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Weak topology 18

Symbol index

${}^*f^*$, the restricted biconjugate of the function f 26

$\mathcal{CC}(E)$, the set of all real convex continuous functions on the Banach space E 129

χ_S, a certain convex function determined by the multifunction S 53

$\mathcal{CLB}(E)$, the set of all convex functions that are Lipschitz on the bounded subsets of the Banach space E 133

$\mathcal{CO}(G)$, the big convexification of G 32

$D(S)$, the domain of the multifunction S 29

$\delta_{(y,y^*)}$, the point mass at (y, y^*) 32

$\partial f(x)$, the subdifferential of the function f at the point x 31

E^*, the dual space of the Banach space E 18

E^{**}, the bidual of the Banach space E 18

f^*, the conjugate of the function f 25

f^{**}, the full biconjugate of the function f 129

$F^\perp$, the subspace of E^* orthogonal to the subspace F of the Banach space E 59

$G(S)$, the graph of the multifunction S 29

$H(T)$, a set determined by the positive linear map T 141

I_C, the indicator function of the set C 31

J, the duality map on the Banach space E 37

N_C, the normality multifunction of the set C 31

p, a certain linear operator from $\mathbb{R}^{(E\times E^*)}$ into E 32

$\mathcal{PCLSC}(E)$, the set of all somewhere finite convex lower semicontinuous functions on the Banach space E 31

ψ_S, a certain convex function determined by the multifunction S 53

q, a certain linear operator from $\mathbb{R}^{(E\times E^*)}$ into E^* 32

r, a certain linear operator from $\mathbb{R}^{(E\times E^*)}$ into $\mathbb{R}$ 32

$R(S)$, the range of the multifunction S 29

S^{-1}, the inverse of the multifunction S 29

$\overline{S}$, a multifunction defined by Gossez 97
$\mathcal{T}_{CC}(E^{**})$, a certain topology on the bidual E^{**} of the Banach space E 135
$\mathcal{T}_{C\mathcal{LB}}(E^{**})$, a certain topology on the bidual E^{**} of the Banach space E 135
$\mathcal{T}_{\| \ \|}$, the norm topology of 132
$w(E^*, E)$, the weak* topology of the dual, E^*, of the Banach space E 18
$w(E, E^*)$, the weak topology of the Banach space E 18
ξ_S, a certain convex function determined by the multifunction S 71
$\widehat{x}$, the canonical image of x in the bidual 18