APPENDIX

A. INDUCED REPRESENTATIONS

It is our purpose here to provide a brief summary of the main ideas and results on induced representations which are fundamental to the subject matter of this entire treatise. The idea of forming induced representations -- that is, of inducing a representation of a group G from a representation of a subgroup H -- goes back to Frobenius in the last century. His work of course was in the case G finite. The possibility of forming induced representations for infinite groups was later explored (usually in primitive or ad hoc fashion) by many authors, most notably by the Russians Gelfand, Namiark, Graev, etc. The subject was finally givena firm footing by Mackey in the 1940's and early 1950's. It was a fundamental achievement, without which the theory of group representations might still be crawling around on its hands and knees.

We begin with the notion of a quasi-invariant measure. Let G be a locally compact group and suppose X is a right Borel G-space. That is, there is a Borel map $X \times G \to X$, $(x,g) \to x \cdot g$, such that:

$$x \cdot e = x, \quad \forall x \in X, \quad \text{and} \quad x \cdot (g_1 g_2) = (x \cdot g_1) \cdot g_2$$

or said otherwise $g \to (g\colon x \to x \cdot g)$ is an anti-homomorphism of G into the group of Borel automorphisms of X. Next suppose that X carries a σ finite positive Borel measure μ. We say that μ is *quasi-invariant* under the action of G if for every $g \in G$, the measures μ and $\mu \cdot g$ are equivalent. By $\mu \cdot g$ we mean the measure that assigns to a Borel set $E \subseteqq X$ the value $(\mu \cdot g)(E) = \mu(E \cdot g)$, $E \cdot g = \{x \cdot g\colon x \in E\}$. If μ is quasi-invariant, then there is a non-negative Borel function $\alpha(x,g)$ such that

$$\int_X f(x)d\mu(x) = \int_X f(x\cdot g)\alpha(x,g)d\mu(x), \qquad f \in L_1(X,\mu).$$

EXERCISE. Check that α must satisfy

$$\alpha(x,g_1g_2) = \alpha(x,g_1)\alpha(x\cdot g_1,g_2).$$

In such a case we can define a unitary representation U of G as follows: the Hilbert space is $L_2(X,\mu)$ and G acts via

$$U_g f(x) = f(x\cdot g)\alpha(x,g)^{\frac{1}{2}}, \qquad f \in L_2(X,\mu).$$

We leave it to the reader as a simple exercise to check that this defines a continuous unitary representation of G.

We now specialize the space X somewhat (in essence, we take the case G transitive in the preceding). Let H be a closed subgroup of the locally compact group G. The space $X = G/H$ of right cosets is then a right Borel G-space, $(Hg,g_1) \to Hgg_1$. We often write $\overline{x} = Hx$, $x \in G$. The point is that G/H carries a quasi-invariant measure. To see that we begin with a lemma.

LEMMA 1. *There exists a strictly positive continuous function* q *on* G *such that* $q(e) = 1$, $q(hx) = \Delta_H(h)\Delta_G(h)^{-1}q(x)$, $h \in H$, $x \in G$.

Proof. It is possible to find a Borel cross-section $s\colon G/H \to G$, $s(\overline{e}) = e$, and s takes compact sets of G/H to relatively compact sets in G (see Mackey [3]). Then define $q(hs(\overline{x})) = \Delta_H(h)\Delta_G(h)^{-1}$, $h \in H$, $\overline{x} \in G/H$. Since every $x \in G$ is written uniquely $x = hs(\overline{x})$, $h \in H$, we see that q is well-defined and Borel. To see that q can actually be chosen continuous see e.g. Bruhat [1].

Now for $f \in C_0(G)$, set $f'(\overline{x}) = \int_H f(hx)dh$, dh = right Haar measure.

EXERCISE. Show that the map $f \to f'$, $C_0(G) \to C_0(G/H)$ is surjective.

Then define a measure (which we write $\overline{dg}$) on G/H by the formula

$$\int_{G/H} f'(\overline{g})d\overline{g} = \int_G f(x)q(x)dx, \quad dx = \text{right Haar measure.}$$

Of course to justify that $d\overline{g}$ is well-defined one needs to show that: $f \in C_0(G)$, $\int_H f(hx)dh = 0$, $\forall x \in G \Rightarrow \int_G f(x)q(x)dx = 0$ -- for that, see Mackey [3].

