

A

Appendix

This supplementary appendix contains well-known definitions and results used in this book which—to provide reading fluency—are not stated before.

The first section of this appendix is devoted to fundamental facts about ordinary differential equations. In Section A.2, some useful lemmata are stated, and in the last section, basic properties of projective spaces are treated.

A.1 Ordinary Differential Equations

We begin with the definition of an ordinary differential equation in the Euclidean space \mathbb{R}^N .

Definition A.1 (Ordinary differential equation). *For given $N, M \in \mathbb{N}$, let $D \subset \mathbb{R} \times \mathbb{R}^N \times \mathbb{R}^M$ be an open set and $f : D \rightarrow \mathbb{R}^N$ be a function. Then the equation*

$$\dot{x} = f(t, x, \alpha) \tag{A.1}_\alpha$$

is called (nonautonomous) ordinary differential equation which depends on a parameter α . For fixed $\hat{\alpha} \in \mathbb{R}^M$, we say, a differentiable function $\mu : \mathbb{I} \rightarrow \mathbb{R}^N$, \mathbb{I} an open interval, is a solution of $(A.1)_{\hat{\alpha}}$ if $(t, \mu(t), \hat{\alpha}) \in D$ for all $t \in \mathbb{I}$ and

$$\dot{\mu}(t) := \frac{d\mu}{dt}(t) = f(t, \mu(t), \hat{\alpha}) \quad \text{for all } t \in \mathbb{I}$$

is fulfilled. The combination of the differential equation $(A.1)_{\hat{\alpha}}$ and an initial value condition $x(\tau) = \xi$ is called initial value problem. We say, a solution μ of $(A.1)_{\hat{\alpha}}$ solves this initial value problem if $\mu(\tau) = \xi$.

For the uniqueness of solutions of ordinary differential equations, the concept of Lipschitz continuity is appropriate.

Definition A.2 (Lipschitz continuous function). For given $N, M \in \mathbb{N}$, let $D \subset \mathbb{R}^{1+N+M}$ and $g : D \rightarrow \mathbb{R}^N$ be a function. We say that g is (globally) Lipschitz continuous if there exists a constant $L \geq 0$ with

$$\|g(t, x, \alpha) - g(t, y, \alpha)\| \leq L\|x - y\| \quad \text{for all } (t, x, \alpha), (t, y, \alpha) \in D.$$

g is called locally Lipschitz continuous if for all $(t, x, \alpha) \in D$, there exist neighborhoods V of t and W of α such that the restriction of g to $V \times W \times \{\alpha\}$ is globally Lipschitz continuous.

The proof of the following proposition can be found, e.g., in AULBACH [14, Definition 2.6.2, Satz 7.2.2].

Proposition A.3 (General solution). Let $N, M \in \mathbb{N}$, $D \subset \mathbb{R} \times \mathbb{R}^N \times \mathbb{R}^M$ be open and $f : D \rightarrow \mathbb{R}^N$ be a locally Lipschitz continuous function, and consider the nonautonomous differential equation $(A.1)_\alpha$. Then there exist an open set $\Omega \subset \mathbb{R} \times \mathbb{R} \times \mathbb{R}^N \times \mathbb{R}^M$ and a continuous function $\lambda : \Omega \rightarrow \mathbb{R}^N$ such that for fixed $(\tau, \xi, \hat{\alpha}) \in D$, the function $\lambda(\cdot, \tau, \xi, \hat{\alpha})$ is a non-continuable solution of the initial value problem $(A.1)_{\hat{\alpha}}$, $x(\tau) = \xi$. The function λ is called the general solution of $(A.1)_\alpha$.

Remark A.4. In case the differential equation $(A.1)_\alpha$ does not depend on α , the fourth argument of the general solution is omitted.

