

---

## Appendix A: Proof of the Thinning Algorithm

Consider the problem of generating samples of a counting process  $N(t) = \sum_{i \geq 1} \mathbf{1}_{\{\tau_i \leq t\}}$  with (possibly random) intensity  $\lambda(s)$ . This amounts to sampling jump times  $\tau_i$ ,  $i \geq 1$ , from the knowledge of  $\lambda(s)$ . The thinning method requires to (1) select  $\lambda^*$  as an upper bound for  $\lambda(s)$  on its domain; (2) sample jump times  $\tau_i^*$  of a Poisson process  $N^*$  with constant intensity  $\lambda^*$ ; (3) perform an acceptance–rejection test on each  $\tau_i^*$ , which consists of sampling the random variable  $\mathbf{1}_{\{\lambda^* U_i \leq \lambda(\tau_i^*)\}}$ , where  $U_i \stackrel{\text{i.i.d.}}{\sim} \mathcal{U}[0, 1]$ ; (4) accept  $\tau_i^*$  as a jump time of  $N$  provided that the test succeeds, namely the independent uniform variable  $\lambda^* U_i$  on  $[0, \lambda^*]$  is smaller than  $\lambda(\tau_i^*)$ . It is assumed that  $\lambda$ ,  $N^*$  and the  $U_i$ 's are all statistically independent.

To prove this, we need to check that the process the counting process of the accepted jump times  $N(t) = \sum_{i: 0 < \tau_i^* \leq t} \mathbf{1}_{\{\lambda^* U_i \leq \lambda(\tau_i^*)\}}$  has intensity  $\lambda$ , that is:

$$\mathbb{E}(N(t) - N(s) | \mathcal{F}_s^N) = \mathbb{E}\left(\int_s^t \lambda(u) du \middle| \mathcal{F}_s^N\right), \quad (\text{A.1})$$

where  $\mathcal{F}_s^N$  is the completed filtration generated by  $N$ . This formula has a clear interpretation in the deterministic case: the expected number of jump times occurring over an interval  $(s, t]$  is given by accruing the jump intensity  $\lambda$  over the same interval. For the sake of clarity, we assume that the function  $\lambda$  is superiorly bounded and set  $\lambda^* = \sup \lambda(t, \omega)$ . It is left to the reader to prove that the intensity process  $\lambda$  is  $\mathcal{F}^N$ -adapted. Using the linearity property and the rule of iterated conditional expectation, we have

$$\begin{aligned} & \mathbb{E}(N(t) - N(s) | \mathcal{F}_s^N) \\ &= \mathbb{E}\left(\sum_{i:s < \tau_i^* \leq t} \mathbf{1}_{\{\lambda^* U_i \leq \lambda(\tau_i^*)\}} \middle| \mathcal{F}_s^N\right) \\ &= \mathbb{E}\left(\sum_{i:s < \tau_i^* \leq t} \mathbb{E}(\mathbf{1}_{\{\lambda^* U_i \leq \lambda(\tau_i^*)\}} | \mathcal{F}_{\tau_i^*}^N) \middle| \mathcal{F}_s^N\right) \quad \text{as } \mathcal{F}_{\tau_i^*}^N \supset \mathcal{F}_s^N \end{aligned}$$

$$\begin{aligned}
 &= \mathbb{E} \left( \sum_{i:s < \tau_i^* \leq t} \mathbb{P}(U_i \leq \lambda(\tau_i^*)/\lambda^* | \mathcal{F}_{\tau_i^*}^N) \middle| \mathcal{F}_s^N \right) \quad \text{as } \mathbb{P}(\mathcal{U}[0, 1] \leq X | X = u) = u \\
 &= \frac{1}{\lambda^*} \mathbb{E} \left( \sum_{i=N^*(s)+1}^{N^*(t)} \lambda(\tau_i^*) \middle| \mathcal{F}_s^N \right) \quad \text{as } \lambda(\tau_i^*) \text{ is } \mathcal{F}_{\tau_i^*}^N\text{-adapted} \\
 &= \frac{1}{\lambda^*} \mathbb{E} \left( \mathbb{E} \left( \sum_{i=N^*(s)+1}^{N^*(t)} \lambda(\tau_i^*) \middle| \mathcal{F}_t^N \right) \middle| \mathcal{F}_s^N \right) \quad \text{as } t > s.
 \end{aligned}$$

Recall that the jump times are uniformly distributed on the interval  $(s, t]$  conditional on the number of jumps having occurred over the same interval, that is  $\tau_{N^*(s)+1}, \dots, \tau_{N^*(t)} | \mathcal{F}_t \stackrel{\text{i.i.d.}}{\sim} \tau \sim \mathcal{U}(s, t]$ . The Wald theorem leads to:

$$\mathbb{E}(N(t) - N(s) | \mathcal{F}_s^N) = \frac{1}{\lambda^*} \mathbb{E}(\mathbb{E}[N^*(t) - N^*(s) | \mathcal{F}_t^N] \times \mathbb{E}[\lambda(\tau) | \mathcal{F}_t^N] | \mathcal{F}_s^N).$$

Given  $\mathcal{F}_s^N$ , the increment  $N^*(t) - N^*(s)$  and the random variable  $\tau$  are mutually independent. Moreover,  $\tau$  is uniformly distributed in  $[s, t]$  given  $\mathcal{F}_t^N$ . The last term then becomes equal to:

$$\begin{aligned}
 &\frac{1}{\lambda^*} \mathbb{E}(N^*(t) - N^*(s) | \mathcal{F}_s^N) \times \mathbb{E}(\lambda(\tau, \omega) | \mathcal{F}_s^N) \\
 &= \frac{1}{\lambda^*} \int_s^t \lambda^* \, du \mathbb{E} \left( \int_s^t \lambda(u) \frac{du}{t-s} \middle| \mathcal{F}_s^N \right) \\
 &= \mathbb{E} \left( \int_s^t \lambda(u) \, du \middle| \mathcal{F}_s^N \right),
 \end{aligned}$$

which is the claim to be proved.

---

## Appendix B: Sample Problems for Monte Carlo

1. Let the state variable  $X$  be the instantaneous short rate  $r$ . Fix three increasing times  $t < T_1 < T_2$  and let  $P_{T_2}$  be the process describing the value of a default free zero-coupon bond maturing at time  $T_2$ . We choose  $T_2 \geq T_1$  because the asset needs to exist at time  $T_1$ . We consider a call option on  $P_{T_2}$ , expiring at  $T_1$  and strike at  $K$  Euros. We want to design a procedure to evaluate the fair value of this option at time  $t$  by making use of Monte Carlo methods. This option is a  $T_1$ -maturing contingent claim with pay-off function  $(x - K)_+$  on a  $T_2$ -maturing contingent claim with pay-off function equal to 1 in all states of the sample world:

$$\begin{aligned} V(t) &= \mathbb{E}_t^* \left( e^{-\int_t^{T_1} r(s) ds} (P_{T_2}(T_1) - K)_+ \right) \\ &= \mathbb{E}_t^* \left( e^{-\int_t^{T_1} r(s) ds} \left( \mathbb{E}_{T_1}^* \left( e^{-\int_{T_1}^{T_2} r(s) ds} \right) - K \right)_+ \right). \end{aligned}$$

To simulate this value, we may follow two routes. The *first algorithm* is naive. We set a small  $\Delta t$  and evolve  $r$  over time points  $t, t + \Delta t, t + 2\Delta t, \dots, T_1 - \Delta t, T_1, T_1 + \Delta t, \dots, T_2 - \Delta t$ . This is done by discretizing the s.d.e. for  $r$  over that partition. For each simulated interest rate path, we compute the option pay-off. Summing up the resulting payoff over  $n$  sampled paths and dividing by  $n$ , results in the following approximated value:

$$C(t) \approx \frac{1}{n} \sum_{i=1}^n \left[ e^{-\sum_{t=0}^{T_1-\Delta t} r^{(i)}(t)\Delta t} \left( e^{-\sum_{t=T_1}^{T_2-\Delta t} r^{(i)}(t)\Delta t} - K \right)_+ \right].$$

The *second algorithm* is more effective. For each  $i = 1, \dots, N$ :

- (1) generate a path  $(r^{(i)}(t + \Delta t), \dots, r^{(i)}(T_1 - \Delta t))$  up to time  $T_1 - \Delta t$ ;
- (2) generate  $M$  “continuations”:

$$\{(r^{(i,k)}(T_1), r^{(i,k)}(T_1 + \Delta t), \dots, r^{(i,k)}(T_2 - \Delta t)), k = 1, \dots, M\},$$

after time  $T_1$ , until time  $T_2 - \Delta t$  is reached;

- (3) use these continuations to obtain a Monte Carlo estimate for the time  $T_1$  value of the bond:

$$P_{T_2}^{(i)}(T_1) \approx \frac{1}{M} \sum_{k=1}^M \left[ e^{-\sum_{t=T_1}^{T_2-\Delta t} r^{(i,k)}(t)\Delta t} \right];$$

- (4) plug this value into the call option pay-off;  
 (5) discount between  $T_1$  and  $t$  using the path  $r^{(i)}(t + \Delta t), \dots, r^{(i)}(T_1 - \Delta t)$  generated at the first step; finally  
 (6) sum all these terms up and divide by  $n$ . The corresponding Monte Carlo estimate is:

$$C(t) \approx \frac{1}{n} \sum_{i=1}^n \left[ e^{-\sum_{t=0}^{T_1-\Delta t} r^{(i)}(t)\Delta t} \left( \left( \frac{1}{M} \sum_{k=1}^M \left[ e^{-\sum_{t=T_1}^{T_2-\Delta t} r^{(i,k)}(t)\Delta t} \right] \right) - K \right)_+ \right].$$

Write and run a computer code implementing the two procedures above. Elaborate and apply a test to compare their performance.

**2.** Suppose a model for the underlying factor dynamics  $X$  depends on a parameter. For instance, in the Black–Scholes model,  $X$  is the underlying security and we may take the volatility  $\sigma$  as a parameter. We wish to evaluate the rate of variation of the current fair value  $V(t)$  of a given contingent claim resulting from a small change in the parameter  $\delta$ . In the Black–Scholes example, we look for the Vega of an option. We may perform two Monte Carlo estimations: one under dynamics for the underlying factor corresponding to a parameter value  $\delta + \varepsilon$ ; the other for a parameter value  $\delta - \varepsilon$ , where  $\varepsilon$  is a small positive constant. To reduce computations, in both cases we may use the same sequence of drawn r.v.’s needed to generate a path (this is one of the variance reduction techniques we will develop in the last section). We come up with estimations  $V(t; \delta + \varepsilon)$  and  $V(t; \delta - \varepsilon)$ . The sensitivity can be approximated by a first-order difference  $\partial_\delta V(t) \approx (V(t; \delta + \varepsilon) - V(t; \delta - \varepsilon))/(2\varepsilon)$ . This method lets us compute hedge ratios too: indeed, the call option hedger is required to take a  $\Delta$ -position in the underlying asset  $S$ , where  $\Delta$  is just the sensitivity  $\partial_S V(t, S(t))$ . Write and run a computer code for computing sensitivities of the options detailed in the examples of Section 1 with respect to the parameters involved in a Black–Scholes model.

**3.** We wish to generate a simulated random path for a short rate model so that a given set of observed discount bond prices is perfectly matched. Suppose time  $t$  discount function is given by  $N$  observed zero-coupon bond prices  $P_{T_1}(t) = p_1, \dots, P_{T_N}(t) = p_N$ , where  $T_k = t + k\Delta t$ . Recall the theoretical value for a discount bond price is:  $P_T(t) = \mathbb{E}_t^*(\exp(-\int_t^T r(s) ds))$ . Let  $r(t) = r_0$ . If we simulate a number  $n$  of discrete trajectories

$$\{r_{iT}^{(i)} = (r^{(i)}(T_1), \dots, r^{(i)}(T_{N-1})), i = 1, \dots, n\}$$

for the risk-neutral dynamics of the short rate process  $r$ , the theoretical value of any discount bond  $P_{T_i}(t)$  need not match the corresponding observed price  $p_i$ :

$$p_i = P_{T_i}(t) \neq P_{T_i}^{\text{sampled}}(t) = \frac{1}{n} \sum_{i=1}^n e^{-\Delta t \sum_{k=1}^{N-1} r^{(i)}(T_k)}.$$

We want to bias each simulated path  $r_{iT}^{(i)}$  so as to obtain a new path  $\hat{r}_{iT}^{(i)}$  which is compatible with observed prices  $p_1, \dots, p_N$  in that:

$$p_i = \frac{1}{n} \sum_{i=1}^n e^{-\Delta t \sum_{k=1}^{N-1} \hat{r}^{(i)}(T_k)}.$$

To each sample  $r^{(i)}(T_k)$ , we

- (1) add the continuously compounded forward rate spanning  $[T_k, T_{k+1}]$  computed from the simulated path  $r_{iT}^{(i)}$  by:

$$\begin{aligned} \{r_{iT}^{(1)}, \dots, r_{iT}^{(n)}\} &\rightarrow f_{T_k T_{k+1}}(t) = \frac{1}{\Delta t} \lg \left( \frac{P_{T_k}(t)}{P_{T_{k+1}}(t)} \right) \\ &= \frac{1}{\Delta t} \lg \left( \frac{\frac{1}{n} \sum_{i=1}^n e^{-\Delta t \sum_{j=1}^{k-1} r^{(i)}(T_j)}}{\frac{1}{n} \sum_{i=1}^n e^{-\Delta t \sum_{j=1}^k r^{(i)}(T_j)}} \right), \end{aligned}$$

and

- (2) subtract the continuously compounded forward rate spanning  $[T_k, T_{k+1}]$  implied in the observed discount function by:

$$\{p_1, \dots, p_n\} \rightarrow \hat{f}_{T_k T_{k+1}}(t) = \frac{1}{\Delta t} \lg \left( \frac{p_k}{p_{k+1}} \right).$$

Then the resulting paths

$$\{\hat{r}_{iT}^{(i)} = (\hat{r}^{(i)}(T_1), \dots, \hat{r}^{(i)}(T_{N-1})), i = 1, \dots, n\}, \quad (\text{B.1})$$

defined by

$$\hat{r}^{(i)}(T_k) = r^{(i)}(T_k) + f_{T_k T_{k+1}}(t) - \hat{f}_{T_k T_{k+1}}(t),$$

satisfy the matching property. That is bond prices estimated by Monte Carlo over the modified random paths in (B.1) exactly equal the observed prices  $p_1, \dots, p_N$ :

$$p_i = P_{T_i}^{\text{sampled}}(t) = \frac{1}{n} \sum_{i=1}^n e^{-\Delta t \sum_{k=1}^{N-1} \hat{r}^{(i)}(T_k)}.$$

Write a program to implement this fitting procedure.

**4.** Give a formal proof for generating jump times by using the algorithm detailed in Sect. 2.3.4.

**5.** The recursive formula for simulating  $r$  in the Vasicek model is:

$$r(t_{i+1}) = \mu(t_{i+1}; t_i, r(t_i)) + \sigma(t_{i+1}; t_i, r(t_i)) \sqrt{t_{i+1} - t_i} \mathcal{N}(0, 1)$$

starting at  $(0, r_0)$ . For the CIR short rate model we have  $dr = \alpha(b - r)dt + \sigma\sqrt{r}dW(t)$  and:

$$r(t_{i+1}) = r(t_i) + \frac{\sigma^2}{4\alpha}(1 - e^{-\alpha\Delta t})\chi^2\left(\frac{4\alpha\beta}{\sigma^2}, \frac{\sigma^2}{\alpha}(1 - e^{-\alpha\Delta t})e^{-\alpha\Delta t}r(t_i)\right),$$

where  $\chi^2(d, c)$  denotes a random sample from a non-central chi-square distribution with  $d$  degrees of freedom and non-centrality parameter  $c$ . See Johnson and Kotz (1995) for details on this family of distributions. For each of these models, compare Monte Carlo simulated European bond option values over increasing samples to the theoretical values given by closed form formula.

**6.** Prove that the approximating process  $X^{(n)}$  in the Algorithm “Sampling theorem method for stationary Gaussian processes” is not stationary by verifying that its covariance function  $c^{(n)}(t, s) = \mathbb{E}(X^{(n)}(t)X^{(n)}(s))$  does not depend on  $t$  and  $s$  through  $t - s$ . By applying the sampling theorem stated in the same section, show that  $c^{(n)}(t, s)$  converges to a function of  $t - s$  as  $n \rightarrow \infty$ . This justifies the approximation made in the above-mentioned algorithm.

---

## Appendix C: The Matlab<sup>®</sup> Solver

Here we present the PDE solver implemented in a Matlab<sup>®</sup> environment. This tool aims at solving initial-boundary value problems for systems of parabolic and elliptic partial differential equations (PDEs) in one space variable. The syntax to be used in the command windows is as follows:

```
sol = pdepe(m, pdefun, icfun, bcfun, xmesh, tspan).
```

Here,  $m$  is parameter corresponding to the symmetry of the problem and can be set equal to 0, 1, or 2;  $pdefun$  defines the PDE;  $icfun$  sets up initial conditions;  $bcfun$  states boundary conditions;  $xmesh$  is a vector  $[x_0, x_1, \dots, x_n]$  of increasing entries specifying the points at which a numerical solution is to be returned for each time in  $tspan = [t_0, t_1, \dots, t_f]$ . Entries  $xmesh(1)$  and  $xmesh(end)$  are equal to  $z_L$  and  $z_U$ , respectively. Moreover, the dimensions  $n$  and  $f$  of the vectors  $xmesh$  and  $tspan$  must be greater or equal to 3. Entries  $tspan(1)$  and  $tspan(end)$  are the starting and final maturities, respectively. Function `pdepe` performs the time integration with an ODE solver that selects the time step in a dynamic manner.<sup>1</sup> Second-order approximations to the solution of the PDE are made on the mesh specified in  $xmesh$ . Notice that the function `pdepe` does not select the input mesh automatically. Therefore, it must be provided by the final user, who may decide to use an unevenly spaced grid. Therefore, it is a good practice to stagger mesh points by smaller amounts on the region of the domain where the solution changes rapidly. It is usually convenient to refine the mesh near the strike price and to use a sparse grid away from this region. The computational cost of the resulting routine strongly depends on the length of  $xmesh$ .

Function `pdepe` solves PDEs of the following type:

$$c(z, \tau, u, \partial_z u) \partial_\tau u = z^{-m} \partial_z (z^m f(z, \tau, u, \partial_z u)) + s(z, \tau, u, \partial_z u). \quad (C.1)$$

The initial condition at  $\tau = \tau_0$  is  $u(\tau_0, z) = u_0(z)$ . For all  $\tau$  and either  $z = z_L$ , or  $z = z_U$ , the solution components satisfy a boundary condition:

---

<sup>1</sup> For reference, see Shampine and Reichelt (1997) and Skeel and Berzins (1990).

$$p(z, \tau, u) + q(z, \tau) f(z, \tau, u, \partial_z u) = 0. \tag{C.2}$$

Elements in  $q(z, \tau)$  are either all zero or none of them is null. Nonzero values for  $q$  are associated to Neumann and Robin boundary conditions, whereas Dirichlet boundary conditions lead to a vanishing  $q(z, \tau)$ . Note that boundary conditions are expressed in terms of the function  $f(z, \tau, u, \partial_z u)$  rather than  $\partial_z u$ . Notice that  $p(z, \tau, u)$  depends on  $u$  whereas  $q(z, \tau)$  is independent of it.

