## Appendix : The Geometric Setting of Gauge Theories.

The characteristic feature of gauge theories is the fact that at each point $p$ of the space-time manifold $M$ we have an internal symmetry space $I$. $I$ is either a Lie group $G$, the gauge group (if we are dealing with the gauge field itself), or a vector space on which $G$ is acting (if we are dealing with matter fields). So if we consider an open neighborhood $U$ of a point $p \in M$, the space $B$ on which the fields live looks like a direct product $U \times I$. This local direct product decomposition is called a local trivialization or in the language of physics a choice of a gauge. Different patches $U$, $U'$ require a transition function $g_{U,U'}$ that relates the gauge choices in $U$ and $U'$ whenever $U \cap U' \neq \phi$ : if $p \in U \cap U'$ we consider $(p,f) \in U \times I$ and $(p,f') \in U' \times I$ to be the same point in $B$ provided $f$ is related to $f'$ by the action of $g_{U,U'}$. A patching (i.e. a covering by open sets) of $M$ together with all transition functions defines $B$ as a fiber bundle; $I$ is called the fiber, $M$ is called the base manifold. If $I = G$ we speak of a principal bundle, if $I$ is a vector space we speak of an associated vector bundle. If $M$ is not contractible, for instance if it has the topology of a sphere or a torus, $B$ may turn out not to be homeomorphic to $M \times I$ ; in this case we say that $B$ is nontrivial. In our context this is relevant for the case of the so-called (anti-) periodic boundary conditions corresponding to a torus as the base manifold $M$.

The concept of a gauge field corresponds geometrically to the concept of a connection in a principal bundle. A connection gives a prescription in which direction to move in the bundle $B$ when a certain direction is given in the base space $M$, or put differently : A connection is a prescription to lift curves from $M$ into $B$ provided a starting point in $B$ has been chosen. An equivalent description is the following: The tangent space of a point in $B$ splits into a direct sum of the tangent space to the fiber $I$ (the vertical directions) and its algebraic complement (the horizontal directions) which is isomorphic to the tangent space of a point in $M$ ; a connection is just a (smooth) choice of horizontal subspace for each point. This horizontal subspace can be obtained as the null space of a Lie algebra valued 1-form $\omega$, the connection form. In a local trivialization $\omega$ is determined by a Lie algebra valued 1- form $A = \Sigma A_\mu dx^\mu$ on $M$ through the equation $\omega = g^{-1}Ag+g^{-1}dg$ ($g \in I = G$ ; more precisely $g^{-1}Ag+g^{-1}dg$ is the pullback of $\omega$ with respect to the local trivialization $\varphi : U \times I \to B$ , $U \subset M$ ). The components $A_\mu$ are known as the Yang-Mills vector potential in the language of physics.

Normally a closed curve $C$ in $M$ will not lift to a closed curve in $B$. The endpoint will however lie on the same fiber as the starting point $\tilde{p}_o$ and thereby define an element $g_{\tilde{p}_o}(C)$ of $G$. $g_{\tilde{p}_o}(C)$ is called the holonomy operator corresponding to $C$ and $\tilde{p}_o$. If a local trivialization (gauge) has been chosen such that $\tilde{p}_o = (p_o, \mathbb{1})$ the suggestive physicist's notation

$$g_{\tilde{p}_o}(C) = P \exp \int_C A$$

(where P stands for path ordering) can be used.

If $g_{\tilde{p}_o}(C)$ is not always the identity $\mathbb{1}$ we call the connection nontrivial; if this is the case for a contractible C we say the connection possesses <u>curvature</u>. The <u>curvature form</u> is a (horizontal) 2-form with values in the Lie algebra on B ; in a local trivialization it is determined by a Lie algebra valued 2-form F on M :

$$F = dA + \frac{1}{2}[A,A] .$$

The components of F are the Yang-Mills <u>field strength tensor</u>. If S(C) is a smooth surface bordered by C

$$\exp \int_{S(C)} F \cong g_{\tilde{p}_o}(C)$$

in leading order in $|S(C)|$ . $g_{\tilde{p}_o}(C) = \mathbb{1} \in G$ for all C implies $F = 0$ and triviality of the principal bundle; on the other hand $F = 0$ implies $g_{\tilde{p}_o}(C) = \mathbb{1}$ for all contractible closed curves C .

Matter fields are given as <u>sections</u> of vector or spinor bundles. A section is simply a smooth assignment of a point $\tilde{p}$ in B to each $p \in M$ such that in a local trivialization $\tilde{p} = (p,f)$ . So locally a section is just a function from M to $\mathcal{I}$ .

An important concept is the notion of topological charge density which is given by the mathematical concept of Chern classes. In a simple minded way the Chern classes $c_n$ are defined by [I,90]

$$\det(1 + \frac{\lambda}{2\pi} F) = \Sigma \lambda^n c_n$$

so

$$c_1 = \frac{1}{2\pi} \text{tr } F , \quad c_2 = \frac{1}{8\pi^2} \text{tr } F \wedge F .$$

It can be shown that $\int_{M_{2n}} c_n$ is always an integer if $M_{2n}$ is a compact 2n-dimensional submanifold of M without boundary (i.e. a <u>2n-cycle</u>). $\int_{M_{2n}} c_n$ is called <u>Chern number</u> or topological charge.

The concepts of principal bundles, connection and curvature can easily be visualized by the following simple "classical" example : Consider a ball that is allowed to roll on a surface M ; assume that there are some patterns on the ball so its orientation can be observed. Its configuration space is locally just the direct product of its position space M and the space of orientations. The space of orientations may be taken as the space of orthonormal 3-frames fixed to the ball and can be identified with SO(3) . So the configuration space may be viewed as a principal bundle B with fiber SO(3) and base M .

A connection is now given by the "rolling" constraint : Moving the ball along a given curve will change its orientation in a definite way. This connection is non-trivial; the curvature F corresponds to infinitesimal rotations about the axis determined by the ball's center and the point of contact with M . For a flat surface and a "natural" choice of gauge

$$A = \frac{1}{R}(L_1 dx_2 - L_2 dx_1) \ ; \quad F = -\frac{1}{R^2} L_3 dx_1 dx_2$$

where $L_1$, $L_2$ , $L_3$ are standard skew adjoint SO(3) generators and R is the ball's radius. We leave it to the reader to work out various holonomy operators in this example; unfortunately this example does not allow for non-vanishing Chern numbers.

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The following nonstandard abbreviations are used :

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JMP = Journal of Mathematical Physics

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