THEOREM 2. $d\overline{g}$ *is quasi-invariant under* G. *Moreover any other quasi-invariant measure on* G/H *is equivalent to* $d\overline{g}$. *Finally we have the equation*

$$\int_{G/H} f(\overline{x})d\overline{x} = \int_{G/H} f(\overline{x}\cdot g)\left[\frac{q(xg)}{q(x)}\right] d\overline{x}, \qquad f \in C_0(G/H).$$

Proof. The equation of the theorem is easily derived from the equation which defines the measure $d\overline{x}$. The first statement follows immediately. The justification of the second statement may be found in Mackey [3].

EXERCISE. Show that G/H has an invariant measure if and only if $\Delta_G|_H = \Delta_H$.

In light of the previous results, it follows that the formula

$$U_g f(\overline{x}) = f(\overline{x}\cdot g)\left[\frac{q(xg)}{q(x)}\right]^{\frac{1}{2}} d\overline{x}, \qquad f \in L_2(G/H)$$

defines a unitary representation of G. In fact this is the representation of G induced by the trivial representation of H. However we want to induce representations other than the trivial one. For that it is customary and convenient to realize the Hilbert spaces as function spaces on G rather than on G/H.

DEFINITION. Let γ be a representation of H on $\mathcal{H}_\gamma$. Consider the space $\mathcal{H}(\gamma)$ of all functions from G to $\mathcal{H}_\gamma$ satisfying

(1) $x \to (f(x),\xi)$ is Borel $\forall \xi \in \mathcal{H}_\gamma$,

(2) $f(hx) = \gamma(h)f(x)$, $h \in H$, a.a. $x \in G$

(3) $\int_{G/H} \|f(\bar{x})\|^2 d\bar{x} < \infty$,

where we identify functions which differ on sets of measure zero. We set

$$\pi(g)f(x) = f(xg)\left[\frac{q(xg)}{q(x)}\right]^{\frac{1}{2}}, \qquad f \in \mathcal{H}(\gamma),$$

and call π the representation of G *induced* from H by γ. We write $\pi = \mathrm{Ind}_H^G\, \gamma$.

EXERCISES. (1) Check that π is a continuous unitary representation of G.

(2) In case $\gamma = 1_H$, show that the map $\mathcal{H}(\gamma) \to L_2(G/H)$, $f \to F$, $F(\bar{x}) = f(s(\bar{x}))$, $\bar{x} \in G/H$, sets up an equivalence between $\mathrm{Ind}_H^G\, 1_H$ and the representation U defined previously. If $H = \{e\}$, the resulting representation is the right regular representation of G.

(3) If we begin the procedure with another function q' satisfying the conditions of Lemma 1 and call the resulting representation π', show that $\pi \cong \pi'$.

(4) Let $D = \{\phi \in C_0^+(G): \int_H \phi(hx)dh \leq 1\}$ and set $\mathcal{H}(\gamma)'$ to be the space of functions from G to $\mathcal{H}_\gamma$ satisfying

(a) $x \to (f(x),\xi)$ is Borel $\forall \xi \in \mathcal{H}_\gamma$,

(b) $f(hx) = q^{\frac{1}{2}}(h)\, \gamma(h)f(x)$, $h \in H$, a.a. $x \in G$

(c) $\sup_{\phi \in D} \int_G \|f(x)\|^2\, \phi(x)\, dx < \infty$,

identifying functions equal almost everywhere as usual. Show that the unitary map

$$\mathcal{H}(\gamma) \to \mathcal{H}(\gamma)'$$

$$f \to F, \quad F(hs(\bar{x})) = q^{\frac{1}{2}}\,(h)F(hs(\bar{x})), \quad h \in H, \quad \bar{x} \in G/H$$

converts $\pi = \mathrm{Ind}_H^G\, \gamma$ into an equivalent representation π' on $\mathcal{H}(\gamma)'$ which acts via

$$\pi'(g)F(x) = F(xg), \qquad F \in \mathcal{H}(\gamma)'.$$

(5) Reformulate the definition of an induced representation so as to be phrased in terms of left actions.

We conclude this section by stating some of the elementary properties of these induced representations. Although elementary, the actual proofs often involve tedious measure-theoretic considerations (see Mackey [3]). The notation is $\pi = \mathrm{Ind}_H^G \gamma$.

(1) π in general fails to be irreducible, even when γ is. However if π is irreducible, then γ must also be irreducible (see number (5)).

(2) If T is an automorphism of G whose restriction to H is an automorphism of H, then $T\pi \cong \mathrm{Ind}_H^G T\gamma$. By $T\pi$, we mean the representation $T\pi(x) = \pi(T^{-1}x)$, $x \in G$.

