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Standard notation

Here we list some standard notation that we use in the thesis. Non standard symbols which are defined in the thesis can be looked up in the Index.

Set theory

1. \mathbb{N} : the set of natural numbers $\{1, 2, 3, \dots\}$.
2. $\complement S$: complement of the set S .

Linear algebra

Throughout this thesis, unless otherwise specified, when considering a vector space, the underlying field is \mathbb{C} . In a matrix, blank entries are always to be understood to be zeros.

1. $\dim(\mathcal{V})$, where \mathcal{V} is a finite-dimensional vector space: The dimension of \mathcal{V} .
2. $\ker(T)$ and $\text{ran}(T)$, where $T : \mathcal{V}_1 \rightarrow \mathcal{V}_2$ is a linear transformation from the vector space \mathcal{V}_1 to the vector space \mathcal{V}_2 : the null space and the range of T , respectively.

Linear analysis

1. $\mathcal{L}(\mathcal{H}_1, \mathcal{H}_2)$, where \mathcal{H}_1 and \mathcal{H}_2 are Hilbert spaces: the Banach space of all bounded linear operators with the norm $\|T\| = \sup_{(0 \neq) x \in \mathcal{H}_1} \frac{\|Tx\|_{\mathcal{H}_2}}{\|x\|_{\mathcal{H}_1}}$.
2. $\rho(A)$, $\sigma(A)$, $\sigma_p(A)$, $\sigma_c(A)$, where $A : D(A) (\subset \mathcal{H}) \rightarrow \mathcal{H}$ is a closed linear operator, and \mathcal{H} is a Hilbert space: the resolvent set, spectrum, point spectrum and continuous spectrum, respectively, of A . If $D(A)$ is dense in \mathcal{H} , then A^* denotes the adjoint of A .

3. $\mathcal{C}(X_1, X_2)$, where X_1 and X_2 are topological spaces: the space of continuous maps from X_1 to X_2 . If X_2 is \mathbb{C} , we simply write $\mathcal{C}(X_1)$ instead of $\mathcal{C}(X_1, \mathbb{C})$.

4. $\ell_2(\mathbb{N})$ ($\ell_2(\mathbb{Z})$): the Hilbert space of square summable complex sequences indexed by \mathbb{N} (respectively \mathbb{Z}), with the inner product

$$\langle a, b \rangle_2 = \sum_{n=1}^{\infty} a_n \overline{b_n} \quad \left(\text{respectively } \langle a, b \rangle_2 = \sum_{n=-\infty}^{\infty} a_n \overline{b_n} \right).$$

5. ℓ_1 : the Banach space of summable complex sequences indexed by \mathbb{N} with the norm

$$\|a\|_1 = \sum_{n=1}^{\infty} |a_n|.$$

6. ℓ_{∞} : the Banach space of complex sequences indexed by \mathbb{N} and bounded in absolute value with the norm

$$\|a\|_{\infty} = \sup_{n \in \mathbb{N}} |a_n|.$$

7. $L_2^{\text{loc}}(\Omega, X)$, where $\Omega \subset \mathbb{R}$ is measurable and X is a Banach space:

$$\left\{ f : \Omega \rightarrow X \mid \begin{array}{l} f \text{ is Bochner-Lebesgue measurable and} \\ \int_K \|f(\omega)\|^2 d\omega < \infty \text{ for any compact set } K \subset \Omega \end{array} \right\}.$$

$L_p(\Omega, X)$, where $1 \leq p < \infty$:

$$\left\{ f : \Omega \rightarrow X \mid \begin{array}{l} f \text{ is Bochner-Lebesgue measurable and} \\ \int_{\Omega} \|f(\omega)\|^p d\omega < \infty \end{array} \right\}$$

with the norm $\|f\|_p = \left(\int_{\Omega} \|f(\omega)\|^p d\omega \right)^{\frac{1}{p}}$. If X is \mathbb{C} , we simply write $L_p(\Omega)$ instead of $L_p(\Omega, \mathbb{C})$.

If $p = 2$ and X is a Hilbert space, then it is also a Hilbert space with the inner product $\langle f, g \rangle_2 = \int_{\Omega} \langle f(\omega), g(\omega) \rangle d\omega$.

$L_{\infty}(\Omega, X)$:

$$\left\{ f : \Omega \rightarrow X \mid \begin{array}{l} f \text{ is Bochner-Lebesgue measurable and} \\ \text{esssup}_{\omega \in \Omega} \|f(\omega)\| < \infty \end{array} \right\}$$

with the norm $\|f\|_{\infty} = \text{esssup}_{\omega \in \Omega} \|f(\omega)\|$.

8. $H_{\infty}(X)$, where X is a Banach space:

$$\left\{ f : \mathbb{C}_0^+ \rightarrow X \mid f \text{ is analytic and } \sup_{s \in \mathbb{C}_0^+} \|f(s)\| < \infty \right\}.$$