Appendix A: Theorems and Lemmas

This appendix collects the theorems and lemmas needed for the proof of several theorems which have been established in the previous chapter. Each lemma will be denoted by **A.i-j**, where **i** represents the chapter's number and **j** the order of classification in the considered chapter. The references cited in these appendices are given the bibliography section of chapter 1.

Theorem *(Kall's theorem, 1966). For the linear equation*

(K1)

$$Ax = b \ \left(A \in R^{m\times n}, b \in R^m, n \geq m+1\right)$$

where the matrix A has the first m independent columns, written $A_1, ..., A_m$, to have a nonnegative solution $x \in R^n$ for all $b \in R^m$, it is necessary and sufficient that there exist

(K2)

$$\mu_j \geq 0, \ \lambda_j < 0$$

such that

(K3)

$$\sum_{j=m+1}^{n} \mu_j A_j = \sum_{i=1}^{m} \lambda_j A_j$$

where A_j is the j^{-th} column of the matrix A.

Remark 5 *In fact, the matrix A must have more than m columns, because it is impossible that some $b = b_0$ and also $-b_0$ (such situation is possible) be represented as a nonnegative linear combination the same m independent columns. Hence, $n \geq m+1$.*

Proof.

a) *Necessity.*
The vectors $A_1, ..., A_m$ are independent, then there exit a numbers βi $(i = 1, ..., m)$ such that
(K4)

$$b = \sum_{i=1}^{m} \beta_j A_j$$

Then, for any $x \in R^n$ $(x_i \geq 0)$ satisfying (K1) we can write:

$$Ax = \sum_{j=1}^{n} x_j A_j = \sum_{i=1}^{m} \beta_j A_j$$

or

$$\sum_{i=1}^{m} (\beta_i - x_i) A_i = \sum_{j=m+1}^{n} x_j A_j$$

It is clear that if we want to prove this statement for any $b \in R^m$ there exist such b that for the given matrix A the numbers β_j will be nonpositive and hence we can identify μ_j in (K3) with the coefficients $(\beta_i - x_i)$ of the left side of the last inequality and λ_j in (K3) with the coefficients in the right side, i.e.,

$$\begin{aligned} \mu_j &= \beta_i - x_i \\ \lambda_j &= x_j \end{aligned}$$

So, the necessity is proved.

2) *Sufficiency.*

Taking into account the presentation (K4) and the linear independence of the vectors $A_1, ..., A_m$ we can conclude the for any fixed b the corresponding values β_j are unique. If all of them are nonnegative then the equation (K1) has the solution

$$x_j = \begin{cases} \beta_j & \text{for } j = 1, ..., m \\ 0 & \text{for } j = m+1, ...n \end{cases}$$

Let us now suggest that one at least of the β_j is negative. Notice that the largest of the ratios β_j / λ_j $(j \leq m)$ is positive, if at least one of the β_j founded above is negative. Let us define γ_0 as follows

$$\gamma_0 := \max_j |\beta_j| / |\lambda_j|$$

If relation (K3) is true then the following relation

$$\sum_{j=m+1}^{n} \gamma_0 \mu_j A_j = \sum_{i=1}^{m} \gamma_0 \lambda_j A_j$$

is also true, and hence, taking into account the representation (K4) we derive:

$$\sum_{i=1}^{m} (\gamma_0 \lambda_j - \beta_j + \beta_j) A_j = \sum_{i=1}^{m} (\gamma_0 \lambda_j - \beta_j) A_j + b =$$

$$= \sum_{j=m+1}^{n} \gamma_0 \mu_j A_j$$

or

$$b = \sum_{i=1}^{m} (\gamma_0 |\lambda_j| + \beta_j) A_j + \sum_{j=m+1}^{n} \gamma_0 \mu_j A_j =$$

$$= \sum_{i=1}^{m} |\lambda_j| \left(\gamma_0 + \frac{\beta_j}{|\lambda_j|} \right) A_j + \sum_{j=m+1}^{n} \gamma_0 \mu_j A_j$$

