R.E., 520, 523  
 Euler, L., 13, 30, 31, 35, 36, 40, 41, 45–47, 289, 315, 535  
 Even, S., 398, 399, 434, 476, 488
- Faigle, U., 335  
 Fakcharoenphol, J., 155, 163  
 Falikman, D.I., 252, 255  
 Farkas, G., 63, 64, 69, 87, 106  
 Feder, T., 229, 255  
 Feige, U., 396, 420, 422, 423, 432, 435, 576, 597  
 Feinstein, A., 169, 195  
 Feng, Q., 497, 523  
 Fernández-Baca, D., 426, 428, 435  
 Fernandez de la Vega, W., 456–458, 462–464  
 Fischetti, M., 299, 303, 357  
 Fleischer, L.K., 220, 224, 347, 355, 356, 475, 488, 520, 524  
 Floyd, R.W., 157, 158, 162, 163, 448  
 Fonlupt, J., 122, 125  
 Ford, L.R., 154, 156–158, 163, 167–169, 172, 189, 191, 194, 195, 201, 202, 208, 219, 221, 224, 229, 472, 483, 488, 521  
 Fortune, S., 145, 148, 477, 488  
 Fourier, J.B.J., 67, 69  
 Francis, R., 596  
 Frank, A., 88, 97, 143, 147, 148, 187, 193–195, 302, 323, 329, 330, 334, 335, 344, 354, 356, 479, 487, 488, 557, 560  
 Fredman, M.L., 131, 133, 149, 154, 163, 164  
 Fremuth-Paeger, C., 249, 255  
 Friesen, D.K., 417, 435  
 Frieze, A., 557, 560  
 Frobenius, G., 229, 234, 255  
 Fuchs, B., 495, 524  
 Fujishige, S., 174–176, 192, 195, 347, 352, 355, 356  
 Fujito, T., 430, 435
- Fulkerson, D.R., 108, 115, 125, 139, 149, 167–169, 172, 189, 191, 194, 195, 201, 202, 208, 219, 221, 224, 229, 251, 255, 316, 333, 335, 409, 435, 472, 483, 488, 521, 545, 560  
 Furer, M., 366, 391, 412, 435
- Gabow, H.N., 133, 137, 147, 149, 187, 194–196, 275, 282, 327, 334, 335, 505, 506, 509, 520, 521, 524  
 Gabriel, R., 562  
 Gács, P., 87, 97  
 Galbati, G., 557, 560  
 Gale, D., 61, 69, 220, 225  
 Galil, Z., 133, 149, 174, 195  
 Gallai, T., 169, 196, 249, 250, 254, 255, 261, 265  
 Gallo, G., 163  
 Gambosi, G., 391, 434  
 Garcia-Diaz, A., 194  
 Garey, M.R., 385, 391, 397, 399, 434–436, 446–448, 450, 453, 454, 464, 465, 493, 498, 524, 532, 560  
 Garg, N., 473, 488, 597  
 Gärtner, B., 68  
 Gauss, C.F., 77  
 Gavril, F., 397, 398  
 Geelen, J.F., 232, 255  
 Geiger, D., 431, 434  
 Gens, G.V., 445, 448  
 Geoffrion, A.M., 120, 126  
 Gerards, A.M.H., 125, 254, 282, 299, 302  
 Ghouila-Houri, A., 112, 126, 343  
 Gilbert, E.N., 495, 523, 524  
 Giles, R., 23, 46, 107, 108, 125, 126, 355, 356  
 Gilmore, P.C., 459, 464  
 Goemans, M.X., 95, 124, 126, 400, 403, 404, 417, 418, 420, 432, 434–436, 491, 503, 505, 506, 509, 510, 513, 521, 522, 524, 525  
 Goffin, J.L., 120, 126  
 Goldberg, A.V., 155, 156, 163, 164, 176, 180, 194, 196, 206, 223–225, 249, 255, 524  
 Goldfarb, D., 80, 97  
 Goldreich, O., 422, 435  
 Goldwasser, S., 435

- Gomory, R.E., 108, 115, 116, 119, 126, 180–183, 186, 188, 192, 193, 196, 282, 295, 296, 299, 302, 303, 459, 464, 504–506, 521
- Gondran, M., 147, 163, 194, 224, 335
- Gonzalez, T., 527, 562
- Gowen, P.J., 207, 224
- Graham, R.L., 46, 194, 225, 254, 282, 334, 355, 463–465, 493, 524, 532, 560
- Graver, J.E., 103, 126
- Graves, R.L., 126
- Graves, S.C., 465
- Grigoriadis, M.D., 475, 488
- Gröpl, C., 502, 524
- Grötschel, M., 46, 76, 80, 82, 85, 89, 90, 93, 95–97, 194, 223, 225, 254, 281–283, 302, 334, 345, 347, 355, 356, 411, 436, 459–461, 463, 479, 523, 546, 549–551, 560
- Guan, M., 289, 303
- Gubeladze, J., 125
- Guha, S., 564, 573, 575, 576, 585, 597
- Gusfield, D., 186, 196
- Gutin, G., 559
- Guy, R., 148, 303, 335, 356
- Hadlock, F., 46, 300, 303
- Hajnal, A., 488
- Haken, W., 411, 434
- Hall, M., 196, 225
- Hall, P., 228, 229, 251, 255
- Halldórsson, M.M., 430, 436
- Halmos, P.R., 228, 255
- Hamburger, M.J., 563, 597
- Hammer, P.L., 46, 125, 283, 335, 356, 436, 464
- Han, Y., 11, 12
- Hanan, M., 493, 524
- Hanani, H., 148, 303, 335, 356
- Hao, J., 187, 196
- Harvey, W., 119, 126
- Hassin, R., 222, 225
- Håstad, J., 405, 422, 423, 432, 436
- Hausmann, D., 309, 318, 319, 333, 335
- Heawood, P.J., 411, 436
- Held, M., 551–553, 555, 556, 560–562
- Hell, P., 335
- Henk, M., 125
- Henzinger, M.R., 154, 164, 194, 196
- Hetzl, A., 493, 524
- Hierholzer, C., 30, 47
- Hitchcock, F.L., 195, 200, 225
- Hochbaum, D.S., 123, 126, 399, 431, 434, 436, 463, 464, 489, 522, 565, 597
- Hoey, D., 145, 149
- Hoffman, A.J., 52, 69, 107, 110, 112, 126, 190, 196, 221, 225, 259
- Holyer, I., 406, 436
- Hopcroft, J.E., 8, 12, 40, 47, 229, 249, 252, 255, 366, 391, 392, 477, 488
- Hoppe, B., 220, 225
- Horowitz, E., 391, 434, 463, 464
- Hougardy, S., 422, 436, 496, 524
- Hsu, W.L., 431, 436
- Hu, T.C., 180–183, 186, 188, 191–193, 196, 295, 296, 299, 302, 487, 488, 504–506, 521
- Hurkens, C.A.J., 528, 561
- Hwang, F.K., 496, 522–524
- Hwang, R.Z., 556, 561
- Ibaraki, T., 187, 188, 194–196, 353, 356, 477, 488
- Ibarra, O.H., 444, 445, 448
- Iri, M., 32, 46, 207, 225
- Itai, A., 476, 488
- Iudin, D.B., 80, 97
- Iwama, K., 434
- Iwata, S., 232, 255, 347, 351, 355, 356
- Iyengar, G., 476, 488
- Jain, K., 514–516, 519, 520, 522, 524, 565, 567–570, 572, 576, 586, 595, 597
- Jansen, K., 596, 597
- Jarník, V., 130, 149
- Jenkyns, T.A., 309, 319, 335
- Jensen, P.M., 354, 356
- Jewell, W.S., 207, 225
- Jin, Z., 218, 224
- John, F., 81
- Johnson, D.B., 154, 164
- Johnson, D.S., 195, 385, 391, 394, 396, 397, 399, 417, 420, 434–436, 446–448, 450, 453, 454, 464, 465, 493, 498, 524, 532, 553, 558, 560, 561

