
Notes

Chapter 1

Section 1.1 contains the basic differential calculus on Banach spaces; the material can be found in any nonlinear functional analysis book, for instance, Schwartz [Scw], Nirenberg [Ni 1], Deimling [De], Zeidler [Zei], Berger [Ber 1] etc. The discussion on Nemytski operator can be found in Vainberg [Va], but the proof is much simpler than that in the reference.

The presentations of the implicit function theorem and the inverse function theorem are standard. Interesting applications are scattered in the literature, for instance, Nirenberg [Ni 4], Kazdan [Ka], Chow, Hale [CH], Mawhin [Maw 3] etc. The continuity method is extensively used in the existence proof of differential equations. The global implicit function theorem is due to Hadamard [Ha], Caccioppoli [Cac 1]; extensions can be found in Browder [Bd 1] and Plastock [Pl]. However, the proof presented here is very different. The application of the continuity method to an a priori bound as a simpler proof for semi-linear elliptic equations with quadratic growth was given by Amann and Crandall [AC]; the result for quasi-linear elliptic equations can be found in Ladyzenskaya and Ural'zeva [LU].

The Lyapunov–Schmidt reduction is extensively used in nonlinear problems. The application to the study of bifurcation problems introduced here is due to Crandall and Rabinowitz [CR 1], [CR 2]. The references on bifurcation theory are recommended to Chow and Hale [CH].

Sections 1.3.3–1.3.4 provide examples on gluing, which is an important technique in symplectic geometry. The material here is taken from Floer and Weinstein [FW] and Oh [Oh]. Further references are Floer [Fl 1] [Fl 2], Hofer and Zehnder [HZ 2]. Parallel to the implicit function theorem method, a gluing technique via the variational method was developed by Sere [Se]; it has been applied to the homoclinic orbits in Hamiltonian systems, multi-bump solutions, and multi-peak solutions for elliptic differential equations, see also Coti Zelati and Rabinowitz [CR], Li [Li 1], Gui [Gu] etc. Section 1.3.5 is another important technique in applying the implicit function theorem. The

notion of transversality is taken from differential geometry. Combining with the Sard theorem, it provides a method for proving various generic type results. The finite-dimensional form of the transversality theorem can be found in Guillemin and Podollak [GP]. The Sard–Smale theorem is taken from Smale [Sm 3]. The simplicity of eigenvalues of the Laplacian on generic domains is due to Uhlenbeck [Uh].

Section 1.4 is on the Nash–Moser technique. See Nash [Na 2] and Moser [Mos 1] [Mos 2]. KAM theory is due to Kolmogorov [Ko], Arnold [Ar 2] and Moser [Mo 3]. The presentation here can be found in Hormander [Hor 2]. Applications in differential geometry can be found in Hamilton [Ham]; a version setting up on Frechet spaces is given therein. Other versions can be found in Nirenberg [Ni 1], see also Zehnder [Ze]. For further reading on recent developments of KAM theory to partial differential equations, Bourgain [Bou], Kuksin [Kuk], Wayne [Way], and Poschel [Po] are recommended.

Chapter 2

The order method is very different from other methods in this book. In concrete problems, once the assumptions are met, the method is simple and powerful. Our discussions start with the Bourbaki–Kneser principle [Bo], [Kn], see also Tarski [Ta]. The Amann theorem [Am 1] is a version that is easy to apply. The sub- and super-solutions method is extensively applied in ODE and PDE whenever the maximum principle is applicable. However, the constructions of sub- and super-solutions require special knowledge and techniques. We are satisfied introducing the method by an example. The Caristi fixed-point theorem [Ca] is among a few fixed-point theorems without assuming the continuity of the nonlinear mappings, applications can be found in [Lie]. Also, an equivalent version of this theorem is the very important Ekeland variational principle [Ek 1]; various applications in different branches of analysis can be found in [Ek 1] and de Figueiredo [dF].

There are lots of books on convex analysis; in Sect. 2.2, we only present very briefly the necessary material for the sequel discussions. References can be found in Ekeland and Teman [ET], Aubin and Ekeland [AE] etc.

