## BIBLIOGRAPHICAL NOTES

The theory of analytic sets, especially of analytic sets in $\mathbb{C}^n$, is developed in the books of M. Hervé [19] and S.S. Abhyankar [1] and in the seminar notes of H. Cartan, 1953/54 [10] and 1960/61, Exposés 18-21 by C. Houzel [12]. In less detail, it is treated in Gunning - Rossi [14]. The treatment in Hervé, although very different in detail, is based on the same ideas. That in Abhyankar is quite different and treats analytic sets over arbitrary completely valuated, non-discrete fields (mostly algebraically closed). The ideas in the treatment of Houzel in [12] are drawn from Grothendieek's work in algebraic geometry.

Chapter I. Most of the results stated here without proof are proved in Hervé [19] (and/or Abhyankar [1], Gunning - Rossi [14]). F. Hartogs' theorem stated on page 3, is proved in [18].

The rank theorem, as stated here, is proved e.g. in [25].

Chapter II. Theorem 1, which is one form of the preparation theorem, is proved by C. Houzel in [12]; he ascribes the proof to J.P. Serre. The third proof we have given for the preparation theorem uses ideas of Malgrange [24]; the fourth, due to H. Grauert and R. Remmert, is unpublished, see [17].

For an analogue of the preparation theorem for differentiable functions and applications, see Malgrange [24].

Chapter III. The general remarks made at the beginning of these bibliographical notes apply, above all, to this chapter. Theorem 7 is due to Grauert - Remmert [16]. Their proof, which uses the normalization theorem, is difficult. The idea of the proof given here is due to L. Bungart - H. Rossi [7], although the presentation is different.

Chapter IV. All known proofs of Oka's theorem, Theorem 3, are based on the ideas of Oka [26]. The first published proof of the coherence of the ideal sheaf of an analytic set, Theorem 5, is that of H. Cartan [9], although Oka refers to this theorem in [26], and Cartan himself says, in a footnote in [9], that he understands that Oka also has a proof. Oka's version, which is not very different from that of Cartan, is given in [27]. We have followed Cartan's presentation.

Theorem 7 is proved in Grauert - Remmert [16], and is a very special case of a theorem of Grauert [15]. These proofs are, however, very different from the one we have given. Another proof, quite different from all these, is given in [12].

Chapter V. This chapter follows closely the papers of Cartan [11], Bruhat - Cartan [4, 5] and Bruhat - Whitney [6]. The unproved results concerning C-analytic sets are in Cartan [11] (Proposition 15, Example 2) and in Bruhat - Whitney [6] (Proposition 16, 17, 18).

Many very interesting properties of real analytic sets and functions, and applications are given in the book by Malgrange [24]. Further interesting metric properties and the triangulability of real analytic sets are given in the paper of S. Łojasiewicz [23].

Chapter VI. The original proof of Oka [27] is given in Cartan [10] where, however, there is an error. A complete version of this proof is given in Rossi [33]. Other proofs are due to Abhyankar [1] and Kuhlmann [21]. The proof of Grauert - Remmert is unpublished, see [17].

The proof of Theorem 2 given here is due to Kuhlmann [21]; it is a direct generalization of corresponding results in algebraic geometry.

Theorem 2 can also be proved "geometrically" if one already has Theorem 3 (or Theorem 5). For another algebraic proof of Theorem 2, see Abhyankar [1].

<u>Chapter VII</u>. The Remmert - Stein theorem is in [32], see also [10]. The details are more complicated than in the proof we have given. These articles deal simultaneously with a more general form of the theorem. We have given this simple proof in this special case because this is the form most often used.

Theorem 2 is due to Remmert [28, 30]; Grauert deduces it from his theorem on the direct image [15] (just as we obtained Chapter IV, Proposition 5 from Chapter IV, Theorem 7).

The formulation given in Theorem 3 for Remmert's theorem was suggested by M.S. Rajwade.

The theorem [due to E.E. Levi [22] (see also H. Kneser [20]) for domains in $\mathbb{C}^n$] does not seem to have been proved in general in the literature, although it has been used. We have included a proof for this reason. It can be deduced (in the case of domains in $\mathbb{C}^n$) from an analogue of Chapter I, Proposition 12 for meromorphic functions; see H. Kneser [20].

Special cases of Theorem 5, and the theorem on the field of meromorphic functions stated at the end, were proved by C.L. Siegel [36, 37] who used a very elementary method. Theorem 5 is due to Thimm [38]. The proof given here is due to Remmert [29]; see also [31]. The proof by Siegel has led to very important applications to the theory of automorphic functions. See in particular Andreotti [2], Andreotti - Grauert [3]. A. Borel has obtained a very far-reaching generalization of the early work of Siegel on modular functions as an application of the methods of [3]; this work of Borel is still unpublished.

REFERENCES


1. Abhyankar, S.S. Local analytic geometry, Academic Press, New York, 1964

2. Andreotti, A. Théorèmes de dépendance algébrique sur les espaces complexes pseudo-concaves, Bull. Soc. math. France, 91(1963), 1-38.

3. Andreotti, A. - Grauert, H. Algebraische Körper von automorphen Funktionen, Göttinger Nachr. 1961, 39-48.

4. Bruhat, F. - Cartan, H. Sur la structure des sous-ensembles analytiques-réels, C.R. Acad. Sci. Paris, 244(1957), 988-990.

