List of Symbols

Sets

\( \in \) element
\( \subseteq \) subset
\( \subset \) proper subset
\( \supseteq \) superset
\( \supset \) proper superset
\( \cup \) union
\( \cap \) intersection
\( \setminus \) set difference
\( \Delta \) symmetric difference
\( \emptyset \) empty set
\( |S| \) cardinality of set \( S \)
\( 2^S \) power set (set of all subsets) of set \( S \)
\( S \times T \) Cartesian product of sets \( S \) and \( T \)
\( S^n \) \( n \)-fold Cartesian product of set \( S \)
\( \mathbb{N} \) the set of natural numbers
\( \mathbb{Q} \) the set of rational numbers
\( \mathbb{R} \) the set of real numbers

Logic

\( \land \) conjunction (and)
\( \lor \) disjunction (or)
\( \neg \) negation
\( \rightarrow \) implication
\( \leftrightarrow \) implication (in program clauses)
\( \equiv \) equivalence
\( \forall \) universal quantifier
\( \exists \) existential quantifier
\( T \) true
\( F \) false
LIST OF SYMBOLS

|= logical implication
⇔ logical equivalence
∈ empty substitution
S set of negations of formulas in set S
MII least Herbrand model of definite program II
FII SLD finite failure set of definite program II
comp(II) completion of normal program II
Γr (unconstrained) derivation
Γd (unconstrained) deduction
Γlr linear derivation
Γld linear deduction
Γir input derivation
Γid input deduction
Γsr SLD-derivation
Γsd SLD-deduction
Γsnf SLDNF-resolution
R computation rule

Languages and quasi-orders

A set of all atoms in a language
H Horn language
C clausal language
Cnewsize C bounded by newsize
Ch hypothesis language in model inference
Co observational language in model inference
Cl part of Co that is true under interpretation I
R set of all reduced clauses in C
Rnewsize R bounded by newsize
S set of all theories from C
Cpos clause consisting of all positive literals in clause C
Cneg clause consisting of all negative literals in clause C
C+ head of program clause C
C− body of program clause C
⊤ top element in lattice
⊥ bottom element in lattice
□ empty clause
≥ arbitrary quasi-order
⟨S, ≥⟩ set S quasi-ordered by ≥
A ⊓ B greatest lower bound of {A, B}
A ⊔ B least upper bound of {A, B}
≥ subsumption
≈ equivalence relation induced by quasi-order (for atoms: variants)
~ subsume-equivalence
19.8. **LIST OF SYMBOLS**

\[
\begin{align*}
\preceq_a & \quad \text{atomic order} \\
\mathcal{B} & \quad \text{background knowledge} \\
\preceq_B & \quad \text{relative subsumption} \\
\models_B & \quad \text{relative implication} \\
\succeq_B & \quad \text{generalized subsumption}
\end{align*}
\]

**Refinement operators**

\[
\begin{align*}
\rho_A & \quad \text{downward refinement operator for atoms} \\
\rho_L & \quad \text{downward refinement operator under subsumption} \\
\rho_r & \quad \text{downward refinement operator for reduced clauses} \\
\rho_I & \quad \text{downward refinement operator under implication} \\
\delta_A & \quad \text{upward refinement operator for atoms} \\
\delta_u & \quad \text{upward refinement operator under subsumption} \\
\delta_r & \quad \text{upward refinement operator for reduced clauses}
\end{align*}
\]

**PAC learning**

\[
\begin{align*}
\Sigma^* & \quad \text{set of all finite strings over alphabet } \Sigma \\
X^n & \quad \text{set of all strings of length at most } n \text{ in domain } X \\
\mathcal{F} & \quad \text{concept class} \\
f^n & \quad \text{projection of concept } f \text{ on } X^n \\
\mathcal{F}^n & \quad \text{projection of concept class } \mathcal{F} \text{ on } X^n \\
\mathcal{P} & \quad \text{probability distribution} \\
\mathcal{D}_{VC} & \quad \text{Vapnik-Chervonenkis dimension} \\
\delta & \quad \text{confidence parameter} \\
\varepsilon & \quad \text{error parameter} \\
\eta & \quad \text{rate of malicious or random classification noise} \\
\eta_b & \quad \text{upper bound on } \eta \\
l_{\text{min}}(f, R) & \quad \text{size (shortest name) of concept } f \text{ in representation } R
\end{align*}
\]
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