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The “\*” indicates that the respective reference has been added to the list of references in the second edition of these notes.

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