For the sake of clarity, let us examine a concrete example of utilization of the function `pdepe`. We consider the initial value problem illustrated in Chapter 4 “Finite Difference Methods”. On the unit interval  $[0, 1]$ , we consider the heat equation:

$$-\partial_\tau u(\tau, z) + \partial_{zz} u(\tau, z) = 0, \tag{C.3}$$

with initial condition:

$$u(0, z) = \begin{cases} 2z, & 0 \leq z \leq \frac{1}{2}, \\ 2(1 - z), & \frac{1}{2} \leq z \leq 1, \end{cases} \tag{C.4}$$

and boundary conditions:

$$u(\tau, 1) = u(\tau, 0) = 0. \tag{C.5}$$

The analytical solution of this *initial value problem* is:

$$u(\tau, z) = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin\left(\frac{n\pi}{2}\right) \sin(n\pi z) e^{-n^2\pi^2\tau}. \tag{C.6}$$

Equation (C.3) can be reduced to the form (C.1) by setting

$$\begin{aligned} m &= 0, & c(z, \tau, u, \partial_z u) &= 1, \\ f(z, \tau, u, \partial_z u) &= \partial_z u, & s(z, \tau, u, \partial_z u) &= 0. \end{aligned} \tag{C.7}$$

The boundary conditions are (C.5) and can be written in the form (C.2) using

$$p(z, \tau, u) = u(\tau, z), \quad q(z, \tau) = 0,$$

where  $z$  can be either 0 or 1.<sup>2</sup> The arguments of function `pdepe` can be built as follows.

Let us examine a Matlab<sup>®</sup> formulation of this example.

---

<sup>2</sup> If, instead, we have opted for boundary condition (C.5), we would have set  $z_L = 0, z_U = 1$ , and

$$\begin{aligned} p(z_U, \tau, u) &= u(\tau, z_U), & q(z_U, \tau) &= 0, \\ p(z_L, \tau, u) &= u(\tau, z_L), & q(z_U, \tau) &= 0. \end{aligned}$$



- `pdefun` is a function that returns the terms  $c$ ,  $f$ , and  $u$ . Input variables are defined by scalars  $x$  and  $t$  and vectors  $u$  and  $DuDx$  that approximate the solution and its partial derivative with respect to  $x$ . Comparing this to formula (C.7), the Matlab<sup>®</sup> code reads as follows:

```
function [c,f,s] = pdefun(x,t,u,DuDx)
c = 1;
f = DuDx;
s = 0;
```

- `icfun` evaluates the initial conditions. It has the form  $u = icfun(x)$ . When called with an argument  $x$ , `icfun` evaluates and returns the initial values of the solution components at  $x$  in the column vector  $u$ . With reference to the initial condition assigned in (C.4), the Matlab<sup>®</sup> code reads as follows:

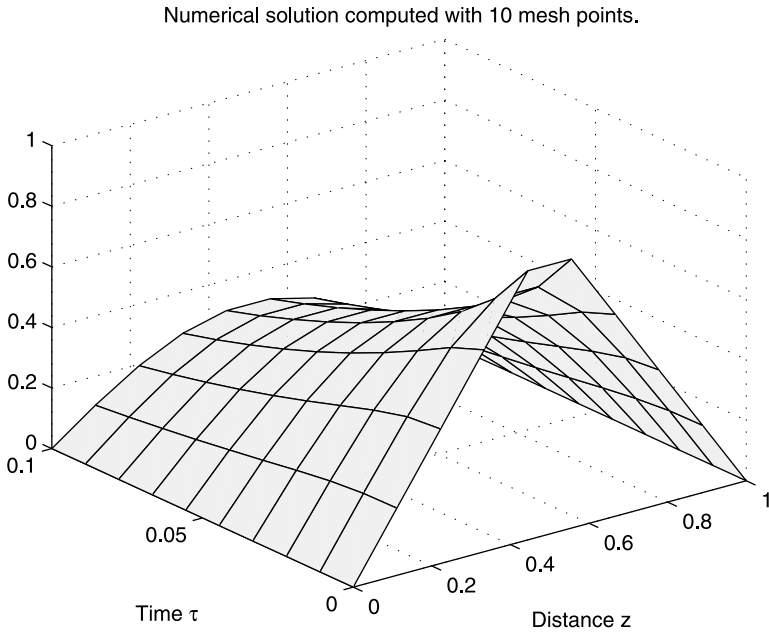
```
function u0 = pdexlic(x)
u0 = 2*(x>=1/2).* (1-x)+2*(x<1/2).*x;
```

- `bcfun` evaluates terms  $p$  and  $q$  on boundary conditions (C.5) and returns the vector  $[p_l, q_l, p_r, q_r]$ , where  $p_l$  and  $q_l$  are scalars corresponding to  $p$  and  $q$  evaluated at  $x_L$ , similarly  $p_r$  and  $q_r$  correspond to  $x_R$ . Therefore `bcfun` assumes form  $[p_l, q_l, p_r, q_r] = bcfun(x_l, u_l, x_r, u_r, t)$ , where  $u_l$  is the solution at the left boundary  $x_L$  and  $u_r$  is the solution at the right boundary  $x_R$ . With reference to our example, the Matlab<sup>®</sup> formulation given in (C.2) becomes:

```
function [p_l,q_l,p_r,q_r] = pdex1bcfun(x_l,u_l,x_r,u_r,t)
p_l = u_l;
q_l = 0;
p_r = u_r;
q_r = 0;
```

Below, we use subfunctions to place all the functions required by `pdepe` in a single M-file that is named `pdeexample.m`. This file can be run from the Matlab<sup>®</sup> command window. The proposed function provides a solution for the example discussed above using, for illustrative purposes, 10 points for the  $x$ -mesh and 24 points for the  $t$ -mesh. It calls function `pdepe` using the command `sol = pdepe(m,@pdefun,@pdexlic,@pdex1bcfun,x,t)`. The resulting numerical solution, generated by the line `surf(x,t,sol)`, is illustrated in Fig. C.1.

```
function pdeExample
m = 0;
```



**Fig. C.1.** Numerical solution of the PDE given in the example.

```

x = linspace(0,1,10);
t = linspace(0,0.1,10);
%computes the numerical solution of the pde
sol = pdepe(m,@pdefun,@pdexlic,@pdex1bcfun,x,t);
% A surface plot is often a good way
to study a solution.
surf(x,t,sol)
title('Numerical solution computed
with 10 mesh points.')
xlabel('Distance z')
ylabel('Time t')
% Defining the PDE-----
function [c,f,s] = pdefun(x,t,u,DuDx)
c = 1;
f = DuDx;
s = 0;
% Defining the IC-----
function u0 = pdexlic(x)

```

```

%u0 = 2*(x>=1/2).*(x-1/2);
u0 = 2*(x>=1/2).*(1-x)+2*(x< 1/2).*x;
% Defining the BC-----
function [pl,ql,pr,qr] = pdex1bcfun(xl,ul,xr,ur,t)
pl = ul;
ql = 0;
pr = ur;
qr = 0;
% The analytical solution-----
function ansolution = solutionpde(x, t)
sumseries = 0
for n = 1:20
    term = sin(n*pi/2)*sin(n*pi*x)*exp(-t*(n*pi)^2)/(n*n);
    sumseries = sumseries+term;
end
ansolution = sumseries*8/pi^2.

```

---

## Appendix D: Optimal Control

### D.1 Setting up the Optimal Stopping Problem

We begin by replacing the performance measure (3.4) with the equivalent  $\sum_{s=t}^{\tau} F(s, X^{t,x}(s))$  and then redefine  $\mathbb{X}$  by adding one singleton  $\{k\}$ , with  $k \notin \mathbb{X}$ , which plays the role of a “flag” for the stopping time  $\tau$ . The set of controls  $\mathcal{U}_{t,T}$  is given by the canonical basis in  $\mathbb{R}^{T-t}$ , i.e.,  $u = (u_t, \dots, u_{T-1}) \in \mathcal{U}_{t,T}$  and a single entry  $u_s$  is equal to 1 (“stopping at time  $s$ ”) and all the others equal 0. Then, we set dynamics as

$$p(dx; s, y, u, \omega) = \begin{cases} p(dx; s, X(s, \omega), \omega) & \text{if } u = 0, \\ \delta_k(dx) \text{ (Dirac delta mass on } \{k\}) & \text{if } u = 1 \text{ or } y = k, \end{cases}$$

or, alternatively, as

$$X^{\mathbf{u}}(s+1, \omega) = \begin{cases} f(s, X^{\mathbf{u}}(s, \omega), W(s, \omega)) & \text{if } u = 0, \\ k & \text{if } u = 1, \end{cases}$$
$$f(s, k, w) = k \quad \text{for all } s \text{ and } k.^1$$

This means that the system is trapped into a steady state  $k$  once the control has been activated ( $u = 1$ ). Finally, the cost/reward function vanishes on the steady state  $\{k\}$ :

$$F(s, y, u) = \begin{cases} F(s, y) & \text{if } y \neq k, \\ 0 & \text{if } y = k. \end{cases}$$

The equivalence of this formulation to an optimal stopping problem can now be easily proven as an exercise. Notice that the artificial state  $k$  signals the “death” of the system. For practical purposes, there is no need to cast an optimal stopping problem in this framework.

---

<sup>1</sup> We remark the difference between symbols “ $\omega$ ” and “ $w$ ”. The former denotes a generic elementary event in a sample space  $\Omega$ . You may think of it as a sample path itself, as is the case of the so-called canonical process  $X(t, \omega) = \omega(t)$ , defined on  $[0, T] \times \mathbb{R}^{[0, T]}$ . The latter is a real value assumed by the random noise  $W(t)$ .

## D.2 Proof of the Bellman Principle of Optimality

The two control policies  $\mathbf{u}$  and  $\mathbf{u}'$  give rise to the same exact dynamics on  $\{t, \dots, s\}$ . Therefore their performance coincides on  $\{t, \dots, s-1\}$ . Note that they need not be equal at  $s$  because the performance index in general depends on the control and the two control policies  $\mathbf{u}$  and  $\mathbf{u}'$  may not match at time  $s$ . This observation reduces the problem to the one of showing that the performance stemming from the *residual* dynamics  $(X^{t,x,\mathbf{u}}(s), \dots, X^{t,x,\mathbf{u}}(T))$  resulting from applying  $\mathbf{u}$  is no greater than the reward deriving from the residual dynamics obtained by using  $\mathbf{u}'$  instead. Control  $\mathbf{u} = (u(t, \cdot), \dots, u(T, \cdot))$  applied to the system dynamics for the remaining period gives rise to a reward assessment:

$$J^* := \sum_{i=s}^{T-1} F(i, X^{t,x,\mathbf{u}}(i), u(i, X^{t,x,\mathbf{u}}(i))) + \Psi(X^{t,x,\mathbf{u}}(T)).$$

The uniqueness of the solution of a dynamic system implies the flow property:

$$X^{t,x,\mathbf{u}}(i) = X^{s,X^{t,x,\mathbf{u}}(s),\mathbf{u}|_s}(i),$$

where  $t \leq s \leq i \leq T$ , and  $\mathbf{u}|_s$  is the truncated control  $(u(s), \dots, u(T))$  resulting from restricting  $\mathbf{u}$  on the remaining period  $\{s, \dots, T\}$ . By applying, in order, the flow property, the optimality of  $\hat{\mathbf{u}}^s$  on  $\{s, \dots, T\}$ , and the flow property again, we have:

$$\begin{aligned} J^* &= \sum_{i=s}^{T-1} F(i, X^{s,X^{t,x,\mathbf{u}}(s),\mathbf{u}|_s}(i), u(i, X^{s,X^{t,x,\mathbf{u}}(s),\mathbf{u}|_s}(i))) \\ &\quad + \Psi(X^{s,X^{t,x,\mathbf{u}}(s),\mathbf{u}|_s}(T)) \\ &\leq \sum_{i=s}^{T-1} F(i, X^{s,X^{t,x,\mathbf{u}}(s),\hat{\mathbf{u}}^s}(i), \hat{u}(i, X^{s,X^{t,x,\mathbf{u}}(s),\hat{\mathbf{u}}^s}(i))) \\ &\quad + \Psi(X^{s,X^{t,x,\mathbf{u}}(s),\hat{\mathbf{u}}^s}(T)) \\ &= \sum_{i=s}^{T-1} F(i, X^{t,x,\mathbf{u}'}(i), \hat{u}(i, X^{t,x,\mathbf{u}'}(i))) + \Psi(X^{t,x,\mathbf{u}'}(T)). \end{aligned}$$

This last term represents the performance generated by residual dynamics  $(X^{t,x,\mathbf{u}'}(s), \dots, X^{t,x,\mathbf{u}'}(T))$  corresponding to the control policy  $\mathbf{u}'$ .

## D.3 Proof of the Dynamic Programming Algorithm

We need to prove that  $\mathbf{u}^B$  dominates any other control policy in  $\mathcal{U}_{t,T}$ . We apply the Bellman principle of optimality at each step in the preceding backward induction. At time  $T$ , if the reached state is  $y$ , the generated performance is uncontrollable and

matches  $\Psi(y)$ . This holds for any control policy and thus for the optimal one. At time  $T - 1$ , whatever is the control policy  $\mathbf{u} = (u(t), \dots, u(T - 2))$  adopted on the elapsed period  $\{t, \dots, T - 2\}$ , the policy

$$(u(t, \cdot), \dots, u(T - 2, \cdot), \hat{u}^{T-1}(T - 1, \cdot)),$$

with  $\hat{u}^{T-1}(T - 1, \cdot) = \arg \max_{\mathcal{U}_{T-1, T}} J(T - 1, \cdot, \mathbf{u})$ , dominates any other control policy sharing the same first  $T - 2$  components. This is because each entry of the control policy contributes to the overall performance  $J$  *additively*. Therefore, the optimal control policy must have  $\hat{u}(T - 1, \cdot)$  as its  $(T - 1)$ th component. At time  $T - 2$ , the Bellman principle of optimality states that the control policy:

$$(u(t, \cdot), \dots, \hat{u}^{T-2}(T - 2, \cdot), \hat{u}^{T-2}(T - 1, \cdot)),$$

with  $(\hat{u}^{T-2}(T - 2, \cdot), \hat{u}^{T-2}(T - 1, \cdot)) = \arg \max_{\mathcal{U}_{T-2, T}} J(T - 2, \cdot, \mathbf{u})$ , dominates any other control policy sharing the same first  $T - 3$  components. But the previous step says that the optimal control policy must have  $\hat{u}^{T-1}(T - 1, \cdot)$  as its  $(T - 1)$ th component, i.e.,

$$\begin{aligned} & (u(t, \cdot), \dots, \hat{u}^{T-2}(T - 2, \cdot), \hat{u}^{T-1}(T - 1, \cdot)) \\ & \geq (u(t, \cdot), \dots, u(T - 3, \cdot), \hat{u}^{T-2}(T - 2, \cdot), \hat{u}^{T-2}(T - 1, \cdot)) \\ & \geq (u(t, \cdot), \dots, u(T - 3, \cdot), u(T - 2, \cdot), u(T - 1, \cdot)). \end{aligned}$$

Proceeding this way, we come up to the control policy:

$$\begin{aligned} \mathbf{u}^B &= (\hat{u}^t(t, \cdot), \dots, \hat{u}^{T-2}(T - 2, \cdot), \hat{u}^{T-1}(T - 1, \cdot)) \\ &\geq (\hat{u}^t(t, \cdot), \dots, \hat{u}^t(T - 2, \cdot), \hat{u}^t(T - 1, \cdot)) = \hat{\mathbf{u}}. \end{aligned}$$

But  $\hat{\mathbf{u}} = \arg \max_{\mathcal{U}_{t, T}} J(t, \cdot, u)$  dominates all control policy, in particular  $\mathbf{u}^B$ , i.e.,  $\hat{\mathbf{u}} \geq \mathbf{u}^B$ . Therefore,  $\hat{\mathbf{u}} = \mathbf{u}^B$ .

---

## Bibliography

- Abate, J., Choudhury, G.L., Whitt, W. (1996). On the Laguerre Method for Numerically Inverting Laplace Transforms. *Inform. Comput.* 8, 413–427.
- Abate, J., Choudhury, G.L., Whitt, W. (1998). Numerical Inversion of Multidimensional Laplace Transforms by the Laguerre Method. *Performance Evaluation*, 31, 229–243.
- Abate, J., Whitt, W. (1992a). Numerical Inversion of Probability Generating Functions. *Operations Research Letters* 12, 245–251.
- Abate, J., Whitt, W. (1992b). The Fourier-Series Method for Inverting Transforms of Probability Distributions. *Queueing Systems Theory Appl.* 10, 5–88.
- Abken, P.A. (2000). An Empirical Evaluation of Value at Risk by Scenario Simulation. *Journal of Derivatives* 7, 12–30.
- Abramowitz, M., Stegun, I.A. (1965). *Handbook of Mathematical Functions*. National Bureau of Standards, Washington, DC. (Reprinted by Dover, New York.)
- Abramowitz, M., Stegun, I.A., Kampen, J. (1993). *Handbook of Mathematical Functions* (3rd ed.). John Wiley.
- Acworth, P., Broadie, M., Glasserman, P. (1998). A Comparison of Some Monte Carlo and Quasi-Monte Carlo Methods for Option Pricing. In: Hellekaed, P., Larcher, G., Niederreiter, H., Zinterhof, P. (Eds.), *Monte Carlo and Quasi-Monte Carlo Methods*. Springer-Verlag.
- Aitsahlia, F., Lai, T. (1997). Valuation of Discrete Barrier and Hindsight Options. *The Journal of Financial Engineering* 6(2), 169–177.
- Ait-Sahalia, Y., Lo, A.W. (1998). Non-Parametric Estimation of State-Price Densities Implicit in Financial Asset Prices. *Journal of Finance* 53(2), 499–548.
- Akahori, J. (1995). Some Formulae for a New Type of Path-Dependent Options. *Annals of Applied Probability* 5, 383–388.
- Akesson, F., Lehoczky, L. (2000). Path Generation for Quasi-Monte Carlo Simulation of Mortgage-Backed Securities. *Management Science* 46, 1171–1187.
- Albrecher, H., Mayer, P., Schoutens, W., Tistaert, J. (2007). The Little Heston Trap. *Wilmott Magazine*, January, 83–92.
- Alexander, C. (2001). *Market Models: A Guide to Financial Data Analysis*, Wiley Series in Financial Engineering. John Wiley & Sons.
- Altman, E., Resti, A., Sironi, A. (2004). Default and Recovery Rates in Credit Risk Modeling: A Review of the Literature and Empirical Evidence. *Journal of Finance Literature* 1.