- Johnson, E.L., 46, 125, 283, 287, 291, 294, 300, 301, 303, 335, 356, 436, 464
- Johnson, S., 115, 125, 545, 560
- Jothi, R., 520, 524
- Jünger, M., 488, 556, 559, 561
- Jungnickel, D., 194, 224, 249, 255, 559
- Kahn, A.B., 29, 47
- Kaibel, V., 488
- Kann, V., 391, 434
- Kannan, R., 119, 125, 126
- Kaplan, H., 557, 561
- Karakostas, G., 476, 488
- Karel, C., 561
- Karger, D.R., 133, 149, 187, 193, 196
- Karloff, H., 540, 560
- Karmarkar, N., 88, 97, 459, 461–465
- Karp, R.M., 89, 97, 135, 137, 149, 155, 159, 160, 163, 164, 172–174, 192, 195, 208–210, 224, 229, 249, 252, 255, 371, 375, 376, 379–381, 391, 392, 459, 461–465, 479, 488, 492, 524, 532, 551–553, 555, 556, 560–562
- Karpinski, M., 496, 524, 532, 560
- Karzanov, A.V., 174, 196, 249, 255, 485, 489
- Kayal, N., 383, 391
- Kellerer, H., 445, 448
- Kelner, J.A., 56, 69
- Kenyon-Mathieu, C., 464, 493, 523
- Kern, W., 524
- Kernighan, B.W., 541–543, 551, 557, 558, 560, 561
- Khachiyan, L.G., 80, 86, 87, 97, 368, 462, 475, 488
- Khandekar, R., 597
- Khanna, S., 408, 436
- Khintchine, A., 76, 97
- Khuller, S., 520, 524, 525, 564, 575, 576, 596, 597
- Kim, C.E., 444, 445, 448
- King, V., 133, 149, 176, 196
- Klee, V., 56, 69
- Klein, M., 202, 203, 225
- Klein, P.N., 133, 149, 164, 191, 195, 493, 502, 503, 523, 525, 539, 561
- Klinz, B., 220, 225
- Knuth, D.E., 12, 29, 47, 145, 149, 403, 436
- Koch, J., 411, 434
- Kolliopoulos, S.G., 565, 597
- Könemann, J., 473, 488
- König, D., 26, 32, 47, 122, 228, 229, 250, 251, 255, 256, 280, 406, 436
- Koopmans, T.C., 69
- Korst, J., 545, 561
- Korte, B., 46, 125, 130, 149, 163, 194, 224, 283, 302, 309, 318, 319, 321, 333, 335, 336, 341, 354–356, 436, 446, 448, 464, 487, 493, 525, 560
- Kortsarz, G., 520, 525
- Korupolu, M., 579, 587, 597
- Kou, L., 495, 496, 525
- Krauthgamer, R., 520, 525
- Krentel, M.W., 558, 561
- Kroghdahl, S., 334
- Kruskal, J.B., 52, 69, 110, 112, 126, 129, 138, 139, 145, 149, 259, 318, 322, 339
- Kuehn, A.A., 563, 597
- Kuhn, H.W., 61, 67, 69, 126, 195, 229, 255, 258, 281, 283
- Kuich, W., 197
- Kumar, M.P., 174, 196
- Kuratowski, K., 37, 38, 40, 45, 47
- Ladner, R.E., 383, 392
- Lagergren, J., 426, 428, 435
- Land, A.H., 554, 561
- Lasserre, J.B., 105, 126
- Lau, L.C., 412, 437
- Laurent, M., 432, 435
- Lawler, E.L., 163, 224, 254, 275, 282, 330, 335, 445, 448, 463–465, 559
- Lee, J.R., 520, 525
- Lee, R.C.T., 556, 561
- Legendre, A.M., 35, 47
- Lehman, A., 316, 336
- Leiserson, C.E., 12, 147, 163, 194
- Lenstra, H.W., 457, 465
- Lenstra, J.K., 464, 465, 545, 559
- Letchford, A.N., 299, 303
- Levi, R., 587, 597
- Levin, A., 123, 126
- Levine, M.S., 187, 196
- Levner, E.V., 445, 448



- Lewenstein, M., 561  
 Lewis, H.R., 366, 392  
 Lewis, P.M., 528, 562  
 Lieberherr, K., 433, 436  
 Lin, S., 541–543, 551, 557, 558, 560, 561  
 Linial, N., 408, 436  
 Lipton, R.J., 275, 283  
 Little, J.D.C., 554, 561  
 Liu, T., 136, 148  
 Liu, X., 435  
 Lomonosov, M.V., 485, 489  
 Lovász, L., 46, 76, 80, 82, 85, 87, 89, 90, 93, 95–97, 143, 148, 149, 194, 223–225, 231, 232, 235, 236, 244, 254, 255, 280–283, 302, 334, 341, 345, 347, 354–356, 394, 405, 409, 411, 434–436, 459–461, 463, 479, 487–489, 525  
 Löwner, K., 81  
 Lucchesi, C.L., 478, 479, 487, 489  
 Lueker, G.S., 456–458, 462–464  
 Lund, C., 434  
  
 Mader, W., 140, 188, 191, 196  
 Maffioli, F., 557, 560  
 Magnanti, T.L., 145–148, 163, 194, 224, 254, 282, 302, 523, 559  
 Mahajan, S., 405, 420, 437  
 Mahdian, M., 572, 575, 585–588, 596, 597  
 Maheshwari, S.N., 174, 180, 195, 196  
 Malhotra, V.M., 174, 196  
 Mangaserian, O., 448  
 Manne, A.S., 563, 597  
 Manu, K.S., 147, 149  
 Marchetti-Spaccamela, A., 391, 434  
 Markakis, E., 597  
 Markowsky, G., 495, 496, 525  
 Marsh, A.B., 275, 276, 279, 282, 286, 299, 303  
 Martello, S., 447  
 Martin, A., 125, 493, 525  
 Matoušek, J., 68  
 Matsumoto, K., 481, 489  
 Matsuyama, A., 495, 525  
 Matula, D.W., 473, 489  
 McCormick, S.T., 355  
 McGeoch, L.A., 553, 561  
  
 Megiddo, N., 161, 164  
 Mehlhorn, K., 163, 180, 195, 254, 275, 283, 483, 488, 496, 525  
 Meinardus, G., 336  
 Melkonian, V., 520, 525  
 Mendelsohn, N.S., 251, 256  
 Menger, K., 166, 170–172, 191, 193, 196, 197, 228, 301, 467, 477  
 Meyer, R.R., 100, 101, 126, 448  
 Meyerson, A., 597  
 Micali, S., 249, 256  
 Michiels, W., 545, 561  
 Middendorf, M., 480, 489  
 Mihail, M., 525  
 Miklós, D., 302  
 Milková, E., 127, 149  
 Miller, D.J., 335  
 Miller, R.E., 392, 488, 524  
 Minkowski, H., 53, 64, 65, 69  
 Minoux, M., 147, 163, 194, 224, 335  
 Minty, G.J., 19, 47, 56, 69  
 Mirchandani, P., 596  
 Mitchell, J., 532, 561  
 Mölle, D., 524  
 Monge, G., 258, 283  
 Monma, C.L., 148, 254, 282, 302, 318, 321, 336, 520, 523, 559  
 Moore, E.F., 26, 47, 154, 156–158, 164, 208, 495  
 Motwani, R., 229, 255, 434, 557, 560  
 Motzkin, T.S., 67, 69  
 Mucha, M., 232, 256  
 Mulmuley, K., 232, 256  
 Munagala, K., 597  
 Munkres, J., 258, 283  
 Murty, K.G., 561  
 Murty, U.S.R., 46  
  
 Naddef, D., 556, 561  
 Nagamochi, H., 187, 188, 194–196, 353, 356  
 Näher, S., 283  
 Namaad, A., 174, 195  
 Nash-Williams, C.S.J.A., 140, 141, 143, 149, 327, 336, 477, 487, 489  
 Nemhauser, G.L., 46, 122, 125, 148, 254, 282, 302, 303, 355, 356, 431, 436, 489, 523, 559, 596–598  
 Nemirovskii, A.S., 80, 97

- Nešetřil, J., 127, 130, 149  
 Nešetřilová, H., 127, 149  
 Nicholls, W., 145, 149  
 Nierhoff, T., 524  
 Nishizeki, T., 481, 486, 489  
 Nolles, W., 556, 562  
  
 Okamura, H., 481, 486, 487, 489  
 Orden, A., 55, 69, 200, 225  
 Ore, O., 221, 225  
 Orlin, J.B., 147, 162–164, 176, 187, 194–  
 196, 211–214, 218, 223–225, 357  
 Oxley, J.G., 335  
  
 Padberg, M.W., 68, 89, 96, 97, 296, 299,  
 303, 546, 549, 551, 559, 560  
 Pál, M., 585, 587, 588, 597, 598  
 Pallottino, S., 163  
 Pandit, V., 597  
 Papadimitriou, C.H., 89, 97, 195, 254,  
 282, 366, 379, 389, 391, 392, 396,  
 405, 415, 425, 426, 428–430, 433,  
 434, 437, 447, 451, 465, 530, 532,  
 544, 551, 558, 559, 561  
 Pardalos, P.M., 164  
 Paul, M., 254  
 Petersen, J., 253, 256, 515, 522  
 Pettie, S., 154, 157, 164  
 Pfeiffer, F., 480, 489  
 Pferschy, U., 445, 448  
 Phillips, D.T., 194  
 Phillips, S., 190, 196  
 Picard, J., 190, 196  
 Pisinger, D., 444, 448  
 Plassmann, P., 492, 523  
 Plaxton, C., 579, 587, 597  
 Plotkin, S.A., 212, 213, 218, 225, 462,  
 465, 524  
 Plummer, M.D., 236, 244, 254, 302  
 Poljak, S., 477, 488  
 Pollak, H.O., 495, 523, 524  
 Polyak, B.T., 120, 126  
 Pomerance, C., 383, 391  
 Pratt, V., 383, 392, 448  
 Prim, R.C., 130, 131, 133, 145, 149, 153,  
 339, 496  
 Prömel, H.J., 194, 224, 422, 436, 487,  
 493, 496, 523–525  
 Protasi, M., 391, 434  
  
 Prüfer, H., 144, 149  
 Pulleyblank, W.R., 108, 125, 126, 194,  
 224, 254, 281, 282, 287, 302, 303,  
 334, 550, 551, 559, 560  
 Punnen, A.P., 559  
  