(3) *Induction in Stages.* If $K \subseteqq H \subseteqq G$ are closed subgroups and σ is a unitary representation of K, then

$$\mathrm{Ind}_K^G \sigma \cong \mathrm{Ind}_H^G(\mathrm{Ind}_K^H \sigma).$$

Taking $K = \{e\}$, σ trivial, we see that inducing the regular representation of H to G yields the regular representation of G.

(4) If A,B are closed subgroups of C,D respectively, and π_1,π_2 are representations of A,B respectively, then

$$\mathrm{Ind}_A^C \pi_1 \times \mathrm{Ind}_B^D \pi_2 \cong \mathrm{Ind}_{A\times B}^{C\times D} \pi_1 \times \pi_2.$$

In particular if $G = HN$ is a direct product of subgroups and $K \subseteqq H$, then for representations σ of K we have

$$\mathrm{Ind}_K^G \sigma \cong \mathrm{Ind}_K^H \sigma \times \lambda_N,$$

λ_N = the regular representation of N.

(5) If the unitary representation γ of H is a direct integral of representations ω

$$\gamma = \int_\Omega^\oplus \omega \, d\nu(\omega)$$

then

$$\mathrm{Ind}_H^G\ \gamma \cong \int_\Omega^\oplus \mathrm{Ind}_H^G\,\omega\, d\nu(\omega).$$

Finally we note that a multitude of examples of induced representations occur throughout the entire body of these notes.

B. THE IMPRIMITIVITY THEOREM

The definition of a system of imprimitivity and the imprimitivity theorem that we state here are not nearly as general as is possible. However, they will suffice for our purposes.

Let π be a unitary representation of the locally compact group G. Suppose also that we have a right Borel G-space X on which there is a projection-valued measure P taking values in the space of π. We say that P is a *system of imprimitivity* for π (based on X) if

$$\pi(x)P_E\pi(x)^{-1} = P_{E\cdot x^{-1}}, \quad x \in G, \quad E \in \mathcal{B}(X).$$

EXAMPLE. Let $H \subseteq G$ be a closed subgroup and γ a unitary representation of H. Set $\pi = \mathrm{Ind}_H^G\,\gamma$ and take $X = G/H$. G acts in the usual way. Then for Borel sets $E \subseteq X$, set

$$(P_E^\gamma f)(x) = \chi_E(\bar{x})f(x), \quad f \in \mathcal{H}_\pi.$$

The simple calculation verifying that P^γ is a system of imprimitivity for π (based on G/H) is left to the reader.

In the preceding example, the Borel space on which the system of imprimitivity is based is a homogeneous space of G. Such a system is quite naturally called a *transitive* system of imprimitivity. The content of the imprimitivity theorem is that this example is the most general type of transitive system of imprimitivity.

THEOREM 1. (Mackey [1]) *Let π be a unitary representation of G, and let P be a system of imprimitivity for π based on G/H, H a closed subgroup. Then there exists a representation γ*

of H, *uniquely determined up to equivalence, and a unitary map*

$$U: \mathcal{H}_\pi \to \mathcal{H}(\gamma)$$

such that for all $g \in G$ *and Borel sets* $E \subseteq G/H$ *we have*

$$U\pi(g) = (\mathrm{Ind}_H^G \gamma)(g)U \qquad UP_E = P_E^\gamma U.$$

EXERCISE. In Chapter III, where we use the imprimitivity theorem in a crucial way, the group actions are written on the left. Using a left action formulation of induced representations (see Appendix A, Exercise 5), rework the imprimitivity theorem so as to accommodate left actions.

NOTATION AND TERMINOLOGY

I have only one carte blanche convention -- all groups throughout the text are assumed to be locally compact Hausdorff and *separable*. It saves me trouble.

1. The following symbols denote the sets as indicated:

$\mathbb{R}$ = the real numbers

$\mathbb{C}$ = the complex numbers

$\mathbf{T}$ = $\{z \in C: |z| = 1\}$

$\mathbb{Z}$ = the integers

$\mathbb{Z}_n$ = the integers mod n, $n \geqq 1$

$\mathbb{Q}$ = the rational numbers

$\mathbb{H}$ = the quaternions

$\mathbb{Q}_p$ = the p-adic numbers

k^* = the multiplicative group of non-zero elements in a field k

$\mathbb{R}_+^*$ = the positive real numbers

$\mathbb{Z}_+$ = the positive integers

2. If V is a vector space over a field K, we write

End(V) = the space of K-endomorphisms of V

GL(V) = the non-singular endomorphisms in End(V).