- Altman, E., Resti, A., Sironi, A. (2005). The Link Between Default and Recovery Rates: Theory, Empirical Evidence and Implications. *The Journal of Business* 78(6), 2203–2228.
- Amin, K.I. (1993). Jump Diffusion Option Valuation in Discrete Time. *Journal of Finance* 48(5), 1883–1863.
- Andersen, L.B.G. (2007). Efficient Simulation of the Heston Stochastic Volatility Model. (Available at: <http://ssrn.com/abstract=946405>).
- Andersen, T.G. (1995). Simulation and Calibration of the HJM Model. *Working Paper*, General Re Financial Products, New York.
- Andersen, T.G., Broadie, M. (2001). A Primal–Dual Simulation Algorithm for Pricing Multi-Dimensional American Options. *Working Paper*, Columbia Business School, New York.
- Andersen, L., Brotherton-Ratcliffe, R. (1996). Exact Exotics. *Risk* 9, 85–89.
- Andricopoulos, A.D., Widdicks, M., Duck, P.W., Newton, D.P. (2003). Universal Option Valuation Using Quadrature Methods. *Journal of Financial Economics* 67, 447–471.
- Antia, H.M. (2002). *Numerical Methods for Scientists and Engineers* (2nd ed.). H.M. Birkhäuser.
- Antonov, I.A., Saleev, V.M. (1980). An Economic Method of Computing  $l_{p\tau}$ -Sequences. *USSR Comput. Maths. Math. Phys.* 19, 252–256.
- Aparicio, S., Hodges, S. (1998). Implied Risk-Neutral Distribution: A Comparison of Estimation Methods. *FORC preprint* PP98-95, Warwick University.
- Appel, G., Hitschler, F. (1980). *Stock Market Trading Systems*. Traders Press.
- Applebaum, D. (2004). *Lévy Processes and Stochastic Calculus*. Cambridge University Press.
- Asmussen, S., Rosinski, J. (2001). Approximations of Small Jumps of Lévy Processes with a View Towards Simulation. *Journal of Applied Probability* 38, 482–493.
- Atkinson, K.E. (1989). *An Introduction to Numerical Analysis*. John Wiley and Sons.
- Atkinson, C., Fusai, G. (2004). Discrete Extrema of Brownian Motion and Pricing of Look-back Options. *Working Paper*, Dipartimento SEMEQ, Università del Piemonte Orientale.
- Atlan, M., Geman, H., Yor, M. (2005). Options on Hedge Funds under the High Water Mark Rule. *Working Paper*.
- Avellaneda, M., Buff, R., Friedman, C., Grandchamp, N., Kruk, L., Newman, J. (2001). Weighted Monte Carlo: A New Technique for Calibrating Asset-Pricing Models. *International Journal of Theoretical and Applied Finance* 4(1), 1–29.
- Avellaneda, M., Gamba, R. (2000). Conquering the Greeks in Monte Carlo: Efficient Calculation of the Market Sensitivities and Hedge-Ratios of Financial Assets by Direct Numerical Simulation. In: Avellaneda, M. (Ed.), *Quantitative Analysis in Financial Markets, Vol. III*. World Scientific, Singapore, 336–356.
- Avellaneda, M., Paras, A. (1994). *Dynamic Hedging with Transaction Costs: From Lattice Models to Nonlinear Volatility and Free-Boundary Problems*. Unpublished manuscript.
- Avellaneda, M., Scherer, K. (2002). All for One and One for All: A Principal Components Analysis of the Latin American Brady Bond Market from 1994 to 2000. *International Journal of Theoretical and Applied Finance* 5(1), 79–107.
- Avellaneda, M., Wu, L. (1999). Pricing Parisian-Style Options with a Lattice Method. *International Journal of Theoretical and Applied Finance* 2(1), 1–17.
- Baccara, M., Battauz, A., Ortu, F. (2005). Effective Securities in Arbitrage-Free Markets with Bid-Ask Spreads at Liquidation: A Linear Programming Characterization. *Journal of Economic Dynamics and Control* 30(1), 55–79.
- Bahra, B. (1997). Implied Risk-Neutral Probability Density Functions from Option Prices: Theory and Application. *Working Paper* 66, Bank of England, 1368–5562.
- Bakshi, G.S., Cao, C., Chen, Z.W. (1997). Empirical Performance of Alternative Option Pricing Models. *Journal of Finance* 52, 2003–2049.



- Baldi, P., Caramellino, L., Iovino, G. (1999). Pricing General Barrier Options: a Numerical Approach Using Sharp Large Deviations. *Mathematical Finance* 9(4), 293–321.
- Baldick, R., Kolos, S., Tompaidis, S. (2003). Valuation and Optimal Interruption for Interruptible Electricity Contracts. *Working Paper*, University of Texas.
- Ballotta, L. (2001). Lévy Processes, Option Valuation and Pricing of the Alpha-Quantile Option. *Ph.D. Thesis*, Università Cattolica Sacro Cuore, Milan.
- Ballotta, L., Kyprianou, A. (2001). A Note on the Alpha-Quantile Option. *Applied Mathematical Finance* 8, 137–144.
- Bandi, F.M., Nguyen, T.H. (1999). Fully Nonparametric Estimators for Diffusions: A Small Sample Analysis. *Working Paper*, University of Chicago.
- Bandi, F.M., Nguyen, T.H. (2003). On the Functional Estimation of Jump-Diffusion Processes. *Journal of Econometrics* 116, 293–328.
- Barbieri, A., Garman, M.B. (1996). Putting a Price on Swings. *Energy and Power Risk Management* 1(6).
- Barbieri, A., Garman, M.B. (1997). Ups and Downs of Swings. *Energy and Power Risk Management* 2(1).
- Barlow, M.T. (2002). A Diffusion Model for Electricity Prices. *Mathematical Finance* 12, 287–298.
- Barone-Adesi, G. (2005). The Saga of the American Put. *Journal of Banking and Finance* 29(11), 2909–2918.
- Barone-Adesi, G., Whaley, R. (1987). Efficient Analytic Approximation of American Option Values. *Journal of Finance* 42(2), 301–320.
- Barrett, R., Berry, M., Chan, T.F., Demmel, J., Donato, J., Dongarra, J., Eijkhout, V., Pozo, R., Romine, C., Van der Vorst, H. (1994). *Templates for the Solution of Linear Systems: Building Blocks for Iterative Methods*. SIAM, Philadelphia (<http://www.netlib.org/templates/Templates.html>).
- Barton, D.E., Dennis, K.E.R. (1952). The Conditions Under Which Gram–Charlier and Edgeworth Curves are Positive Definite and Unimodal. *Biometrika* 39, 425–427.
- Barucci, E. (2003). *Financial Markets Theory: Equilibrium, Efficiency and Information*. Springer Finance. Springer-Verlag, Berlin–Heidelberg–New York.
- Bates, D.S. (1991). The Crash of '87: Was It Expected? The Evidence from Options Markets. *Journal of Finance* 46(3), 1009–1044.
- Bates, D. (1996). Jumps and Stochastic Volatility: Exchange Rate Processes in Deutschemark Options. *Review of Financial Studies* 9, 69–108.
- Battauz, A. (2002). Change of Numéraire and American Options. *Stochastic Analysis and Applications* 20(4), 709–730.
- Baxter, M., Rennie, A. (1996). *Financial Calculus: An Introduction to Derivative Pricing*. Cambridge University Press.
- Baz, J., Das, S.R. (1996). Analytical Approximations of the Term Structure for Jump-Diffusion Processes: A Numerical Analysis. *Journal of Fixed Income* 78–86.
- Bazaraa, M.S., Sherali, H.D., Shetty, C.M. (1993). *NonLinear Programming, Theory and Algorithms* (2nd ed.). Wiley.
- Beaglehole, D., Dybvig, P., Zhou, G. (1997). Going to Extremes: Correcting Simulation Bias in Exotic Option Valuation. *Financial Analysts Journal* 53, 62–68.
- Beamon (1998). Competitive Electricity Prices. *Working Paper*, Energy Information Administration.
- Bedendo, M., Anagnou, I., Hodges, S.D., Tompkins, R. (2005). Forecasting Accuracy of Implied and GARCH-Based Probability Density Functions. *Review of Futures Markets* 14(1), Summer.

- Bellman, R. (1957). *Dynamic Programming*. Dover Publications.
- Bellman, R., Kalaba, R.E., Lockett, J. (1966). *Numerical Inversion of the Laplace Transform. Application to Biology, Economics, Engineering and Physics*. American Elsevier, New York.
- Benth, F.E., Dahl, L.O., Karlsen, K.H. (2003). Quasi Monte Carlo Evaluation of Sensitivities of Options in Commodity and Energy Markets. *International Journal of Theoretical and Applied Finance* 6(8), 865–884.
- Benth, F.E., Kallsen, J., Meyer-Brandis, T. (2007). A Non-Gaussian Ornstein–Uhlenbeck Process for Electricity Spot Price Modeling and Derivatives Pricing. *Applied Mathematical Finance* 14(2), 153–169.
- Benth, F.E., Saltyte-Benth, J. (2006). Analytical Approximation for the Price Dynamics of Spark Spread Options. *Studies of Nonlinear Dynamics & Econometrics* 10(3).
- Bertsekas, D.P. (2005). *Dynamic Programming and Optimal Control*, Vol. 1 and 2 (2nd ed.). Athena Scientific.
- Bessembinder, H., Chan, K. (1995). The Profitability of Technical Trading Rules in the Asian Stock Markets. *Pacific-Basin Finance Journal* 3, 257–284.
- Bessembinder, H., Chan, K. (1998). Market Efficiency and the Returns to Technical Analysis. *Financial Management* 27(2), 5–17.
- Best, M.J., Grauer, R. (1991). On the Sensitivity of Mean-Variance Efficient Portfolios to Changes in Asset Means: Some Analytical and Computational Results. *Review of Financial Studies* 4, 315–342.
- Bhanot, K. (2000). Behavior of Power Prices. *Journal of Risk* 2, 43–62.
- Biffis, E., Millossovich, P. (2006). The Fair Value of Guaranteed Annuity Options. *Scandinavian Actuarial Journal* 1, 23–41.
- Björk, T. (2004). *Arbitrage Theory in Continuous Time* (2nd ed.). Oxford University Press.
- Block, S.B. (1999). A Study of Financial Analysts: Practice and Theory. *Financial Analysts Journal* 55(4), 86–95.
- Blume, M.E. (1975). Betas and Their Regression Tendencies. *Journal of Finance* 30(3), 785–789.
- Borodin, A.N., Salminen, P. (2002). *Handbook of Brownian Motion: Facts and Formulae* (2nd ed.). Birkhäuser.
- Botterud, A., Bhattacharyya, B., Ilic, M. (2002). Futures and Spot Prices, an Analysis of the Scandinavian Electricity Market. *Working Paper*, MIT.
- Bouchard, B., Ekeland, I., Touzi, N. (2004). On the Malliavin Approach to Monte Carlo Approximation of Conditional Expectations. *Finance and Stochastics* 8(1), 45–71.
- Bouchaud, J.P., Potters, M., Sestovic, D. (2000). Hedged Monte Carlo: Low Variance Derivative Pricing with Objective Probabilities. *Working Paper*, Science & Finance Capital Fund Management, France.
- Boudoukh, J., Whitelaw, R.F., Richardson, M., Stanton, R. (1997). Pricing Mortgage-Backed Securities in a Multifactor Interest Rate Environment: A Multivariate Density Estimation Approach. *Review of Financial Studies* 10, 405–446.
- Box, G.E.P., Muller, M.E. (1958). A Note on the Generation of Random Normal Deviates. *Annals of Mathematical Statistics* 29, 610–611.
- Boyle, P. (1977). Options: A Monte Carlo Approach. *Journal of Financial Economics* 4(3), 323–338.
- Boyle, P., Broadie, M., Glasserman, P. (1997). Monte Carlo Methods for Security Pricing. *Journal of Economic Dynamics and Control* 21, 1267–1321.
- Boyle, P., Evnine, J., Gibbs, S. (1989). Numerical Evaluation of Multivariate Contingent Claims. *Review of Financial Studies* 2(2), 241–250.

- Boyle, P., Kolkiewicz, A. (2002). Pricing American Derivatives Using Simulation: A Biased Low Approach. In: Fang, K.T., Hickernell, F.J., Niederreiter, H. (Eds.), *Monte Carlo and Quasi-Monte Carlo Methods*. Springer-Verlag, Berlin–Heidelberg–New York.
- Boyle, P.P., Lau, S.H. (1994). Bumping Up Against the Barrier with the Binomial Method. *Journal of Derivatives* 1, 6–14.
- Boyle, P., Tan, K.S. (1997). Quasi-Monte Carlo Methods. *Working Paper*, University of Waterloo.
- Boyle, P., Tian, Y.S. (1998). An Explicit Finite Difference Approach to the Pricing of Barrier Options. *Applied Mathematical Finance* 5, 17–43.
- Brandt, M.W. (2006). Portfolio Choice Problems. In: Ait-Sahalia, Y., Hansen L.P. (Eds.), *Handbook of Financial Econometrics*. Elsevier, forthcoming.
- Brandt, M.W., Santa Clara, P. (2002). Simulated Likelihood Estimation of Diffusions with an Application to Exchange Rate Dynamics in Incomplete Markets. *Journal of Financial Economics* 63, 161–212.
- Breedon, D., Litzenberger, R.H. (1978). Prices of State-Contingent Claims Implicit in Option Prices. *Journal of Business* 51(4), 621–651.
- Brennan, M., Schwartz, E. (1977). The Valuation of American Put Options. *Journal of Finance* 32(2), 449–463.
- Brennan, M., Schwartz, E. (1978). Finite Difference Methods and Jump Processes and the Pricing of Contingent Claims: A Synthesis. *Journal of Financial and Quantitative Analysis* 13, 461–474.
- Brennan, M., Schwartz, E. (1985). Determinants of GNMA Mortgage Prices. *Journal of the American Real Estate and Urban Economics Association* 13, 209–228.
- Brigo, D., Mercurio, F. (2006). *Interest Rate Models – Theory and Practice: With Smile, Inflation and Credit* (2nd ed.). Springer Finance. Springer-Verlag, Berlin–Heidelberg–New York.
- Brigo, D., Mercurio, F., Rapisarda, F. (2004). Smile at the Uncertainty. *Risk* 97–100.
- Britten-Jones, M. (1999). The Sampling Error in Estimates of Mean-Variance Efficient Portfolio Weights. *Journal of Finance* 54(2), 655–671.
- Briys, E., Bellalah, M., Mai, H.M., De Varenne, F. (1998). *Options, Futures and Exotic Derivatives*. Wiley, England.
- Broadie, M., Detemple, J. (1996). American Option Valuation: New Bounds, Approximations, and a Comparison of Existing Methods. *Review of Financial Studies* 9, 1211–1250.
- Broadie, M., Detemple, J. (1997). Valuation of American Options on Multiple Assets. *Review of Financial Studies* 7, 241–286.
- Broadie, M., Glasserman, P. (1996). Estimating Security Price Derivatives Using Simulation. *Management Science* 42, 269–285.
- Broadie, M., Glasserman, P. (1997). Pricing American-Style Securities Using Simulation. *Journal of Economic, Dynamics and Control* 21, 1323–1352.
- Broadie, M., Glasserman, P. (1997). A Stochastic Mesh Method for Pricing High-Dimensional American Options. *Working Paper*, Columbia University.
- Broadie, M., Glasserman, P., Jain, G. (1997). Enhanced Monte Carlo Estimates of American Options Prices. *Journal of Derivatives* 4, 25–44.
- Broadie, M., Glasserman, P., Kou, S. (1997). A Continuity Correction for Discrete Barrier Options. *Mathematical Finance* 7(4), 325–349.
- Broadie, M., Glasserman, P., Kou, S. (1999). Connecting Discrete and Continuous Path-Dependent Options. *Finance and Stochastics* 3, 55–82.
- Broadie, M., Kaya, Ö. (2006). Exact Simulation of Stochastic Volatility and Other Affine Jump Diffusion Processes. *Operations Research* 54, 217–231.

- Brock, W., Lakonishok, J., LeBaron, B. (1992). Simple Technical Trading Rules and the Stochastic Properties of Stock Returns. *Journal of Finance* 47(5), 1731–1764.
- Brooks, R.D., Faff, R.W., McKenzie, M.D. (1998). Time-Varying Beta Risk of Australian Industry Portfolios: A Comparison of Modelling Techniques. *Australian Journal of Management* 23, 1–22.
- Bruner, R.F., Eades, K.M., Harris, R.S., Higgins, R.C. (1998). Best Practices in Estimating the Cost of Capital: Survey and Synthesis. *Financial Practice and Education* 27, 13–28.
- Bruti-Liberati, N., Platen, E. (2006). Strong Approximations of Stochastic Differential Equations with Jumps. *Journal of Computational and Applied Mathematics* 205(2), 982–1001.
- Bruti-Liberati, N., Platen, E. (2007). Approximation of Jump Diffusions in Finance and Economics. *Computational Economics* 29(3/4), 283–312.
- Bruti-Liberati, N., Nikitopoulos-Sklibosios, C., Platen, E. (2006). First Order Strong Approximations of Jump Diffusions. *Monte Carlo Methods and Applications* 12(3), 191–209.
- Burlisch, R., Stoer, J. (1992). *Introduction to Numerical Analysis* (2nd ed). Springer-Verlag, Berlin–Heidelberg–New York.
- Buser, S.A. (1986). Laplace Transforms as Present Value Rules: A Note. *Journal of Finance* XLI(1), 243–247.
- Cai, N., Kou, S.G. (2007). Pricing Asian Options via a Double-Laplace Transform. Working paper, Columbia University.
- Cairns, A.J. (2004). *Interest Rate Models: An Introduction*. Princeton University Press.
- Campa, J.M., Chang, K., Reider, R. (1997). ERM Bandwidths for EMU and After: Evidence from Foreign Exchange Options. *Economic Policy* 24, 55–89.
- Campa, J.M., Chang, K., Reider, R. (1998). Implied Exchange Rate Distributions: Evidence from OTC Option Markets. *Journal of International Money and Finance* 17(1), 117–160.
- Campbell, J.Y., Lo, A.W., MacKinlay, A.C. (1997). *The Econometrics of Financial Markets*. Princeton University Press, Princeton, NJ.
- Carmona, R., Dayanik, S. (2003). Optimal Multiple Stopping of Linear Diffusions and Swing Options. *Working Paper*, Princeton University.
- Carr, P. (1998). Randomization and the American Put. *Review of Financial Studies* 11(3), 597–626.
- Carr, P. (2000). Deriving Derivatives of Derivative Securities. *Journal of Computational Finance* 4(2), 5–29.
- Carr, P., Geman, H., Madan, D.H., Yor, M. (2003). Stochastic Volatility for Lévy processes, *Mathematical Finance* 13(3), 345–382.
- Carr, P., Geman, H., Madan, D.H., Wu, L., Yor, M. (2005). Option Pricing Using Integral Transforms. *Working Paper*.
- Carr, P., Hirska, A. (2003). Why Be Backward? Forward Equations for American Options. *Risk* 16(1), 103–107.
- Carr, P., Jarrow, R., Myneni, R. (1992). Alternative Characterizations of American Put Options. *Mathematical Finance* 2(4).
- Carr, P., Madan, D.H. (1999). Option Evaluation Using the Fast-Fourier Transform. *Journal of Computational Finance* 2(4), 61–73.
- Carr, P., Schröder, M. (2004). Bessel Processes, the Integral of Geometric Brownian Motion and Asian Options. *Theory of Probability and its Applications* 71(1), 113–141.
- Carr, P., Yang, G. (1997). Simulating Bermudan Interest Rate Derivatives. *Working Paper*, Morgan Stanley, New York.
- Carr, P., Yang, G. (1998). Simulating American Bond Options in an HJM Framework. *Working Paper*, Morgan Stanley, New York.