 Queyranne, M., 189, 190, 196, 351, 352,  
 356, 357  
  
 Rabin, M.O., 232, 256  
 Radhakrishnan, J., 430, 436  
 Rado, R., 320, 322, 334, 336  
 Radzik, T., 161, 164  
 Raghavachari, B., 412, 435, 520, 524  
 Raghavan, P., 418, 437, 485, 489, 565,  
 596  
 Rajagopalan, R., 577, 596  
 Rajaraman, R., 579, 587, 597  
 Ramachandran, V., 154, 164  
 Ramaiyer, V., 496, 499, 523  
 Raman, R., 154, 164  
 Ramesh, H., 405, 420, 437  
 Rao, A., 557, 560  
 Rao, M.R., 89, 97, 296, 299, 303  
 Rao, S., 155, 163, 164, 176, 194, 196,  
 539, 562, 565, 596, 597  
 Rauch, M., 133, 148  
 Ravi, R., 502, 503, 523, 525  
 Raz, R., 396, 437  
 Recski, A., 335  
 Rédei, L., 44, 47  
 Reed, B.A., 436  
 Reinelt, G., 299, 303, 559  
 Reingold, O., 597  
 Richards, D.S., 522  
 Richter, S., 524  
 Rinaldi, G., 559  
 Rinnooy Kan, A.H.G., 464, 465, 559  
 Ripphausen-Lipa, H., 487  
 Rivest, R.L., 12, 147, 163, 194, 448  
 Rizzi, R., 250, 256, 302, 303, 353, 357  
 Robbins, H.E., 44, 47, 487, 488  
 Robertson, N., 44, 45, 47, 411, 412, 435,  
 437, 481, 486, 489  
 Robins, G., 496, 497, 500, 502, 525  
 Robinson, S.M., 448  
 Röck, H., 355, 356  
 Röglin, H., 540, 560  
 Rohe, A., 275, 282, 543, 560

- Rolim, J.D.P., 597  
Rose, D.J., 193, 196  
Rosenberg, I.G., 126  
Rosenkrantz, D.J., 528, 562  
Rosenstiehl, P., 336  
Rossmanith, P., 524  
Rothberg, E.E., 553, 561  
Rothschild, B., 484, 489  
Rubinstein, J.H., 522, 525  
Ruhe, G., 194, 224  
Rumely, R.S., 383, 391  
Rustin, R., 148
- Saber, A., 597  
Safra, S., 396, 397, 408, 422, 434–437  
Sahni, S., 391, 434, 445, 448, 463, 464, 527, 562  
Saito, N., 481, 489  
Sales, C.L., 436  
Sanders, D.P., 437  
Sanders, P., 413, 437  
Sankowski, P., 232, 256  
Sauer, N., 148, 303, 335, 356  
Saxena, N., 383, 391  
Schäfer, G., 275, 283  
Scheffler, P., 486, 489  
Schietke, J., 163  
Schönhage, A., 366, 392  
Schonheim, J., 148, 303, 335, 356  
Schrader, R., 341, 355, 446, 448  
Schrijver, A., 65, 68, 76, 77, 80, 82, 85, 89, 90, 93, 95–97, 109, 115, 117–119, 121, 122, 125, 126, 148, 163, 194, 224, 225, 253, 254, 256, 280, 283, 300, 302, 334, 335, 345, 347–350, 355–357, 411, 436, 459–461, 463, 479, 487, 488, 525, 559  
Schulz, A.S., 123, 125, 223, 225, 598  
Sebő, A., 122, 126, 294, 303  
Seiden, S.S., 455, 465  
Seymour, P.D., 44, 45, 47, 115, 126, 195, 293, 303, 333, 336, 434, 435, 437, 480, 481, 485–487, 489  
Shafir, N., 561  
Shahrokhi, F., 473, 489  
Shamir, A., 476, 488  
Shamos, M.I., 145, 149  
Shannon, C.E., 169, 195  
Shenoy, N., 145, 149  
Shiloach, Y., 174, 191, 197  
Shioura, A., 176, 197  
Shisha, O., 69  
Shmoys, D.B., 431, 434, 436, 462–465, 473, 489, 524, 553, 559, 562, 565–567, 577, 587, 595–598  
Shor, N.Z., 80, 97  
Silverberg, E.B., 222, 224  
Simchi-Levi, D., 453, 465  
Singh, M., 412, 437  
Skutella, M., 124, 126, 220, 224  
Slavík, P., 396, 437  
Sleator, D.D., 174–176, 197  
Smith, J.M., 522, 525  
Smith, W.D., 539, 562  
Sós, V.T., 148, 302, 356, 488  
Specker, E., 433, 436  
Spencer, T., 133, 149  
Sperner, E., 251, 256  
Spielman, D.A., 56, 69  
Spirakis, P., 225  
Stearns, R.E., 528, 562  
Steger, A., 422, 436, 493, 523, 525  
Steiglitz, K., 254, 282, 379, 391, 396, 437, 447, 544, 558, 559, 561  
Stein, C., 12, 147, 163, 193, 194, 196  
Steinitz, E., 65, 69, 96, 97  
Steuere, D., 413, 437  
Stirling, J., 3, 12  
Stockmeyer, L., 385, 391, 399, 406, 436, 437  
Stoer, M., 187, 188, 197, 523  
Stollsteimer, J.F., 563, 598  
Strassen, V., 366, 392  
Su, X.Y., 191, 197  
Subramanian, S., 164  
Sudan, M., 420, 422, 434, 435  
Sviridenko, M., 464, 561, 564, 567, 576, 598  
Swamy, C., 587, 597  
Swamy, M.N.S., 195  
Sweeny, D.W., 561  
Szegedy, B., 253, 256  
Szegedy, C., 253, 256  
Szegedy, M., 434, 435  
Szigeti, Z., 253, 256  
Szőnyi, T., 302
- Takada, H., 434

- Takahashi, M., 495, 525  
Tang, L., 194, 195  
Tardos, É., 88, 97, 125, 194, 205, 212, 213, 218, 220, 224, 225, 334, 335, 462, 465, 487, 488, 520, 524, 525, 565–567, 587, 588, 595, 597, 598  
Tarjan, R.E., 29, 40, 47, 130, 131, 133, 145, 148, 149, 154, 163, 175, 176, 180, 194–197, 206, 223–225, 254, 275, 283, 448  
Tarry, G., 26  
Taub, A.H., 69  
Teng, S.-H., 56, 69  
Teo, C., 491, 514, 523  
Thatcher, J.W., 392, 488, 524  
Theis, D.O., 299, 303  
Thimm, M., 496, 525  
Thomas, R., 435, 437  
Thomassen, C., 33, 37, 38, 47  
Thompson, C.D., 418, 437, 485, 489  
Thorup, M., 154, 164  
Thulasiraman, K., 195  
Tindell, R., 487, 488  
Todd, M.J., 80, 97  
Tolstoï, A.N., 202, 225  
Tomizawa, N., 208, 226  
Toth, P., 447  
Tovey, C., 540, 560  
Trémaux, C.P., 26  
Trevisan, L., 430, 433, 435, 437, 496, 523  
Triesch, E., 556, 560, 562  
Tsitsiklis, J.N., 68  
Tucker, A.W., 61, 69, 126, 195  
Tunçel, L., 180, 197  
Turing, A.M., 359–362, 365–368, 371, 372, 387–389, 392  
Tutte, W.T., 35, 37, 47, 140, 141, 149, 230–234, 244, 253, 256, 280, 281, 289, 300, 303, 489  
Ullman, J.D., 8, 12, 366, 391, 392, 465  
Vaidya, P.M., 95, 97  
van der Waerden, B.L., 253, 255  
van Emde Boas, P., 366, 391  
van Leeuwen, J., 391  
van Vliet, A., 455, 465  
Varadarajan, K.R., 275, 283  
Varadarajan, S., 520, 524  
Vaughan, H.E., 228, 255  
Vazirani, U.V., 232, 256  
Vazirani, V.V., 194, 197, 232, 249, 256, 434, 523, 525, 567, 568, 570, 576, 586, 595, 597  
Veinott, A.F., 111, 126, 520, 523  
Vempala, S., 532, 561  
Vetta, A., 520, 523  
Vishkin, U., 520, 525  
Vizing, V.G., 407, 408, 413, 437  
Vöcking, B., 540, 560  
von Neumann, J., 61, 69, 259, 280, 283  
von Randow, R., 335  
Voronoi, G., 145  
Vušković, K., 435  
Vygen, J., 163, 191, 197, 214, 226, 350, 357, 440, 448, 476, 477, 480, 486, 488, 489, 556, 560, 562, 565, 576, 588, 590, 594, 596, 598  
Wagner, D., 283, 483, 487, 489  
Wagner, F., 187, 188, 197  
Wagner, H.M., 200, 226  
Wagner, K., 37, 40, 47  
Wagner, R.A., 493–495, 520, 523  
Wang, X., 524  
Warme, D.M., 496, 525  
Warshall, S., 157, 158, 162, 164  
Weber, G.M., 276, 283  
Wegener, I., 391  
Weihe, K., 191, 197, 483, 487, 489  
Weismantel, R., 125, 223, 225, 355  
Welsh, D.J.A., 335  
Werners, B., 562  
Wetterling, W., 336  
Wexler, T., 587, 588, 598  
Weyl, H., 64, 65, 69  
Whinston, A., 484, 489  
White, N., 335, 355  
Whitney, H., 29, 42, 46, 47, 171, 197, 314, 315, 336  
Wigderson, A., 432, 437  
Willard, D.E., 133, 149, 154, 164  
Williamson, D.P., 357, 400, 403, 404, 417, 418, 420, 434, 436, 503, 505, 506, 509–512, 520–522, 524, 525, 553, 562  
Wilson, R.J., 46

- Winter, P., 496, 522, 525  
Woeginger, G.J., 220, 225, 446, 448, 528,  
529, 556, 560–562  
Wolfe, P., 55, 69, 126, 563, 597  
Wolsey, L.A., 104, 122, 125, 126, 145,  
146, 148, 553, 562, 596  
Wu, B.Y., 148  
Wyllie, J., 477, 488  
  
Xu, Y., 327, 335  
  
Yamashita, Y., 434  
Yannakakis, M., 194, 195, 197, 391, 392,  
405, 415, 417, 425, 426, 428–430,  
433, 437, 530, 532, 558, 561  
Yao, A.C., 464  
Ye, Y., 572, 575, 586, 587, 596–598  
Young, N., 473, 489  
  
