

## BIBLIOGRAPHY

Some abbreviations which are not standard or self-explanatory:

- HB           for: Handbook of Mathematical Logic.  
              J. Barwise (editor), NHPC, Amsterdam, 1977.
- IPT          for: Intuitionism and Proof Theory. Proceedings of the  
              summer conference at Buffalo, N.Y., 1968.  
              A. Kino, J. Myhill, R.E. Vesley (editors)  
              Amsterdam-London, 1970.
- LMPS        for: Logic, Methodology and Philosophy of Science, III.  
              B. van Rootselaar, J.F. Staal (editors)  
              Amsterdam, 1968.
- LMN         for: Lecture Notes in Mathematics.
- SR          for: Stanford Report on the Foundations of Analysis.  
              (Mimeographed) Stanford, 1963.
- JSL         for: Journal of Symbolic Logic.
- NHPC        for: North-Holland Publishing Company.

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INDICES FOR THE INDIVIDUAL CHAPTERS. We decided to have separate indices for the individual chapters, as those may be more useful for the reader than a longish combined index. After all, the chapters II through VI are relatively independent and make use of some special notation. The basic framework is given in Chapter I; so the index for that chapter should be consulted for notations and definitions not listed in the other indices.

INDEX FOR CHAPTER I.

General notation and abbreviations.

$\alpha, \beta, \gamma, \dots$	2 0
$\mu, \nu, \rho, \sigma, \dots$	3 0
$i, j, k, \ell, m, n, \dots$	1 7, 2 5
$f, g, h, \dots$	3 8
$s, t, \dots$	2 5, 3 8
$x, y, z, \dots$	2 5
$A, B, \dots, F, G, \dots$	2 6, 3 8
$X, Y, Z, \dots$	1 7
$(f_j)_{j \in \mathbb{N}}$	2 5
$\langle \cdot, \cdot \rangle, ( )_0, ( )_1$	2 5
$\mathfrak{L}, \mathfrak{L}[P], \mathfrak{L}[P, Q]$	2 5, 2 6, 3 0
$\mathfrak{L}_1, \mathfrak{L}_\leftarrow$	2 9, 3 1
$\mathfrak{L}^2$	3 8
$\Sigma_k^i, \Pi_k^i, \Delta_k^i$	4 2
$\mathfrak{I}, I_{\mathfrak{I}}$	1 8
$\Gamma, I_\Gamma, I_\Gamma^\alpha$	1 9, 2 0
$I_\mu^{\mathfrak{U}}, I_\nu^{\mathfrak{U}}$	2 7, 3 0
$ n _\Gamma,  \Gamma $	2 1
$ ID_\mu(\mathfrak{K}) ,  ID_\leftarrow(\mathfrak{K}) $	3 7

Names of special formulas and axioms.

$AC_0$ , AC , DC	4 0
$\mathfrak{U}(P, x)$	2 7
$\mathfrak{U}(P, Q, x, y)$	3 0
$P^{\mathfrak{U}}_y x$ , $P^{\mathfrak{U}}_{< y} x$ , $\mathfrak{U}_y(P, x)$	3 1
$(P^{\mathfrak{U}.1})$ , $(P^{\mathfrak{U}.2})$	3 4
$(P^{\mathfrak{U}.1})_{\mu}$ , $(P^{\mathfrak{U}.2})_{\mu}$	3 5
CA	3 8
$TI_R(F)$	4 0
$WF_R$	4 0
$TI(\mu)$	6 4
$(TI)_{\mu}$	3 5
$TI^*$	2 8
Prog	2 8
$\perp$	2 5
$\mathfrak{G}$ , $\mathfrak{G}_p$	2 1, 2 7, 3 2
$W$ , $W_p$	2 7, 3 2, 6 9
$F \subseteq G$	2 8

Names of theories.

