APPENDIX

NEGATIVE SOLUTION TO POST'S PROBLEM

It is fair to say that the study of Post's problem, in the general setting of recursion-theoretic structures, provided a powerful thrust to the impressive development of higher recursion theory in the past thirty years. Many would even argue that it is the principal guiding force. In each instance where a recursion theory on a structure is introduced, the question of the existence of two incomparable r.e. sets over the structure naturally arises. The implications of a positive solution (if exists) are far reaching. On the one hand the technique used in the construction would provide a deeper insight into the inherent properties of the structure under study. The use or the absence of the priority method in the proof becomes especially significant. On the other hand, the result would lead immediately to a long list of questions related to the structure theory of r.e. degrees, from splitting, to density and beyond.

Thus the viability of a particular notion of reducibility depends on whether, based on that notion, Post's problem has a positive solution. It was for this reason that the relation 'hyperarithmetical in' was rejected as a proper generalization of 'recursive in' for ω_1^{CK}-recursion theory (on the reals), since every Π_1^1 set of natural numbers is either complete or hyper-arithmetical. This eventually led to the discovery of the notion of admissible ordinals, where positive solutions to Post's problem were obtained. Indeed a positive solution to Post's problem exists on an admissible set A whenever there is a 'nice' recursive well-ordering of A. The subsequent introduction of recursion theory on inadmissible ordinals, where incomparable r.e. degrees were obtained for some of them (S. Friedman [1980]), showed in fact that the assumption of admissibility on a structure is not needed to answer Post's problem positively. As it turned out, neither is it sufficient. The first result along this line was obtained by Simpson ([1974b]), who showed that under the Axiom of Determinacy, the smallest admissible set M containing the reals R solves Post's problem negatively, i.e. every $\Sigma(M)$ set is either $\Delta(M)$ or complete. The hypothesis of determinacy was later removed by Harrington (unpublished) using a difficult forcing argument. We present here Harrington's proof following S. Friedman [1981a].

THEOREM There exists an admissible set A for which Post's problem has a negative solution.

Proof. Let C be a complete $\Sigma_1(A)$ set. We will show that for every $B \subseteq A$ which is $\Sigma_1(A)$, either B is $\Delta_1(A)$ or there is a $\Sigma_1(A)$ predicate $\phi(x,y)$ such that for all x in A

$$(1) \quad x \notin C \quad \leftrightarrow \quad (\exists y)(\phi(x,y) \;\&\; y \notin B).$$

The structure A we construct shall be a generic extension of L_{ω_2}. To do this we state without proof the following result of Jensen and Solovay:

LEMMA 1 There is a Σ_1 formula ψ such that for all $\beta < \omega_2$, and functions $f\colon \beta \to L_{\omega_2}$, there is an admissible set A_f such that

$$(2) \quad A_f \cap \mathrm{ORD} = \omega_2$$

$$(3) \quad A_f = L_{\omega_2}[\langle w_i \mid i < \beta\rangle], \text{ where } w_i \text{ is a set of witnesses for } \psi(x),\ x \in f(i).$$

$$(4) \quad A_f \models \psi(x) \quad \leftrightarrow \quad (\exists\, i)(x \in f(i))$$
$$\leftrightarrow \quad (\exists\, z \varepsilon w_i)(A_f \models \psi'(z,x), \text{ where}$$

$\psi(x)$ is $(\exists z)\psi'(z,x)$, and ψ' is $\Delta_0(A)$.

Unless otherwise indicated, we let $\alpha = \omega_2$. For each $\beta < \alpha$, let $\psi_\beta(x)$ be $\psi(<\beta,x>)$. The association $\psi_\beta \leftrightarrow \psi(<\beta,x>)$ is clearly $\Delta_0(L_\alpha)$. Let $C_\beta = <w_i | i < \beta>$.

For each Σ_1 formula $\theta(t)$, we have the following two types of requirement:

$$(I)_\beta \qquad (\forall x)[\sim\theta(x) \leftrightarrow \psi_\beta(x)].$$

The objective here is to make θ 'recursive' by proving its complement to be Σ_1. To achieve this, we introduce an operator O_β such that $O_\beta(A) \subseteq A$, and O_β is monotone, i.e. $A_1 \subset A_2$ implies that $O_\beta(A_1) \subseteq O_\beta(A_2)$. We think of O_β as a guess at $\sim\theta$.

$$(II)_\beta \quad (\forall x)[x \not\in C \leftrightarrow (\exists y)(\psi_\beta(<x,y>) \ \& \sim\theta(y)].$$

In this case we ensure that if $\psi_\beta(<x,y>)$ holds, then either y is in (the set defined by) θ or x is not in (or 'forced' to be out of) C. This would force ψ to be complete.