Taking into account that

$$\gamma_j := \gamma_0 + \frac{\beta_j}{|\lambda_j|} \geq 0$$

Finally, we can write

$$b = \sum_{i=1}^{m} |\lambda_j| \gamma_j A_j + \sum_{j=m+1}^{n} \gamma_0 \mu_j A_j =$$

$$= \sum_{j=1}^{n} x_j A_j$$

with

$$x_j = \begin{cases} |\lambda_j| \gamma_j & \text{for } j = 1, ..., m \\ \gamma_0 \mu_j & \text{for } j = m+1, ..., n \end{cases} \geq 0$$

Theorem is proved. ■

Lemma A.3-1. *Let $\{\mathcal{F}_n\}$ be a sequence of σ-algebras and η_n, θ_n, λ_n, and ν_n are $\mathcal{F}_n$-measurable nonnegative random variables such that*

1. $\mathbf{E}(\eta_n) < \infty$
2. $\sum_{n=1}^{\infty} \mathbf{E}(\theta_n) < \infty$
3. $\sum_{n=1}^{\infty} \lambda_n \overset{a.s.}{=} \infty, \quad \sum_{n=1}^{\infty} \nu_n \overset{a.s.}{<} \infty$
4. $\mathbf{E}(\eta_{n+1}/\mathcal{F}_n) \overset{a.s.}{\leq} (1 - \lambda_{n+1} + \nu_{n+1})\eta_n + \theta_n$

Then,

$$\lim_{n \to \infty} \eta_n \overset{a.s.}{=} 0$$

Proof.

In view of the assumptions of the previous theorem and Robbins-Siegmund theorem (Robbins and Siegmund, 1971), it follows that

$$\eta_n \underset{n \to \infty}{\overset{a.s.}{\to}} \eta^*$$

and

$$\sum_{n=1}^{\infty} \lambda_{n+1} \eta_n \overset{a.s.}{<} \infty$$

As

$$\sum_{n=1}^{\infty} \lambda_n \overset{a.s.}{=} \infty$$

Then, a subsequence η_{n_k} which tends to zero with probability 1 exists.

Hence $\eta^* \overset{a.s.}{=} 0$. ■

Lemma A.3-2. *Let $\{u_n\}$ be a sequence of nonnegative random variables u_n measurable with respect to the σ-algebra $\mathcal{F}_n$, for all $n = 1, 2...,$. If*

1. $\mathbf{E}(u_n/\mathcal{F}_n)\ \forall\ n = 1, 2, ...$ *exists*
2. *the following inequality holds*

$$\mathbf{E}(u_{n+1}/\mathcal{F}_n) \leq u_n(1 - \alpha_n) + \beta_n$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences of non random variables such that

$$\alpha_n \in (0, 1], \quad \beta_n \geq 0$$

$$\sum_{n=1}^{\infty} \alpha_n = \infty, \sum_{n=1}^{\infty} \beta_n \nu_n < \infty$$

for some nonnegative sequence $\{\nu_n\}$ $(\nu_n > 0,\ n = 1, 2, ...)$

3. *the limit*

$$\lim_{n \to \infty} \frac{\nu_{n+1} - \nu_n}{\alpha_n \nu_n} := \mu < 1$$

exists,

then,

$$u_n = o_\omega\left(\frac{1}{\nu_n}\right) \overset{a.s.}{\to} 0$$

with probability 1 when $\nu_n \to \infty$.

Proof.

Let $\widetilde{u}_n$ be the sequence defined as

$$\widetilde{u}_n := u_n \nu_n$$

Then, using assumption (2), we obtain

$$\mathbf{E}(\widetilde{u}_{n+1}/\mathcal{F}_n) \overset{a.s.}{\leq} \widetilde{u}_n(1 - \alpha_n)\left(\frac{\nu_{n+1}}{\nu_n}\right) + \nu_{n+1}\beta_n =$$

$$= \widetilde{u}_n(1 - \alpha_n)\left(\frac{\nu_{n+1} - \nu_n}{\nu_n} - 1\right) + \nu_{n+1}\beta_n$$