Younger, D.H., 478, 479, 487, 489  
Yue, M., 454, 465  
  
Zachariasen, M., 496, 525  
Zelikovsky, A.Z., 496, 497, 499, 500,  
502, 524, 525  
Zhang, G., 462, 465  
Zhang, J., 572, 575, 586, 587, 596–598  
Zhang, Y., 497, 523  
Zheng, H., 417, 435  
Zhou, H., 145, 149  
Zhou, X., 486, 489  
Ziegler, G.M., 223, 225, 355  
Zimmermann, U., 355, 356  
Zipkin, P.H., 465  
Zuckerman, D., 408, 423, 437  
Zwick, U., 157, 164, 597

# Subject Index

- $\epsilon$ -DOMINANCE PROBLEM, 446
- $\epsilon$ -dominance, 445
- $\gamma$ -expander, 426
- 0-1-string, 11, 360
- 1-tree, 551, 552
- 2-connected graph, 29
- 2-edge-connected graph, 44
- 2-matching inequalities, 549, 555, 558
- 2-opt, 540
- 2-polymatroid, 354
- 2SAT, 374, 389
- 3-connected graph, 37
- 3-connected planar graph, 96
- 3-cut, 141, 194
- 3-DIMENSIONAL MATCHING (3DM), 379
- 3-dimensional polytope, 96
- 3-MATROID INTERSECTION, 390
- 3-OCCURRENCE MAX-SAT PROBLEM, 426, 428
- 3-OCCURRENCE SAT, 389
- 3-OPT ALGORITHM, 543
- 3-opt, 540, 543
- 3DM, 379
- 3SAT, 374
  
- absolute approximation algorithm, 393, 408, 412, 413, 444
- abstract dual graph, 42, 46, 315
- accessible set system, 337
- Ackermann's function, 130, 133
- active vertex (network flows), 177
- active vertex set (network design), 506, 521
- acyclic digraph, 19, 20, 28, 46
  
- ADD, 588
- adjacency list, 25, 367
- adjacency matrix, 25
- adjacent edges, 13
- adjacent vertices, 13
- admissible edge (network flows), 177
- affinely independent, 66
- algebraic numbers, 366
- algorithm, 5, 384
- algorithm for a decision problem, 367
- ALL PAIRS SHORTEST PATHS PROBLEM, 157, 158, 292, 496
- almost satisfying edge set (network design), 506
- alphabet, 360
- alternating ear-decomposition, 236–238
- alternating forest, 242, 243
- alternating path, 229, 263, 324
- alternating walk, 541
  - closed, 541, 543
  - proper, 541, 543
  - proper closed, 541
- ANOTHER HAMILTONIAN CIRCUIT, 557, 558
- anti-arborescence, 27
- antiblocker, 433
- antichain, 251
- antimatroid, 338, 353
- approximation algorithm
  - absolute, 393, 412, 413, 444
  - asymptotic  $k$ -factor, 413
  - $k$ -factor, 393
- approximation ratio, 413
  - asymptotic, 413

- approximation scheme, 413, 415, 423, 425, 426, 493, 532, 539  
   asymptotic, 413, 415  
   fully polynomial, 414, 445, 446, 473, 474  
   fully polynomial asymptotic, 414, 459, 462  
 arborescence, 18, 24, 139, 142, 143, 147, 339, *see* MINIMUM WEIGHT ARBORESCENCE PROBLEM, *see* MINIMUM WEIGHT ROOTED ARBORESCENCE PROBLEM  
 arborescence polytope, 147  
 arc, 13  
 ARORA'S ALGORITHM, 537–539  
 articulation vertex, 17  
 ASSIGNMENT PROBLEM, 258, 281  
 associated with the ear-decomposition, 236, 272  
 asymmetric traveling salesman problem, 527  
 ASYMMETRIC TSP, 557  
 asymptotic approximation ratio, 413  
 asymptotic approximation scheme, 413, 415, 457  
 asymptotic  $k$ -factor approximation algorithm, 413  
 asymptotic performance ratio, 413  
 augment (network flows), 167  
 augmenting cycle, 201, 202, 204  
 augmenting path, 167, 168, 172, 229, 230, 241, 251, 264  
 average running time, 6, 56  
  
*b*-factor, 285  
*b*-flow, 199, 220  
*b*-flow associated with a spanning tree structure, 216  
*b*-matching, 285, 286, *see* MAXIMUM WEIGHT  $b$ -MATCHING PROBLEM  
*b*-matching polytope, 286, 287, 296  
 backtracking, 3  
 balance, 199  
 BAR-YEHUDA-EVEN ALGORITHM, 398  
 barrier, 233  
 base of a blossom, 241  
 base polyhedron, 347  
 basic solution, 51, 515, 516  
 basis, 59, 305, 309, 332  
  
 basis-superset oracle, 318  
 Bellman's principle of optimality, 152  
 Berge's Theorem, 230, 241, 242, 251  
 Berge-Tutte formula, 234, 244, 280, 281  
 BEST-IN-GREEDY ALGORITHM, 317–322, 333, 339, 394  
 BFS, 26, 29, 152, 330, 496  
 BFS-tree, 26  
 BIN-PACKING PROBLEM, 449–458, 461, 462  
 binary clutter, 333  
 binary representation, 6  
 binary search, 166, 345, 387  
 binary string, 360, 367  
 bipartite graph, 32, 42  
 bipartition, 32, 44  
 bipartition inequalities, 550  
 Birkhoff-von-Neumann Theorem, 259, 280  
 bit, 360  
 Bland's pivot rule, 55  
 block, 29, 44  
 blocking clutter, 315, 316, 333  
 blocking flow, 174, 175, 191  
 blocking polyhedron, 333  
 blossom, 241, 242, 262, 264, 272  
   inner, 244  
   out-of-forest, 262  
   outer, 244  
 blossom forest  
   general, 244, 262, 273  
   special, 244, 353  
 Blossom Shrinking Lemma, 242, 244  
 BLOSSOMPATH, 264  
 Boolean formula, 371  
 Boolean variable, 371  
 bottleneck edge, 168  
 bottleneck function, 333, 341, 353  
 bottleneck matching problem, 281  
 bounded LP, 101  
 bounded polyhedron, 87  
 BRANCH-AND-BOUND, 554, 555  
 branch-and-bound tree, 555, 556  
 branch-and-cut, 556  
 branching, 18, 134, 147, 390, *see* MAXIMUM WEIGHT BRANCHING PROBLEM  
 branching greedoid, 339

- branching polytope, 146
- BREADTH-FIRST SEARCH, *see* BFS
- bridge, 17, 42
- capacity, 165, 167
  - residual, 167
- CAPACITY SCALING ALGORITHM, 209, 210
- Carathéodory's theorem, 68
- CARDINALITY MATCHING PROBLEM, 227, 229, 231, 232, 249, 252
- Cayley's theorem, 144
- certificate, 368, 382
- certificate-checking algorithm, 368, 420, 421
- chain, 251
- CHINESE POSTMAN PROBLEM, 289, 390
  - DIRECTED, 222
- Cholesky factorization, 96
- chordal graph, 193, 432
- CHRISTOFIDES' ALGORITHM, 522, 529, 553, 557
- chromatic index, 406
- chromatic number, 406
  - edge-, 406
- Church's thesis, 362
- Chvátal rank, 119
- circuit, 16, 41, 305, 312
  - Hamiltonian, 16, 333
  - undirected, 18–20
- circulation, 165, 201, 202
- city, 386
- clause, 371
- CLIQUE, 376, 390
- clique, 15, 16, *see* MAXIMUM CLIQUE PROBLEM, *see* MAXIMUM WEIGHT CLIQUE PROBLEM
- clique polytope, 433
- clique tree inequalities, 550, 555, 558
- closed alternating walk, 541, 543
- closed Jordan curve, 33
- closed walk, 16
- closure, 305, 311
- closure operator, 338
- closure oracle, 318
- clutter, 315, 316, 333
- cocycle basis, 20, 21
- cocycle space, 20
- colour, 405
- colouring
  - edge-, 405, 407
  - vertex-, 405, 408, 411
- column generation, 60, 459, 472
- comb inequalities, 549, 550, 555
- comet, 591
- commodity, 467
- complement of a decision problem, 382
- complement of a graph, 15
- complementary slackness, 62, 512, 567
- complete graph, 15
- component
  - connected, 17, 25, 130
  - strongly connected, 19, 27–29
- computable in polynomial time, 6, 361
- computation, 360, 362
- compute a function, 6, 361
- cone, 53
  - finitely generated, 53, 54, 64
  - polyhedral, 53, 54, 64, 101, 102, 122
- conjunctive normal form, 371
- connected component, 17, 25, 130
- connected digraph, 18
- connected region, 33, 34
- connected undirected graph, 17
- connected vertex set, 17
- connectivity
  - edge-, 29, 181, 187, 193
  - vertex-, 29, 188, 193
- connectivity requirements, 503
- connector, 491
- coNP*, 382
- coNP*-complete, 382
- conservative weights, 151, 155, 290
- containing a subgraph, 14
- CONTINUED FRACTION EXPANSION, 74–76, 95
- contraction, 14
- convex combination, 64
- convex function, 354
- convex hull, 64, 65
- convex set, 65, 89
- cost, 384
  - reduced, 155
- covered vertex, 227
- critical path method (CPM), 190
- cross-free family, 21–23, 114
- crossing submodular function, 355