We introduce the following set of forcing conditions:

$$\mathbb{P} = \{<R,A> | (\exists\beta<\alpha)[A = L_\alpha[C_\beta] \ \& \ R = <(X)_\gamma | X = I \text{ or } II \ \& \ \gamma < \beta>\}.$$

Elements of $\mathbb{P}$ are sometimes denoted p. An ordering $\leq$ will be defined on $\mathbb{P}$ so that $<R',A'> \leq <R,A>$ if and only if $R \subseteq R'$, $A \subseteq A'$ and

(1) If $(I)_\beta \in R$, and $A' \models \psi_\beta(x)$, $A \not\models \psi_\beta(x)$, then $x \in O_\beta(A')$.

(2) If $(II)_\beta \in R$, $A' \models \psi_\beta(<x,y>)$, $A \not\models \psi_\beta(<x,y>)$, then either $A' \models \theta(y)$ or $<R',A'> \Vdash \sim C(x)$.

There is an obvious circularity inherent in the definition of

$\leq$ with regard to (2) above, namely $\leq$ is defined in terms of $||-$, which is on the other hand supposed to be defined in terms of $\leq$. To resolve this we introduce the following notion:

DEFINITION Let $\langle R,A\rangle$, $\langle R',A'\rangle$ be fixed. Then $\langle R',A'\rangle$ is a safe extension of $\langle R,A\rangle$, written $\langle R',A'\rangle \leq_S \langle R,A\rangle$ if and only if $R \subseteq R'$, $A \subseteq A'$ and

(3) If $(I)_\beta \in R$, and $A' \models \psi_\beta(x)$, $A \not\models \psi_\beta(x)$, then $x \in O_\beta(A')$.

(4) If $(II)_\beta \in R$, $A' \models \psi_\beta(\langle x,y\rangle)$ and $A \not\models \psi_\beta(\langle x,y\rangle)$, then $A' \models \theta(y)$.

If $\langle R',A'\rangle$ is a safe extension of $\langle R,A\rangle$, we also say that A' is safe over $\langle R,A\rangle$.

We now define the notion of forcing for Σ_1 and Π_1 formulas: If ϕ is Σ_1, let

(5) $\langle R,A\rangle ||-_S \phi(x) \quad \leftrightarrow \quad A \models \phi(x)$

(6) $\langle R,A\rangle ||-_S \sim\phi(x) \quad \leftrightarrow \quad A' \models \sim\phi(x)$ for all A' which is safe over $\langle R,A\rangle$.

The relation $\leq$ is now defined to be

$\langle R',A'\rangle \leq \langle R,A\rangle \quad \leftrightarrow$

(7) $R \subseteq R'$ and $A \subseteq A'$.

(8) If $(I)_\beta \in R$, $A' \models \psi_\beta(x)$ and $A \not\models \psi_\beta(x)$, then $x \in O_\beta(A')$.

(9) If $(II)_\beta \in R$, $A' \models \psi_\beta(\langle x,y\rangle)$ and $A \not\models \psi_\beta(\langle x,y\rangle)$, then either $A' \models \theta(y)$ or $\langle R',A'\rangle ||-_S \sim C(x)$.

(10) There is a $B \supseteq A'$ such that B is safe over $\langle R,A\rangle$.

The definition of forcing now proceeds as follows: Let ϕ be Σ_1. Then

$$(11)\quad \langle R,A\rangle \Vdash \phi(x) \quad \leftrightarrow \quad A \models \phi(x)$$

$$(12)\quad \langle R,A\rangle \Vdash \sim\phi(x) \quad \leftrightarrow \quad \text{for all } \langle R',A'\rangle \leqslant \langle R,A\rangle,\ \langle R',A'\rangle \nVdash \phi(x).$$

We first discuss some basic properties of this notion of forcing.

LEMMA 2 If ϕ is either Σ_1 or Π_1, then $\langle R,A\rangle \Vdash \phi(x)$ if and only if $\langle R,A\rangle \Vdash_S \phi(x)$.