Definition A.5 (Transition operator). Let $\mathbb{I} \subset \mathbb{R}$ be an interval, and consider the nonautonomous linear differential equation

$$\dot{x} = A(t)x \tag{A.2}$$

with a continuous function $A : \mathbb{I} \rightarrow \mathbb{R}^{N \times N}$. The (uniquely determined) function $\Lambda : \mathbb{I} \times \mathbb{I} \rightarrow \mathbb{R}^{N \times N}$ with

$$\Lambda(t, \tau)\xi = \lambda(t, \tau, \xi) \quad \text{for all } t, \tau \in \mathbb{I} \text{ and } \xi \in \mathbb{R}^N,$$

where λ denotes the general solution of (A.2), is called transition operator of (A.2). In case (A.2) is autonomous, i.e., $A = A(t)$ for all $t \in \mathbb{I} = \mathbb{R}$ with a matrix $A \in \mathbb{R}^{N \times N}$, the matrix exponential function $e^{A \cdot} : \mathbb{R} \rightarrow \mathbb{R}^{N \times N}$ is defined by

$$e^{At} := \Lambda(t, 0) \quad \text{for all } t \in \mathbb{R}.$$

Inhomogeneous linear differential equations are solved by the variation of constants formula.

Proposition A.6 (Variation of constants formula). Let $\mathbb{I} \subset \mathbb{R}$ be an interval, and consider the nonautonomous inhomogeneous linear differential equation

$$\dot{x} = A(t)x + b(t) \tag{A.3}$$

with continuous functions $A : \mathbb{I} \rightarrow \mathbb{R}^{N \times N}$ and $b : \mathbb{I} \rightarrow \mathbb{R}^N$. Let λ denote the general solution of (A.3) and Λ denote the transition operator of $\dot{x} = A(t)x$. Then we have the representation

$$\lambda(t, \tau, \xi) = \Lambda(t, \tau)\xi + \int_{\tau}^t \Lambda(t, s)b(s) ds \quad \text{for all } t, \tau \in \mathbb{I} \text{ and } \xi \in \mathbb{R}^N.$$

This equation is called the variation of constants formula.

Proof. See, e.g., COPPEL [54, p. 45]. □

For the analysis in the vicinity of a given reference solution, the differential equation of perturbed motion is of great importance.

Proposition A.7 (Differential equation of perturbed motion). For given $D \subset \mathbb{R} \times \mathbb{R}^N$, let $f : D \rightarrow \mathbb{R}^N$ be a locally Lipschitz continuous function, and consider the nonautonomous differential equation

$$\dot{x} = f(t, x) \tag{A.4}$$

with a solution $\lambda : \mathbb{I} \rightarrow \mathbb{R}^N$, \mathbb{I} an interval. Then the so-called differential equation of perturbed motion

$$\dot{x} = f(t, x + \lambda(t)) - f(t, \lambda(t)) \tag{A.5}$$

has the following properties:

- (i) If $\nu : \mathbb{J} \rightarrow \mathbb{R}^N$ is a solution of (A.4) and $\mathbb{J} \subset \mathbb{I}$, then $\mu := \nu - \lambda$ is a solution of (A.5) on \mathbb{J} .
- (ii) If $\mu : \mathbb{J} \rightarrow \mathbb{R}^N$ is a solution of (A.5) and $\mathbb{J} \subset \mathbb{I}$, then $\nu := \mu + \lambda$ is a solution of (A.4) on \mathbb{J} .

A.2 Useful Lemmata

The following lemma, which goes back to GRONWALL [71], plays a central role in obtaining estimates for solutions of differential equations.

Lemma A.8 (Gronwall's inequality). Let $a \geq 0$ and $u, b : [\tau_-, \tau_+] \rightarrow \mathbb{R}_0^+$ be continuous functions, and suppose that

$$u(t) \leq a + \int_{\tau_-}^t b(s)u(s) ds \quad \text{for all } t \in [\tau_-, \tau_+]$$

is fulfilled. Then

$$u(t) \leq a \exp \left(\int_{\tau_-}^t b(s) ds \right) \quad \text{for all } t \in [\tau_-, \tau_+].$$

Proof. See, e.g., ABRAHAM & MARSDEN & RATIU [1, Theorem 4.1.7, p. 242]. \square

The following lemma provides a triangle inequality for the Hausdorff-semi distance, which has been introduced in Section 2.1.

Lemma A.9 (Triangle inequality for the Hausdorff semi-distance). *Let X be a metric space and d denote the Hausdorff semi-distance. Then, for all nonempty sets $A, B, C \subset X$, the relation*

$$d(A|C) \leq d(A|B) + d(B|C)$$

is fulfilled.