- Carrière, J. (1996). Valuation of the Early-Exercise Price for Options Using Simulations and Nonparametric Regression. *Insurance: Mathematics and Economics* 19(1), 19–30.
- Cartea, Á., Figueroa, M.G. (2005). Pricing in Electricity Markets: A Mean Reverting Jump Diffusion Model with Seasonality. *Applied Mathematical Finance* 12(4), 313–335.
- Cartea, Á., Williams, T. (2007). UK Gas Markets: the Market Price of Risk and Applications to Multiple Interruptible Supply Contracts, forthcoming in *Energy Economics*.
- Carverhill, A., Pang, K. (1998). Efficient and Flexible Bond Option Valuation in the Heath, Jarrow and Morton Framework. In: Dupire, B. (Ed.), *Monte Carlo: Methodologies and Applications for Pricing and Risk Management*. Risk Publications, London.
- Cathcart, L. (1998). The Pricing of Floating Rate Instruments. *Journal of Computational Finance* 1(4), 31–51.
- Cerny, A. (2003). *Mathematical Techniques in Finance: Tools for Incomplete Markets*. Princeton University Press.
- Chan, K.C., Karoly, G.A., Longstaff, F.A., Sanders, A.B. (1992). The Volatility of Short Term Interest Rates: An Empirical Comparison of Alternative Models of the Term Structure of Interest Rates. *Journal of Finance* 47, 1209–1227.
- Chapman, D., Pearson, N. (2000). Is the Short Rate Drift Actually Non Linear? *Journal of Finance* 55(1), 355–388.
- Chen, R.R., Scott, L. (1993). Maximum Likelihood Estimation for a Multifactor Equilibrium Model of the Term Structure of Interest Rates. *Journal of Fixed Income* 3, 14–31.
- Chen, R., Scott, L. (1995). Interest Rate Options in Multifactor Cox–Ingersoll–Ross Models of the Term Structure. *Journal of Derivatives* 3, 53–72.
- Cherubini, U., Luciano, E. (2001). Value-at-Risk Trade-Off and Capital Allocation with Copulas. *Economic Notes* 30(2), 235–256.
- Cherubini, U., Luciano, E., Vecchiato, W. (2004). *Copula Methods in Finance*. Wiley Finance Series. John Wiley & Sons.
- Chesney, M., Cornwall, J., Jeanblanc-Picqué, M., Kentwell, G., Yor, M. (1997). Parisian Pricing. *Risk* 10(1), 77–80.
- Chesney, M., Jeanblanc-Picqué, M., Yor, M. (1995). Brownian Excursion and Parisian Barrier Options. *Advances in Applied Probability* 29, 165–184.
- Cheuk, T., Vorst, T. (1996). Complex Barrier Options. *Journal of Derivatives*, Fall, 8–22.
- Chopra, V., Ziemba, T. (1993). The Effect of Errors in Means, Variances and Covariances on Optimal Portfolio Choice. *Journal of Portfolio Management* 19(3).
- Choudhury, G.L., Lucantoni, D.M., Whitt, W. (1994). Multidimensional Transform Inversion with Applications to the Transient M/G/1 Queue. *Annals of Applied Probability* 4(3), 719–740.
- Christoffersen, P.F. (1998). Evaluating Interval Forecast. *International Economic Review* 39, 841–862.
- Christoffersen, P., Jacobs, K. (2004). The Importance of Loss Function in Option Evaluation. *Journal of Financial Economics* 72, 291–318.
- Churchill, R.V., Brown, J.W. (1989). *Complex Variables and Applications* (5th ed.). McGraw-Hill Companies.
- Clelow, L., Carverhill, A. (1994). On the Simulation of Contingent Claims. *Journal of Derivatives* 2, 66–74.
- Clelow, L., Strickland, C. (1997). Monte Carlo Valuation of Interest Rate Derivatives Under Stochastic Volatility. *Journal of Fixed Income* 7(3), 35–45.
- Clelow, L., Strickland, C. (1998). *Implementing Derivative Models*. Wiley & Sons, London.
- Clelow, L., Strickland, C. (2000). *Energy Derivatives: Pricing and Risk Management*. Lacima Group Publications.

- Clewlou, L., Strickland, C., Kaminski, V. (2001). Risk Analysis of Swing Contracts. *Energy and Power Risk Management*.
- Cochrane, J.H. (2001). *Asset Pricing*. Princeton University Press, Princeton, NJ.
- Connor, G., Herbert, N. (1999). Estimation of the European Equity Model. *Horizon: The Barra Newsletter* 169.
- Cont, R., Tankov, P. (2004). *Financial Modelling with Jump Processes*. Chapman & Hall/CRC Press.
- Conze, A., Viswanathan, R. (1991). Path-Dependent Options: The Case of Lookback Options. *Journal of Finance* 46(5), 1893–1907.
- Cook, R.D., Weisberg, S. (1982). *Residuals and Influence in Regression*. Chapman and Hall, New York.
- Corielli, F. (2006). Hedging with Energy. *Mathematical Finance* 16(3), 495–517.
- Courtadon, G. (1982). A More Accurate Finite Difference Approximation for the Valuation of Options. *Journal of Financial and Quantitative Analysis* 17(5), 697–703.
- Cox, J.C. (1996). The Constant Elasticity of Variance Option Pricing Model. *Journal of Portfolio Management*, Special Issue, 15–17.
- Cox, J.C., Ingersoll, J.E., Ross, S.A. (1985). A Theory of the Term Structure of Interest Rates. *Econometrica* 53(2), 385–407.
- Cox, J.C., Ross, S.A. (1976). The Valuation of Options for Alternative Stochastic Processes. *Journal of Financial Economics* 3(1-2), 145–166.
- Cox, J.C., Ross, S.A., Rubinstein, M. (1979). Option Pricing: A Simplified Approach. *Journal of Financial Economics* 7, 229–264.
- Craddock, M., Heath, D., Platen, E. (2000). Numerical Inversion of Laplace Transforms: A Survey of Techniques With Applications to Derivatives Pricing. *Journal of Computational Finance* 4(1).
- Craig, I., Thompson, A.M. (1994). Why Laplace Transforms are Difficult to Invert Numerically? *Computers in Physics* 8(6), 648–654.
- Crump, K. (1976). Numerical Inversion of Laplace Transform using Fourier Series Approximation. *J. Assoc. Comp. Mach.* 23, 89–96.
- Cryer, C.W. (1971). The Solution of a Quadratic Programming Problem Using Systematic Overrelaxation. *SIAM J. Control* 9, 385–395.
- Dahl, L.O., Benth, F.E. (2002). Fast Evaluation of Asian Basket Option by Singular Value Decomposition. In: *Monte Carlo and Quasi-Monte Carlo Methods*. Springer, 201–213.
- Das, S.R. (1997a). A Direct Discrete-Time Approach to Poisson–Gaussian Bond Option Pricing in the Heath–Jarrow–Morton Model. *Working Paper*, Harvard Business School.
- Das, S.R. (1997b). Poisson–Gaussian Processes and the Bond Markets. *Working Paper*, Harvard University.
- Das, S.R., Sundaram, R.K. (1999). Of Smiles and Smirks: A Term Structure Perspective. *Journal of Financial and Quantitative Analysis* 34(2), 211–239.
- Dassios, A. (1995). The Distribution of the Quantile of a Brownian Motion with Drift and the Pricing of Related Path-Dependent Options. *Annals of Applied Probability* 4, 719–740.
- Davies, B., Martin, B.L. (1970). Numerical Inversion of Laplace Transforms: A Critical Evaluation and Review of Methods. *Journal of Computational Physics* 33, 1–32.
- Davis, P., Rabinowitz, P. (1975). *Methods of Numerical Integration*. Academic Press, New York.
- Davydov, D., Linetsky, V. (2001a). Pricing and Hedging Path-Dependent Options under the CEV Process. *Management Science* 47, 949–965.
- Davydov, D., Linetsky, V. (2001b). Structuring, Pricing and Hedging Double Barrier Step Options. *Journal of Computational Finance* 5(2), 55–87.

- D'Ecclesia, R.L., Zenios, S.A. (1994). Risk Factor Analysis and Portfolio Immunization in the Italian Bond Market. *Journal of Fixed Income*, 51–60.
- De Jong, C., Huisman, R. (2002). Option Formulas with Mean Reverting Power Prices with Spikes. *Working Paper*, Erasmus University, Rotterdam.
- Demange, G., Rochet, J.-C. (1997). *Methodes Mathématiques de la Finance*. Economica.
- Dempster, M.A., Hutton, J.P. (1999). Pricing American Options by Linear Programming. *Mathematical Finance* 9(3).
- Deng, S. (1999). Financial Methods in Competitive Electricity Markets. *Ph.D. Thesis*, University of California at Berkeley.
- Deng, S., Johnson, B., Sogomonian, A. (1999). Spark Spread Options and the Valuation of Electricity Generation Assets. *Proceedings of the 32nd Hawaii International Conference on System Sciences*.
- Den Iseger, P. (2006). Numerical Inversion of Laplace Transforms Using a Gaussian Quadrature for the Poisson Summation Formula. *Probability in the Engineering and Informational Sciences* 20(1), 1–44.
- Derman, E., Kani, I. (1994). Riding on a Smile. *Risk* 7(2), 32–39.
- Detry, P.J., Grégoire, P. (2001). Other Evidences of the Predictive Power of Technical Analysis: The Moving Average Rules on European Indexes. *Working Paper*, EFMA 2001 Lugano Meeting.
- Devroye, L. (1986). *Non-Uniform Random Variate Generation*. Springer-Verlag, Berlin–Heidelberg–New York.
- D'Halluin, Y., Forsyth, P.A., Vetzal, K.R. (2003). Robust Numerical Methods for Contingent Claims under Jump Diffusion Processes. *IMA Journal on Numerical Analysis* 25, 87–112.
- D'Halluin, Y., Forsyth, P.A., Labahn, G. (2005). A Semi-Lagrangian Approach for American Asian Options Under Jump Diffusion. *SIAM Journal on Scientific Computing*, 27, 315–345.
- Di Graziano, G., Rogers, L.C.G. (2005). A New Approach to the Modelling and Pricing of Correlation Credit Derivatives. *Working Paper*, Cambridge University.
- Doetsch, G. (1970). *Introduction to the Theory and Application of the Laplace Transformation*. Springer-Verlag, Berlin–Heidelberg–New York.
- Dongarra, J., Bunch, J., Moler, C., Stewart, G. (1979). *LINPACK User's Guide*. SIAM Pub., Philadelphia.
- Douady, R. (1998). Model Calibration in the Monte Carlo Framework. In: Dupire, B. (Ed.), *Monte Carlo: Methodologies and Applications for Pricing and Risk Management*. Risk Publications, London.
- Douglas, M., Simin, T. (2003). Outlier-Resistant Estimates of Beta. *Financial Analysts Journal* 59(5), 56–69.
- Duan, J. (1995). The Garch Option Pricing Model. *Mathematical Finance* 5, 13–32.
- Duan, J. (1996). Cracking the Smile. *Risk* 9, 55–59.
- Duan, J.-C., Dudley, E., Gauthier, G., Simonato, J.-G. (2003). Pricing Discretely Monitored Barrier Options by a Markov Chain. *Journal of Derivatives* 10(4), Summer, 9–32.
- Duan, J.-C., Gauthier, G., Simonato, J.-G. (2001). Asymptotic Distribution of the EMS Option Price Estimator. *Management Science* 47(8), 1122–1132.
- Duan, J., Simonato, J.G. (1998). Empirical Martingale Simulation for Asset Prices. *Management Science* 44(9), 1218–1233.
- Dubner, H., Abate, J. (1968). Numerical Inversion of Laplace Transforms by Relating them to the Finite Fourier Cosine Transform. *J. ACM* 15(1), 115–123.

- Duffy, D.A. (1993). On the Numerical Inversion of Laplace Transforms: Comparison of Three new Methods on Characteristic Problems from Applications. *ACM Trans. on Math. Soft.* 19(3), 333–359.
- Duffie, D. (2001). *Dynamic Asset Pricing Theory* (3rd ed.). Princeton University Press.
- Duffie, D., Glynn, P. (1995). Efficient Monte Carlo Simulation of Security Prices. *Annals of Applied Probability* 5, 897–905.
- Duffie, D., Pan, J., Singleton, K.J. (1998). Transform Analysis and Asset Pricing for Affine Jump-Diffusions. *Econometrica* 68(6), 1343–1376.
- Duffie, D., Singleton, K.J. (1993). Simulated Moments Estimation of Markov Models of Asset Prices. *Econometrica* 61, 929–952.
- Dumas, B., Luciano, E. (1991). An Exact Solution to a Dynamic Portfolio Choice Problem with Transaction Costs. *Journal of Finance* 46(2), 577–595.
- Dunn, K.B., McConnell, J.J. (1981a). A Comparison of Alternative Models for Pricing GNMA Mortgage-Backed Securities. *Journal of Finance* 36(2), 471–490.
- Dunn, K.B., McConnell, J.J. (1981b). Valuation of GNMA Mortgage-Backed Securities. *Journal of Finance* 36(3), 599–617.
- Dumas, B., Fleming, J., Whaley, R. (1998). Implied Volatility Functions: Empirical Test. *Journal of Finance* 53, 2059–2106.
- Dupire, B. (1994). Pricing with a Smile. *Risk* 7(1), 18–20.
- Dupire, B. (Ed.) (1998). *Monte Carlo: Methodologies and Applications for Pricing and Risk Management*. Risk Publications, London.
- Dupire, B., Savine, A. (1998). Dimension Reduction and Other Ways of Speeding Monte Carlo Simulation. In: *Risk Handbook*. Risk Publications, 51–63.
- Dyke, P.P. (1999). *An Introduction to Laplace Transforms and Fourier Series*. Springer-Verlag, Berlin–Heidelberg–New York.
- Efron, B. (1979). Bootstrap Methods: Another Look at the Jackknife. *Annals of Statistics* 7, 1–26.
- Efron, B., Tibshirani, R. (1986). Bootstrap Methods for Standard Errors, Confidence Intervals, and Other Measures of Statistical Accuracy. *Statistical Science* 1, 54–77.
- Emanuel, D., MacBeth, J. (1982). Further Results on the Constant Elasticity of Variance Call Option Pricing Model. *Journal of Financial and Quantitative Analysis* 17, 533–554.
- Embrechts, P., Klüppelberg, C., Mikosch, T. (1997). *Modelling Extremal Events for Insurance and Finance*. Springer-Verlag, Berlin–Heidelberg–New York.
- Embrechts, P., Lindskog, F., McNeil, A. (2001). Modeling Dependence with Copulas and Applications to Risk Management. *Working Paper*, ETH, Zurich.
- Engle, R.F., Ng, V. (1993). Measuring and Testing the Impact of News on Volatility. *Journal of Finance* 48, 1749–1779.
- Escribano, Á., Peña, J.I., Villaplana, P. (2002). Modeling Electricity Prices: International Evidence. *Working Paper* 02-27, Economic Series 08, Departamento de Economía, Universidad Carlos III de Madrid.
- Evans, M., Swartz, T. (2000). *Approximating Integrals via Monte Carlo and Deterministic Methods*. Oxford Statistical Science Series 20. Oxford University Press.
- Eydeland, A., Wolyniec, K. (2002). *Energy and Power Risk Management: New Developments in Modeling, Pricing and Hedging*. Wiley, Chicago.
- Falloon, W., Turner, D. (1999). The Evolution of a Market. In: *Managing Energy Price Risk*, RiskBooks, London.
- Fama, E. (1969). Efficient Capital Markets: A Review of Theory and Empirical Work. *Journal of Finance* 25(2) 383–417.



- Fama, E., Blume, M. (1966). Filters Rules and Stock-Market Trading. *Journal of Business* 39(1), 226–241.
- Fama, E., French, K. (1992). The Cross-Section of Expected Stock Returns. *Journal of Finance* 47, 427–465.
- Feller, W. (1951). Two Singular Diffusion Problems. *Annals of Mathematics* 54(1), 173–182.
- Fermanian, J.D., Scaillet, O. (2004). Some Statistical Pitfalls in Copula Modeling for Financial Applications. *Research Paper* 108, FAME, Université de Genève.
- Fermanian, J.D., Wegkamp, M. (2004). Time-Dependent Copulas. *Working Paper*, CREST, Paris.
- Figlewski, S., Gao, B. (1999). The Adaptive Mesh Model: A New Approach to Efficient Option Pricing. *Journal of Financial Economics* 53, 313–351.
- Fiorenzani, S. (2005). Load-Based Models for Electricity Prices. *Working Paper*, EDISON Trading.
- Fiorenzani, S. (2006a). Financial Optimization and Risk Management in Refining Activities. *International Journal of Global Energy*. Special Issue on Energy Finance 26(1), 62–82.
- Fiorenzani, S. (2006b). Pricing Illiquidity in Energy Markets. *Energy Risk* (May), 65–75.
- Fiorenzani, S. (2006c). *Quantitative Methods for Electricity Trading and Risk Management: Advanced Mathematical and Statistical Methods for Energy Finance*. Palgrave Macmillan Trading.
- Fishman, G.S. (1996). *Monte Carlo: Concepts, Algorithms, and Applications*. Springer-Verlag, Berlin–Heidelberg–New York.
- Fleming, W.H., Rishel, R.W. (1975). *Deterministic and Stochastic Optimal Control*. Springer-Verlag, Berlin–Heidelberg–New York.
- Fournié, E., Lasry, J.-M., Touzi, N. (1997). Monte Carlo Methods for Stochastic Volatility Models. In: Rogers, L.C.G., Talay, D. (Eds.), *Numerical Methods in Finance*. Cambridge University Press.
- Fournié, E., Lasry, J.M., Lebuchoux, J., Lions, P.L., Touzi, N. (1999). Applications of Malliavin Calculus to Monte Carlo Methods in Finance. *Finance and Stochastics* 3, 391–412.
- Fréchet, M. (1951). Sur les Tableaux de Corrélation dont les Marges sont Données. *Annales Universitaires Lyon Sc.* 4, 53–84.
- Freedman, D.A., Peters, S.C. (1984). Bootstrapping a Regression Equation: Some Empirical Results. *Journal of the American Statistical Association* 79, 97–106.
- Fu, M.C., Madan, D., Wang, T. (1998). Pricing Continuous Asian Options: A Comparison of Monte Carlo and Laplace Transform Inversion Methods. *Journal of Computational Finance* 2(1), 49–74.
- Fusai, G. (2000). Corridor Options and Arc-Sine Law. *Annals of Applied Probability* 10(2), 634–663.
- Fusai, G. (2001). Applications of Laplace Transform for Evaluating Occupation Time Options and Other Derivatives. PhD. Thesis, University of Warwick.
- Fusai, G. (2004). Pricing Asian Options via Fourier and Laplace Transforms. *Journal of Computational Finance* 7(3).
- Fusai, G., Abrahams, I.D., Sgarra, C. (2006). An Exact Analytical Solution for Discrete Barrier Options. *Finance and Stochastics* 10, 1–26.
- Fusai, G., Recchioni, M.C. (2001). Analysis of Quadrature Methods for Pricing Discrete Barrier Options. *Working Paper*, Financial Options Research Center Preprint, 2001/119, Warwick Business School, to appear in *Journal of Economics Dynamics and Control*.
- Fusai, G., Sanfelici, S., Tagliani, A. (2002). Practical Problems in the Numerical Solution of PDE's in Finance. *Rendiconti per gli Studi Economici Quantitativi*, Università Ca' Foscari Venezia, 105–132.