Proof. Suppose that ϕ is Σ_1. Then

$$\begin{aligned} \langle R,A\rangle \Vdash \phi(x) \quad &\leftrightarrow \quad A \models \phi(x) \\ &\leftrightarrow \quad \langle R,A\rangle \Vdash_S \phi(x). \end{aligned}$$

If ϕ is $\sim\zeta$, where ζ is Σ_1, one concludes from the fact that $\leqslant_S$ is extended by $\leqslant$ that

$$\langle R,A\rangle \Vdash \phi(x) \quad \rightarrow \langle R,A\rangle \Vdash_S \phi(x).$$

Thus suppose that $\langle R,A\rangle \Vdash_S \phi(x)$ but $\langle R,A\rangle \nVdash \phi(x)$. Then there is a pair $\langle R_1,A_1\rangle \leqslant \langle R,A\rangle$ such that $\langle R_1,A_1\rangle \Vdash \zeta(x)$. This means, by the definition of $\Vdash$, that $A_1 \models \zeta(x)$. Since Σ_1 formulas persist upwards, this implies that there is a B which is safe over $\langle R,A\rangle$ such that $B \models \zeta(x)$. We then have $\langle R,B\rangle \Vdash_S \zeta(x)$. But as $\langle R,B\rangle \leqslant_S \langle R,A\rangle$, we have $\langle R,B\rangle \Vdash_S \sim\zeta(x)$. This is a contradiction.

The proof of the next two lemmas are left to the reader.

LEMMA 3 (a) $\leqslant$ is transitive.

(b) $\langle \mathbf{P}, \leqslant \rangle$ is ω_1-closed, i.e. if $\langle R_\gamma, A_\gamma \rangle_{\gamma < \omega_1}$ is a sequence of compatible elements in $\mathbf{P}$, then $\bigcup \langle R_\gamma, A_\gamma \rangle$, $\gamma < \omega_1$, is also an element of $\mathbf{P}$.

LEMMA 4 If G is $\mathbf{P}$-generic, and ϕ is either Σ_1 or Π_1, then $L_\alpha[G] \models \phi(x)$ if and only if there is a $p \in G$ such that $p \Vdash \phi(x)$.

Thus it follows that if θ is Σ_1, then in the structure $L_\alpha[G]$ we have

(13) $x \in \sim\theta$ (the set defined by $\sim\theta$) $\leftrightarrow$

$(\exists \langle R,A \rangle \varepsilon G)(\langle R,A \rangle \Vdash \sim\theta(x)) \quad \leftrightarrow$

$(\exists \langle R,A \rangle \varepsilon G)(\forall \langle R',A' \rangle \leqslant \langle R,A \rangle)(\langle R',A' \rangle \nVdash \theta(x)) \quad \leftrightarrow$

$(\exists \langle R,A \rangle \varepsilon G)(\forall \langle R',A' \rangle \leqslant \langle R,A \rangle)(A' \models \sim\theta(x)) \quad \leftrightarrow$

$(\exists \langle R,A \rangle \varepsilon G)(\forall A' \supseteq A)[A' \text{ safe over } \langle R,A \rangle \rightarrow A' \models \sim\theta(x)]$

Given a condition $\langle R,A \rangle$, we define as result of this the operator O_β (for requirement $(I)_\beta$) on extensions of $\langle R,A \rangle$ by

(14) $$O_\beta(B) = \{x \mid x \in B \ \& \ (\forall A')[A' \text{ safe over } \langle R,A \rangle \rightarrow A' \models \sim\theta(x)\}.$$

Observe that $O_\beta(B)$ is defined in terms of $\langle R,A \rangle$ and θ. Let G be generic. If we set

(15) $$O_\beta(G) = \bigcup \{O_\beta(A') \mid \langle R',A' \rangle \leqslant \langle R,A \rangle \ \& \ \langle R',A' \rangle \in G\},$$

then we have $\langle R,A \rangle$ in G implies $L_\alpha[G] \models (\forall x \varepsilon O_\beta(G))(\sim\theta(x))$. Now the intention here is to make $\sim\theta$ equivalent to ψ_β (i.e. to make θ recursive). This can be achieved by choosing a generic G such that ψ_β defines $O_\beta(G)$, and $O_\beta(G)$ is defined by $\sim\theta$. There is however a difficulty. Namely, it may happen that for some x, x is forced to be in $\sim\theta$, but is not an element of O_β. More specifically, for any $\langle R',A'\rangle \leq \langle R,A\rangle$, there is an $\langle R'',A''\rangle \leq \langle R',A'\rangle$ such that for some x in A", $\langle R'',A''\rangle$ forces $\sim\theta(x)$, but $x \notin O_\beta(A'')$ for the O_β which is defined over $\langle R',A'\rangle$. In other words, the following situation may arise:

