APPENDIX

(1) Proposition. If E is an admissible vector space, then the scalar multiplication: $\mathbb{R} \times E \longrightarrow E$ is equably continuous.

Proof. Let $\mathfrak{X} \downarrow E$. Then by (2.1.1), (2.4.2) and (7.1.1)

$$\mathfrak{Y} = (\mathfrak{X} \vee (-\mathfrak{X}))^{\wedge} \downarrow E.$$

If $Y \in \mathfrak{Y}$, then $Y \supset (X \cup (-X))^{\wedge}$, $X \in \mathfrak{X}$. Thus for any $x \in X$ the segment $[-x,x] \subset (X \cup (-X))^{\wedge}$ and consequently $\lambda x \in (X \cup (-X))^{\wedge}$ for each $|\lambda| \leq 1$ and $x \in X$, hence $I_1 X \subset Y$ where $I_1 = [-1,1]$, and therefore $I_1 \mathfrak{X} \leq \mathfrak{Y} \downarrow E$. By (2.4.2) also $\beta . I_1 \mathfrak{X} = I_\beta \mathfrak{X} \downarrow E$ for each $\beta \neq 0$. This proves the equable continuity of the scalar multiplication by (2.8.8) and (2.5.1).

(2) Proposition. Let $\mathfrak{X}$ be a filter on a vector space. Then the filter $\mathfrak{X}^* = \mathfrak{V} \cdot \sup_{\delta \neq 0} (\delta \cdot \mathfrak{X})$ is the finest one among all equable filters coarser then $\mathfrak{X}$.

Proof. Obviously $\mathfrak{X}^*$ is equable. Let $M \in \mathfrak{X}^*$. Then $M \supset V \cdot A$, where $V \in \mathfrak{V}$, $A \in \sup_{\delta \neq 0} (\delta \mathfrak{X})$. Choose $\alpha \in V$, $\alpha \neq 0$. Since $A \in \delta \mathfrak{X}$ for all $\delta \neq 0$, we have $\alpha \cdot A \in \mathfrak{X}$, and therefore $M \in \mathfrak{X}$, because $M \supset \alpha A$. This proves that $\mathfrak{X}^*$ is coarser then $\mathfrak{X}$.

Let now $\mathfrak{Y}$ be any equable filter coarser then $\mathfrak{X}$: $\mathfrak{X} \leq \mathfrak{Y} = \mathfrak{V}\mathfrak{Y}$

Then, for $\delta \neq 0$:

$$\delta \mathfrak{X} \leq \delta \mathfrak{Y} = \delta (\mathfrak{V} \mathfrak{Y}) = (\delta \mathfrak{V}) \mathfrak{Y} = \mathfrak{V} \cdot \mathfrak{Y} = \mathfrak{Y}$$; hence

$\sup_{\lambda\neq 0}(\lambda\mathfrak{X}) \leq \mathfrak{Y}$ and thus $\mathfrak{X}^* = \mathcal{V}\cdot\sup_{\lambda\neq 0}(\lambda\mathfrak{X}) \leq \mathcal{V}\mathfrak{Y} = \mathfrak{Y}$,

which completes the proof.

<u>Corollary</u>. For any pseudo-topological vector space E one has:

$$\mathfrak{X}\downarrow E^{\#} \Longleftrightarrow \mathfrak{X}^*\downarrow E.$$

(3) <u>Proposition</u>. Let E be any pseudo-topological vector space. Then

$$E \text{ equable} \Longleftrightarrow (\mathfrak{X}\downarrow E \text{ implies } \sup_{\lambda\neq 0}\lambda\mathfrak{X}\downarrow E).$$

<u>Proof</u>. (a) Suppose E equable, and let $\mathfrak{X}\downarrow E$. Then there exists $\mathfrak{Y}$ with $\mathfrak{X} \leq \mathfrak{Y} = \mathcal{V}\cdot\mathfrak{Y}\downarrow E$. Since $\mathfrak{X}^*$ is coarser then $\mathfrak{X}$ and equable, $\lambda\cdot\mathfrak{X} \leq \lambda\cdot\mathfrak{X}^* = \mathfrak{X}^*$ for $\lambda\neq 0$; hence $\sup_{\lambda\neq 0}(\lambda\mathfrak{X}) \leq \mathfrak{X}^* \leq \mathfrak{Y}$, showing that $\sup_{\lambda\neq 0}\lambda\mathfrak{X}\downarrow E$.

(b) Suppose the condition satisfied, and let $\mathfrak{X}\downarrow E$. Then we have $\mathfrak{X} \leq \mathfrak{X}^* = \mathcal{V}\cdot\mathfrak{X}^*\downarrow E$, which proves that E is equable.

(4) <u>Proposition</u>. Let E_i, $i\in I$ be a projective system of equable and admissible vector spaces (cf.(2.3.5)(c)).

Then $E' = \operatorname{proj.lim}_{i\in I} E_i$ is equable and admissible.

<u>Proof</u>. The admissibility is proved in (7.3.2). Let $\mathfrak{X}\downarrow E$. Then $i_k(\mathfrak{X})\downarrow E_k$ for each $k\in I$. But $\sup_{\lambda\neq 0}\lambda\, i_k(\mathfrak{X}) = i_k(\sup_{\lambda\neq 0}\lambda\mathfrak{X})$, since one has $\bigcup_{\lambda\neq 0}\lambda\, i_k(X_\lambda) = \bigcup_{\lambda\neq 0} i_k(\lambda X_\lambda) = i_k(\bigcup_{\lambda\neq 0}\lambda X_\lambda)$ for each such union. Hence $\sup_{\lambda\neq 0}\lambda\mathfrak{X}\downarrow E$ and thus E is equable by the preceding proposition.