- Fusai, G., Tagliani, A. (2001). Pricing of Occupation Time Derivatives: Continuous and Discrete Monitoring. *Journal of Computational Finance* 5(1), 1–37.
- Gabbi, G., Sironi, A. (2005). Which Factors Affect Corporate Bonds Pricing: Empirical Evidence from Eurobonds Primary Market Spreads. *The European Journal of Finance* 11(1), 59–74.
- Galiani, S. (2003). Copula Functions and Their Application in Pricing and Risk Managing Multiname Credit Derivative Products. *MSc Dissertation*, King's College, University of London.
- Galluccio, S., Le Cam, Y. (2006a). Implied Calibration of Stochastic Volatility Jump Diffusion Models. *Working Paper* (downloadable at [ssrn.com](http://ssrn.com)).
- Galluccio, S., Le Cam, Y. (2006b). Modelling Hybrids with Jumps and Stochastic Volatility. *Working Paper* (downloadable at [ssrn.com](http://ssrn.com)).
- Galluccio, S., Roncoroni, A. (2006). A New Measure of Cross-Sectional Risk and Its Empirical Implications for Portfolio Risk Management. *Journal of Banking and Finance*, forthcoming. (Preprint, available on [www.ssrn.com](http://www.ssrn.com)).
- Gander, W., Gautschi, W. (2000). Adaptive Quadrature-Revisited. *BIT* 40(1), 84–101.
- Garbow, B.S., Giunta, G., Lyness, J.N., Murli, A. (1988a). Software for an Implementation of Weeks' Method for the Inverse Laplace Transform Problem. *ACM Trans. Math. Software* 14, 163–170.
- Garbow, B.S., Giunta, G., Lyness, J.N., Murli, A. (1988b). Algorithm 662: A FORTRAN Software Package for Numerical Inversion of the Laplace Transform Based on Weeks' Mmethod. *ACM Trans. Math. Software*, 14, 171–176.
- Garcia, D. (2003). Convergence and Biases of Monte Carlo Estimates of American Option Prices Using a Parametric Exercise Rule. *Journal of Economic Dynamics and Control* 27, 1855–1879.
- Gardner, D., Zhuang, Y. (2000). Valuation of Power Generation Assets: A Real Options Approach. *Algo Research Quarterly* 3, 2–20.
- Gatheral, J. (2006). *The Volatility Surface: A Practitioner's Guide*. Wiley Finance.
- Gatti, S., Rigamonti, A., Saita, F., Senati, M. (2006). Measuring Value at Risk in Project Finance Transactions. *European Financial Management* (forthcoming).
- Gaver, D.P. Jr. (1966). Observing Stochastic Processes and Approximate Transform Inversion. *Operations Research* 14(3), 444–459.
- Geman, H., El Karoui, N., Rochet, J.C. (1995). Changes of Numéraire, Changes of Probability Measure and Option Pricing. *Journal of Applied Probability* 32, 443–458.
- Geman, H., Eydeland, A. (1995). Domino Effect. *Risk* 8(4), 65–67.
- Geman, H., Roncoroni, A. (2006). Understanding the Fine Structure of Electricity Prices. *Journal of Business* 79(3), forthcoming (Preprint available on [www.ssrn.com](http://www.ssrn.com)).
- Geman, H., Yor, M. (1993). Bessel Processes, Asian Options and Perpetuities. *Mathematical Finance* 3(4), 349–375.
- Geman, H., Yor, M. (1996). Pricing and Hedging Double Barrier Options: A Probabilistic Approach. *Mathematical Finance* 6(4), 365–378.
- Gentle, J.E. (1998). *Random Number Generation and Monte Carlo Methods*. Springer-Verlag, Berlin–Heidelberg–New York.
- Gerber, H.U., Shiu, E.S. (1994). Option Pricing by Esscher Transforms. *Transactions of the Society of Actuaries* XLVI, 99–140.
- Gibson, M.S., Pristker, M. (2000). Improving Grid-Based Methods for Estimating Value at Risk of Fixed-Income Portfolios. *Working Paper*, Federal Reserve Board, Washington.
- Gihman, I.I., Skorohod, A.V. (1979). *Controlled Stochastic Processes*. Springer-Verlag, Berlin–Heidelberg–New York.

- Gitman, L.J., Mercurio, V.A. (1982). Cost of Capital Techniques Used by Major U.S. Firms: Survey and Analysis of Fortune's 1000. *Financial Management* 14(4), 21–29.
- Glasserman, P. (2004). *Monte Carlo Methods in Finance*. Springer-Verlag, Berlin–Heidelberg–New York.
- Glasserman, P., Heidelberger, P., Shahabuddin, P. (1999a). Importance Sampling in the Heath–Jarrow–Morton Framework. *Journal of Derivatives* 6, 32–50.
- Glasserman, P., Heidelberger, P., Shahabuddin, P. (1999b). Stratification Issues in Estimating Value-At-Risk. In: *Proceedings of the Winter Simulation Conference*. IEEE Press, New York.
- Glasserman, P., Heidelberger, P., Shahabuddin, P. (2000). Variance Reduction Techniques for Estimating Value-at-Risk. *Management Science* 46, 1349–1364.
- Glasserman, P., Zhao, X. (1999). Fast Greeks by Simulation in Forward LIBOR Models. *Journal of Computational Finance* 3, 5–39.
- Glynn, P.W., Iglehart, D.L. (1989). Importance Sampling for Stochastic Simulations. *Management Science* 35, 1367–1392.
- Glynn, P.W., Whitt, W. (1992). The Efficiency of Simulation Estimators. *Operations Research* 40, 505–520.
- Gobet, E., Munos, R. (2002). Sensitivity Analysis Using Itô–Malliavin Calculus and Martingales: Application to Stochastic Optimal Control. *Report 498*, Centre de Mathématiques Appliquées, Ecole Polytechnique, Palaiseau, France.
- Gocharov, Y., Pliska, S.R. (2003). Optimal Mortgage Refinancing with Endogenous Mortgage Rates. *Working Paper*, University of Illinois at Chicago.
- Goldenberg, D. (1991). A Unified Method for Pricing Options on Diffusion Processes. *Journal of Financial Economics* 29(1), 3–34.
- Goldman, M.B., Sosin, H.B., Gatto, M.A. (1979). Path-Dependent Options: Buy at the Low, Sell at the High. *Journal of Finance* 34, 1111–1127.
- Golub, G., Van Loan, C. (1996). *Matrix Computations*. John Hopkins Studies in Mathematical Sciences, Baltimore.
- van den Goorbergh, R.W.J., Genest, C., Werker, B. (2003). Multivariate Option Pricing Using Dynamic Copula Models. *Working Paper 2003-122*, Center, Tilburg University.
- Gourieroux, C., Monfort, A. (1996). *Simulation Based Econometric Methods*. Oxford University Press.
- Greene, W.H. (2002). *Econometric Analysis* (5th ed.). Prentice Hall, New Jersey.
- Grigoriu, M. (2003). *Stochastic Calculus*. Birkhäuser.
- Grüne, L., Semmler, W. (2004). Solving Asset Pricing Models with Stochastic Dynamic Programming. *Working Paper 54*, CEM, Bielefeld University.
- Guiotto, P., Roncoroni, A. (2001). Theory and Calibration of HJM with Shape Factors. In: Geman et al. (Eds.), *Mathematical Finance – Bachelier Congress 2000*. Springer-Verlag, Berlin–Heidelberg–New York, 407–426.
- Hageman, L.A., Young, D.M. (1981). *Applied Iterative Methods*. Academic Press, New York.
- Hampel, F.R. (1986). *Robust Statistics*. Wiley, New York.
- Harvey, A.C. (1994). *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge University Press.
- Harvey, D.I., Leybourne, S.J., Newbold, P. (1999). Forecast Evaluation Tests in the Presence of ARCH. *Journal of Forecasting* 18, 343–445.
- Herold, U., Maurer, R. (2002). Portfolio Choice and Estimation Risk: A Comparison of Bayesian Approaches to Resampled Efficiency. *Working Paper 94*, Johann Wolfgang Goethe Universität, Frankfurt.

- Hestenes, M.R., Stiefel, E. (1952). Methods of Conjugate Gradients for Solving Linear Systems. *Journal Research National Bureau of Standard* 49, 409–436.
- Heston, S. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Application to Bond and Currency Options. *Review of Financial Studies* 6, 327–343.
- Heynen, R.C., Kat, H.M. (1995). Lookback Options with Discrete and Partial Monitoring of the Underlying Price. *Applied Mathematical Finance* 2, 273–284.
- Hinz, Y. (2003). Modelling Day-Ahead Electricity Prices. *Applied Mathematical Finance* 10, 149–161.
- Hirsa, A., Madan, D. (2003). Pricing American Options under Variance Gamma. *Journal of Computational Finance* 7(2), 63–80.
- Hoeffding, W. (1940). Massstabinvariante Korrelationstheorie. *Schriften der Mathematischen Seminars und Instituts für Angewandte Mathematik der Universität Berlin* 5, 181–233.
- Hörfelt, P. (2003). Extension of the Corrected Barrier Approximation by Broadie, Glasserman, and Kou. *Finance and Stochastics* 7, 231–243.
- Hong, H.S., Hickernell, F.J. (2000). Implementing Scrambled Digital Nets. *Unpublished Technical Report*, Hong Kong Baptist University.
- Hu, T., Müller, A., Scarsini, M. (2003). Some Counterexamples in Positive Dependence. *Applied Mathematics Working Paper*, Series 28/2003, ICER, Torino.
- Huber, P. (1981). *Robust Statistics*. Wiley, New York.
- Hudson, R., Dempsey, M., Keasey, K. (1996). A Note on the Weak Form Efficiency of Capital Markets: The Application of Simple Technical Trading Rules to UK Stock Prices – 1935 to 1994. *Journal of Banking and Finance* 20, 1121–1132.
- Hugonnier, J.N. (1999). The Feynman–Kac Formula and Pricing of Occupation Time Derivatives. *International Journal of Theoretical and Applied Finance* 2(2), 153–178.
- Hui, C.H., Lo, C.F., Yuen, P.H. (2000). Comment on Pricing Double Barrier Options Using Laplace Transforms by Antoon Pelsser. *Finance and Stochastic* 4, 105–107.
- Huisman, R., Mahieu, R. (2003). Regime Jumps in Electricity Prices. *Energy Economics* 25, 425–434.
- Hull, J.C. (2005). *Options, Futures and Other Derivatives* (6th ed.). Prentice-Hall.
- Hull, J.C., White, A. (1987). The Pricing of Options on Assets with Stochastic Volatilities. *Journal of Finance* 42(2), 281–300.
- Hull, J.C., White, A. (1990). Valuing Derivative Securities Using the Explicit Finite Difference Method. *Journal of Financial and Quantitative Analysis* 25(1), 87–100.
- Hull, J.C., White, A. (2003). Valuation of a CDO and an N-th to Default CDS without Monte Carlo Simulation. *Working Paper*, University of Toronto.
- Hsu, M. (1998). Spark Spread Options Are Hot! *Journal of Electricity* 11, 28–39.
- Imai, J., Tan, K.S. (2002). Enhanced Quasi-Monte Carlo Method with Dimension Reduction. *Proceedings of the 2002 Winter Simulation Conference*, 1502–1510.
- Ince, E.L. (1964). *Ordinary Differential Equations*. Dover Publications, Inc., New York.
- Ingersoll, J.E. (1986). *Theory of Financial Decisions Making*. Rowman & Littlefield Publishers, Inc.
- Isakov, D., Hollistein, M. (1999). Application of Simple Technical Rules to Swiss Stock Prices: Is it profitable? *Finanzmarkt and Portfolio Management* 13(1), 9–26.
- Jackwerth, J. (1999). Option Implied Risk-Neutral Distributions and Implied Binomial Trees: A Literature Review. *Journal of Derivatives* 7(2), 66–82.
- Jackwerth, J., Rubinstein, M. (1996). Recovering Probability Distributions from Option Prices. *Journal of Finance* 51(5), 1611–1631.
- Jacod, J., Protter, P. (1998). Asymptotic Error Distributions for the Euler Method for Stochastic Differential Equations. *Annals of Probability* 26, 267–307.

- Jacod, J., Shiryaev, A. (1988). *Limit Theorems for Stochastic Processes*. Springer-Verlag, Berlin–Heidelberg–New York.
- Jaillet, P., Ronn, E., Tompaidis, S. (2003). Valuation of Commodity Based Swing Options. *Management Science* 50(7), 909–921.
- James, W., Stein, C. (1961). Estimation with Quadratic Loss. *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*. University of California Press, Berkeley, 361–379.
- James, J., Webber, N. (2000). *Interest Rate Modelling*. Wiley Series in Financial Engineering, John Wiley & Sons.
- Jamshidian, F. (1987). Pricing of Contingent Claims in the One-Factor Term Structure Model. *Working Paper*, Merryl Lynch, New York. In: *Vasicek and Beyond* (1996), Risk Publications.
- Jamshidian, F. (1989). An Exact Bond Option Formula. *Journal of Finance* 44, 205–209.
- Jamshidian, F. (1990). The Preference-Free Determination of Bond and Option Prices from the Spot Interest Rate. *Advances in Futures and Options Research* 4, 51–67.
- Jamshidian, F. (1991a). Bond and Options Evaluation in the Gaussian Interest Rate Model. *Research in Finance* 9, 131–170. Appeared also in: *Vasicek and Beyond* (1996), Risk Publications.
- Jamshidian, F. (1991b). Forward Induction and Construction of Yield Curve Diffusion Models. *Journal of Fixed Income* 1, 62–74.
- Jamshidian, F. (1991c). Commodity Option Evaluation in the Gaussian Futures Term Structure Model. *Review of Futures Markets* 10(2), 324–346.
- Jamshidian, F. (1992). An Analysis of American Options. *Review of Futures Markets* 11(1), 73–80.
- Jamshidian, F. (1993). Options and Futures Evaluation with Deterministic Volatility. *Mathematical Finance* 3(2), 149–159.
- Jamshidian, F. (1995). A Simple Class of Square-Root Models. *Applied Mathematical Finance* 2, 61–72.
- Jamshidian, F. (1996). Bond, Futures and Option Evaluation in the Quadratic Interest Rate Model. *Applied Mathematical Finance* 3, 93–115.
- Jamshidian, F. (1997). Libor and Swap Market Model and Measures. *Finance and Stochastics* 1, 293–330.
- Jamshidian, F. (1997). A Note on Analytical Valuation of Double Barrier Options. *Working Paper*, Sakura Global Capital.
- Jamshidian, F. (1999). Libor Market Model with Semimartingales. In: *Option Pricing, Interest Rates and Risk Management* (2001), Cambridge University.
- Jamshidian, F., Zhu, Y. (1997). Scenario Simulation: Theory and Methodology. *Finance and Stochastics* 1, 43–67.
- Jarrow, R.A. (1986). The Pricing of Commodity Options with Stochastic Interest Rates. *Advances in Futures and Options Research* 2, 19–45.
- Jarrow, R.A., Rudd, A. (1982). Approximate Option Valuation for Arbitrary Stochastic Processes. *Journal of Financial Economics* 10, 347–369.
- Jensen, M., Benington, G. (1970). Random Walks and Technical Theories: Some Additional Evidence. *Journal of Finance* 25, 469–482.
- Jobson, J.D., Korkie, B. (1980). Estimation of Markowitz Efficient Portfolios. *Journal of the American Statistical Association* 75, 544–554.
- Joe, H. (1997). *Multivariate Models and Dependence Concepts*. Monographs on Statistics and Applied Probability. Chapman and Hall, London.

- Joe, H., Xu, J.J. (1996). The Estimation Method of Inference Functions for Margins for Multivariate Models. Unpublished *Working Paper*, University of British Columbia.
- Johannes, M. (1999). Jumps in Interest Rates: A Nonparametric Approach. *Working Paper*, University of Chicago.
- Johannes, M. (2004). The Statistic and Economic Role of Jumps in Interest Rates. *Journal of Finance* 59, 227–260.
- Johnson, N.L., Kotz, S. (1995). *Continuous Univariate Distributions*, Vol. 1 and 2 (2nd ed.). Wiley Series in Probability and Statistics.
- Jolliffe, L. (1986). *Principal Components Analysis. Series in Statistics*. Springer-Verlag, Berlin–Heidelberg–New York.
- Joskow, P., Kahn, J. (2001). A Quantitative Analysis of Pricing Behavior in California Wholesale Electricity Market During Summer 2000. *Working Paper*, MIT.
- Jouanin, J.F., Riboulet, G., Roncalli, T. (2003). Financial Applications of Copula Functions. *Working Paper*, Crédit Lyonnais.
- Ju, N. (2002). Pricing Asian and Basket Options via Taylor Expansion. *Journal of Computational Finance* 5(3), 79–103.
- Judge, G.G., Hill, R.C., Griffiths, W.E., Lütkepohl, H., Lee, T.C. (1988). *Introduction to the Theory and Practice of Econometrics* (2nd ed.). Wiley, New York.
- Kahl, C., Jäckel, P. (2005). Not-so-Complex Logarithms in the Heston Model. *Wilmott Magazine*, Sept., 94–103.
- Kahl, C., Jäckel, P. (2006). Fast Strong Approximation Monte Carlo Schemes for Stochastic Volatility Models. *Quantitative Finance* 6(6), 513–536.
- Kallsen, J., Tankov, P. (2004). Characterization of Dependence of Multidimensional Lévy Processes Using Lévy Copulas. *Working Paper*, Ecole Polytechnique, France.
- Karatzas, I., Lehoczky, J.P., Sethi, S.P., Shreve, S.E. (1986). Explicit Solution of a General Consumption/Investment Problem. *Math. Operations Research* 111, 261–294.
- Karatzas, I., Lehoczky, J.P., Shreve, S.E. (1987). Optimal Portfolio and Consumption Decisions for a “Small Investor” on a Finite Horizon. *SIAM Journal of Control and Optimization* 25, 1557–1586.
- Karatzas, I., Shreve, S.E. (1997). *Brownian Motion and Stochastic Calculus* (2nd ed.). GTM Collection, Springer-Verlag, Berlin–Heidelberg–New York.
- Kat, H.M. (2001). *Structured Equity Derivatives: The Definitive Guide to Exotic Options and Structured Notes*. Wiley Finance, London.
- Këllezi, E., Webber, N. (2004). Valuing Bermudian Options when Asset Returns Are Lévy Processes. *Quantitative Finance* 4, 87–100.
- Kendall, M. (1994). *Advanced Theory of Statistics* (6th ed.). Edward Arnold, London, Halsted Press, New York.
- Keppo, J. (2004). Pricing Electricity Swing Options. *Journal of Derivatives* 11, 26–43.
- Kimberling, C.H. (1974). A Probabilistic Interpretation of Complete Monotonicity. *Aequationes Math.* 10, 152–164.
- Kloeden, P.E., Platen, E. (2000). Numerical Solution of Stochastic Differential Equations. *Applications of Mathematics Collection*. Springer-Verlag, Berlin–Heidelberg–New York.
- Knez, P.J., Ready, M.J. (1997). On the Robustness of Size and Book-to-Market in Cross-Sectional Regressions. *Journal of Finance* 52(4), 1355–1382.
- Knittel, C.R., Roberts, M.R. (2001). An Empirical Examination of Deregulated Electricity Prices. *Working Paper*, Boston University.
- Koehler, J.R., Owen, A. (1996). Computer Experiment. In: *Handbook of Statistics, Design and Analysis of Experiments*.

- Kou, S. (2002). A Jump-Diffusion Model for Option Pricing. *Management Science* 48, 1086–1101.
- Krylov, N.V. (1980). *Controlled Diffusion Processes*. Springer-Verlag, Berlin–Heidelberg–New York.
- Kunitomo, N., Ikeda, M. (1992). Pricing Options with Curved Boundaries. *Mathematical Finance* 2, 275–298.
- Kupiec, P. (1995). Techniques for Verifying the Accuracy of Risk Measurement Models. *Journal of Derivatives*, 2, 173–184.
- Kushner, H.J. (1967). *Stochastic Stability and Control*. Academic Press, New York.
- Kushner, H.J., Dupuis, P. (1992). *Numerical Methods for Stochastic Control Problems in Continuous Time*. Springer-Verlag, Berlin–Heidelberg–New York.
- Kwok, Y.-K. (1998). *Mathematical Models of Financial Derivatives*. Springer-Verlag, Berlin–Heidelberg–New York.
- Kwok, Y.-K., Barthez, D. (1989). An Algorithm for the Numerical Inversion of Laplace Transforms. *Inverse Problems* 5, 1089–1095.
- Lacoste, V., El Karoui, N., Jeanblanc, M. (2005). Optimal Portfolio Management with American Capital Guarantee. *Journal of Economic Dynamics and Control* 29, 449–468.
- Lambert, J.D. (1991). *Numerical Methods for Ordinary Differential Systems. The Initial Value Problem*. John Wiley & Sons.
- Lamberton, D., Lapeyre, B. (1996). *Introduction to Stochastic Calculus Applied to Finance*. Chapman & Hall, London.
- Lari Lavassani, A., Simchi, M., Ware, A. (2000). A Discrete Valuation of Swing Options. *Canadian Applied Mathematics* 9, 35–74.
- Lavelly, J., Wakefield, G., Barrett, B. (1980). Toward Enhancing Beta Estimates. *Journal of Portfolio Management* 6(4), 43–46.
- Lax, P.D., Richtmyer, R.D. (1956). Survey of the Stability of Linear Finite Difference Equations. *Comm. Pure Appl. Math.* 9, 267–293.
- L'Ecuyer, P. (1988). Efficient and Portable Combined Random Number Generators. *Communications of the ACM* 31.
- L'Ecuyer, P. (1994). Uniform Random Number Generation. *Annals of Operations Research* 53, 77–120.
- L'Ecuyer, P., Simard, R., Wegenkittl, S. (2002). Sparse Serial Tests of Uniformity for Random Number Generators. *SIAM Journal of Scientific Computing* 24, 652–668.
- Leblanc, B., Scaillet, O. (1998). Path Dependent Options on Yields in the Affine Term Structure Model. *Finance and Stochastics* 2, 349–367.
- Lehmann, E. (1966). Some Concepts of Dependence. *Annals of Mathematical Statistics* 37, 1137–1153.
- Levy, E. (1992). Pricing European Average Rate Currency Options. *Journal of International Money and Finance* 11, 474–491.
- Lewis, A. (2000). *Option Valuation Under Stochastic Volatility*. Finance Press, Newport Beach.
- Lewis, A. (2002). Asian Connections. *Wilmott Magazine* 57–63.
- Lewis, P.A.W., Shedler, G.S. (1979). Simulation of Nonhomogeneous Poisson Processes by Thinning. *Naval Logistics Quarterly* 26, 403–413.
- Li, X.D. (2000). On Default Correlation: A Copula Approach. *Journal of Fixed Income* 9, 43–54.
- Li, A., Ritchken, P., Sankarasubramanian, L. (1995). Lattice Methods for Pricing American Interest Rate Claims. *Journal of Finance* 50, 719–737.
- Linetski, V. (1999). Step Options. *Mathematical Finance* 9(1), 55–96.