$$(*) \quad (\forall\langle R',A'\rangle\leq\langle R,A\rangle)(\exists\langle R'',A''\rangle\leq\langle R',A'\rangle)(\exists x\varepsilon A'')(\exists B\supseteq A'')$$
$$[B \text{ is safe over } \langle R',A'\rangle \ \& \ B\models \theta(x) \ \& \ \langle R'',A''\rangle\Vdash \sim\theta(x)]$$

We will show that if $\sim(*)$ holds, then the attempt to make θ recursive will be successful. Thus suppose that $\sim(*)$ holds at $\langle R,A\rangle$. This means that

$$(16) \quad (\exists\langle R',A'\rangle\leq\langle R,A\rangle)(\forall\langle R'',A''\rangle\leq\langle R',A'\rangle)(\forall x\varepsilon A'')(\forall B)$$
$$[B \supseteq A'' \ \& \ B \text{ is safe over } \langle R',A'\rangle \ \& \ B\models \theta(x) \rightarrow \langle R'',A''\rangle\not\Vdash \sim\theta(x)].$$

Fix such a pair $\langle R',A'\rangle$. Let $\beta < \alpha$ be an ordinal not mentioned in $\langle R',A'\rangle$, and let $(I)_\beta$ be the corresponding requirement for θ, with operator O_β defined from $\langle R',A'\rangle$ and θ. Let $R'' = R'\cup\{(I)_\beta\}$.

<u>LEMMA 5</u> $\langle R'',A'\rangle\Vdash \theta$ is recursive.

Proof. It is sufficient to show that

$$\langle R'',A'\rangle\Vdash (\forall x)(\psi_\beta(x) \leftrightarrow \sim\theta(x)).$$

Let G be generic containing $\langle R'',A'\rangle$. Let x be in $O_\beta(G)$. Then there is an $\langle R_1,A_1\rangle \varepsilon$ G such that $\langle R_1,A_1\rangle \leq \langle R',A'\rangle$ and $x \varepsilon O_\beta(A_1)$. Thus for all $B\supseteq A_1$ which is safe over $\langle R',A'\rangle$, $B\models \sim\theta(x)$.

First suppose that $L_\alpha[G]\models \theta(x)$. Then there is an $\langle R_2,A_2\rangle \varepsilon$ G such that $\langle R_2,A_2\rangle\Vdash \theta(x)$. It follows that there is an

$\langle R_2,A_2\rangle \in G$ such that $\langle R_2,A_2\rangle \leq \langle R_1,A_1\rangle$ and $\langle R_2,A_2\rangle \Vdash \theta(x)$. Since $\langle R_2,A_2\rangle \leq \langle R_1,A_1\rangle$, for all $B \supseteq A_2$ which is safe over $\langle R',A'\rangle$, $B \models \sim\theta(x)$. Also, from the fact that $\langle R_2,A_2\rangle \Vdash \theta(x)$ we have $A_2 \models \theta(x)$ and so for all $B \supseteq A_2$, $B \models \theta(x)$. This of course is a contradiction. Hence we get

$$L_\alpha[G] \models (\forall x)[(\psi_\beta(x) \leftrightarrow x \in O_\beta(G)) \ \& \\ (x \in O_\beta(G) \rightarrow \sim\theta(x))].$$

To prove the converse, suppose that $L_\alpha[G] \models \sim\theta(x)$. Then there is an $\langle R_1,A_1\rangle \in G$ such that $\langle R_1,A_1\rangle \Vdash \sim\theta(x)$ with $\langle R_1,A_1\rangle \leq \langle R'',A'\rangle$, so that $\langle R_1,A_1\rangle \leq \langle R',A'\rangle$. Using $\sim(*)$, we conclude that for all $B \supset A_1$ such that B is safe over $\langle R',A'\rangle$, $B \models \sim\theta(x)$, i.e. $x \in O_\beta(A_1)$. This proves the lemma.