5. Bruhat, F. - Cartan, H. Sur les composantes irréductibles d'un sous-ensemble analytique-réel, C.R. Acad. Sci. Paris, 244(1957), 1123-1126.

6. Bruhat, F. - Whitney, H. Quelques propriétés fondamentales des ensembles analytiques-réels, Comm. Math. Helv. 33(1959), 132-160.

7. Bungart, L. - Rossi, H. On the closure of certain spaces of holomorphic functions, Math. Annalen, 155(1964), 173-183.

8. Cartan, H. Idéaux de fonctions analytiques de n variables complexes, Ann. Ecole Norm. Sup. 61(1944), 149-197.

9. Cartan, H. Idéaux et modules des fonctions analytiques des variables complexes, Bull. Soc. math. France, 78(1950), 29-64.

10. Cartan, H. Séminaire de l'Ecole Norm. Sup. 1953/54.

11. Cartan, H. Variétés analytiques réelles et variétés analytiques complexes, Bull. Soc. math. France, 85(1957), 77-99.

12. Cartan, H. Séminaire de l'Ecole Norm. Sup. 1960/61, (particularly Exposés 18-21 by C. Houzel).

13. Godement, R. Théorie des faisceaux, Hermann, Paris, 1958.

14. Gunning, R. - Rossi, H. Analytic functions of several complex variables, Prentice Hall, N. J. 1965.

15. Grauert, H. Ein Theorem der analytischen Garbentheorie und die Modulräume komplexer Strukturen, Publ. math. Inst. Hautes Etudes, Paris, no.5, 1960.

16. Grauert, H. - Remmert, R. Komplexe Räume, Math. Annalen, 136(1958), 245-318.

17. Grauert, H. - Remmert, R. Forthcoming book on analytic spaces.

18. Hartogs, F. Zur Theorie der analytischen Funktionen mehrerer Veränderlichen,... Math. Annalen, 62(1906), 1-88.

19. Hervé, M. Several complex variables, Local theory, Oxford Univ. Press and Tata Institute of Fundamental Research, 1963.

20. Kneser, H. Ein Satz über die Meromorphiebereiche analytischer Funktionen von mehreren Veränderlichen, Math. Annalen, 106(1932), 648-655. [See also: Der Satz von dem Fortbestehen der wesentlichen Singularitäten einer analytischen Funktion zweier Veränderlichen, Jahresbericht der D.M.V. 41(1932), 164-168.]

21. Kuhlmann, N. Die Normalisierung komplexer Räume, Math. Annalen, 144(1961), 110-125.

22. Levi, E.E. Studii sui punti singolari essenziali delle funzioni analitiche di due o più variabili complesse, Annali di mat. pur. appl. 17, 3(1910), 61-87.

23. Łojasiewicz, S. Triangulation of semi-analytic sets, Ann. Scuola Norm. Sup. Pisa, 18(1964), 449-474.

24. Malgrange, B. Ideals of differentiable functions, Oxford Univ. Press and Tata Institute of Fundamental Research, 1966.

25. Narasimhan, R. Lectures on topics in analysis, Tata Institute of Fundamental Research, 1965.

26. Oka, K. Sur les fonctions analytiques de plusieurs variables. VII. Sur quelques notions arithmétiques, Bull. Soc. math. France, 78(1950), 1-27.

27. Oka, K. Sur les fonctions analytiques de plusieurs variables. VII. Lemme fondamental, Journ. math. Soc. Japan, 3(1951), 204-214; 259-278.

Nine papers of Oka, including the two papers listed above, are published under the title "Sur les fonctions analytiques de plusieurs variables", Iwanami Shoten, Tokyo, 1961.

28. Remmert, R. Projektionen analytischer Mengen, Math. Annalen, 130(1956), 410-441.

29. Remmert, R. Meromorphe Funktionen in kompakten komplexen Räumen, Math. Annalen, 132(1956/57), 277-288.

30. Remmert, R. Holomorphe und meromorphe Abbildungen komplexer Räume, Math. Annalen, 133(1957), 328-370.

31. Remmert, R. Analytic and algebraic dependence of meromorphic functions, Amer. Journ. Math. 82(1960), 891-899.

32. Remmert, R. - Stein, K. Ueber die wesenstlichen Singularitäten analytischer Mengen, Math. Annalen, 126(1953), 263-306.

33. Rossi, H. Analytic spaces, Princeton Univ. lecture notes, 1960.

34. Serre, J.P. Faisceaux alggébriques cohérents, Annals of math. 61(1955), 197-278.

35. Serre, J.P. Géométrie algébrique et géométrie analytique, Annales de l'Inst. Fourier, 6(1955/56), 1-42.

36. Siegel, C.L. Meromorphe Funktionen auf kompakten analytischen Mannigfaltigkeiten, Göttinger Nachr. 1955, 71-77.

37. Siegel, C.L. On meromorphic functions of several variables, Bull. Calcutta math. Soc. 50(1958), 165-168.

38. Thimm, W. Meromorphe Abbildungen von Riemannschen Bereichen, Math. Zeitschr. 60(1954), 435-457.

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