Appendix

A1. The Hardy Inequality

We use at several places in the text the following result.

Proposition A1. *The kernel operator* $\mathbf{1}(x \geq 0, y \geq 0)\frac{1}{x+y}$ *is bounded on* $L^2(\mathbb{R}^+)$.

Proof. We call A and A_1 the kernel operators defined by

$$A(x,y) = \mathbf{1}(x \geq 0, y \geq 0)\frac{1}{x+y}, \quad A_1(x,y) = \mathbf{1}(x \geq y \geq 0)\frac{1}{x+y}$$

We have $A = A_1 + A_1^*$, and it is sufficient to prove that A_1 is bounded on $L^2(\mathbb{R}^+)$. Due to the estimate

$$|A_1 f(x)| = |\int_0^x \frac{f(y)}{x+y} dy| \leq \frac{1}{x}\int_0^x |f(y)|dy$$

it is sufficient to prove that the operator A_2 defined by

$$A_2 f(x) = \frac{1}{x}\int_0^x f(y)dy = \int_0^1 f(tx)dt$$

is bounded on $L^2(\mathbb{R}^+)$. For any $g \in L^2(\mathbb{R}^+)$, we have

$$(A_2 f, g) = \int_0^1 \left(\int_0^\infty f(tx)\overline{g(x)}dx\right) dt \leq ||g||M$$

where the constant M is

$$M = \int_0^1 \left(\int_0^\infty |f(tx)|^2 dx\right)^{1/2} dt = ||f|| \int_0^1 \frac{dt}{\sqrt{t}} = 2||f||$$

which proves the result. ∎

A2. The Hilbert projector

Recall that we call H^+ the Hilbert space of the functions $f(\xi)$, analytic in the lower half-plane and uniformly L^2 on the horizontal lines, *i.e.* (cf Sect.3.3)

$$\sup_{c>0} \int_{\mathbb{R}} |f(\xi - ic)|^2 d\xi < +\infty$$

equipped with the norm

$$||f|| = \left(\int_{\mathbb{R}} |f(\xi)|^2 d\xi \right)^{1/2}$$

Definition A2.1. *The Hilbert projector is the application $H : L^2(\mathbb{R}) \to H^+$ defined for $g \in L^2(\mathbb{R})$ by*

$$Hg(\xi) = \frac{1}{2\pi} \int_{\mathbb{R}} \frac{g(\xi)}{\xi - \zeta} d\zeta$$

We check easily that if the function $g = \mathcal{F}f$ is the Fourier-Plancherel transform of $f \in L^2(\mathbb{R})$, then Hg is defined by $Hg(\xi) = \mathcal{F}(f(x)\mathbf{1}(x \geq 0))(\xi)$. Therefore $||Hg||_{H^+} = ||f||_{L^2(\mathbb{R}^+)} \leq ||f||_{L^2(\mathbb{R})} = ||g||_{L^2(\mathbb{R})}$. This proves the following

Proposition A2.2. *The Hilbert projector is bounded from $L^2(\mathbb{R})$ onto H^+.*

Subject Index

Asymptotics 1, 10
– diffracted wave 22
– high frequency 6, 7, 79

Boundary conditions 4
– Dirichlet condition 1
– Neumann condition 1
Boundary operator 27

Critical angle 10
– longitudinal 98
– Rayleigh 98
– transversal 98

Density
– fluid 3
– solid 3
Diagram of diffraction 2, 97, 100
– wedge of 120° 102, 107
– wedge of 150° 101, 105
– wedge of 25° 103
– wedge of 45° 102, 116
– wedge of 60° 102, 113
– wedge of 90° 102, 110
Diffraction of diagram
– plotted function 25
Displacement 3

Elasticity
– coupled system elasticity/fluid 60
– Dirichlet boundary problem 60
– Neumann boundary problem 60
– operator 6
– symbol 13
Equations
– dimensionless 4
– solid/fluid 4, 33

Fredholm operator 68
Function
– complex cosine function 15
– tempered distribution 13
Functional space
– class $\mathcal{A}$ 13, 27
– Hilbert space 68
– space H^+ 41
– space $\mathcal{H}$ 41

Galerkin method 2, 10
– basis 87, 98
– collocation points 79, 89, 98

Hardy inequality 42
Hilbert projector 42, 58

Impedance 1
Integration contour
– Γ_0 22, 49
– γ_ε 20

Korn inequality 60, 71

Lamé
– parameters of 3

Layer potentials 7

Matrix operator
- DM^θ 27
- TM^θ 27
- dm 27, 33, 40
- tm, tm_L, tm_T, tm_0 27
- operator DM, TM 41
- transfer 55, 90

Maxwell equations 1
Mellin transform 70
Metaharmonic operator 1
Mode
- longitudinal 13
- transversal 13

Numerical accuracy 104

Outgoing solution
- see: Wave, outgoing 19

Paley-Wiener theorem 41
Poles
- generation of 10, 15
- recurrence relation 10

Potential 3

Rayleigh
- equation 39
- velocity 39, 40, 97
- wave 39

Recursive equation 11, 12, 90, 92, 98

Scholte-Stoneley
- characteristic equation 36
- incident wave 81
- velocity 39, 98

Singularities
- localization of 1
- propagation of 1

Spectral function 2, 7
- computation of the regular part X 85, 91
- computation of the singular part y 91
- decomposition 6, 9, 16, 50
- Existence and uniqueness 73
- functional equation 53, 79, 90
- holomorphic part X 52
- integral system 27
- meromorphic part y 9, 51
- numerical computation 90
- physical interpretation 9
- poles 9, 80
- recursive equation 11
- Structure 75
- structure 7, 85
- system 32

Stationnary phase theorem 10, 24
Symbol 2
- elasticity 13

Translation operator 11, 14, 44

Velocity
- longitudinal 3, 4, 18
- potential 3
- Scholte-Stoneley 3, 18
- transversal 3, 4, 18

Wave
- acoustic 1
- boundary problem (Dirichlet) 61
- boundary problem (Neumann) 61
- diffracted 7
- front 10
- incident 4, 80, 97, 100
- interface 1
- longitudinal 1, 10
- outgoing 2, 7, 12, 14, 16, 19, 83, 84
- Rayleigh 1, 39

– reflected 10
– Scholte-Stoneley 1, 35, 97, 100
– solid/fluid interface 33
– transmitted 10
– transversal 1, 10
– velocities 97

Wedge
– angle 2, 3
– boundary of 3
– conducting 1
– elastic 2
– face 6

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