From now on we assume that $(*)$ holds at $\langle R,A\rangle$. In other words, for every $\langle R',A'\rangle \leq \langle R,A\rangle$, there is an $\langle R'',A''\rangle \leq \langle R',A'\rangle$ and an x in A'', a $B \supset A''$ which is safe over $\langle R',A'\rangle$ such that $B \models \theta(x)$ and $\langle R'',A''\rangle \Vdash \sim\theta(x)$. We will show that under this assumption, θ is complete. Let $\hat{R} = R \cup \{(II)_\beta\}$.

<u>Claim 1</u> $\langle \hat{R},A\rangle \Vdash (\forall x)[\sim C(x) \leftrightarrow (\exists y)(\psi_\beta(\langle x,y\rangle) \ \& \ \sim\theta(y))]$.

The claim is established in a number of steps. First of all, we have

$$(17) \quad \langle \hat{R},A\rangle \Vdash (\forall x)(\forall y)[\psi_\beta(\langle x,y\rangle) \ \& \sim\theta(y) \rightarrow \sim C(x)].$$

This fact is immediate and we leave its verification to the reader. Next let $\langle R',A'\rangle \leq \langle \hat{R},A\rangle$ and $x_0 \in A'$ be chosen such that $\langle R',A'\rangle \Vdash \sim C(x_0)$. The goal now is to select an $\langle R'',A''\rangle \leq \langle R',A'\rangle$ and a $y \varepsilon A''$ such that $\langle R'',A''\rangle \|\!- \sim\theta(y)$ & $\psi_\beta(\langle x_0,y\rangle)$. There is however some difficulty in attaining this goal. Let us elaborate. If we only choose a witness w to put it into A", to make $\langle R'',A''[w]\rangle \|\!- \psi_\beta(\langle x_0,y\rangle)$, there is the problem of ensuring that $\langle R'',A''[w]\rangle \leq \langle R',A'\rangle$ and $\langle R'',A''[w]\rangle \|\!- \sim\theta(y)$. This difficulty arose because we are considering requirements of type (II) rather than type (I) whose witnesses can be easily obtained. The idea then is to transform $(II)_\beta$ into a requirement of type (I), and in the process define O_β so that forcing is preserved. This is carried out as follows:

Given $\langle T,D\rangle \leq \langle R',A'\rangle$, let T* be the set obtained from T by changing $(II)_\beta$ to $(I)_\beta$ (for θ). Let

$$O_\beta{}^*(B) = \{\langle x,y\rangle \mid \langle x,y\rangle \varepsilon B \; \& \; (B \models \theta(y) \text{ or } [x = x_0 \; \& \text{ there exists } B' \supseteq B \text{ safe over } \langle R',A'\rangle \; \& \; B' \models \theta(y)])\}.$$

The proof of the next lemma is left to the reader.

<u>LEMMA 6</u> Let $\langle T,D\rangle \leq \langle R',A'\rangle$. Then

(a) If E is safe over $\langle T,D\rangle$, then E is safe over $\langle T^*,D\rangle$.

(b) If E is safe over $\langle R'^*,A'\rangle$, then there exists $F \supseteq E$ such that F is safe over $\langle R',A'\rangle$.

<u>COROLLARY 1</u> (a) If ϕ is Σ_1, then $\langle T,D\rangle \Vdash \phi$ if and only if $\langle T^*,D\rangle \Vdash \phi$.

(b) If ϕ is π_1, then $\langle T^*,D\rangle \Vdash \phi$ implies $\langle T,D\rangle \Vdash \phi$.

(c) If ϕ is π_1, then $\langle R',A'\rangle \Vdash \phi$ implies $\langle R'^*,A'\rangle \Vdash \phi$.

<u>COROLLARY 2</u> (*) holds at $\langle (R')^*,A'\rangle$.

Proof. Observe first of all that since (*) holds at $\langle R,A\rangle$, it holds at all extensions of $\langle R,A\rangle$. It follows that (*) holds at $\langle R',A'\rangle$, and so at $\langle (R')^*,A'\rangle$ as well by (a).