- customer (facility location), 564
- cut, 18, 41, *see* MAXIMUM WEIGHT CUT PROBLEM, *see* MINIMUM CAPACITY CUT PROBLEM
- directed, 18, 192, 478
- $r$ -, 18
- $s$ - $t$ -, 18, 167, 169, 190, 507
- $T$ -, 293, 301
- undirected, 18–20
- cut cancelling algorithm, 222
- cut criterion, 470, 471, 477, 480, 481, 484, 487
- cut semimetric, 432
- cut-incidence matrix
- one-way, 114, 115, 123
- two-way, 114, 115, 123
- cutting plane method, 115, 119, 555
- cutting stock problem, 449
- cycle, 16
- cycle basis, 20, 21, 35, 45
- cycle matroid, 308, 314, 318
- cycle space, 20
- cycle time, 162
- decidable in polynomial time, 361
- decidable language, 361
- decision problem, 367
- decomposition theorem for polyhedra, 67
- degree, 14
- Delaunay triangulation, 145
- demand, 199
- demand edge, 171, 467
- dense graph, 25
- dependent set, 305
- DEPTH-FIRST SEARCH, *see* DFS
- derandomization, 416, 539
- determinant, 72, 77
- DFS, 26, 27, 29, 252
- DFS-forest, 27
- DFS-tree, 26
- digraph, 13
- DIJKSTRA'S ALGORITHM, 26, 153, 154, 157, 161, 209, 472
- Dilworth's Theorem, 251
- dimension, 51
- DINIC'S ALGORITHM, 174, 175
- direct predecessor, 43
- direct successor, 43
- DIRECTED CHINESE POSTMAN PROBLEM, 222
- directed cut, 18, 192, 478
- DIRECTED EDGE-DISJOINT PATHS PROBLEM, 171, 172, 191, 476, 477, 479, 480, 486, 487
- directed graph, *see* digraph
- DIRECTED HAMILTONIAN PATH, 390
- DIRECTED MAXIMUM WEIGHT CUT PROBLEM, 432
- DIRECTED MULTICOMMODITY FLOW PROBLEM, 467, 468
- DIRECTED VERTEX-DISJOINT PATHS PROBLEM, 171, 486
- disconnected, 17
- discrete optimization problem, 384
- DISJOINT PATHS PROBLEM, 171
- DIRECTED EDGE-, 171, 172, 191, 476, 477, 479, 480, 486, 487
- DIRECTED VERTEX-, 171, 486
- EDGE-, 467, 469, 470, 485
- UNDIRECTED EDGE-, 171, 480, 481, 483, 484, 486
- UNDIRECTED VERTEX-, 171, 481, 486
- distance, 16, 386
- distance criterion, 469–471, 479
- distance labeling, 177
- divide and conquer, 9, 275
- dominance relation, 445
- DOMINATING SET, 389
- DOUBLE-TREE ALGORITHM, 529, 557
- doubly stochastic matrix, 252, 259
- DREYFUS-WAGNER ALGORITHM, 494, 495, 520
- DRILLING PROBLEM, 1
- dual complementary slackness conditions, 62
- DUAL FITTING ALGORITHM, 570, 572, 575
- dual graph
- abstract, 42, 46, 315
- planar, 40–42, 300, 314
- dual independence system, 313, 314
- dual LP, 60
- Duality Theorem, 61, 63
- dynamic flows, 218
- dynamic programming, 152, 159, 443, 494, 556

- DYNAMIC PROGRAMMING KNAPSACK ALGORITHM, 443, 444, 460
- dynamic tree, 175, 176
- ear, 29
- ear-decomposition, 29, 44
- alternating, 236–238
  - associated with the, 236, 272
  - $M$ -alternating, 236–238
  - odd, 235, 253
  - proper, 29
- EAR-DECOMPOSITION ALGORITHM, 236, 237
- edge, 13
- edge-chromatic number, 406
- edge-colouring, 405, 407
- EDGE-COLOURING PROBLEM, 406, 408, 413
- edge-connectivity, 29, 181, 187, 193
- edge cover, 15, 250, 253, *see* MINIMUM WEIGHT EDGE COVER PROBLEM
- EDGE-DISJOINT PATHS PROBLEM, 467, 469, 470, 485
- DIRECTED, 171, 172, 191, 476, 477, 479, 480, 486, 487
  - UNDIRECTED, 171, 480, 481, 483, 484, 486
- edge progression, 16, 155, 159
- EDMONDS' BRANCHING ALGORITHM, 136, 137
- EDMONDS' CARDINALITY MATCHING ALGORITHM, 243, 246–250, 261
- EDMONDS' MATROID INTERSECTION ALGORITHM, 324, 326
- EDMONDS-KARP ALGORITHM, 172–174, 229
- Edmonds-Rado Theorem, 320, 322
- efficient algorithm, 6
- elementary step, 5, 365
- ellipsoid, 81, 96
- ELLIPSOID METHOD, 71, 80, 82, 88, 90, 345, 411, 457, 549
- empty graph, 15
- empty string, 360
- endpoints of a path, 16
- endpoints of a simple Jordan curve, 33
- endpoints of an edge, 13
- enumeration, 2, 553
- equivalent problems, 128
- EUCLIDEAN ALGORITHM, 74, 75, 79, 106
- Euclidean ball, 81
- Euclidean norm, 82
- EUCLIDEAN STEINER TREE PROBLEM, 493, 496, 557
- EUCLIDEAN TRAVELING SALESMAN PROBLEM, 532
- EUCLIDEAN TSP, 532, 533, 535, 537, 539, 556
- EULER'S ALGORITHM, 31
- Euler's formula, 35, 36, 315
- Eulerian digraph, 483
- Eulerian graph, 30, 31, 42, 290, 484, 528
- Eulerian walk, 30, 289, 495, 528, 535
- exact algorithm, 384
- expander graph, 426
- extreme point, 65, 68
- $f$ -augmenting cycle, 201, 202
- $f$ -augmenting path, 167, 168
- face of a polyhedron, 51, 52
- face of an embedded graph, 33, 35
- facet, 52, 66
- facet-defining inequality, 52, 551
- facility, 564
- facility cost, 565
- facility location, 124, 563, *see* (METRIC) UNCAPACITATED FACILITY LOCATION PROBLEM, *see* METRIC (SOFT-)CAPACITATED FACILITY LOCATION PROBLEM, *see* UNIVERSAL FACILITY LOCATION PROBLEM
- factor-critical graph, 233, 235–238
- Farkas' Lemma, 63
- fast matrix multiplication, 157
- feasible potential, 155, 156, 202
- feasible solution of an LP, 49
- feasible solution of an optimization problem, 50, 384
- feasible spanning tree structure, 216
- feedback edge set, 316, 479
- feedback number, 479, 485
- feedback vertex set, *see* MINIMUM WEIGHT FEEDBACK VERTEX SET PROBLEM
- FERNANDEZ-DE-LA-VEGA-LUEKER ALGORITHM, 462

- FF, *see* FIRST-FIT ALGORITHM  
 FFD, *see* FIRST-FIT-DECREASING ALGORITHM  
 Fibonacci heap, 131–133, 137, 153, 154, 187  
 Fibonacci number, 95  
 finite basis theorem for polytopes, 65  
 finitely generated cone, 53, 54, 64  
 FIRST-FIT ALGORITHM, 452, 453, 462  
 FIRST-FIT-DECREASING ALGORITHM, 453, 454, 462  
 Five Colour Theorem, 411  
 flow, 165, *see* MAXIMUM FLOW OVER TIME PROBLEM, *see* MAXIMUM FLOW PROBLEM, *see* MINIMUM COST FLOW PROBLEM  
   *b*-, 199, 220  
   blocking, 174, 175, 191  
   *s-t*-, 165, 168, 169  
 flow conservation rule, 165  
 Flow Decomposition Theorem, 169  
 flow over time, 218  
   *s-t*-, 219  
 FLOYD-WARSHALL ALGORITHM, 157, 158, 162  
 forbidden minor, 45  
 FORD-FULKERSON ALGORITHM, 168, 169, 172, 189, 229, 521  
 forest, 17, 143, *see* MAXIMUM WEIGHT FOREST PROBLEM  
 forest polytope, 146  
 Four Colour Problem, 411  
 Four Colour Theorem, 411, 412  
 Fourier-Motzkin elimination, 67  
 FPAS, FPTAS, *see* fully polynomial approximation scheme  
 fractional *b*-matching problem, 222  
 FRACTIONAL KNAPSACK PROBLEM, 439–441  
 fractional matching polytope, 259  
 fractional perfect matching polytope, 259, 280, 282  
 FUJISHIGE'S ALGORITHM, 175, 192  
 full component of a Steiner tree, 497  
 full Steiner tree, 497  
 full-dimensional polyhedron, 51, 87  
 fully polynomial approximation scheme, 414, 445, 446, 473, 474  
 fully polynomial asymptotic approximation scheme, 414, 459, 462  
 fundamental circuit, 20, 21, 43, 217  
 fundamental cut, 21, 296, 505  
 gain of an alternating walk, 541  
 Gallai-Edmonds decomposition, 250, 261, 265  
 Gallai-Edmonds Structure Theorem, 249  
 GAUSSIAN ELIMINATION, 55, 77–80, 87, 95  
 general blossom forest, 244, 262, 273  
 GENERALIZED STEINER TREE PROBLEM, 502, 505  
 girth, 36, 292  
 GOEMANS-WILLIAMSON ALGORITHM FOR MAX-SAT, 418, 420  
 GOEMANS-WILLIAMSON MAX-CUT ALGORITHM, 404  
 Gomory's cutting plane method, 116  
 Gomory-Chvátal-truncation, 115, 282  
 GOMORY-HU ALGORITHM, 183, 186, 521  
 Gomory-Hu tree, 181–183, 186, 295, 296, 299, 504–506, 521  
 good algorithm, 6  
 good characterization, 234, 382  
 graph, 9, 13  
   directed, *see* digraph  
   mixed, 483, 487  
   simple, 13  
   undirected, 13  
 GRAPH SCANNING ALGORITHM, 24, 25  
 graphic matroid, 308, 315, 320, 332  
 greatest common divisor, 74  
 greedoid, 337–341, 353  
 greedy algorithm, 129, 317, 394  
 GREEDY ALGORITHM FOR GREEDOIDS, 339, 340, 353  
 GREEDY ALGORITHM FOR SET COVER, 394  
 GREEDY ALGORITHM FOR VERTEX COVER, 396  
 greedy augmentation, 573, 574, 595  
 GREEDY COLOURING ALGORITHM, 408, 432  
 grid graphs, 493  
 GRÖTSCHEL-LOVÁSZ-SCHRIJVER ALGORITHM, 90, 93, 459–461, 479