PRA , $PRA_{\epsilon_0}$	5 9, 6 4
Z , HA , $Z^{\infty}$	3 3, 6 3
$ID_1(\mathfrak{U})$ , $ID_1(\mathfrak{G})$ , $ID_1$	3 3, 3 4
$ID_{\mu}(\mathfrak{U})$ , $ID_{< \mu}$ , $ID_{<}$	3 5
$ID_{\alpha}$ , $ID_{< \alpha}$	3 5
(CA)	3 8
$(\mathfrak{F} - CA)$ , $(\mathfrak{F} - CA)^{\uparrow}$ , $(\mathfrak{F} - CA)^{-}$	3 8, 4 8
$(\Pi_1^1 - CA)_{< \mu}$ for $i = 0, 1$	4 6, 5 0,
$(RA_{\mu})$	5 7, 6 6
(BI) , $(BI_{pr})$	4 0, 4 3

Definitions.

accessible part	2 2, 2 3
bar induction	4 0, 4 3
choice principles	4 0
comprehension principle	3 8
double-negation translation (DNT)	5 5
fixed point	2 0
formula	
- arithmetic	4 2
- essentially prenex	4 2
- negative (Neg)	2 6
- positive (Pos)	2 7
- PR	3 9
- weak	5 0
graph principle (GP)	3 8
i.d. class	1 7
- generalized	3 0
inductive definition	
- accessibility	2 2
- elementary	2 5, 2 7
- iterated (elementary)	2 9, 3 1
Markov's rule	5 7
operator (form)	2 7
- definable	2 5, 3 2
- monotone	1 9
- positive	2 7, 3 0
- strictly positive	3 6
- superpositive	3 6
reflection principle (partial)	5 4, 5 9, 7 6
rule set	1 8
- deterministic	2 2, 2 3
truth definition (partial)	6 0
$\alpha$ - stage	2 0
$\bar{\varphi}$ - proof	2 1
$\omega$ - rule	6 3

INDEX FOR CHAPTER II.Notation and abbreviations.

$a, b, c, \dots$	8 1
$x, y, z, \dots$	8 1
$A, B, C, \dots$	8 1
$A^\mu, B^\mu, C^\mu, \dots$	9 1
$X, Y, Z, \dots$	8 1
$X^\mu, Y^\mu, Z^\mu, \dots$	9 1
$\kappa, \lambda, \mu, \nu, \dots$	8 6
$\mathbb{N}, 0, s_{\mathbb{N}}, p_{\mathbb{N}}$	8 1
$k, s, d, p, p_1, p_2$	8 1
$r_{\mathbb{N}}$	8 5
$j, i, c_n \quad (n < \omega)$	8 1
$=, \text{App}, e$	8 1
$\omega, \oplus, o, \exp_\omega$	8 6
$v_0, v_1, v_2$	1 0 9, 1 1 3
$j^<(a, b, h), j^<(a, h)$	8 7
$\mathfrak{L}(T_0)$	8 1
$\mathfrak{L}_\lambda$	9 1
$\mathfrak{L}^2$	1 0 0
$\mathfrak{L}(OT^0), \mathfrak{L}(OT^\omega), \mathfrak{L}(OT^{\mathbb{N}})$	1 0 9, 1 1 3
$\mathfrak{L}(OT_\infty^0), \mathfrak{L}(OT_\infty^\omega)$	1 1 9, 1 2 6
$L, A_L, L^\infty, A_L^\infty$	1 0 4, 1 2 0, 1 2 5
$\Gamma, \Delta, \dots$	1 0 0
$\Gamma[F]$	1 0 0
$QF, \mathbb{E}_1, \mathbb{V}_i$	1 0 2
$ F ,  \mathfrak{L} $	1 2 0

Names for special formulas and axioms.

App	8 3
(BR)	1 2 6
(CA <sub>λ</sub> )	9 2
D <sub>Y</sub>	8 4
(ECA)	8 3
$\mathfrak{F}$ -CA, $\langle \mathfrak{F}$ -CA $\rangle$	1 0 5
$\mathfrak{F}$ -AC, $\langle \mathfrak{F}$ -AC $\rangle$	1 0 5
Ind, $\langle \mathfrak{F}$ -Ind $\rangle$	1 0 4
(IG), (IG) $\uparrow$	8 3
(J), (JR)	8 3, 8 5
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