By (*) there is a pair $\langle (R'')^*,A''\rangle \leq \langle (R')^*,A'\rangle$, a y_0 in A", such that for some $D \supseteq A''$ which is safe over $\langle (R')^*,A'\rangle$, $D \models \theta(y_0)$ and $\langle (R'')^*,A''\rangle \Vdash \sim\theta(y_0)$. By Lemma 6(b) there is a $D' \supseteq D$ which is safe over $\langle R'',A''\rangle$, and since θ is Σ_1 we have $D' \models \theta(y_0)$. This implies that $\langle x_0,y_0\rangle \in O_\beta^*(A'')$. Suppose now that w is a witness to $\psi_\beta(\langle x_0,y_0\rangle)$ obtained generically (so that it will not interfere with other ψ_δ, for $\delta \neq \beta$), then we have $\langle (R'')^*,A''[w]\rangle \leq_s \langle (R'')^*,A''\rangle$. This implies that

$$\langle (R'')^*,A''[w]\rangle \Vdash \sim\theta(y_0) \ \& \ \psi_\beta(\langle x_0,y_0\rangle).$$

By Corollary 1, we get

$$\langle R'',A''[w]\rangle \Vdash \sim\theta(y_0) \ \& \ \psi_\beta(\langle x_0,y_0\rangle).$$

Claim 2. $\langle R'',A''[w]\rangle \leq \langle R',A'\rangle$.

If $\beta \neq \beta'$, one observes that the required facts hold for $\langle R''^*,A''[\omega]\rangle$, and so for $\langle R'',A''[\omega]\rangle$ as well by Corollary 1. If $\beta = \beta'$, we have by assumption $\langle R',A\rangle \Vdash \sim C(x_o)$, so that $\langle R'^*,A'\rangle \Vdash \sim C(x_o)$ by Corollary 1 (c). As $\langle R''^*,A''[\omega]\rangle \leq \langle R'^*,A'\rangle$, we have $\langle R''^*,A''[\omega]\rangle \Vdash \sim C(x_o)$. By Corollary 1 (b), we have $\langle R'',A''[\omega]\rangle \Vdash \sim C(x_o)$. This proves Claim 2 and completes the proof of the Theorem.

BIBLIOGRAPHY

C. T. Chong [1976], An α-finite injury method of the unbounded type, J. Symbolic Logic 41, 1-17

C. T. Chong [1979], Generic sets and minimal α-degrees, Trans. Amer. Math. Soc. 254, 157-169

C. T. Chong [1979a], Major subsets of α-recursively enumerable sets, Israel J. Math. 34, 106-114

C. T. Chong [1982], Global and local admissibility, in: Proc. Patras Logic Symposium (Ed. G. Metakides), North Holland

C. T. Chong [1983], Global and local admissibility: II. Major subsets and automorphisms, Annals Pure and Applied Logic 24, 99-111

C. T. Chong and S. D. Friedman [1983], Degree theory on $\aleph_\omega^L$, Annals Pure and Applied Logic 24, 87-97

C. T. Chong and M. Lerman [1976], Hyperhypersimple α-r.e. sets, Annals Math. Logic 9, 1-48

K. J. Devlin [1974], An introduction to the fine structure of the constructible hierarchy, in Generalized Recursion Theory (Ed. J. Fenstad et. al), North Holland

K. J. Devlin [1974a], Aspects of Constructibility, Lecture Notes in Math Vol. 354, Springer Verlag

G. Driscoll [1968], Metarecursively enumerable sets and their metadegrees, J. Symbolic Logic 33, 389-411

R. Epstein and D. Posner [1978], Diagonalization in degree constructions, J. Symbolic Logic 43, 280-283

R. M. Friedberg [1957], Two recursively enumerable sets of incomparable degrees of unsolvability, Proc. Nat. Acad. Sci. (USA) 43, 236-238

R. M. Friedberg [1957a], A criterion for completeness of degrees of unsolvability, J. Symbolic Logic 22, 159-160

R. M. Friedberg [1958], Three theorems on recursive enumeration: I. Decomposition, II. Maximal set, III. Enumeration without duplication, J. Symbolic Logic 23, 309-316

S. D. Friedman [1976], Recursion on Inadmissible Ordinals, Ph.D. Thesis, Massachusetts Institute of Technology

S. D. Friedman [1978], Negative solutions to Post's problem, I. in: Generalized Recursion Theory II (Ed. J. Fenstad et. al.), North Holland

S. D,. Friedman [1979], β-recursion theory, Trans. Amer. Math. Soc. 255, 173-200

S. D. Friedman [1981], Lecture Notes in Higher Recursion Theory, Massachusetts Institute of Technology (unpublished)

S. D. Friedman [1981a], Natural α-r.e. degrees, in: Logic Year 1979 - 1980 (Ed. M. Lerman et. al.), Lecture Notes in Math. Vol. 859, Springer Verlag