- half-ellipsoid, 81, 84
- Hall condition, 228
- Hall's Theorem, 228, 229
- HALTING PROBLEM, 387
- HAMILTONIAN CIRCUIT, 367, 368, 376
- Hamiltonian circuit, 16, 318, 333
- Hamiltonian graph, 16, 43
- HAMILTONIAN PATH, 390
- Hamiltonian path, 16
- handle, 550
- head, 13
- heap, 131
- heap order, 132
- Held-Karp bound, 552, 553, 555
- hereditary graph property, 45
- Hermite normal form, 106
- Hilbert basis, 102, 122
- HITCHCOCK PROBLEM, 200
- Hoffman's circulation theorem, 190
- Hoffman-Kruskal Theorem, 110, 112, 259
- HOPCROFT-KARP ALGORITHM, 229, 252
- Hungarian method, 258, 281
- hypergraph, 21
  
- in-degree, 14
- incidence matrix, 24, 114
- incidence vector, 65, 66
- incident, 13
- independence oracle, 317, 318, 330
- independence system, 305, *see* MAXIMIZATION PROBLEM FOR INDEPENDENCE SYSTEMS, *see* MINIMIZATION PROBLEM FOR INDEPENDENCE SYSTEMS
  - dual, 313, 314
- independent set, 305
- induced subgraph, 14
- infeasible LP, 49, 62, 63
- inner blossom, 244
- inner vertex, 242, 244
- input size, 6
- instance, 367, 384
- integer hull, 99
- INTEGER LINEAR INEQUALITIES, 368, 369, 381
- INTEGER PROGRAMMING, 99, 101, 367, 386, 457
- Integral Flow Theorem, 169
- integral polyhedron, 107, 108, 110, 390
  
- integrality constraints, 99
- interior point algorithms, 71, 88, 403
- intersection of independence systems, 322
- intersection of matroids, 322, 323
- interval graph, 432
- interval matrix, 123
- interval packing, 123, 222
- inverse of a matrix, 77
- isolated vertex, 14
- isomorphic graphs, 14
  
- JAIN'S ALGORITHM, 519
- JAIN-VAZIRANI ALGORITHM, 568
- JOB ASSIGNMENT PROBLEM, 2, 8, 121, 165, 199, 227
- JOHNSON'S ALGORITHM FOR MAX-SAT, 417, 420
- Jordan curve theorem, 33
  
- $k$ -CENTER PROBLEM, 430, 431
- $k$ -connected graph, 29, 171
- $k$ -edge-connected graph, 29, 171
  - strongly, 191, 487
- $k$ -edge-connected subgraph, 503, *see* MINIMUM WEIGHT  $k$ -EDGE-CONNECTED SUBGRAPH PROBLEM
- $k$ -FACILITY LOCATION PROBLEM, 576
- $k$ -factor approximation algorithm, 393
  - asymptotic, 413
- $k$ -MEDIAN PROBLEM, 576
- $k$ -OPT ALGORITHM, 539–541
- $k$ -opt tour, 540, 558
- $k$ -regular graph, 14
- $k$ -restricted Steiner tree, 497
- $K$ -TH HEAVIEST SUBSET, 390
- $K_{3,3}$ , 36, 40
- $K_5$ , 36, 40
- KARMARKAR-KARP ALGORITHM, 459–462, 464
- Karp reduction, 371
- key, 131
- Khachiyan's theorem, 86, 87
- KNAPSACK APPROXIMATION SCHEME, 444, 445
- KNAPSACK PROBLEM, 306, 439, 442–445, 447, 459, 460
- Königsberg, 30
- König's Theorem, 122, 228, 251, 280

- KOU-MARKOWSKY-BERMAN ALGORITHM, 496
- KRUSKAL'S ALGORITHM, 129, 138, 139, 145, 318, 322, 339
- Kuratowski's Theorem, 37, 38, 40, 45
- L-reducible, 425
- L-reduction, 424, 425, 492, 530
- $\ell_1$ -distance, 1
- $\ell_\infty$ -distance, 1
- Lagrange multipliers, 120, 552
- Lagrangean dual, 120, 124, 552
- Lagrangean relaxation, 119–121, 123, 124, 447, 551, 552, 576
- laminar family, 21–23, 114, 262, 272, 515
- language, 360, 367
- Las Vegas algorithm, 133, 369
- leaf, 17, 18
- length (of a path or circuit), 16
- length (of a string), 360
- level (EUCLIDEAN TSP), 534
- level graph, 174
- lexicographic rule, 55
- lexicographical order, 3, 12
- light Steiner tour, 535
- LIN-KERNIGHAN ALGORITHM, 542, 543, 557
- line, 13
- line graph, 16
- LINEAR INEQUALITIES, 367, 368, 383
- linear inequality system, 63, 67, 86
- linear program, *see* LP
- LINEAR PROGRAMMING, 49, 53, 54, 71, 86–88, 367, 383
- linear reduction, 128
- linear time, 4
- linear-time algorithm, 6
- linear-time graph algorithm, 25
- literal, 371
- local edge-connectivity, 181, 192
- local optimum, 558
- local search, 539, 545, 558, 579, 585
- loop, 14, 40, 42
- loss of a Steiner tree, 498
- lower rank function, 308
- Löwner-John ellipsoid, 81
- LP, 8, 49
  - dual, 60
  - primal, 60
- LP Duality Theorem, *see* Duality Theorem
- LP relaxation, 101
- Lucchesi-Younger Theorem, 478, 479
- $M$ -alternating ear-decomposition, 236–238
- $M$ -alternating path, 229
- $M$ -augmenting path, 229, 230, 251
- MA order, 187, 193
- MANHATTAN STEINER TREE PROBLEM, 493, 496
- Marriage Theorem, 229
- matching, 9, 15, 228, 277, *see* CARDINALITY MATCHING PROBLEM, *see* MAXIMUM WEIGHT MATCHING PROBLEM
  - $b$ -, 285, 286
  - perfect, 227, 277
- matching polytope, 277
- matrix norm, 82
- matroid, 307, 309–311, 313, 320
- matroid intersection, 322, *see* WEIGHTED MATROID INTERSECTION PROBLEM
- MATROID INTERSECTION PROBLEM, 323, 326, 328
- MATROID PARITY PROBLEM, 354
- MATROID PARTITIONING PROBLEM, 327, 328
- matroid polytope, 139, 320, 341, 344
- MAX-2SAT, 385, 433
- MAX-3SAT, 420, 423, 426
- MAX-CUT, 399, *see* MAXIMUM WEIGHT CUT PROBLEM
- Max-Flow-Min-Cut property, 316, 333
- Max-Flow-Min-Cut Theorem, 169, 317, 507
- MAX-SAT, 417, 418, 420, *see* MAXIMUM SATISFIABILITY
- maximal, 16
- MAXIMIZATION PROBLEM, 317–322
- MAXIMIZATION PROBLEM FOR INDEPENDENCE SYSTEMS, 306, 445, 446
- maximum, 16
- MAXIMUM CLIQUE PROBLEM, 422, 423, 433