(5) <u>Proposition</u>. For each projective system of admissible vector spaces, $(\operatorname{proj.lim}_{i \in I} E_i)^{*} = \operatorname{proj.lim}_{i \in I} E_i^{*}$.

<u>Proof</u>. (a) Let $\mathfrak{X} \downarrow (\operatorname{proj.lim}_{i \in I} E_i)^{*}$. Then there is $\mathfrak{Y}$ with $\mathfrak{X} \blacktriangleleft \mathfrak{Y} = V\mathfrak{Y} \downarrow \operatorname{proj.lim}_{i \in I} E_i$. Since $i_k(\mathfrak{X}) \blacktriangleleft i_k(\mathfrak{Y}) = i_k(V\mathfrak{Y}) = Vi_k(\mathfrak{Y})$, one gets $i_k(\mathfrak{X}) \downarrow E_k^{*}$ for each $k \in I$ and thus $\mathfrak{X} \downarrow \operatorname{proj.lim}_{i \in I} E_i^{*}$.

(b) If $\mathfrak{X}$ $\operatorname{proj.lim}_{i \in I} E_i^{*}$, then $i_k(\mathfrak{X}) \downarrow E_k$ for each $k \in I$ by (2.6.3). But by (4) and (2.6.1) we can suppose that $\mathfrak{X}$ is equable, hence $\mathfrak{X} \downarrow (\operatorname{proj.lim}_{i \in I} E_i)^{*}$.

NOTATIONS

$\mathfrak{A}, \mathfrak{F}, \mathfrak{X}$	filters 1.1
Φ	empty set 1.1
$[B],[A],[a]$	generated filters 1.1
$\mathfrak{X}_1 \leq \mathfrak{X}_2$	comparison of filters (1.2.1)
$\mathfrak{X}\downarrow_x \; E$	$\mathfrak{X}$ converges to x on E 2.1
$\mathfrak{X} \downarrow E$	$\mathfrak{X}$ converges to zero on E 2.4
$\{\mathfrak{X}_i\}_{i\in I}$	family of filters 1.2
$\sup_{i\in I} \mathfrak{X}_i$	least upper bound of filters 1.2
$\mathfrak{X}_1 \vee \mathfrak{X}_2$	sup $(\mathfrak{X}_1, \mathfrak{X}_2)$ 1.2
$\mathfrak{U}_x$	$\sup_{\mathfrak{X}\downarrow_x E} \mathfrak{X}$ (2.4.3)
$\mathfrak{X}^o$	2.7
$\hat{\mathfrak{X}}, (\mathfrak{X})\hat{}$	2.7
$\bar{\mathfrak{X}}, (\mathfrak{X})^-$	(5.3.3)
E, E_1, E_2	pseudo topological spaces
$\underline{E}$	underlying set
E^*	equable space associated to E 2.6
E^o	locally convex space associated to E 2.7
$E_1 \leq E_2$	comparison of structures 2.3
$E_1 \times E_2, \times_{i\in I} E_i$	direct product of pseudo-topological spaces 2.3

$\mathfrak{X}_1 \times \mathfrak{X}_2$	direct product of filters (on a direct product of two sets) 1.4
$\mathbb{R}$	the reals (with the natural topology)
$\mathbb{V}$	neighborhood filter of zero in $\mathbb{R}$
$[\alpha , \beta]$	closed interval in $\mathbb{R}$
I_δ	closed interval $[-\delta , \delta]$, $\delta > 0$
$\mathbb{N}$	$\{1,2,3,\ldots\}$
$\mathbb{N}^0$	$\{0,1,2,\ldots\}$
$\text{proj.lim}_{i \in I} E_i$	projective limit 2.3
$R(E_1;E_2)$	set of remainders (3.1.2)
$L_n(E_1;E_2)$	space of n-linear maps (6.1.6)
$C_k(E_1;E_2)$	space of C_k-mappings 10.2
$C_\infty(E_1;E_2)$	space of C_∞-mappings 10.3
$L_n^*(E_1;E_2)$	instead of $(L_n(E_1;E_2))^*$ (6.1.7)
$C_k^*(E_1;E_2)$	instead of $(C_k(E_1;E_2))^*$ (6.1.7)
$C_\infty^*(E_1;E_2)$	instead of $(C_\infty(E_1;E_2))^*$
$f : E_1 \longrightarrow E_2$	mapping f of E_1 into E_2
$x \longmapsto y$	x is sent into y under the considered (anonymous) map
$[f_1,f_2]$, $\prod_{i \in I} f_i$	1.3
$f_1 \times f_2$, $\times_{i \in I} f_i$	1.3

π_k	k-th projection 2.3
c	composition map
e	evaluation map
$\tilde{u}$	map associated to u (11.1.1)
$u \cdot x$	instead of $u(x)$
$\Delta f(x,y)$	abbreviation of $f(x+y)-f(x)$
$\Theta f(\lambda,x)$	abbreviation of $\begin{cases} \frac{f(\lambda x)}{\lambda} & \text{for } \lambda \neq 0 \\ 0 & \text{for } \lambda = 0 \end{cases}$
$f^{\cdot}(a)$	differential quotient (4.3.2)
$f'(a)$, $Df(a)$	derivative of f at the point a (3.2.2)
$Rf(a)$	remainder belonging to a map f which is differentiable at a (3.2.4)
$f^{(k)}(a)$	see 9.2
$D_1f(a_1,a_2), D_2f(a_1,a_2)$	partial derivatives 8.1
f^*	$f^*(g) = g \circ f$
f_*	$f_*(g) = f \circ g$
$\Longrightarrow$	implies
$\Longleftrightarrow$	if and only if
$\approx$	linearly homeomorphic

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