S. D. Friedman [1981b], Negative solutions to Post's problem, II., Annals Math. 113, 25-43

S. D. Friedman [1983], Some recent developments in higher recursion theory, J. Symbolic Logic 48, 629-642

S. D. Friedman [1984], Fine structure theory and its applications, in: Proc. Summer Research Institute in Recursion Theory, Proc. Symposia in Pure Math., Amer. Math. Soc., to appear

S. D. Friedman and G. E. Sacks [1977], Inadmissible recursion theory, Bull. Amer. Math. Soc. 83, 255-256

R. B. Jensen [1972], The fine structure of the constructible universe, Annals Math. Logic 4, 229-308

C. G. Jockusch [1981], Degrees of generic sets, in Proc. Leeds Logic Conference (Ed. F. R. Drake et. al.), Cambridge Univ. Press

S. C. Kleene [1938], On notation for ordinal numbers, J. Symbolic Logic 3, 150-155

G. Kreisel [1971], Some reasons for generalizing recursion theory, in: Logic Colloquium '69 (Ed. C. E. M. Yates et. al.), North Holland

G. Kreisel and G. E. Sacks [1965], Metarecursive sets, J. Symbolic Logic 30, 318-336

S. Kripke [1964], Transfinite recursion on admissible ordinals, I, II (abstracts), J. Symbolic Logic 29, 161-162

A. H. Lachlan [1966], Lower bounds for pairs of recursively enumerable degrees, Proc. London Math. Soc. 16, 537-569

A. H. Lachlan [1968], On the lattice of recursively enumerable sets, Trans. Amer. Math. Soc. 130, 1-37

A. H. Lachlan [1968a], The elementary theory of recursively enumerable sets, Duke Math. J. 35, 123-146

A. H. Lachlan [1976], A recursively enumerable degree which will not split over all lesser ones, Annals. Math. Logic 9, 337-366

A. Leggett [1974], Maximal α-r.e. sets and their complements, Annals Math. Logic 6, 293-357

A. Leggett and R. A. Shore [1976], Types of simple α-recursively enumerable sets, J. Symbolic Logic 41, 681-694

M. Lerman [1972], On suborderings of the α-recursively enumerable degrees, Annals Math. Logic 4, 369-392

M. Lerman [1973], Admissible ordinals and priority arguments, in: Cambridge Summer School on Math. Logic (Ed. A. R. D. Mathias et. al.), Lecture Notes in Math Vol. 337, Springer Verlag

M. Lerman [1974], Maximal α-r.e. sets, Trans. Amer. Math. Soc. 188, 341-386

M. Lerman [1976], Types of simple α-recursively enumerable sets, J. Symbolic Logic 41, 419-426

M. Lerman [1983], Degrees of Unsolvability, Perspectives in Math. Logic, Springer Verlag

M. Lerman and G. E. Sacks [1972], Some minimal pairs of α-recursively enumerable degrees, Annals Math. Logic 4, 415-442

M. Lerman and S. G. Simpson [1973], Maximal sets in α-recursion theory, Israel J. Math. 4, 236-247

A. Levy [1965], A hierarchy of formulas in set theory, Memoirs Amer. Math. Soc. Vol. 57, 1-76

W. Maass [1977], Minimal pairs and minimal degrees in higher recursion theory, Z. Math. Logik 18, 169-186

W. Maass [1978a], Inadmissibility, tame r.e. sets and the admissible collapse, Annals Math. Logic 13, 149-170

W. Maass [1978b], The uniform regular set theorem in α-recursion theory, J. Symbolic Logic 43, 270-279

W. Maass [1978c], High α-recursively enumerable degrees, in: Generalized Recursion Theory II (Ed. J. Fenstad et. al.), North Holland

W. Maass [1978d], Fine structure of the constructible universe in α and β recursion theory, in: Higher Set Theory (Ed. G. H. Müller et. al.), Lecture Notes in Math. Vol. 669, Springer Verlag

J. W. MacIntyre [1973], Minimal α-recursion theoretic degrees, J. Symbolic Logic 38, 18-28

A. A. Muchnik [1957], On the unsolvability of the problem of reducibility in the theory of algorithms (Russian), Doklady Akad. Nauk SSSR 108, 194-197