- MAXIMUM CUT PROBLEM, 399, 431, 433  
 MAXIMUM FLOW OVER TIME PROBLEM, 219  
 MAXIMUM FLOW PROBLEM, 165–168, 172–175, 177, 180, 479  
 MAXIMUM MATCHING PROBLEM, 248  
 MAXIMUM MULTICOMMODITY FLOW PROBLEM, 472  
 MAXIMUM SATISFIABILITY (MAX-SAT), 416  
 MAXIMUM STABLE SET PROBLEM, 422, 423, 429, 430  
 MAXIMUM WEIGHT  $b$ -MATCHING PROBLEM, 285, 287, 299, 302  
 MAXIMUM WEIGHT BRANCHING PROBLEM, 133, 134, 136, 307  
 MAXIMUM WEIGHT CLIQUE PROBLEM, 411  
 MAXIMUM WEIGHT CUT PROBLEM, 300, 399, 400, 432  
 MAXIMUM WEIGHT FOREST PROBLEM, 128, 306  
 MAXIMUM WEIGHT MATCHING PROBLEM, 257, 307  
 MAXIMUM WEIGHT STABLE SET PROBLEM, 306, 411  
 MAXSNP, 426  
 MAXSNP-hard, 426, 428–430, 433, 492, 520, 530, 532  
 median, *see* WEIGHTED MEDIAN PROBLEM  
     weighted, 440  
 Menger's Theorem, 170–172, 191, 228, 301  
 MERGE-SORT ALGORITHM, 10, 11  
 method of conditional probabilities, 416  
 METRIC BIPARTITE TSP, 557  
 METRIC CAPACITATED FACILITY LOCATION PROBLEM, 586, 587, 595, 596  
 metric closure, 157, 162, 495  
 METRIC  $k$ -FACILITY LOCATION PROBLEM, 576, 579  
 METRIC  $k$ -MEDIAN PROBLEM, 580, 581  
 METRIC SOFT-CAPACITATED FACILITY LOCATION PROBLEM, 586, 587, 595  
 METRIC TSP, 528–530, 532, 544, 553  
 METRIC UNCAPACITATED FACILITY LOCATION PROBLEM, 564, 567, 568, 572, 575, 583, 586  
 minimal, 16  
 minimal face, 52, 53  
 MINIMIZATION PROBLEM, 318, 321  
 MINIMIZATION PROBLEM FOR INDEPENDENCE SYSTEMS, 306, 316  
 minimum, 16  
 MINIMUM CAPACITY CUT PROBLEM, 180, 181, 188  
 MINIMUM CAPACITY  $T$ -CUT PROBLEM, 295, 296, 299, 302  
 MINIMUM COST FLOW PROBLEM, 200, 203–207, 210, 212–215, 217, 221–223, 355  
 MINIMUM MEAN CYCLE ALGORITHM, 160, 301  
 MINIMUM MEAN CYCLE PROBLEM, 159, 160, 162  
 MINIMUM MEAN CYCLE-CANCELLING ALGORITHM, 204–206  
 MINIMUM SET COVER PROBLEM, 394, 405  
 MINIMUM SPANNING TREE PROBLEM, 128–130, 133, 137, 145, 306, 495, 496, 529, 552  
 MINIMUM VERTEX COVER PROBLEM, 396, 397, 422, 430, 431, 433, 530  
 MINIMUM WEIGHT ARBORESCENCE PROBLEM, 134  
 MINIMUM WEIGHT EDGE COVER PROBLEM, 281, 396  
 MINIMUM WEIGHT FEEDBACK VERTEX SET PROBLEM, 431  
 MINIMUM WEIGHT  $k$ -EDGE-CONNECTED SUBGRAPH PROBLEM, 520  
 MINIMUM WEIGHT PERFECT MATCHING PROBLEM, 257–259, 267, 275, 290, 291  
 MINIMUM WEIGHT ROOTED ARBORESCENCE PROBLEM, 134, 139, 140, 145, 191  
 MINIMUM WEIGHT SET COVER PROBLEM, 394, 398  
 MINIMUM WEIGHT  $T$ -JOIN PROBLEM, 290–292, 294, 295, 300, 522

- MINIMUM WEIGHT VERTEX COVER PROBLEM, 394, 399, 431  
 minor, 36, 45  
 mixed graph, 483, 487  
 mixed integer program, 99  
 mixed integer programming, 119, 121  
 modular function, 15, 16, 305, 340, 344  
 monotone set function, 341  
 Monte Carlo algorithm, 193, 369  
 MOORE-BELLMAN-FORD ALGORITHM, 154, 156, 208  
 MULTICOMMODITY FLOW APPROXIMATION SCHEME, 473  
 MULTICOMMODITY FLOW PROBLEM, 468, 469, 471  
 multicut, 141, 146  
 multigraph, 13  
 multiplication, 366  
 MULTIPROCESSOR SCHEDULING PROBLEM, 463  
  
 near-perfect matching, 233, 236  
 nearest neighbour heuristic, 528  
 negative circuit, 156, 158  
 neighbour, 13, 14  
 nested family, 21  
 network, 165  
 network matrix, 115, 123  
 NETWORK SIMPLEX ALGORITHM, 215, 217, 218  
 NEXT-FIT ALGORITHM, 451, 452  
 NF, *see* NEXT-FIT ALGORITHM  
 no-instance, 367  
 node, 13  
 nondeterministic algorithm, 369  
 nonnegative weights, 384  
 nonsaturating push, 179  
 NP, 368, 369, 389, 422  
 NP-complete, 371, 373  
     strongly, 386  
 NP-easy, 384, 385  
 NP-equivalent, 384  
 NP-hard, 384  
     strongly, 386, 446  
  
 $O$ -notation, 4  
 odd circuit, 32, 44  
 odd cover, 31, 32  
 odd cycle cover, 300  
  
 odd ear-decomposition, 235, 253  
 odd join, 31, 32  
 Okamura-Seymour Theorem, 481, 486, 487  
 one-sided error, 369  
 one-way cut-incidence matrix, 114, 115, 123  
 online algorithms, 455  
 optimization problem, 384  
     discrete, 384  
 optimum basic solution, 86, 87  
 optimum solution of an LP, 49  
 optimum solution of an optimization problem, 384  
 oracle, 89, 306, 317, 333, 345, 348, 505, 508  
 oracle algorithm, 89, 370, 384  
 oracle Turing machine, 366  
 orientation, 14, 483, 484, 487  
 ORLIN'S ALGORITHM, 211–214  
 out-degree, 14  
 out-of-forest blossom, 262  
 outer blossom, 244  
 outer face, 35, 45, 481, 495, 520  
 outer vertex, 242, 244  
  
 $P$ , 368  
 Padberg-Rao Theorem, 296  
 parallel edges, 13  
 partially ordered set, 250  
 PARTITION, 381, 386  
 partitionable, 327, 333, 334  
 Patching Lemma, 535, 536  
 path, 16  
     undirected, 18  
 PATH ENUMERATION ALGORITHM, 3, 5  
 PCP Theorem, 421, 422  
 PCP( $\log n, 1$ ), 421, 422  
 peak (network simplex), 217  
 perfect  $b$ -matching, 285, 289  
 perfect graph, 409, 432  
 perfect matching, 227, 229, 230, 233, 234, 277, *see* MINIMUM WEIGHT PERFECT MATCHING PROBLEM  
 perfect matching polytope, 277, 280, 282, 295  
 perfect simple 2-matching, 285, 299, 548, 555

- performance guarantee, 393
- performance ratio, 393
- permanent of a matrix, 252
- permutation, 1, 3, 73
- permutation matrix, 252, 259
- Petersen graph, 515
- PIVOT, 589
- pivot rule, 55
- planar dual graph, 40–42, 300, 314
- planar embedding, 33, 40, 45
- planar graph, 33, 40, 46, 314, 315
- plant location problem, 563
- Platonic graphs, 45
- Platonic solids, 45
- PLS*, 558
- point, 13
- pointed polyhedron, 53
- polar, 93, 94, 96
- polygon, 33, 34, 532
- polygonal arc, 33, 34
- polyhedral combinatorics, 65
- polyhedral cone, 53, 54, 64, 101, 102, 122
- polyhedron, 8, 51
  - bounded, 87
  - full-dimensional, 51, 87
  - integral, 107, 108, 110, 390
  - rational, 51
- polyhedron of blocking type, 333
- polymatroid, 341, 345, 346, 354
- POLYMATROID GREEDY ALGORITHM, 341, 342, 345, 346, 354
- polymatroid intersection theorem, 342
- POLYMATROID MATCHING PROBLEM, 354
- polynomial reduction, 370, 371, 384
- polynomial transformation, 370, 371
- polynomial-time algorithm, 6, 361, 366
- polynomial-time Turing machine, 361, 366
- polynomially equivalent oracles, 318, 333
- polynomially equivalent problems, 384
- polytope, 51, 64, 65
- portal (EUCLIDEAN TSP), 535
- poset, 250
- positive semidefinite matrix, 96
- potential associated with a spanning tree structure, 216
- power set, 15
- predecessor, 43
  - direct, 43
- preflow, 192
  - s-t*-, 177
- PRIM'S ALGORITHM, 130, 131, 133, 145, 339, 496
- primal complementary slackness conditions, 62
- primal LP, 60
- primal-dual algorithm, 260, 329, 473, 509, 567
- PRIMAL-DUAL ALGORITHM FOR NETWORK DESIGN, 508, 509, 513, 514, 521, 522
- PRIME, 383
- printed circuit boards, 1
- priority queue, 131
- probabilistic method, 416
- probabilistically checkable proof (*PCP*), 421
- problem
  - decision, 367
  - optimization, 384
- program evaluation and review technique (*PERT*), 190
- proper alternating walk, 541, 543
- proper closed alternating walk, 541
- proper ear-decomposition, 29
- proper function, 503–505
- pseudopolynomial algorithm, 385, 386, 442, 444, 450, 462
- PTAS, *see* approximation scheme
- PUSH, 177, 179
- push
  - nonsaturating, 179
  - saturating, 179
- PUSH-RELABEL ALGORITHM, 177, 179, 180, 192, 223
- quickest transshipment problem, 220
- r*-cut, 18
- radix sorting, 12
- RAM machine, 366, 388
- randomized algorithm, 133, 193, 231, 369, 416, 418
- randomized rounding, 418, 485
- rank function, 305, 310



- lower, 308
- rank of a matrix, 51, 77
- rank oracle, 318
- rank quotient, 308, 309, 332
- rate of flow, 219
- rate of growth, 4
- rational polyhedron, 51
- reachable, 16
- realizable demand edge, 471
- realizing path, 171
- recursive algorithms, 9
- recursive function, 362
- reduced cost, 155
- region (EUCLIDEAN TSP), 534
- regular expression, 8
- RELABEL, 177, 178
- relative performance guarantees, 393
- relatively prime, 72
- representable matroid, 308, 332
- residual capacity, 167
- residual graph, 167
- RESTRICTED HAMILTONIAN CIRCUIT, 544
- restriction of a problem, 386
- reverse edge, 167
- revised simplex, 60
- ROBINS-ZELIKOVSKY ALGORITHM, 500, 502
- root, 18, 43, 242
- running time, 6
- running time of graph algorithms, 25
  