Taking into account assumption (3), we derive

$$\mathbf{E}(\widetilde{u}_{n+1}/\mathcal{F}_n) \overset{a.s.}{\leq} \widetilde{u}_n \left[1 - \alpha_n(1 - \mu + o(1))\right] + \nu_{n+1}\beta_n$$

From this inequality and Robbins-Siegmund theorem (Robbins and Siegmund, 1971), we obtain

$$\widetilde{u}_n \overset{a.s.}{\to} 0$$

which is equivalent to

$$u_n = o_\omega(\frac{1}{\nu_n}) \overset{a.s.}{\to} 0$$

Lemma is proved. ■

Lemma A.3-3 (see ref. 31 of chapter 2). *For the sequence* $\{\gamma_n\}$ *where*

$$\gamma_n = \frac{\gamma}{n+a},$$

the following inequalities are fulfilled

a) for $\gamma \in (0,1)$ *and* $a > \gamma$, *then*

$$\left(\frac{1+a}{n+a-\gamma+1}\right)^\gamma \geq \prod_{k=1}^{n} (1-\gamma_k) \geq \left(\frac{a-\gamma}{n+a}\right)^\gamma$$

b) for $\gamma = 1$ *and* $a > 0$, *then*

$$\prod_{k=1}^{n} (1-\gamma_k) = \frac{a}{n+a}$$

Proof.

The proof of assertion a) is evident. Using the convexity property of the function $x \ln x$, it follows that

$$(x + \Delta x)\ln(x + \Delta x) - x\ln x \geq \Delta x(1 + x\ln x)$$

Taking into account this inequality, we obtain

$$\left(\frac{1+a}{n+a-\gamma+1}\right)^\gamma = \exp\left\{\int_1^{n+1} \ln\left(1 - \frac{\gamma}{n+a}\right) dx\right\} \geq$$

$$\geq \prod_{k=1}^{n} (1-\gamma(k)) = \exp\left\{\sum_{k=1}^{n} \ln\left(1 - \frac{\gamma}{k+a}\right) dx\right\} \geq$$

$$\geq \exp\left\{\int_0^{n+1} \ln\left(1 - \frac{\gamma}{x+a}\right) dx\right\} = \left(\frac{a-\gamma}{n+a}\right)^{\gamma}$$

Lemma is proved. ■

Lemma A.4-1. *Under assumptions (H1) and (H2)*

1) *the random variables $\Phi^j(\omega)$ $(j = 0, ..., m)$ are the partial limits of the sequences $\{\Phi_n^j\}$ $(j = 0, ..., m)$ for almost all $\omega \in \Omega$ if and only if they can be written in the following form*

$$\Phi^0(\omega) = \sum_{i=1}^{N} v_i^0 p(i) := V_0(p)$$

$$\Phi^j(\omega) = \sum_{i=1}^{N} v_i^j p(i) := V_j(p), \quad (j = 1, ..., m)$$

where the random vector $p = p(\omega) \in S^N$ (defined by equation (4.11)) is a limit point of the vector sequence $f_n = (f_n(1), ..., f_n(N))^T$ defined by (equation (4.13)).

2) *for almost all $\omega \in \Omega$*

$$\Phi^j(\omega) \in \left[\min_i v_i^j, \max_i v_i^j\right] \quad (j = 0, ..., m)$$

Proof.

Let us rewrite Φ_n^j (equation (4.1)) in the following form

$$\Phi_n^j = \sum_{i=1}^{N} f_n(i) \overline{\zeta}_n^j(u_n, \omega)$$

where

$$\overline{\zeta}_n^j(u_n, \omega) = \begin{cases} \dfrac{\sum_{t=1}^{n} \chi(u_t = u(i)) \zeta_n^j(u(i), \omega)}{\sum_{t=1}^{n} \chi(u_t = u(i))} & , \quad \sum_{t=1}^{n} \chi(u_t = u(i)) > 0 \\ 0 & , \quad \sum_{t=1}^{n} \chi(u_t = u(i)) = 0 \end{cases}$$

are the current average loss functions for $u_t = u(i)$ $(i = 1, ..., N)$. According to lemma A.12 (Najim and Poznyak, 1994) (see also Ash, 1972), for almost all