R. Platek [1966], Foundations of Recursion Theory, Ph.D. Thesis, Stanford Univ.

W. Richter [1968], Constructively accessible ordinal numbers, J. Symbolic Logic 33, 43-55

H. Rogers Jr. [1967], Theory of Recursive Functions and Effective Computability, McGraw-Hill

G. E. Sacks [1963a], Degrees of Unsolvability, Annals Math. Studies, Princeton Univ. Press

G. E. Sacks [1963b], On the degrees less than **0'**, Annals Math. 77, 211-231

G. E. Sacks [1964], The recursively enumerable degrees are dense, Annals Math. 80, 300-312

G. E. Sacks [1966a], Post's problem, admissible ordinals and regularity, Trans. Amer. Math. Soc. 124, 1-23

G. E. Sacks [1966b], Metarecursively enumerable sets and admissible ordinals, Bull. Amer. Math. Soc. 72, 59-64

G. E. Sacks [1967], Metarecursion theory, in Sets, Models and Recursion Theory (Ed. J. Crossley), North Holland

G. E. Sacks [1971], Forcing with perfect closed sets, in Axiomatic Set Theory, Proc. Symposia in Pure Math. Vol 13, Amer. Math. Soc.

G. E. Sacks [1982], E-recursion, 1982 Summer Research Institute in Recursion Theory, Amer. Math. Soc. (mimeographed notes)

G. E. Sacks and S. G. Simpson [1972], The α-finite injury method, Annals Math. Logic 4, 323-367

J. R. Shoenfield [1972], Degrees of Unsolvability, North Holland

R. A. Shore [1972], Minimal α-degrees, Annals Math. Logic 4, 393-414

R. A. Shore [1975a], Splitting an α-recursively enumerable set, Trans. Amer. Math. Soc. 204, 65-78

R. A. Shore [1975b], The irregular and non-hyperregular α-r.e. degrees, Israel J. Math. 22, 28-41

R. A. Shore [1976], The recursively enumerable α-r.e. degrees are dense, Annals Math. Logic 9, 123-155

R. A. Shore [1976a], On the jump of an α-recursively enumerable set, Trans. Amer. Math. Soc. 217, 351-363

R. A. Shore [1977], α-recursion theory, in: Handbook of Mathematical Logic (Ed. K. J. Barwise et. al.), North Holland

R. A. Shore [1978], Some more minimal pairs of α-r.e. degrees, Z. Math. Logik und Grund. Math. 24, 409-418

R. A. Shore [1978a], On the AE-sentences of α-recursion theory, in: Generalized Recursion Theory II (Ed. J. Fenstad et. al.), North Holland

J. Silver [1974], On the singular cardinals problem, in: Proc. Int. Congress Math. (Vancouver)

S. G. Simpson [1971], Recursion Theory on Admissible Ordinals, Ph.D. Thesis, Massachusetts Institute of Technology

S. G. Simpson [1974], Degree theory on admissible ordinals, in: Generalized Recursion Theory (Ed. J. Fenstad), North Holland

S. G. Simpson [1974a], Post's problem for admissible sets, ibid.

S. G. Simpson [Notes], Introduction to admissible recursion theory (mimeographed notes)

S. G. Simpson [1978], Short course on admissible recursion theory, in : Generalized Recursion Theory II (Ed. J. Fenstad), North Holland

R. I. Soare [1976], The infinite injury priority method, J. Symbolic Logic 41, 513-530

R. I. Soare [1984], Recursively Enumerable Sets and Degrees, Perspectives in Math. Logic, Springer Verlag

C. Spector [1956], On degrees of recursive unsolvability, Annals Math. 64, 581-592

J. Stillwell [1969], Reducibility in Generalized Recursion Theory, Ph.D. Thesis, Massachusetts Institute of Technology

J. Sukonick [1969], Lower Bounds for Pairs of Metarecursively Enumerable Degrees, Ph.D. Thesis, Massachusetts Institute of Technology

G. Takeuti [1960], On the recursive functions of ordinal numbers, J. Math. Soc. Japan 12, 119-128

G. Takeuti [1965], Recursive functions and arithmetical functions of ordinal numbers, in: Logic, Methodology and Philosophy of Science, North Holland

D. Yang [1984], On the embedding of α-recursive presentable lattices into α-recursive degrees below **0'**, J. Symbolic Logic (to appear)

C. E. M. Yates [1966], A minimal pair of recursively enumerable degrees, J. Symbolic Logic 31, 159-168

INDEX