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INDEX OF SYMBOLS

$B(x; r)$	4
$C(p)$	84
D_β	62
df	3
$d^+ f$	2
d_C	1
δ_C	39
$D(T)$	17
$\text{dom}(f)$	38
$\text{epi}(f)$	38
$G(T)$	26
K_λ	43
p_C	1
σ_A	39