- Linetsky, V. (2004). Spectral Expansions for Asian (Average Price) Options. *Operations Research* 52, 856–867.
- Lipton, A. (1999). Similarities via Self-Similarities. *Risk* 12(9), 101–105.
- Lipton, A. (2001). *Mathematical Methods for Foreign Exchange*. World Scientific.
- Litterman, R., Scheinkman, J. (1991). Common Factors Affecting Bond Returns. *Journal of Fixed Income* 1, 54–61.
- Lo, A., MacKinlay, A.C., Zhang, J. (1997). Econometric Models of Limit Order Executions. *Working Paper* 6257, NBER.
- Longin, F. (1996). The Asymptotic Distribution of Extreme Stock Market Returns. *Journal of Business* 69, 383–408.
- Longin, F. (2000). From VaR to Stress Testing: The Extreme Value Approach. *Journal of Banking and Finance* 24, 1097–1130.
- Longin, F., Bouyé, E., Legras, J., Soupé, F. (2001). Correlation and Dependence in Financial Markets. HSBC CCF. *Quants* 41.
- Longin, F., Solnik, B. (2001). Correlation Structure of International Equity Markets During Extremely Volatile Periods. *Journal of Finance* 46, 649–676.
- Longstaff, F.A. (1993). The Valuations of Options on Coupon Bonds. *Journal of Banking and Finance* 17(1), 27–42.
- Longstaff, F.A. (2002). Optimal Recursive Refinancing and the Valuation of Mortgage-Backed Securities. *Working Paper*, UCLA.
- Longstaff, F.A., Schwartz, E.S. (2001). Valuing American Options by Simulation: A Simple Least Squares Approach. *Review of Financial Studies* 14, 113–147.
- Lord, R., Koekoek, R., Van Dijk, D. (2006). Comparison of Biased Simulation Schemes for Stochastic Volatility Models. Discussion Paper No. 06-046/4, Tinbergen Institute.
- Lucia, F., Schwartz, E. (2002). Electricity Prices and Power Derivatives. *Review of Derivative Research* 5, 5–50.
- Luciano, E., Marena, M. (2002). Copulae as a New Tool in Financial Modelling. *Operational Research: An International Journal* 2, 139–155.
- Luenberger, D.G. (1989). *Linear and Nonlinear Programming* (2nd ed.). Addison-Wesley.
- Lund, A., Ollmar, F. (2003). Analyzing Flexible Load Contracts. *Working Paper*.
- MacMillan, L.W. (1986). An Analytical Approximation for the American Put Prices. *Advances in Futures and Options Research* 1, 119–139.
- Maddala, G.S., Li, H. (1996). Bootstrap Based Tests in Financial Models. In: Maddala, G.S., Rao, C.R. (Eds.), *Handbook of Statistics. Statistical Methods in Finance* 14. Elsevier.
- Manoliu, M., Tompaidis, S. (2002). Energy Futures Prices: Term Structure Models with Kalman Filter Estimation. *Applied Mathematical Finance* 9, 21–43.
- Martin, R.D., Simin, T. (1999). Robust Estimation of Beta. *Technical Report* 350, Department of Statistics, University of Washington.
- Marsaglia, G. (1972). The Structure of Linear Congruential Generators. In: Zaremba, S.K. (Ed.), *Applications of Number Theory to Numerical Analysis*, 249–286. Academic Press, New York.
- Marsaglia, G., Bray, T.A. (1964). A Convenient Method for Generating Normal Variables. *SIAM Review* 6, 260–264.
- Maspero, D., Saita, F. (2005). Risk Measurement for Asset Managers: A Test of Relative VaR. *Journal of Asset Management* 5(5), 338–350.
- McCauley, R., Melick, W. (1996a). Risk Reversal. *Risk* 9(11), 54–57.
- McCauley, R., Melick, W. (1996b). Propensity and Density. *Risk* 9(12), 52–54.
- McKean, H.P. (1967). Appendix: A Free Boundary Problem for the Heath Equation Arising From a Problem in Mathematical Economics. *Industrial Management Review* 6, 32–39.



- Melick, W., Thomas, C.P. (1997). Recovering an Asset's Implied PDF from Option Prices: An Application to Crude Oil During the Gulf Crisis. *Journal of Financial and Quantitative Analysis* 32(1), 91–115.
- Melino, A., Turnbull, S. (1990). Pricing Foreign Currency Options with Stochastic Volatility. *Journal of Econometrics* 45, 239–265.
- Meneguzzo, D., Vecchiato, W. (2004). Copula Sensitivity in Collateralized Debt Obligations and Basket Default Swaps. *Journal of Futures Markets* 24(1), 37–70.
- Merton, R. (1971). Optimum Consumption and Portfolio Rules in a Continuous-Time Model. *Journal of Economics Theory* 3, 373–413. Erratum: *ibidem* 6 (1973), 213–214.
- Merton, R. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance* 29, 449–470.
- Merton, R. (1976). Option Pricing when Underlying Stock Returns Are Discontinuous. *Journal of Financial Economics* 3(1/2), 125–144.
- Meucci, A. (2005). *Risk and Asset Allocation*. Springer-Verlag, Berlin–Heidelberg–New York.
- Michaud, R.O. (1998). *Efficient Asset Management*. Harvard Business School Press, Boston.
- Mikusinski, P., Sherwood, H., Taylor, M.D. (1992). Shuffles of Min. *Stochastica* 13, 61–74.
- Milevsky, M.A., Posner, S.E. (1998). Asian Options, The Sum of Lognormals and the Reciprocal Gamma Distribution. *Journal of Financial and Quantitative Analysis* 33(3), 409–422.
- Miltersen, K. (1999). Pricing Interest Rate Contingent Claims: Implementing a Simulation Approach. *Working Paper*, Odense University.
- Mitchell, A.R., Griffiths, D.F. (1980). *The Finite Difference Method in Partial Differential Equations*. John Wiley (Corrected reprinted edition, 1994).
- Moorthy, M. (1995a). Numerical Inversion of Two-Dimensional Laplace Transforms Fourier Series Representation. *Applied Numerical Mathematics* 17, 119–127.
- Moorthy, M.V. (1995b). Inversion of the Multi-Dimensional Laplace Transform – Expansion by Laguerre Series. *Z. Angew. Math. Phys.* 46, 793–806.
- Morton, K.W., Mayers, D.F. (1994). *Numerical Solution of Partial Differential Equations*. Cambridge University Press.
- Murphy, J. (1999). Technical Analysis of the Financial Markets. *Report*, New York Institute of Finance.
- Musiela, M., Rutkowski, M. (1997). *Martingale Methods in Financial Modelling*. Applications of Mathematics 36. Springer-Verlag, Berlin–Heidelberg–New York.
- Nahum, E. (1998). On the Distribution of the Sumpremum of the Sum of a Brownian Motion with Drift and a Marked Point Process, and the Pricing of Lookback Options. *Technical Report* N. 516, Dept. of Statistics, Berkeley.
- Nelsen, R.B. (1999). *An Introduction to Copulas*. Lectures Notes in Statistics. Springer-Verlag, Berlin–Heidelberg–New York.
- Niederreiter, H. (1992). *Random Number Generation and Quasi-Monte Carlo Methods*. CBMS-NSF 63, SIAM.
- Oksendal, B. (2003). *Stochastic Differential Equations: An Introduction with Applications*. Springer-Verlag, Berlin–Heidelberg–New York.
- Oksendal, B., Sulem, A. (2004). *Applied Stochastic Control of Jump Diffusions*. Springer-Verlag, Berlin–Heidelberg–New York.
- Owen, A. (1998). Latin Supercube Sampling for Very High-Dimensional Simulations. *ACM Transaction on Modelling and Computer Simulation* 8, 71–102.
- Owen, A. (2002). Variance and Discrepancy with Alternative Scramblings. *ACM Transactions on Computational Logic*, Vol. V.
- Pacelli, G., Recchioni, M.C., Zirilli, F. (1999). A Hybrid Method for Pricing European Options Based on Multiple Assets. *Applied Mathematical Finance* 6, 61–85.

- Pelsser, A. (2000). Pricing Double Barrier Options Using Laplace Transforms. *Finance and Stochastics* 4, 95–104.
- Pedersen, A. (1995). A New Approach to Maximum Likelihood Estimation for Stochastic Differential Equations Based on Discrete Observations. *Scandinavian Journal of Statistics* 22, 55–71.
- Piazzesi, M. (2001). An Econometric Model of the Yield Curve with Macroeconomic Jumps Effects. *Working Paper*, University of California, Los Angeles.
- Picoult, E. (1999). Calculating Value-at-Risk with Monte Carlo Simulation. In: Dupire, B. (Ed.). *Monte Carlo: Methodologies and Applications for Pricing and Risk Management* 209–229. Risk Publications, London.
- Pilipovich, D., Wengler, J. (1998). Getting into the Swing. *Energy and Power Risk Management* 2(10).
- Pitsianis, N., Van Loan, C. (1993). Approximation with Kronecker Products. In: *Linear Algebra for Large Scale and Real Time Application*. Kluwer Academic Publishers, 293–314.
- Platzman, L.K., Ammons, J.C., Bartholdi, J.J. (1988). A Simple and Efficient Algorithm to Compute Tail Probabilities from Transforms. *Oper. Res.* 26, 137–144.
- Poncet, P., Gesser, V. (1997). Volatility Patterns: Theory and Some Evidence from the Dollar-Mark Option Market. *Journal of Derivatives* 5(2).
- Portait, R., Bajeux-Besnainou, I., Jordan, J. (2001). An Asset Allocation Puzzle: Comment. *American Economic Review* 91(4), 1170–1180.
- Portait, R., Bajeux-Besnainou, I., Jordan, J. (2003). Dynamic Asset Allocation for Stocks, Bonds and Cash. *Journal of Business* 76(2), 263–287.
- Portait, R., Nguyen, P. (2002). Dynamic Mean Variance Efficiency and Asset Allocation with a Solvency Constraint. *Journal of Economics Dynamics and Control*.
- Prakasa-Rao, B.L.S. (1999). *Semimartingales and Their Statistical Inference*. Chapman & Hall/CRC.
- Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P. (1992). *Numerical Recipes in C: The Art of Scientific Computing*. Cambridge University Press.
- Protter, P. (2005). *Stochastic Integration and Differential Equations* (2nd ed.). Springer-Verlag, Berlin–Heidelberg–New York.
- Quarteroni, A., Sacco, R., Saleri, F. (2000). *Numerical Mathematics*. Springer-Verlag, Berlin–Heidelberg–New York.
- Rebonato, R. (1998). *Interest Rate Option Models* (2nd ed.). Wiley & Sons.
- Rebonato, R. (1999). *Volatility and Correlation in the Pricing of Equity, FX and Interest-Rate Options*. Wiley Series in Financial Engineering, John Wiley & Sons.
- Ribeiro, C., Webber, N. (2003). Valuing Path Dependent Options in the Variance-Gamma Model by Monte Carlo with a Gamma Bridge. *Journal of Computational Finance* 7.
- Ribeiro, C., Webber, N. (2005). Correcting for Simulation Bias in Monte Carlo Methods to Value Exotic Options in Models Driven by Lévy Processes. *Applied Mathematical Finance*.
- Rich, D.R. (1994). The Mathematical Foundations of Barrier Option Pricing Theory. *Advances in Futures and Options Research* 7, 267–371.
- Ritchken, P. (1995). On Pricing Barrier Options. *Journal of Derivatives* 3, 19–28.
- Richtmyer, R.D., Morton, K.W. (1967). *Difference Methods for Initial Value Problems* (2nd ed.). Wiley-Interscience, New York.
- Rogers, C. (2000). Evaluating First-Passage Probabilities for Spectrally One-Sided Lévy Processes. *Journal of Applied Probability* 37(4), 1173–1180.
- Rogers, L.C.G. (2002). Monte Carlo Valuation of American Options. *Mathematical Finance* 12, 271–286.

- Rogers, L.C.G., Shi, Z. (1992). The Value of an Asian Option. *Journal of Applied Probability* 32, 1077–1088.
- Rogers, L.C.G., Talay, B. (Eds.) (1997). *Numerical Methods in Finance*. Cambridge University Press.
- Rogers, L.C.G., Williams, D. (1987). *Diffusions, Markov Processes and Martingales*, Vol. 2, Ito Calculus. Wiley.
- Roncoroni, A. (1995). A Trade-off Optimal Choice Problem arising in the Financial Economic Policy of Developing Countries: Private Sector Credit Demand Incentives under Constrained Debt Recovery Policy. “*Laurea*” Degree Dissertation, Bocconi University, Milan.
- Roncoroni, A. (1997). Principal Component Analysis for Finite and Infinite Dimensional Dynamical Models. *Working Paper*, Courant Institute of Mathematical Sciences, New York.
- Roncoroni, A. (1999). Infinite Dimensional HJM Dynamics for the Term Structure of Interest Rates. *Working Paper* 9903, ESSEC.
- Roncoroni, A. (2000). The S Option – An Alternative to the Surrender Option in Mortgage Backed Securities. *Working Paper*, CEREG, Université Paris Dauphine.
- Roncoroni, A. (2002). Essays in Quantitative Finance: Modelling and Calibration in Interest Rate and Electricity Markets. *Ph.D. Dissertation*, Université Paris IX Dauphine, France.
- Roncoroni, A. (2004). Models for Risk Management in the Energy Markets and the Italian “Nuovo Mercato Elettrico”. *Technical Report*, The Italian Stock Exchange, Milan.
- Roncoroni, A., Galluccio, S., Guiotto, P. (2003). Shape Factors and Cross-Sectional Risk. *Working Paper*, ESSEC Business School, France.
- Roncoroni, A., Moro, A. (2006). Flexible-Rate Mortgages. *International Journal of Business* 11(2).
- Roncoroni, A., Zuccolo, V. (2004). The Optimal Exercise Policy of Volumetric Swing Options with Penalty Constraints. *Working Paper*, ESSEC Business School.
- Ross, S. (1997). *Simulation*. Academic Press, San Diego.
- Rousseeuw, P.J., Leroy, A.M. (1987). *Robust Regression and Outlier Detection*. Wiley, New York.
- Rubinstein, R. (1981). *Simulation and the Monte Carlo Method*. John Wiley & Sons, New York.
- Rubinstein, M. (1994). Implied Binomial Trees. *Journal of Finance* 49(3), 771–818.
- Rubinstein, M., Reiner, E. (1991). Breaking Down the Barriers. *Risk* 8, 28–35.
- Sankaran, M. (1963). Approximations to the Non-Central Chi-Square Distribution. *Biometrika* 50, 199–204.
- Salminen, P., Wallin, O. (2005). Perpetual Integral Functionals of Diffusions and Their Numerical Computations. *Working Paper*.
- Salopek, D.M. (1997). *American Put Options*. Chapman & Hall, CRC.
- Samuelson, P.A. (1967). Rational Theory of Warrant Pricing. *Industrial Management Review* 6, 13–31.
- Sankaran, M. (1963). Approximations to the Non Central Chi-Square Distribution. *Biometrika* 50, 199–204.
- Sato, K.I. (2000). *Lévy Processes and Infinitely Divisible Distributions*. Cambridge University Press.
- Sbuelz, A. (1999). A General Treatment of Barrier Options and Semi-Static Hedges of Double Barrier Options. *Working Paper*, London Business School.
- Sbuelz, A. (2005). Hedging Double Barriers with Singles. *International Journal of Theoretical and Applied Finance* 8, 393–407.

- Scaillet, O. (2000). Nonparametric Estimation of Copulas for Time Series. *Journal of Risk* 5(4), 25–54.
- Scherer, B. (2002). Portfolio Resampling: Review and Critique. *Financial Analysts Journal* 58(6), 98–109.
- Schonbucher, P.J. (2003). *Credit Derivatives Pricing Models*. Wiley Finance, London.
- Schoutens, W. (2003). *Levy Processes in Finance*. Wiley.
- Schroder, M. (1989). Computing the CEV Option Pricing Formula. *Journal of Finance* 44, 211–219.
- Schwager, J.D. (1996). *Schwager on Futures: Technical Analysis*. Wiley & Sons, New York.
- Schwartz, E.S. (1997). The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging. *Journal of Finance* 52, 923–973.
- Schwartz, E.S., Torous, W.N. (1989). Prepayment and the Valuation of Mortgage-Backed Securities. *Journal of Finance* 44, 375–392.
- Schwartz, E.S., Torous, W.N. (1992). Prepayment, Default, and the Valuation of Mortgage Pass-Through Securities. *Journal of Business* 65, 221–239.
- Schweizer, B., Wolff, E. (1981). On Non Parametric Measures of Dependence for Random Variables. *Annals of Statistics* 9, 879–885.
- Selby, M.J.P. (1983). The Application of Option Theory to the Evaluation of Risky Debt. *Ph.D. Thesis*, London Business School.
- Seydel, R.U. (2006). *Tools for Computational Finance* (3rd ed.). Springer Universitext.
- Shampine, L.F., Reichelt, M.W. (1997). The MATLAB ODE Suite. *SIAM Journal on Scientific Computing*, 18, 1–22.
- Sharpe, W.F., Alexander, G.J., Bailey, J.V. (1999). *Investments* (6th ed.). Prentice-Hall International.
- Shimko, D. (1991). *Finance in Continuous Time: A Primer*. Kolb Publishing Company.
- Shimko, D. (1993). Bounds of Probability. *Risk* 6(4), 33–37.
- Singhal, K., Vlach, J. (1975). Computation of Time Domain Response by Numerical Inversion of the Laplace Transform. *Journal of the Franklin Institute* 2, 110–127.
- Shiryayev, A.N. (1978). *Optimal Stopping Rules*. Springer-Verlag, Berlin–Heidelberg–New York.
- Shreve, S. (2004). *Stochastic Calculus for Finance II: Continuous-Time Models*. Springer-Verlag, Berlin–Heidelberg–New York.
- Siegmund, D. (1976). Importance Sampling in the Monte Carlo Study of Sequential Tests. *Annals of Statistics* 4, 673–684.
- Silverman, B.W. (1986). *Density Estimation for Statistics and Data Analysis*. Chapman & Hall.
- Singhal, K., Vlach, J. (1975). Computation of Time Domain Response by Numerical Inversion of the Laplace transform. *Journal of the Franklin Institute* 2, 110–127.
- Singhal, K., Vlach, J., Vlach, M. (1975). Numerical Inversion of Multidimensional Laplace Transforms. *Proc. IEEE* 63, 1627–1628.
- Skeel, R., Berzins, M. (1990). A Method for the Spatial Discretization of Parabolic Equations in One Space Variable. *SIAM Journal on Scientific and Statistical Computing* 11, 1–32.
- Sklar, A. (1959). Fonctions de Repartition à n Dimensions et leurs Marges. *Publications de l'Institut de Statistique de l'Université de Paris* 8, 229–231.
- Smith, G.D. (1985). *Numerical Solution of Partial Differential Equations: Finite Difference Methods*. Oxford University Press.
- Söderlind, P., Svensson, L. (1997). New Techniques to Extract Market Expectations from Financial Instruments. *Journal of Monetary Economics* 2(40), 383–429.