$$\omega \in \mathcal{B}_i := \left\{\omega \mid \sum_{t=1}^{n} \chi(u_t = u(i)) = \infty\right\}$$

we have

$$\lim_{n\to\infty} \overline{\zeta}_n^j(u_n,\omega) = v^i$$

For almost all $\omega \notin \mathcal{B}_i$, we have:

$$\lim_n \left|\overline{\zeta}_n^j(u_n,\omega)\right| < \infty$$

and

$$\lim_{n\to\infty} f_n(i) = 0$$

The vector f_n also belongs to the simplex S^N. It follows that any partial limit $\Phi^j(\omega)$ of a sequence $\{\Phi_n^j\}$ can be expressed in the following form:

$$\Phi^j(\omega) = \sum_{i=1}^{N} v_i^j p(i) := V_j(p), \quad (j = 0, ..., m)$$

where p is a partial limit of the sequence $\{f_n\}$ and consequently,

$$\Phi^j(\omega) \in \left[\min_i v_i^j, \max_i v_i^j\right] \quad (j = 0, ..., m)$$

Lemma is proved. ■

Lemma A.4-2. *Let us assume that*

1. *the control strategy $\{u_n\}$ is stationary, i.e.,*

$$\mathbf{P}\ \{\omega : u_n = u(i)|\mathcal{F}_{n-1}\} \overset{a.s.}{=} p(i)$$

2. *the random variables $\zeta_n^j(u(i),\omega)$ $(j = 0, ..., m; i = 1, ..., N)$ have stationary conditional mathematical expectation and uniformly bounded conditional second moment, i.e.,*

$$\mathbf{E}\left\{\zeta_n^j(u(i),\omega) \mid u_n = u(i) \wedge \mathcal{F}_{n-1}\right\} \overset{a.s.}{=} v_i^j$$

$$\sup_n \mathbf{E}\left\{\left(\zeta_n^j(u(i),\omega)\right)^2 \mid u_n = u(i) \wedge \mathcal{F}_{n-1}\right\} \overset{a.s.}{<} \infty$$

3.

$$\sum_{t=1}^{\infty} \chi(u_t = u(i)) \overset{a.s.}{\to} \infty$$

Then,

$$s_n^{ij} := \frac{\sum_{t=1}^{n} \zeta_t^j(u(i),\omega)\chi(u_t = u(i))}{\sum_{t=1}^{n} \chi(u_t = u(i))} - v_i^j \overset{a.s.}{\to} 0$$

Proof.

From the recurrent form of s_n^{ij}

$$s_n^{ij} = s_{n-1}^{ij}\left(1 - \frac{\chi(u_n = u(i))}{\sum\limits_{t=1}^{n}\chi(u_t = u(i))}\right) +$$

$$+\frac{\chi(u_n = u(i))}{\sum\limits_{t=1}^{n}\chi(u_t = u(i))}\left(\zeta_n^j(u(i),\omega) - v_i^j\right)$$

we derive

$$\mathbf{E}\left\{\left(s_n^{ij}\right)^2 \mid u_n = u(i) \wedge \mathcal{F}_{n-1}\right\} \overset{a.s.}{\leq} \left(s_{n-1}^{ij}\right)^2 \times$$

$$\times\left(1 - \frac{2\chi(u_n = u(i)) + o_\omega(1)}{\sum\limits_{t=1}^{n}\chi(u_t = u(i))}\right) + \frac{\chi(u_n = u(i))}{\left[\sum\limits_{t=1}^{n}\chi(u_t = u(i))\right]^2}\times$$

$$\times\mathbf{E}\left\{\left(\zeta_n^j(u(i),\omega)\chi(u_n = u(i)) - v_i^j\right)^2 \mid u_n = u(i) \wedge \mathcal{F}_{n-1}\right\} \leq$$

$$\leq \left(s_{n-1}^{ij}\right)^2\left(1 - \frac{\chi(u_n = u(i))\left(2 + o_\omega(1)\right)}{\sum\limits_{t=1}^{n}\chi(u_t = u(i))}\right) +$$

$$+\frac{\chi(u_n = u(i))}{\left[\sum\limits_{t=1}^{n}\chi(u_t = u(i))\right]^2}Const(\omega)$$

where $o_\omega(1)$ is a random sequence tending to zero with probability one, and $Const(\omega)$ is an almost surely bounded and positive random variable .

