

Summary

Let us briefly summarize main results of this work.

- One of the main conclusions is that the solution to the Stokes two-boundary-value problem cannot be guaranteed for all wavelengths. The spectral analysis of the boundary operator for harmonic functions regular at infinity has revealed that the eigenvalue spectrum may contain a null eigenvalue or an eigenvalue of a very small size. Consequently, the inverse operator is ill-posed or even does not exist. Once this ill-posed case occurs, the Stokes two-boundary-value problem must be regularized such that this null eigenvalue and its vicinity are excluded from the solution.
- For a mountainous terrain, such as the Rocky Mountains, it is not sufficient to model then topographical density by a constant value ρ_0 equal to a mean crustal density 2.67 g/cm^3 when a 'decimetre geoid' is to be compiled. The topographical density corrections due to lateral density inhomogeneities may reach several decimetres, and, hence, they must be considered in the final computation. On the other hand, in a flat low-land terrain, the approximation of the actual topographical density by ρ_0 is acceptable for a 'decimetre geoid' computation.
- Although it is a common belief that there is not significant for 'decimetre geoid' whether spherical or planar approximation of the geoid is used to compute the topographical effects, we beg to differ. The type of approximations significantly influences the Bouguer parts of topographical terms; the bias due to planar approximation of the geoid may reach a few metres.
- The Taylor series expansion of the Newton kernel does not converge in a rugged mountainous terrain such as the Rocky Mountains. The divergency of the Taylor series may cause errors of the order of several decimetres in terms of geoidal heights. The formulae derived herein overcome the problem of divergency of the Taylor series; they may be used to evaluate the gravitational effect of a very rugged terrain.
- We recommend to carry out the spectral analysis of gravity anomalies for different compensation/condensation models before geoid height computation and to choose that compensation model which reduces a high frequency

part of surface gravity anomalies in a most efficient way. The experience with the spectral analysis of gravity anomalies from the region of the Canadian Rocky Mountains indicates that rather the Airy-Heiskanen model than Helmert's 2nd condensation technique should be used to reduce high-frequency oscillations of surface gravity data.

- There is a number of possibilities how to take into account a satellite gravitational model as a reference in the Stokes two-boundary-value problem. Herein, we have formulated the Stokes two-boundary-value problem for a high-frequency part of the anomalous gravitational potential such that low-frequency reference potential does not depend on the way of compensation of topographical masses but only on a satellite gravitational model and the gravitational field generated by topographical masses.
- The discrete downward continuation problem for geoid determination is undoubtedly stable and its solution may be carry out by Jacobi's iterative method once the grid step size of the surface observations as well as of the discrete solution is not smaller than 5 arcmin. This conclusion drawn for the Canadian Rocky Mountains will be valid anywhere else in the world, with perhaps the exception of Himalayas, since the Rockies represent one of the highest and roughest mountainous terrain pattern.
- The solution to the Stokes boundary-value problem on an ellipsoid of revolution with the accuracy of the order up $O(e_0^2)$ (i.e., omitting terms of $O(e_0^4)$ and higher) has been found in a closed form. The ellipsoidal Stokes function describing the effect of the ellipticity of boundary on the solution to Stokes's boundary-value problem can be approximated by function $1/\psi$ in a neighbourhood of its singular point $\psi = 0$. Hence, the degree of singularity of the ellipsoidal Stokes function at the point $\psi = 0$ is the same as that of the spherical Stokes function.
- Green's function to the external Dirichlet boundary-value problem for the Laplace equation with data distributed on an ellipsoid of revolution with the accuracy of the order up $O(e_0^2)$ has been constructed in a closed form. The ellipsoidal Poisson kernel describing the effect of the ellipticity of the boundary on the solution to this boundary-value problem has been expressed as a finite sum of elementary functions which describe analytically the behaviour of ellipsoidal Poisson kernel at the singular point $\psi = 0$. We have shown that the degree of singularity of the ellipsoidal Poisson kernel in the vicinity of its singular point is of the same degree as that of the original spherical Poisson kernel.
- Green's function to the boundary-value problem of Stokes's type with ellipsoidal corrections in the boundary condition for anomalous gravity has constructed in a closed form. The 'spherical-ellipsoidal' Stokes function describing the effect of two ellipsoidal correcting terms occurring in the bound-

ary condition for anomalous gravity is expressed in $O(e_0^2)$ -approximation as a finite sum of elementary functions analytically representing the behaviour of the integration kernel at the singular point $\psi = 0$. We show that 'spherical-ellipsoidal' Stokes's function has only a logarithmic singularity in the vicinity of its singular point. The constructed Green function enables us to avoid applying an iterative approach to solve Stokes's boundary-value problem with ellipsoidal correction terms involved in boundary condition for anomalous gravity. A new Green function approach is more convenient from numerical point of view since the solution to the boundary-value problem is determined in one step by computing a Stokes-type integral. The question on the convergency of an iterative scheme recommended so far to solve this boundary-value problem is thus irrelevant.

- We demonstrate that the least-squares solution to the discrete altimetry-gravimetry problem is stable provided that the boundary functionals are discretized in an equal angular grid, the cut-off degree of a global gravity model is not greater than 500, and the covariance matrices of the boundary data are set up in accordance with today's accuracy of geodetic measurements. Moreover, provided that the number of observations is greater than the cut-off degree of the potential series, the approximation error of the least-squares solution does not depend on the number of observations. We paper also investigate the numerical stability of the discrete altimetry-gravimetry problem overdetermined by extra gravity data measured by ships over a part of the sea surface. Overlapping boundary data does not have, however, a significant impact on the conditionality of the matrix of normal equations due to the low accuracy of sea-borne gravity observations.

In all of the discussions above, we have not addressed the question of height and gravity data accuracy. The effects of spatial data distribution (irregularity and sparseness) and data accuracy are probably very significant. This also prevented us to investigate the effect of various Stokes's kernel modification schemes which differ in the weighting the near-zone and far-zone contributions to the Stokes integral. Only the stochastic model of surface gravity data errors can be used as an objective function to perform such weighting. There exists surely a number of other questions connected with a precise geoid determination which have not been sorted out in this work. The research reported here should be thus understood as a part of our ultimate goal.

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