- Stanton, R. (1995). Rational Prepayment and the Valuation of Mortgage-Backed Securities. *Review of Financial Studies* 8, 677–708.
- Stanton, R. (1997). A Nonparametric Model of Term Structure Dynamics and the Market Price of Interest Rate Risk. *Journal of Finance* 7(5), 1973–2002.
- Steeley, J.M. (1990). Modelling the Dynamics of the Term Structure of Interest Rates. *Economic and Social Review* 21, 337–361.
- Stehfest, H. (1970). Algorithm 368: Numerical inversion of Laplace Transform. *Communication of the ACM* 13(1), 47–49.
- Stevenson, T. (2001). Filtering and Forecasting Spot Electricity Prices. *Working Paper*, UTS, Sydney.
- Stewart, G. (1973). *Introduction to Matrix Computations*. Academic Press, New York.
- Stoer, J., Bulirsch, R. (1980). *Introduction to Numerical Analysis*. Springer-Verlag, Berlin–Heidelberg–New York.
- Strauss, W.A. (1992). *Partial Differential Equations: An Introduction*. Wiley & Sons, Chichester, England.
- Sullivan, M.A. (2000). Pricing Discretely Monitored Barrier Options. *Journal of Computational Finance* 3(4) 35–52.
- Sullivan, R., Timmermann, A., White, H. (1999). Data-Snooping, Technical Trading Rule Performance, and the Bootstrap. *Journal of Finance* 54(5), 1647–1691.
- Sweeney, R.J. (1988). Some New Filter Rule Tests: Methods and Results. *Journal of Financial and Quantitative Analysis* 23(3), 285–300.
- Talay, D. (1982). How to Discretize Stochastic Differential Equations. In: *Lecture Notes in Mathematics* 972. Springer-Verlag, Berlin–Heidelberg–New York, 276–292.
- Talay, D. (1984). Efficient Numerical Schemes for the Approximation of Expectations of Functionals of the Solutions of a S.D.E., and Applications. In: *Lecture Notes in Control and Information Sciences* 61. Springer-Verlag, Berlin–Heidelberg–New York, 294–313.
- Talay, D. (1995). Simulation and Numerical Analysis of Stochastic Differential Systems: A Review. In: Kree, P., Wedig, W. (Eds.), *Probabilistic Methods in Applied Physics, Lecture Notes in Physics* 451, Springer-Verlag, Berlin–Heidelberg–New York, 63–106.
- Talbot, A. (1979). The Accurate Numerical Inversion of Laplace Transforms. *J. Inst. Math. Appl.* 23(1), 97–120.
- Tankov, P. (2005). Simulation and Option Pricing in Lévy Copula Model. In: Avellaneda, M., Cont, R. (Eds.), *Mathematical Modelling of Financial Derivatives*, IMA volumes in Mathematics and Applications, Springer-Verlag, Berlin–Heidelberg–New York.
- Tavella, D., Randall, C. (2000). *Pricing Financial Instruments: The Finite Difference Method*. Financial Engineering, Wiley.
- Thompson, A.C. (1995). Valuation of Path-Dependent Contingent Claims with Multiple Exercise Decisions Over Time: The Case of Take-or-Pay. *Journal of Financial and Quantitative Analysis* 30(2), 271–293.
- Thompson, G.W.P. (1998). Fast Narrow Bounds on the Value of Asian Options. *Working Paper*, University of Cambridge.
- Thorp, W.A. (2000). The MACD: A Combo of Indicators for the Best of Both Worlds. *AAII Journal* 30–34.
- Tian, Y. (1999). Pricing Complex Barrier Options Under General Diffusion Processes. *Journal of Derivatives*, Winter, 11–30.
- Titman, S., Tompaidis, S., Tsyplakov, S. (2004). Market Imperfections, Investment Flexibility and Default Spreads. *Journal of Finance* 59(1), 165–205.
- Tompkins, R., D’Ecclesia, R.L. (2006). Unconditional Return Disturbances: A Non Parametric Simulation Approach. *Journal of Banking and Finance* 30(1), 287–314.

- Topper, J. (2005). *Financial Engineering with Finite Elements*. The Wiley Finance Series.
- Trigeorgis, L. (1991). A Log-Transformed Binomial Numerical Analysis Method for Valuing Complex Multi-Option Investments. *Journal of Financial and Quantitative Analysis* 26(3), 309–326.
- Turnbull, S., Wakeman, L. (1991). A Quick Algorithm for Pricing European Average Options. *Journal of Financial and Quantitative Analysis* 26, 377–389.
- Varga, R. (1962). *Matrix Analysis*. Prentice-Hall, Englewood Cliffs, NJ.
- Vasicek, O.A. (1973). A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Betas. *Journal of Finance* 28(5), 1233–1239.
- Vasicek, O. (1977). An Equilibrium Characterization of the Term Structure. *Journal of Financial Economics* 5, 177–188.
- Vecer, J. (2001). A New PDE Approach for Pricing Arithmetic Average Asian Options. *Journal of Computational Finance* 4(4), 105–113.
- Vetzal, K.R. (1998). An Improved Finite Difference Approach to Fitting the Initial Term Structure. *Journal of Fixed Income* 7 (March), 62–81.
- Villeneuve, S., Zanette, A. (2002). Parabolic ADI Methods for Pricing American Options on Two Stocks. *Math. Oper. Res.* 27(1), 121–149.
- Vlach, J., Singhal, K. (1993). *Computer Methods for Circuit Analysis and Design* (2nd ed.). Van Nostrand Reinhold Company, New York.
- Wang, S.S. (1999). Aggregation of Correlated Risk Portfolios: Models and Algorithms. Preprint, CAS Committee on Theory of Risk.
- Webber, N., Kuan, G. (2003). Valuing Barrier Options in One-factor Interest Rate Models. *Journal of Derivatives* 10, 33–50.
- Weeks, W. (1966). Numerical Inversion of Laplace Transforms Using Laguerre Functions. *Journal ACM* 13(3), 419–429.
- Weideman, J.A.C. (1999). Algorithms for Parameter Selection in the Weeks Method for Inverting the Laplace Transform. *SIAM J. Sci. Comput.* 21(1), 111–128.
- Wilmott, P., Dewynne, J.N., Howison, S. (1993). *Option Pricing: Mathematical Models and Computation*. Oxford Financial Press.
- Yohai, V.J., Stahel, W.A., Zamar, R.H. (1991). A Procedure for Robust Estimation and Inference in Linear Regression. In: Stahel, W., Weisberg, S. (Eds.), *Directions in Robust Statistics and Diagnostics*. Springer-Verlag, Berlin–Heidelberg–New York, 365–374.
- Yor, M. (1991). *On Exponential Functionals of Brownian Motion and Related Processes*. Springer-Verlag, Berlin–Heidelberg–New York.
- Yor, M. (2001). *Exponential Functionals of Brownian Motion and Related Processes*. Springer-Verlag, New York.
- Young, D.M. (1971). *Iterative Solution of Large Sparse Systems*. Academic Press.
- Zauderer, E. (2006). *Partial Differential Equations of Applied Mathematics*. Pure and Applied Mathematics: A Wiley-Interscience Series of Texts, Monographs and Tracts, 3rd ed.
- Zhang, J.E. (2001). A Semi-Analytical Method for Pricing and Hedging Continuously Sampled Arithmetic Average Rate Options. *Journal of Computational Finance* 5(1), 59–79.
- Zhang, X.L. (1997). Numerical Analysis of American Option Pricing in a Jump-Diffusion Model. *Mathematics of Operations Research* 22, 668–690.
- Zhu, Y.I., Wu, X., Chern, I.L. (2005). *Derivative Securities and Difference Methods*. Springer-Verlag, Berlin–Heidelberg–New York.
- Zvan, R., Forsyth, P.A., Vetzal, K.R. (1998a). Penalty Methods for American Options with Stochastic Volatility. *J. Comput. Appl. Math.* 91, 199–218.

- Zvan, R., Forsyth, P.A., Vetzal, K.R. (1998b). Robust Numerical Methods for PDE Models of Asian Options. *Journal of Computational Finance* 1, 39–78.
- Zvan, R., Vetzal, K.R., Forsyth, P.A. (2000). PDE Methods for Pricing Barrier Options. *Journal of Economic Dynamics and Control* 24, 1563–1590.

---

# Index

## A

$A_0$  stability 115  
absolute normal distribution 22  
acceptance–rejection 11, 20, 21, 25, 27, 61  
adaptive quadrature 188, 193  
admissible  
  control 71, 458, 476  
  stopping rule 477  
  strategy 79  
affine function 532  
Akaike Information Criterion (AIC) 303  
alpha-stable distribution 265  
American  
  call option 472  
  continuation value 77  
  option 66, 77, 148, 149, 156  
  put option 77, 148, 346, 349  
antithetic variables 31–33, 419  
approximate dynamics 45, 46, 59, 60  
arbitrage 4, 9, 48, 77, 83, 86, 119, 157,  
  199, 335, 340, 346, 363, 369, 398,  
  425, 441, 444  
Archimedean copula 245, 264  
Arithmetic Brownian Motion 412, 414  
Asian option 10, 33, 48, 219, 226, 227,  
  374–379, 381, 382, 384, 391, 392,  
  396–398, 406–409  
asset allocation 273, 274, 278, 285  
asymptotically dependent 238  
autoregressive (AR) 311, 316, 321, 326,  
  327, 427

## B

backtesting 289, 304, 305, 308  
backward  
  difference 93, 101  
  induction 66, 480, 570

bandwidth value 536, 537  
bang-bang 457  
barrier options 45, 48, 152, 156, 185, 194,  
  196, 197, 212, 229, 488  
basket  
  default swap 487, 490, 496  
  option 5, 11, 84, 267, 395  
  swap 495, 497, 499  
Bayes' formula 21  
Bayes Information Criterion (BIC) 303  
Bayesian 289, 290, 296–298, 306, 307,  
  309  
Bellman's principle 69, 73, 74, 570, 571  
Bermuda option 346  
Bernoulli number 167  
Bessel function 192, 529  
beta 266, 289–293, 295, 296, 299–301,  
  303, 304, 306–309  
  resistant 293  
binomial  
  model 78  
  tree 149, 462  
Black–Scholes  
  model 32, 34, 144, 149, 185–188, 219,  
  560  
  with jumps 56, 58  
bond  
  default 23  
  option 190, 562  
  prices 152, 154, 191, 490, 560  
bootstrap method 255, 312, 317  
bootstrapping 276, 311, 316  
boundary conditions 86–88, 90, 97, 101,  
  110, 112, 116–118, 125, 150, 152,  
  153, 156, 220, 222, 380, 382, 418,  
  563–565



Box–Müller 20, 25, 26, 28  
 Bromwich 217, 229, 383  
   inversion formula 214

**C**

calibration 3, 66, 331, 333, 335, 336,  
 353, 354, 356, 358–361, 363, 365,  
 367–369, 443, 450  
 call option 10, 37, 43, 84, 86, 87, 119,  
 144–147, 149–152, 154, 186–190,  
 192, 199, 219, 225, 332, 356, 357,  
 413, 425, 460, 461, 546, 559, 560  
 CAPM 289–291  
 cash-and-carry 428  
 Cauchy distribution 29  
 central difference 92, 93  
 Central Limit theorem 6, 24  
 CEV model 147, 212  
 change of numeraire 156, 191, 379  
 characteristic function 29, 197–200, 203,  
 204, 206, 241, 354, 356–358, 494  
 chi-square distribution 30, 31, 154, 212,  
 260  
 Cholesky (decomposition) 258, 396, 397,  
 407, 409, 496  
 Clayton (copula) 238, 239, 247, 249, 250,  
 261, 262  
 Collateralized Debt Obligation (CDO)  
 499, 500  
 commodity 331, 373, 427, 442, 457, 506  
 compensator (of a jump process) 51  
 Composite Newton–Cotes formula 162  
 compound  
   copula 263, 264  
   jump process 50, 51, 56, 533  
 concordance 233–236, 266  
 condition number 127, 139  
 conditional  
   coverage 305  
   default (probability) 492  
   distribution 81, 240, 260, 433  
   expectation 9, 83, 157, 199, 347, 413,  
   459  
   simulation 53, 54  
   transition density 193, 452  
 confidence  
   band 535, 538–540  
   interval 192, 283, 350, 537  
 congruential generator 12  
 Conjugate Gradient Method (CGM) 133  
 conservative term 85  
 consistency 110–112, 354  
 continuous

  diffusion 45, 49, 56, 58, 531–535  
   monitoring 195, 412, 419, 421  
   time process 42, 43, 524  
 control  
   policy 69–71, 73, 75–77, 458, 474–477,  
   480–483, 485, 570, 571  
   variables 33, 72, 73, 80, 431, 458, 475,  
   477  
 controlled dynamic system 69, 71, 75  
 convective term 85, 91  
 convergence 6, 7, 35, 48, 110–112, 119,  
 122, 130–133, 135–139, 174, 218,  
 295, 312, 328, 350, 404, 406, 408,  
 409, 411, 412, 422, 485, 488  
 convolution 215, 493, 496  
   of densities 521  
 copula 67, 231–235, 237–266, 490, 491,  
 496  
   density 241–244, 246, 248–250, 252  
   functions 231–233, 238–240, 245, 252,  
   254, 255, 257–259, 265, 266, 490  
 corporate bonds 499  
 correlated events 491  
 correlation 32, 231, 234, 236, 237, 242,  
 258, 259, 290, 316, 355, 367, 396,  
 407, 408, 488, 496–502, 506, 507  
   matrix 242, 258, 259, 407, 408, 488,  
   496  
 counting process 50, 56, 446, 489, 490,  
 493, 525, 557  
 coupon bond 144, 152–155, 185, 191–193,  
 212, 559, 560  
 covariance matrix 26, 27, 255, 275–278,  
 282, 396, 409, 505, 509, 511, 512  
 Cox–Ingersoll–Ross model 144, 152, 156,  
 190, 212, 519, 523  
 Crank–Nicolson 93, 103, 106–110, 114,  
 118, 120, 121, 124, 126, 137, 145,  
 146, 149, 150, 381, 417, 424  
 credit  
   derivatives 469, 487, 488, 490, 499  
   risk 229, 267, 487  
 cross-sectional data 506, 509, 516  
 cubic copula 237  
 cumulative distribution function 5, 9, 16,  
 17, 42, 240, 242, 491

**D**

day-ahead  
   market 429, 431, 432, 448  
   price 427–429, 431, 432, 434  
 Debye’s function 250

- default
    - probabilities 490, 497
    - time 488, 490–492, 496
    - times 488, 490, 491, 496, 497
  - defaultable bonds 487
  - delivery
    - date 429
    - time (swing option) 458, 463
  - demand peaks 441
  - density function 7, 9, 21, 36, 42, 199, 204, 225, 256, 264, 302, 326, 331–334, 375, 436–438, 520
  - dependence 71, 130, 199, 231–239, 245, 252, 254, 266, 301, 334, 414, 415, 430, 472, 473, 488, 491, 507
  - diffusion coefficients 63, 65, 90, 117, 119, 510, 532, 537, 540
  - digital option 35
  - discounted expected value function 478
  - discrete
    - distribution function 9
    - monitoring 87, 185, 411, 412, 421
  - discretization 11, 45, 48, 58, 71, 78, 91, 111, 117, 121, 166, 218, 355, 360, 363, 386–388, 416, 421, 424, 462, 519
  - scheme 48
  - diversification 273–275, 278, 284, 285
  - downward jump 78
  - drawdowns 314, 315, 325
  - dual problem 345
  - dynamic programming 66, 69, 73, 74, 76, 77, 79–81, 147, 345–347, 457, 459, 463, 471, 477, 570
  - algorithm 74, 76, 79, 477, 570
  - equation 345, 347
- E**
- Edgeworth Expansion 373, 376
  - efficient frontier 274–279, 282, 283
  - eigenfunction 374
  - electricity price 52, 428, 450–454, 525
  - elliptic 85, 563
  - elliptical
    - copula 240
    - distribution 240
  - Empirical Martingale Simulation 546
  - empirical moment 533, 537
  - endogenous random intensity 60, 61
  - energy price 441, 442, 448, 457
  - equity line 326
  - estimation 3, 5, 7, 8, 10, 11, 252–255, 273–277, 282–286, 289–293, 295, 296, 298, 299, 306–309, 401, 404–406, 408, 409, 450, 519, 520, 523, 524, 528–531, 533–535, 540, 541, 554, 555
  - risk 273–276, 284–286
  - estimator 7, 8, 31–39, 253–255, 292–294, 296, 298, 301, 302, 304, 399, 498, 519, 520, 523, 536, 540
- Euler
- discretization 48, 71
  - iterative formula 527
  - scheme 47, 48, 59, 61, 355, 521
- Euler Algorithm 218–220, 224, 227, 228, 385, 386, 389
- European option 48, 212, 362, 398
- European-style derivative 9, 37
- EWMA (Exponentially-weighted moving average) 299, 300, 306, 310
- exogenous random intensity 60
- exotic options 81, 156, 229, 373, 411
- explicit scheme 94, 96, 97, 111, 113–116, 136
- exponential
  - density 22, 51
  - distribution 22, 23, 453
  - moving average 312, 313
  - sampling 17
- F**
- Factor Model 290, 490, 491
  - Fast Fourier Transform (FFT) 185, 197, 201–205, 210, 353, 354, 360, 490, 493–498, 500, 501
  - Feynman–Kac 227, 417
  - filter 289, 290, 300–302, 306, 308–310, 343, 344, 428
  - financial
    - derivatives 9
    - security valuation 3
  - finite
    - difference 47, 83, 84, 93, 94, 101, 103, 109–112, 116, 136, 138, 145, 148, 156, 212, 381, 412, 417, 418, 564
    - element 120
    - first-to-default 501, 502
    - Fisher's information matrix 253
    - Fong–Vasicek model 47
  - forward
    - contract 457
    - difference 92, 94
    - price curve 506
    - provision 430, 431
    - purchase 430–432
  - Fourier
    - methods 64, 493

series expansion 64  
 Fourier Inversion 197, 200, 203, 212, 357, 385, 386  
 Fourier Series Method 217, 218  
 Fourier transform 158, 197, 200–202, 204, 208, 212, 353, 354, 356, 357, 360, 382, 384, 390, 391, 490, 493, 494, 496, 498  
 Frank copula 263  
 Frank  $n$ -copula 249, 263  
 Fréchet–Hoeffding bounds 233  
 free boundary 73

