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## Index of Principal Notation

<u>Page</u>	<u>Notation</u>	<u>Meaning</u>
8	Th	Theory
8	$\equiv$	Elementarily equivalent
9	$A_X$	A with distinguished elements
9	tp	Type
10	<	Elementary substructure
10	UC	Union of a chain
15	Diag	Diagram
18	$V_A(s)$	Relation defined by s on A
18	$A_n, E_n$	Quantifier rank n
19	$<_n$	< restricted to $A_n, E_n$
24	ACF	Algebraically closed field
32	$K^x$	$K - \{0\}$
33	ord	Valuation
34	O	Valuation ring
34	M	Maximal ideal
34	U	O-M (Units)
34	K	Residue field
35	$F((t))$	Power series field
37	$O_p$	p-adic field
52	$\tau(x)$	$x^p - x$
52	$\gamma(x)$	$p^{-1}(\tau(x) - \tau(x)^{-1})^{-1}$
53	$R_T$	Kochen ring
61	$P(F)$	Puiseux series field
69	$E\sum$	Existentially complete
70	$\sum_T$	Models of $T \cap A_1$
74	$T^E$	$Th(E_T)$
75	$T^*$	Model companion
76	$\sum'$	Persistently complete structures
76	$\sum'^{\infty}$	The $\infty$ -companion
84	$\sum^f$	Finitely generic structures
92	CR	Commutative rings
93	Reg	"R/radR is regular"
95	SCR	Semiprime CR
97	$S(A)$	Spectrum of A
97	$B(A)$	Idempotents of A
103	T-rad I	Radical relative to T

<u>Page</u>	<u>Notation</u>	<u>Meaning</u>
108	DR	Division rings
114	$h(x)$	Height of term $x$
120	rk	Rank
124	$\langle\langle X \rangle\rangle$	Division ring generated by $X$
125	$C(x)$	Centralizer
125	$C^2(X)$	$C(C(X))$
133	${}^1N$	First order arithmetic
133	${}^2N$	Second order arithmetic
134	${}^1N(D), {}^2N(D)$	Models of arithmetic in $D$
156	$M_R$	R-modules
156	AG	Abelian groups
159	$M_1 \oplus M_2$	Amalgamated sum
186	$Z/p^\infty$	Prüfer $p$ -group
186	$Z_p$	$Z$ localized at $p$
186	$ a _p$	$p$ -norm
188	$ a $	$\sum 2^{-p}  a _p$
189	$m x$	$m$ divides $x$
195	$\alpha, \beta, \gamma, \delta$	Szmielew invariants
215	Tr	Trace
217	$t(x)$	$x^p - x$