If $\{X_j\}_{1 \leqq j \leqq n}$ is a basis of V, $\dim_K V = n$, then $\{X_j^*\}_{1 \leqq j \leqq n}$ denotes the dual basis of $V^* = \mathrm{Hom}_K(V,K)$.

For $n \geqq 1$

M(n,K) = the $n \times n$ matrices with entries in K

GL(n,K) = the non-singular matrices in M(n,K)

D(n,K) = the diagonal matrices in GL(n,K).

If $X \in M(n,K)$, the symbol ${}^tX$ denotes the transposed matrix.

The letter I usually denotes the identity operator on whatever vector space is under consideration. $\mathcal{H}$ usually denotes a Hilbert space, $\mathcal{U}(\mathcal{H})$ = the unitary operators thereon.

3. If X is a Borel space, we write

$\mathcal{B}(X)$ = the Borel sets of X.

For locally compact Hausdorff topological spaces X,

$C_0(X)$ = the continuous functions of compact support on X

$C_0^+(X) = \{f \in C_0(X): f(x) \geqq 0 \ \forall x \in X\}$.

If X is a C^∞-manifold

$C_0^\infty(X)$ = the infinitely differentiable functions of compact support.

If G is a locally compact group, e always denotes the identity. Also G^0 = the connected component of G containing e, also called the neutral component,

Δ_G = the modular function of G

Z_G = Cent G = the center of G.

If G is a Lie or algebraic group

$\mathfrak{g}$ = LA(G) is the Lie algebra of G.

We write ad, Ad, Ad* respectively for the adjoint representation of $\mathfrak{g}$, adjoint representation of G on $\mathfrak{g}$, and the co-adjoint representation of G on $\mathfrak{g}^*$.

If X is a Borel G-space

$G_x = \{g \in G: g\cdot x = x\}$ = the stabilizer of x

$G\cdot x = \{g\cdot x: g \in G\}$ = the G-orbit containing x.

4. The phrase algebraic group in the text means linear algebraic group. Zariski (as in Zariski-closed) is usually abbreviated by Z-. For subsets S of Lie or algebraic groups or algebras,

$Z(S)$ = the centralizer of S

$N(S)$ = the normalizer of S.

5. The phrase representation when applied to groups means continuous unitary representation (except in Chapter V, section B). If π is

such a representation, $\mathcal{H}_\pi$ denotes the space on which it acts. Also

$\bar{\pi}$ denotes the conjugate representation in $\bar{\mathcal{H}}_\pi$

$\pi \cong \pi'$ means unitary equivalence

$\pi \approx \pi'$ means quasi-equivalence

$\mathcal{I}(\pi,\pi')$ = the dimension of the space of intertwining operators for π, π'

$\mathcal{I}(\pi,\pi') = 0$ means π and π' are disjoint

$\mathcal{I}(\pi,\pi) = 1$ means π is irreducible

$\mathfrak{A}(\pi,\pi')$ = the W*-algebra generated by the algebra of intertwining operators for π and π'

1_G = the trivial representation of G in a space of dimension 1

λ_G = the (left) regular representation of G

$\pi = \mathrm{Ind}_H^G\, \gamma$ = the representation of G induced from H by γ

Rep(G) = the (concrete) space of unitary representations of G

Irr(G) = the subset of Rep(G) consisting of irreducible representations

$\hat{G}$ = (Irr(G)/$\cong$), that is the set of unitary equivalence classes of irreducible unitary representations. We often blur the distinction between a representation $\pi \in$ Irr(G) and its class in $\hat{G}$.

The space $\hat{G}$ carries a natural Borel structure (the Mackey Borel structure) and a natural topology (the Fell or dual topology). These are described in Auslander and Moore [1].

6. Finally if $f \in L_1(G)$, f^* is defined by $f^*(x) = \overline{f(x^{-1})}\Delta(x)^{-1}$, $x \in G$. Also if $\pi \in$ Rep(G), $f \in L_1(G)$, the symbol $\pi(f)$ denotes the operator

$$\pi(f) = \int_G f(g)\, \pi(g)\, dg.$$

If H is a closed subgroup of G, the symbol [G:H] denotes dim $L_2(G/H)$.

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At one point the thought occurred to me that since Mackey's survey article in the 1963 Bulletin, no complete bibliography of articles in representation theory has appeared. That's all -- it just occurred to me. I didn't do anything about it. Nevertheless I do hope the reader finds the following list to be of some use. I have included a few references beyond those quoted in the text.

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