Observe that

$$\sum_{n=n_0}^{\infty}\frac{\chi(u_n = u(i))\left(2 + o_\omega(1)\right)}{\sum\limits_{t=1}^{n}\chi(u_t = u(i))} \overset{a.s.}{=} \infty$$

In view of Robbins-Siegmund theorem (Robbins and Siegmund, 1971) (or Lemma A.9 in Najim and Poznyak, 1994), the previous inequality leads to the desired result. Lemma is proved. ■

The lemma used in chapter 5 is stated and proved in the following.

Lemma A.5-1 *(the matrix version of the Abel's identity).*

$$\sum_{t=n_0}^{n} A_t B_t = A_n \sum_{t=n_0}^{n} B_t - \sum_{t=n_0}^{n} [A_t - A_{t-1}] \sum_{s=n_0}^{t-1} B_s$$

$$A_t \in R^{m \times k}, \quad B_t \in R^{k \times l}$$

Proof.

For $n = n_0$ we obtain:

$$A_{n_0} B_{n_0} = A_{n_0} B_{n_0} - [A_{n_0} - A_{n_0-1}] \sum_{s=n_0}^{n_0-1} B_s = A_{n_0} B_{n_0}$$

The sum $\sum_{s=n_0}^{n_0-1} B_s$ in the previous equality is zero by virtue of the fact that the upper limit of this sum is less than the lower limit.

We use induction. We note that the identity (Abel's identity) is true for n_0. We assume that it is true for n and prove that it is true for $n+1$:

$$\sum_{t=n_0}^{n+1} A_t B_t = \sum_{t=n_0}^{n} A_t B_t + A_{n+1} B_{n+1} =$$

$$= A_n \sum_{t=n_0}^{n} B_t - \sum_{t=n_0}^{n} [A_t - A_{t-1}] \sum_{s=n_0}^{t-1} B_s + A_{n+1} B_{n+1} =$$

$$= \left(A_{n+1} \sum_{t=n_0}^{n} B_t + A_{n+1} B_{n+1} \right) -$$

$$- \left((A_{n+1} - A_n) \sum_{t=n_0}^{n} B_t + \sum_{t=n_0}^{n} [A_t - A_{t-1}] \sum_{s=n_0}^{t-1} B_s \right) =$$

$$= A_{n+1} \sum_{t=n_0}^{n+1} B_t - \sum_{t=n_0}^{n+1} [A_t - A_{t-1}] \sum_{s=n_0}^{t-1} B_s$$

The identity (Abel's identy) is proved. ■

Appendix B: **Stochastic Processes**

In this appendix we shall review the important definitions and some properties concerning stochastic processes.

A stochastic process, $\{x_n, n \in N\}$ is a collection (family) of random variables indexed by a real parameter n and defined on a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ where Ω is the space of elementary events ω, $\mathcal{F}$ the basic σ-algebra and $\mathbf{P}$ the probability measure. A σ-algebra $\mathcal{F}$ is a set of subsets of Ω (collection of subsets). $\mathcal{F}(x_n)$ denotes the σ-algebra generated by the set of random variables x_n. The σ-algebra represents the knowledge about the process at time n. A family $\mathcal{F} = \{\mathcal{F}_n, n \geq 0\}$ of σ-algebras satisfy the standard conditions $\mathcal{F}_s \leq \mathcal{F}_n \leq \mathcal{F}$ for $s \leq n$, $\mathcal{F}_0$ is suggested by sets of measure zero of $\mathcal{F}$, and $\mathcal{F}_n = \bigcap_{s \geq n} \mathcal{F}_s$.