**G**

gamma distribution 30, 377  
 GARCH 311, 316, 317, 321, 326–328, 543–545, 548–555  
 gas spot price 441, 443, 458–460  
 Gauss–Chebyshev 177–179  
 Gauss–Hermite 177, 180  
 Gauss–Laguerre 177, 178, 180  
 Gauss–Legendre quadrature 179, 182, 186, 195, 201, 500  
 Gauss–Seidel 132  
 Gaussian  
 copula 241–244, 253, 258, 259, 491, 496  
 copula simulation 258, 259  
 elimination 122  
 kernel 256, 536  
 process 58, 62  
 quadrature 158, 174, 175, 177, 178, 181, 182, 200, 218, 387–389  
 variable 5, 31, 72  
 Geometric Brownian Motion (GBM) 46, 48, 56, 84, 88–91, 119, 144, 145, 149, 150, 157, 188, 189, 194, 333, 374, 377, 382, 396, 461, 488, 552, 553  
 global  
 minimum variance portfolio 276, 278, 283, 284  
 penalty (swing option) 460, 461  
 Godambe’s information matrix 255  
 Greek 109, 110, 144, 358  
 grid 47, 92, 94, 95, 98, 99, 101, 103, 110, 111, 127, 148, 168, 171, 201, 203, 350, 360, 397, 418, 521, 550–552, 563  
 Gumbel copula 247, 248, 262, 265

**H**

Haar function 44  
 hazard rate 9, 11, 23, 24, 490, 496, 497, 502, 503

heat rate 442, 443, 452, 453, 455  
 hedge funds 229, 505  
 Hessian matrix 524  
 historical probability 206  
 Hoeffding phi 236, 237  
 horizon refinement 460  
 Hull and White recursion 495

**I**

implicit scheme 101, 102, 104, 105, 114, 115, 117, 136, 137  
 implied-tree 333  
 implied volatility 331, 333, 336, 338, 339, 361, 365, 366, 543, 544, 549, 551–554  
 in-the-money (option) 87, 349, 358, 363, 443, 543, 547  
 inference functions for margins 251, 254  
 infinite element 120  
 initial condition 69, 71, 76, 88, 90, 91, 96, 98, 101–103, 109, 113, 116, 150, 152, 153, 221, 222, 227, 380, 382, 388, 476, 563, 565  
 integral equation 412, 414, 415  
 intensity function 52, 446, 447, 490, 525, 526  
 interarrival time 50, 51, 64  
 interruptible contracts 457  
 intrinsic value 196, 346, 348, 358  
 Inverse Fast Fourier Transform 493, 494  
 Inverse Fourier Transform 201, 493  
 Inverse Laplace Transform 213, 222, 264  
 inverse transformation 11, 14, 17, 18, 25, 60  
 iterated expectation theorem 494  
 iterative methods 121, 122, 127, 128, 131, 133, 135, 141, 143, 145, 155  
 Itô formula 5

**J**

Jackknife method 255  
 Jacobi 122, 127–131, 133, 135–141, 143  
 James–Stein estimator 296  
 joint  
 density 241  
 distribution function 231, 235, 238  
 jump  
 intensity process 60, 528  
 process 49–51, 56, 355, 488, 533  
 regime 450–452  
 size 56, 62, 368, 528, 534–536, 540

**K**

Kalman Filter 289, 290, 300–302, 306, 308–310, 428

- Karhounen–Loeve expansion 44  
 Kendall's tau 234–236, 242, 246, 247, 249  
 Kernel regression 333  
 Kimberling theorem 247, 266  
 knock out option 116  
 Kupiec Test 305, 308  
 Kurtosis 316, 324, 340, 353, 355, 366, 376
- L**
- $L_0$  stable  
 Lagrange Interpolation formula 159  
 Laplace Transform 212–217, 219,  
 221–223, 225–230, 263, 264, 374,  
 382–385, 389, 391, 392  
 Lax Equivalence Theorem 111, 112, 114,  
 119  
 Lax–Richtmyer stability 112  
 Least Median of Squares 294, 307, 308  
 Least Trimmed Squares 294, 307, 308  
 Lebesgue measure 42  
 left tail decreasing 240  
 Leverage Effect 544, 545, 550, 552, 553,  
 555  
 Lévy process 29, 45, 67, 185, 206, 207,  
 212  
 likelihood 39, 251–254, 293, 296, 297,  
 301–303, 308, 321, 444, 450, 519–521,  
 523, 524, 528–530, 547  
 linear system 101–103, 107, 117, 121–124,  
 127–129, 136, 140–143, 148, 216,  
 388, 421  
 Lobatto quadrature 183, 184, 420  
 local  
   penalty (swing option) 460  
   volatility 333  
 log-likelihood function 253, 254, 302, 444,  
 524  
 log-normal 25, 331, 333, 339, 343  
 logarithmic likelihood function 253  
 lognormal 78, 146, 225, 237, 343, 373,  
 375, 376, 386, 391, 393, 543, 544  
   mixture 343  
 Longstaff–Schwartz simulation 66, 345,  
 351  
 lookback options 84, 411–413, 418–421,  
 423, 425, 426  
 lower tail dependence 238, 239  
 LU Decomposition 122, 124, 127, 140,  
 144, 382, 421
- M**
- Malliavin derivative 345  
 marginal  
   cost of production 441  
   default probability 494  
 market  
   heat rate 443  
   model 3, 290, 299  
 Markov  
   chain 474, 480  
   control policy 477  
   control problem 476  
 martingale 66, 207, 212, 347–349, 351,  
 379, 380, 546, 547  
 maximum likelihood 251, 253, 293, 301,  
 303, 308, 444, 450, 519, 520, 523,  
 529, 530, 547  
   estimation 301, 519, 529, 547  
   estimator 253, 293, 519, 520, 523  
 mean  
   reversions 427  
   reverting process 58, 444  
 Mean Absolute Error 304, 309  
 Mean Error 304, 309  
 Mean Square Error 5, 103, 105, 107, 108,  
 301, 302, 304, 309, 524  
 mean-variance 273, 276–287  
 measure  
   of concordance 234  
   of dependence 235, 236  
 Midpoint formula 164, 165  
 Milstein scheme 48, 355  
 mixed jump diffusion 49, 56, 66, 531, 533,  
 534, 536, 537, 540  
 mixture 311, 331, 333, 334, 336, 340–343,  
 527  
 moment  
   generating function 30, 225, 226, 493,  
   494  
   matching 375, 386, 391  
 Monte Carlo  
   methods 3, 7, 9, 10, 35, 39, 395, 398,  
   399, 401, 404, 491, 496, 498, 523, 559  
   simulation 3, 9, 11, 154, 212, 231, 257,  
   311, 345, 347, 355, 356, 390, 395,  
   411, 412, 419, 421, 426, 499, 500,  
   519, 520, 545–547, 551–553  
 mortgage-backed securities 471  
 moving average 299, 312, 313  
 Moving Average Convergence Divergence  
 (MACD) 312  
 multidimensional inverse transformation  
 17  
 multinomial approximation 508  
 multiple exercise derivative 457  
 multivariate  
   Gaussian copula 241, 253

- Gaussian distribution 240
  - normal distribution 25, 240, 242, 274, 276, 508, 509
  - optimal stopping rule 476
  - stationary process 255
  - Student's  $t$  distribution 242
- N**
- net present value 443
  - Newton–Cotes formula 158, 161–163, 174, 179, 186
  - NGARCH 545, 547, 548, 550, 552
  - non-central chi-square distribution 154, 212, 562
  - non-linear least squares 335, 360, 545, 549
  - non-parametric 333
  - nonparametric
    - estimation 255, 531, 535
    - kernel 251, 266
  - norm 28, 47, 111, 113, 114, 133, 136, 137, 143
  - normal
    - density 25, 27, 28, 241, 342
    - distribution 6, 22, 24, 25, 43, 46, 240, 242, 274, 276, 305, 316, 324, 508, 509
  - null hypothesis 316, 324, 326
  - numeraire 156, 191, 199, 379, 380
  - numerical inversion 19, 25, 26, 168, 203, 213, 216, 219, 221, 223, 225–230, 354, 374, 384–386, 389
- O**
- OLS regression 548
  - optimal
    - allocation 79
    - control 69, 71, 73–77, 81, 82, 433, 458, 476, 480, 482, 483, 569, 571
    - control policy 73, 75–77, 476, 480, 482, 483, 571
    - control problem 69, 71, 74, 76, 433
    - investment problem 79
    - stopping problem 73, 81, 569
    - stopping time 72, 483
    - strategy 474
  - optimization problem 69, 74, 80, 156, 339
  - option valuation 66, 459
  - options 66, 459
    - American 66, 77, 148, 149, 156
    - Asian 10, 33, 48, 219, 226, 227, 374–379, 381, 382, 384, 391, 392, 396–398, 406–409
    - at-the-money 211, 338, 359
    - barrier 45, 48, 152, 156, 185, 194, 196, 197, 212, 229, 488
    - basket 5, 11, 84, 267, 395
    - Bermuda 346
    - bond 190, 562
    - call 10, 37, 43, 84, 86, 87, 119, 144–147, 149–152, 154, 186–190, 192, 199, 219, 225, 332, 356, 357, 413, 425, 460, 461, 546, 559, 560
    - digital 35
    - European 48, 212, 362, 398
    - exotic 81, 156, 229, 373, 411
    - in-the-money 87, 349, 358, 363, 443, 543, 547
    - knock out 116
    - lookback 84, 411–413, 418–421, 423, 425, 426
    - out-the-money 355
    - path-dependent 5, 10, 37, 38, 84, 395, 399, 411
    - payoff 4, 5, 10, 33, 35, 36, 38, 83, 190, 191, 373, 397, 398, 452, 458, 459, 487, 559, 560
    - put 77, 86, 87, 147, 148, 154, 211, 346, 349, 416, 424, 425, 554
    - real 441, 442
    - spark spread 442–444, 449, 450, 452, 453
    - spread 52, 313, 359, 363, 442–444, 449, 450, 452, 453, 465, 471, 472, 488–490, 496–501
    - swing 457–461, 463–466
    - vega 358, 560
  - Ordinary Differential Equation (ODE) 213, 221
  - ordinary least squares 289, 294, 306, 450
  - orthogonal polynomial 176, 177
  - oscillations 109, 110, 115, 119, 120, 342, 444, 452
  - out-of-sample 274, 278, 280, 281, 284, 285, 287, 368, 472
  - out-the-money 355
  - outliers 290, 293–295
  - outstanding balance 471, 473, 477, 479, 480, 487
- P**
- $p$ -values 306–308, 316, 317, 326–328
  - parabolic 85, 116, 119, 219, 563
  - parametric estimation 519
  - path-dependent 5, 10, 37, 38, 84, 395, 399, 411

- pay-off 4, 5, 10, 33, 35, 36, 38, 83, 190, 191, 373, 397, 398, 452, 458, 459, 487, 559, 560
  - PDE 83–86, 88–92, 95, 110, 111, 116, 119–122, 127, 128, 144, 145, 149, 150, 152–156, 192, 193, 212, 213, 221, 222, 373–375, 379, 388, 411, 412, 417, 418, 420–422, 424–426, 563
  - PDE Solver 149, 563
  - peak price 449
  - Pearson's linear correlation 236, 242
  - penalty 156, 457–461, 463–466
  - performance measure 70, 314, 569
  - periodic intensity function 52
  - Poisson
    - jump time 23, 24, 55
    - process 50, 51, 490, 496, 526
    - random variable 53, 54, 355
    - realization 53
  - Poisson–Gaussian process 58
  - portfolio
    - choice 82
    - turnover 286, 287
  - positive quadrant dependence 239, 266
  - Posterior Distribution 297, 298
  - power
    - plant 427, 429, 442, 443, 448, 449, 452, 453, 455
    - unit value 448
  - present value 9, 10, 335, 443, 458, 459, 461, 496, 497
  - principal components analysis 505, 507–509
  - Prior Distribution 306
  - probability distributions 3, 36, 39, 76, 331, 333, 374, 435, 481, 483, 484, 488, 490, 492, 496, 508
  - profit
    - factors 314, 324, 325
    - & loss (P&L) 304, 348, 431
  - pseudo-random (number) 11, 12, 14
  - pseudo-random samples 9, 11, 511
  - put option 77, 86, 87, 147, 148, 154, 211, 346, 349, 416, 424, 425, 554
- Q**
- quadrature 157, 158, 173–190, 192–197, 200, 201, 211, 212, 216, 218, 354, 387–389, 412, 418, 420, 500
  - quasi-Monte Carlo 66
- R**
- Radon–Nikodym derivative 42
  - ramp-up time 442, 449, 453–455
  - random walk 38, 48, 303, 308, 311, 316, 321, 327, 462, 466
  - rate of convergence 135–137, 406, 523
  - real
    - estate 276, 471
    - options 441, 442
  - reciprocal gamma 375, 377, 386, 391
  - Rectangle Rule 163, 164, 166
  - recursive algorithm 459, 490
  - reducing the variance 3, 31
  - refraction period 459
  - regime-switching behavior 429
  - Relative Strength Index 313
  - replication 8, 82
  - resampled frontiers 279, 283
  - resampling 273, 274, 276, 278–280, 283–286
  - residual capacity 430, 431
  - Reweighted Least Squares 294, 295, 307
  - right-continuous 15
  - risk
    - analysis 505, 516
    - free 35, 549
    - management 3, 505
  - risk-neutral 9, 77, 83, 84, 147, 152, 157, 190, 207, 212, 226, 331–333, 337, 338, 341, 342, 353, 356, 374, 396, 491, 546, 547, 549, 560
    - density 226, 331–333, 338, 342, 353
    - probability 4, 9, 83, 84, 157, 349, 356, 374, 396, 491, 546, 547
  - Robin boundary condition 86, 90, 564
  - robust estimate 289
  - rolling regression 290
  - Romberg Extrapolation 168, 170–172, 181, 187, 195
- S**
- sampling theorem 62, 562
  - Schauder function 44
  - Schweitzer–Wolff's sigma 236
  - seasonality 441, 445
  - self-financing
    - portfolio 80
    - trading strategies 79
  - series expansion 43–45, 48, 64, 104, 118, 376, 391
  - shape factors 505
  - Sharpe ratios 285, 286
  - short-term interest rate 519, 523, 525
  - shrinkage 289, 290, 295, 296
  - simplex
    - method 156, 524

- search method 524
  - Simpson quadrature 173, 500
  - Simpson Rule 158, 172–174, 180, 186
  - simulation 3, 39, 41, 42, 48–51, 53–55, 58, 59, 66, 67, 257–264, 279–281, 317, 321–323, 345, 395, 396, 406–408, 421, 422, 505, 506, 511, 514–516, 528–531, 537–540, 545–547
    - schemes 3, 48, 49
  - skewness 324, 334, 353, 355, 376
  - smile 188, 206, 331, 333, 340, 365, 366, 506, 543, 544
  - smoothing 290, 299, 300, 306, 534, 555
  - Snell envelope 345
  - source term 85
  - spark spread 442–444, 449, 450, 452, 453
  - Spearman's rho 234–236, 242, 249
  - spike 52, 441, 447, 485
  - spot
    - delivery 427
    - market 429–432
    - price 48, 84, 85, 89, 99, 145, 146, 186, 187, 194, 196, 197, 203, 223, 335, 358, 368, 388, 413, 419, 421, 430–436, 442, 443, 457–461, 464
  - spread 52, 313, 359, 363, 442–444, 449, 450, 452, 453, 465, 471, 472, 488–490, 496–501
  - spreadsheet 125, 206, 306, 336, 338
  - square-root model 145, 188, 219, 227, 354, 523, 525, 528, 530
  - stability 110–113, 115, 117, 119, 126, 278, 299, 367
  - standard
    - Brownian motion 38, 42, 44, 48, 56, 374, 435, 532
    - deviation 49, 277, 283, 284, 309, 316, 317, 322, 326, 328, 340, 343, 537, 540, 541
    - normal variable 59, 528
  - state variable 5, 7, 9–11, 41, 46, 51, 60, 69, 71, 72, 79, 80, 84, 86, 301, 412, 458–460, 466, 476, 477, 482, 519, 559
  - stationary 24, 45, 62, 64, 133, 252, 255, 315, 326, 377, 436, 437, 562
    - process 45, 62, 255
  - statistical inference for copulas 251
  - Statistically Equivalent Portfolio 279, 282
  - Steepest Descent Method 133
  - stochastic 11, 41, 42, 45, 49, 56, 66, 69–74, 76, 79, 81, 83, 84, 203, 207, 252, 333, 334, 353–355, 365, 366, 368, 369, 433, 442, 443, 520
    - differential equation 56, 83, 353, 446, 520, 523, 525, 532
    - dynamic programming 76
    - mesh method 345
    - optimal control problem 71, 433
    - volatility 66, 84, 203, 333, 353–355, 365, 366, 368, 369
  - stratified sampling 13, 15, 399, 409
  - strike price 10, 37, 43, 48, 77, 147, 186, 190, 338, 339, 348, 349, 374, 388, 391, 397, 412, 413, 443, 457, 543, 563
  - structural model 488
  - Student  $t$ -copula 242, 244, 245, 260
  - successive over-relaxation (SOR) 122
  - supply
    - & demand 427, 441, 525
    - price 430, 432
  - survival
    - copula 240
    - probability 244, 495
  - swing
    - pay-off 458
    - rights 458, 459, 464–466
  - swing (option) 457–461, 463–466
- T**
- technical analysis 311, 312, 315
  - Technical Rule 312
  - term structure 46, 81, 156, 355, 363, 365, 505–507, 511, 514, 519, 544
  - terminal condition 84, 86, 88, 414
  - thinning
    - method 59–61
    - simulation 55
  - threshold 10, 35, 48, 87, 127, 129, 131, 166, 450, 451, 465, 466, 491
  - time
    - dependent volatility 427, 553
    - horizon 54, 61, 69, 73, 346, 348, 349, 363, 460–462, 471–473, 475, 480
  - time-to-maturity 382, 507, 511, 512, 516, 545, 546, 549, 550, 554
  - Trading Rule 311, 316, 317, 321, 324–328
  - transacted quantities (swing option) 458
  - transaction costs 82, 315, 328, 472, 473, 480
  - transition
    - density 37, 45, 48, 157, 212, 414, 519, 520, 522, 523, 527
    - probabilities 46, 58, 474, 476, 519, 521
  - Trapezoidal Rule 171, 172, 180, 186, 189, 200, 201, 203, 218, 360, 383, 385, 416

trend 25, 311–313, 332, 435, 444, 446,  
450, 451, 544, 555  
tridiagonal system 124, 420  
trinomial discretization 460  
truncation error 110, 111, 201  
 $T$ -statistic 306, 315

**U**

uniform density 13  
univariate  
  standard normal 258, 496  
  Student  $t$  244  
up jump factor 78  
upper tail dependence 247

**V**

value  
  function 74–76, 81, 461, 463, 477–481,  
  485  
  at risk (VAR) 290, 304, 308, 310  
value-at-risk (VaR) 66, 304, 305, 505, 514  
variance–covariance matrix 274–278, 282,  
292, 511  
variance reduction 11, 31, 33, 34, 395,  
398, 399, 455, 560

Vasicek model 46, 47, 307, 561  
vega option 358, 560  
volatility 48, 63–66, 84, 188–190, 203,  
327, 331, 333, 336, 338, 339, 353–356,  
358, 359, 361, 363, 365–369, 425,  
447, 540, 543–546, 549, 551–555  
  smile 188, 333, 366, 506  
  surface 333, 355, 363, 365–367, 544,  
  545, 551, 552, 555

**W**

Wald's theorem 558  
weak solution 532  
Wiener–Hopf 212  
Winsorizing 294

**Y**

yield 338, 339, 363, 507, 509, 511–516  
  curve 507, 511, 512, 514, 516

**Z**

zero-coupon 152, 154, 190, 559, 560  
zero variance estimator 38  
 $Z$ -transform 414–416, 420