- s-t*-cut, 18, 167, 169, 190, 507
- s-t*-flow, 165, 168, 169
- s-t*-flow over time, 219
- s-t*-path, 318
- s-t*-preflow, 177
- SATISFIABILITY, 371
- satisfiable, 371
- satisfied clause, 371
- satisfying edge set (network design), 506
- saturating push, 179
- scalar product, 49
- scaling technique, 176, 209, 223
- scheduling, 447
- scheduling problem, 463
- SCHRIJVER'S ALGORITHM, 348, 350
- SELECTION PROBLEM, 440, 441
- semidefinite program, 71, 401, 403
- separating edge set, 18
- separating hyperplane, 88
- separation oracle, 89, 93
- SEPARATION PROBLEM, 89, 93, 94, 295, 296, 345, 346, 459, 472, 479, 504, 505, 519, 548–550
- separator, 275
- series-parallel graphs, 45
- service cost, 565
- set cover, 394, *see* MINIMUM SET COVER PROBLEM, *see* MINIMUM WEIGHT SET COVER PROBLEM
- SET PACKING PROBLEM, 433
- set system, 21
- shifted grid (EUCLIDEAN TSP), 533
- SHMOYS-TARDOS-AARDAL ALGORITHM, 566
- SHORTEST PATH, 390
- shortest path, 26
- SHORTEST PATH PROBLEM, 151, 153, 154, 290, 292, 306, 472
- shrinking, 14
- sign of a permutation, 73
- simple *b*-matching, 285
- simple graph, 13
- simple Jordan curve, 33
- SIMPLEX ALGORITHM, 54–57, 61, 71, 77, 88, 214, 456, 472, 545
- simplex tableau, 59
- simplicial order, 193
- singleton, 14
- sink, 165, 199
- skew-symmetric, 230
- smoothed analysis, 56
- solution of an optimization problem
  - feasible, 50, 384
  - optimum, 384
- sorting, 9, 11, 12
- source, 165, 199
- spanning subgraph, 14
- spanning tree, 18, 43, 127, 141, 144, 315, 390, 551, *see* MINIMUM SPANNING TREE PROBLEM
- spanning tree polytope, 137, 139, 145, 146
- spanning tree solution, 215, 223
- spanning tree structure, 215
  - feasible, 216

- strongly feasible, 216
- sparse graph, 25
- special blossom forest, 244, 246, 353
- Sperner's Lemma, 251
- STABLE SET, 375, 389
- stable set, 15, 16, 250, *see* MAXIMUM STABLE SET PROBLEM, *see* MAXIMUM WEIGHT STABLE SET PROBLEM
- stable set polytope, 410
- standard embedding, 41
- star, 17, 591
- Steiner points, 491, 520
- Steiner ratio, 496, 497
- Steiner tour, 535
  - light, 535
- Steiner tree, 491, 495, 496, *see* STEINER TREE PROBLEM
- STEINER TREE PROBLEM, 307, 492–496, 500, 502, 520
- Stirling's formula, 3
- string, 360
- strong perfect graph theorem, 409
- strongly connected component, 19, 27–29
- STRONGLY CONNECTED COMPONENT ALGORITHM, 27–29, 507
- strongly connected digraph, 19, 44, 46, 485, 487
- strongly connected graph, 479
- strongly feasible spanning tree structure, 216
- strongly  $k$ -edge-connected graph, 191, 487
- strongly  $NP$ -complete, 386
- strongly  $NP$ -hard, 386, 446
- strongly polynomial-time algorithm, 6, 88
- subdeterminant, 101
- subdivision, 44, 45
- subgradient optimization, 120, 552
- subgraph, 14
  - induced, 14
  - $k$ -edge-connected, 503
  - spanning, 14
- subgraph degree polytope, 300
- submodular flow, 355
- SUBMODULAR FLOW PROBLEM, 355
- submodular function, 15, 189, 310, 341, 342, 344–346, 354
- SUBMODULAR FUNCTION MINIMIZATION PROBLEM, 345, 347, 348, 351
- SUBSET-SUM, 380, 385
- subtour inequalities, 547, 548, 555
- subtour polytope, 547, 548, 553
- SUCCESSIVE SHORTEST PATH ALGORITHM, 207, 208, 258
- successor, 43
  - direct, 43
- sum of matroids, 327
- supermodular function, 15, 344
  - weakly, 503, 515, 516
- supply, 199
- supply edge, 171, 467
- supporting hyperplane, 51
- SURVIVABLE NETWORK DESIGN PROBLEM, 491, 503, 509, 514, 519–521
- symmetric submodular function, 351, 352
- system of distinct representatives, 251
- system of linear equations, 77
- $T$ -cut, 293, 301, *see* MINIMUM CAPACITY  $T$ -CUT PROBLEM
- $T$ -join, 290, 293, 301, 317, *see* MINIMUM WEIGHT  $T$ -JOIN PROBLEM
- $T$ -join polyhedron, 294
- tail, 13
- TDI-system, 108–110, 112, 122, 139, 279, 280, 342, 355, 432, 433, 487
- terminal (DISJOINT PATHS PROBLEM), 171
- terminal (Steiner tree), 491
- test set, 103
- theta-function, 411
- $\Theta$ -notation, 4
- tight edge (weighted matching), 261, 272
- tight set (network design), 515
- time complexity, 6
- time-expanded network, 224
- tooth, 550
- topological order, 20, 28, 29, 507
- totally dual integral system, *see* TDI-system
- totally unimodular matrix, 110–112, 114, 115, 169
- tour, 16, 547

- tournament, 44
- transportation problem, 200
- transshipment problem, 200
- transversal, 332, 334
- traveling salesman polytope, 545, 551
- TRAVELING SALESMAN PROBLEM (TSP), 306, 386, 527, 539, 542, 543, 545, 552, 554, 555
- tree, 17, 24, 339
- tree-decomposition, 44, 481
- tree-representation, 22, 114, 262, 512, 516
- tree-width, 44, 486
- TREEPATH, 264, 273
- triangle inequality, 162, 495, 520, 528, 557
- truth assignment, 371
- TSP, *see* TRAVELING SALESMAN PROBLEM
- TSP FACETS, 551
- Turing machine, 359, 360, 362
- Turing reduction, 371
- Tutte condition, 233, 234
- Tutte matrix, 230
- Tutte set, 234
- Tutte's Theorem, 233, 234, 253
- Two-Commodity Flow Theorem, 487
- two-tape Turing machine, 362, 365
- two-way cut-incidence matrix, 114, 115, 123
  
- unbounded face, 35
- unbounded LP, 49, 62, 63
- UNCAPACITATED FACILITY LOCATION PROBLEM, 124, 565, 566, 568, 570, 576
- undecidable problem, 388
- underlying undirected graph, 14
- UNDIRECTED CHINESE POSTMAN PROBLEM, 290, 300
- undirected circuit, 18–20
- undirected cut, 18–20
- UNDIRECTED EDGE-DISJOINT PATHS PROBLEM, 171, 480, 481, 483, 484, 486
- undirected graph, 13
- UNDIRECTED MINIMUM MEAN CYCLE PROBLEM, 301
  
- UNDIRECTED MULTICOMMODITY FLOW PROBLEM, 468, 487
- undirected path, 18
- UNDIRECTED VERTEX-DISJOINT PATHS PROBLEM, 171, 481, 486
- uniform matroid, 308, 332
- unimodular matrix, 105, 106, 122
- unimodular transformation, 105, 122
- union of matroids, 327
- UNIVERSAL FACILITY LOCATION PROBLEM, 585, 594
- UPDATE, 266, 267
  
- value of an  $s$ - $t$ -flow, 165
- vector matroid, 308
- vertex, 13
- vertex-colouring, 405, 408, 411
- VERTEX-COLOURING PROBLEM, 406, 408, 411
- vertex-connectivity, 29, 188, 193
- VERTEX COVER, 376
- vertex cover, 15, 228, 340, *see* MINIMUM VERTEX COVER PROBLEM, *see* MINIMUM WEIGHT VERTEX COVER PROBLEM
- vertex-disjoint paths, 170
- VERTEX-DISJOINT PATHS PROBLEM DIRECTED, 171, 486
- UNDIRECTED, 171, 481, 486
- vertex of a polyhedron, 51, 53, 56, 65, 68
- violated vertex set (network design), 506
- Vizing's Theorem, 407, 408, 413
- VLSI design, 66, 162, 493
- Voronoi diagram, 145
  
- walk, 16
  - closed, 16
- warehouse location problem, 563
- weak duality, 55
- WEAK OPTIMIZATION PROBLEM, 90, 93, 460
- weak perfect graph theorem, 409
- weak separation oracle, 89
- WEAK SEPARATION PROBLEM, 89, 90, 459, 460
- weakly supermodular function, 503, 515, 516
- weight, 384

WEIGHTED MATCHING ALGORITHM,  
267, 272, 275–277, 281, 289, 292,  
293, 299  
WEIGHTED MATROID INTERSECTION  
ALGORITHM, 329, 330, 334, 354  
WEIGHTED MATROID INTERSECTION  
PROBLEM, 329, 330  
weighted median, 440  
WEIGHTED MEDIAN ALGORITHM, 441

WEIGHTED MEDIAN PROBLEM, 440  
well-rounded EUCLIDEAN TSP instance,  
532, 533  
word, 360  
worst-case running time, 6  
WORST-OUT-GREEDY ALGORITHM, 144,  
318, 321  
yes-instance, 367