Let $\{x_n\}$ be a sequence of random variables with distribution function $\{F_n\}$ we say that:

Definition 7 *$\{x_n\}$ converges in distribution (law) to a random variable with distribution function F if the sequence $\{F_n\}$ converges to F.*

This is written

$$x_n \overset{law}{\longrightarrow} x$$

Definition 8 *$\{x_n\}$ converges in probability to a random variable x if given ε, $\delta > 0$, then there exists $n_0(\varepsilon, \delta)$ such that $\forall n > n_0$ $P(| x_n - x |> \varepsilon) < \delta$*

This is written

$$x_n \overset{prob}{\longrightarrow} x$$

Definition 9 *$\{x_n\}$ converges almost surely (with probability 1) to a random variable x if given ε, $\delta > 0$, then there exists $n_0(\varepsilon, \delta)$ such that $\forall n > n_0$. $P(| x_n - x |< \varepsilon \; \forall n > n_0) > 1 - \delta$*

This is written

$$x_n \overset{a.s.}{\longrightarrow} x$$

Definition 10 *$\{x_n\}$ converges in quadratic mean to a random variable x if*

$$\lim_{n \to \infty} \mathbf{E}\left[(x_n - x)^T (x_n - x)\right] = 0$$

This is written

$$x_n \stackrel{q.m.}{\rightarrow} x$$

The relationships between these convergence concepts are summarized in the following

1. convergence in probability implies convergence in law.
2. convergence in quadratic mean implies convergence in probability.
3. convergence almost surely implies convergence in probability.

In general, the converse of these statements is false.

Stochastic processes as martingales have extensive applications in stochastic problems. They arise naturally whenever one needs to consider mathematical expectations with respect to increasing information patterns. They will be used to state several theoretical results concerning the convergence and the convergence rate of learning systems.

Definition 11 *A sequence of random variables $\{x_n\}$ is said to be adapted to a the sequence of increasing σ-algebras $\{\mathcal{F}_n\}$ if x_n is $\mathcal{F}_n$ measurable for every n.*

Definition 12 *A stochastic process $\{x_n\}$ is a martingale if*

$$E\{| x_n |\} \stackrel{a.s.}{<} \infty$$

and

$$E\{x_{n+1}/\mathcal{F}_n\} \stackrel{a.s.}{=} x_n$$

Definition 13 *A stochastic process $\{x_n\}$ is a supermartingale if*

$$E\{x_{n+1}/\mathcal{F}_n\} \stackrel{a.s.}{\leq} x_n$$

Definition 14 *A stochastic process $\{x_n\}$ is a submartingale if*

$$E\{x_{n+1}/\mathcal{F}_n\} \stackrel{a.s.}{\geq} x_n$$

The following theorems are useful for convergence analysis.

Theorem *(Doob, 1953). Let* $\{x_n, \mathcal{F}_n\}$ *be a nonnegative* $\left(x_n \overset{a.s.}{\geq} 0\right)$ *supermartingale such that*

$$\sup_n E\{x_n\} < \infty.$$

Then there exists a random variable x *(defined on the same probability space) such that*

$$E\{x\} < \infty, \qquad x_n \underset{n\to\infty}{\to} x \ (a.s.).$$

Theorem *(Robbins and Siegmund, 1971). Let* $\{\mathcal{F}_n\}$ *be a sequence of* $\sigma-$*algebras and* x_n, α_n, β_n *and* ξ_n *are* $\mathcal{F}_n$*-measurable nonnegative random variables such that for all* $n = 1, 2, ...$ *there exists* $E\{x_{n+1}/\mathcal{F}_n\}$ *and the following inequality is verified*

$$E\{x_{n+1}/\mathcal{F}_n\} \leq x_n(1+\alpha_n) + \beta_n - \xi_n$$

with probability one.
Then, for all $\omega \in \Omega_0$ *where*

$$\Omega_0 := \left\{\omega \in \Omega | \sum_{n=1}^{\infty} \alpha_n < \infty, \ \sum_{n=1}^{\infty} \beta_n < \infty\right\}$$

the limit

$$\lim_{n\to\infty} x_n = x^*(\omega)$$

exists, and the sum

$$\sum_{n=1}^{\infty} \xi_n < \infty$$

converges.

Since the literature on stochastic processes is extensive we refer the reader to the books written respectively by Doob (1953) and Neveu (1975).

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