Proof. Obviously, for all nonempty sets $M_1, M_2 \subset X$, the Hausdorff semi-distance fulfills

$$d(M_1|M_2) = \inf \{ \delta > 0 : M_1 \subset U_\delta(M_2) \}.$$

Hence, for all $\varepsilon > 0$, we have

$$A \subset U_{d(A|B)+\varepsilon/2}(B) \quad \text{and} \quad B \subset U_{d(B|C)+\varepsilon/2}(C).$$

This implies $A \subset U_{d(A|B)+d(B|C)+\varepsilon}(C)$ and finishes the proof of this lemma. \square

Lemma A.10. *Let A, B, C be linear subspaces of the \mathbb{R}^N such that $A \supset C$. Then the relation*

$$A \cap (B + C) = (A \cap B) + C$$

is fulfilled.

Proof. See SIEGMUND [171, Hilfssatz 2.36, p. 58]. \square

A.3 Projective Spaces

In this section, the real projective space \mathbb{P}^{N-1} of the vector space \mathbb{R}^N is introduced, and some basic properties are derived. Here, the \mathbb{R}^N is equipped with the Euclidean norm $\|\cdot\|$ and the Euclidean scalar product $\langle \cdot, \cdot \rangle$ (cf. Section 2.1). We say, two nonzero elements $x, y \in \mathbb{R}^N$ are equivalent if there exists a real number $c \in \mathbb{R}$ such that $x = cy$. The equivalence class of $x \in \mathbb{R}^N$ is denoted by $\mathbb{P}x$, and we call the set of all equivalence classes the *projective space* \mathbb{P}^{N-1} . Equipped with the metric $d_{\mathbb{P}} : \mathbb{P}^{N-1} \times \mathbb{P}^{N-1} \rightarrow [0, \sqrt{2}]$, given by

$$d_{\mathbb{P}}(\mathbb{P}v, \mathbb{P}w) = \min \left\{ \left\| \frac{v}{\|v\|} - \frac{w}{\|w\|} \right\|, \left\| \frac{v}{\|v\|} + \frac{w}{\|w\|} \right\| \right\} \quad \text{for all } v, w \in \mathbb{R}^N,$$

the projective space is a compact metric space. For any $v \in \mathbb{P}^{N-1}$, we define

$$\mathbb{P}^{-1}v := \{x \in \mathbb{R}^N : \mathbb{P}x = v\} \cup \{0\}.$$

Lemma A.11. *For all $\varepsilon > 0$, there exists a $\delta \in (0, 1)$ such that for all nonzero $v, w \in \mathbb{R}^N$ with*

$$\frac{\langle v, w \rangle^2}{\|v\|^2 \|w\|^2} \geq 1 - \delta,$$

we have

$$d_{\mathbb{P}}(\mathbb{P}v, \mathbb{P}w) \leq \varepsilon.$$

Proof. This is a direct consequence of COLONIUS & KLIEMANN [50, Lemma B.1.17., p. 538]. \square

Lemma A.12. *Let $V, W \subset \mathbb{R}^N$ be linear subspaces of the \mathbb{R}^N with $V \subsetneq W$. Then*

$$d_{\mathbb{P}}(\mathbb{P}W | \mathbb{P}V) = \sqrt{2}.$$

Proof. The linear subspace $V^\perp \cap W$, where

$$V^\perp := \{x \in \mathbb{R}^N : \langle x, v \rangle = 0 \text{ for all } v \in V\},$$

is obviously nontrivial. Let w be a nonzero element of $V^\perp \cap W$. Then, for all $v \in V$, we have

$$\begin{aligned} d_{\mathbb{P}}(\mathbb{P}w, \mathbb{P}v) &= \min \left\{ \left\| \frac{v}{\|v\|} \pm \frac{w}{\|w\|} \right\| \right\} \\ &= \min \left\{ \sqrt{\underbrace{\left\langle \frac{v}{\|v\|}, \frac{v}{\|v\|} \right\rangle}_{=1} + \underbrace{\left\langle \frac{w}{\|w\|}, \frac{w}{\|w\|} \right\rangle}_{=1} \pm 2 \underbrace{\left\langle \frac{v}{\|v\|}, \frac{w}{\|w\|} \right\rangle}_{=0}} \right\} \\ &= \sqrt{2}. \end{aligned}$$

Since $d_{\mathbb{P}}(x, y) \leq \sqrt{2}$ for all $x, y \in \mathbb{P}^{N-1}$, this implies the assertion. \square

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