Fixed points for nonexpansive mappings have been studied by Browder [Bd 2], Goebel [GK], etc. The importance of this class of mappings is that one of the fixed points can be figured out by iteration methods. Many algorithms for finding feasible solutions for convex programming have been studied in recent years, see [B], [BB].

There are many ways to introduce the Schauder fixed-point theorem and related topics. We use the KKM theorem [KKM] and Ky Fan’s inequality [FK 3] as the starting point. The Nash equilibrium [Na 3], the Von Neumann–Sion minimax theorem [VN] [Si], the Schauder fixed-point theorem [Sc], the Schauder–Tychonoff theorem, the Ky Fan–Glicksberg theorem [FK 1], [FK 2], and the existence result of Hartman and Stampacchia on variational inequality [HS] are direct consequences. The variational inequality [LS], [Bd 3], [Stm]

is another direction in convex analysis with many applications in free boundary problems from mathematical physics, see Duvaut and Lions [DL], Kinderlehrer and Stampacchia [KS], Friedman [Fr] etc. However, the approach based on the fixed points of set-valued mappings is due to the author, see [Ch 1], [Ch 2].

The theory of monotone operators and pseudo-monotone operators attracted much attention in the 1960s and 70s. The works of Minty [Min], Browder [Bd 4], Hartman and Stampacchia [HS], H. Brezis [Br 1], etc. constitute the basic content of the theory. Again we use the version of the Ky Fan inequality due to Brezis, Nirenberg and Stampacchia [BNS] to derive the most important results on this topics. For applications to quasi-linear elliptic equations see Leray and Lions [LL], to boundary value problems in nonlinear partial differential equations see Lions [LJ 1], and to nonlinear semigroups of operators see Brezis [Br 1], Crandall and Liggett [CLi].

Chapter 3

The Brouwer degree is a topological invariant; roughly speaking, there are two approaches: algebraic (see for instance Spanier [Sp], Greenberg [GH]) and differential (Milnor [Mi 2]). The Leray–Schauder degree is its extension to compact vector fields [LS] on Banach space.

The analytic presentation can be found in many books and lecture notes, for instance: J. Schwartz [Scw], Nirenberg [Ni 1], Rabinowitz [Ra 2], Guillemin and Podollak [GP], Zeidler [Zei] etc. The materials of Sects. 3.1–3.4 are taken from these references. An application of the transversality theorem to the proof for the Borsuk–Ulam theorem and to the computation of S^1 -invariant degree are due to Nirenberg [Ni 3].

Sections 3.5 and 3.6.3 are adapted from Rabinowitz [Ra 1], [Ra 2]. Section 3.6.2 is based on Dancer [Dan 1]. The material of Sects. 3.6.4 and 3.6.5 is taken from Amann [Am 2], Krasnosel’ski [Kr 2], de Figueiredo, Lions and Nussbaum [FLN], Chang [Ch 3], and Dancer [Dan 2]. The blowing up method in a priori estimates is a useful technique, see for instance, Gidas and Spruck [GS].

Section 7 contains various extensions of the Leray–Schauder degree; for α -set contraction mappings, see Darbo [Dar], Stuart and Toland [ST]; for condensing mappings, see Nussbaum [Nu], Sadovskii [Sa], set-valued mappings see Browder [Bd 5], Cellina [Ce], Ma [Ma], Chang [Ch 2]. There are other directions: Fredholm operators, see Elworthy and Tromba [EIT 1] [EIT 2], Nirenberg [Ni 4], Llyord [Ll], Fitzpatrick, Pejsachowicz and Rabier [FP], [FPR], [PR 1, PR 2]; coincidence degree theory, Mawhin [Maw 1], etc.

Chapter 4

Section 4.1 is an introduction to the calculus of variations. The derivations of the Euler–Lagrange equation, the Legendre–Hadamard condition and the Ljusternik theorem on constraint variational problems can be found in any

standard textbook. For the dual variational principle, i.e., the Legendre–Fenchel transformation, see for instance, Arnold [Ar 1], Ekeland and Temam [ET] or Aubin and Ekeland [AE]. For the Hamiltonian systems, the second version of the dual variational principle, i.e., the Legendre transform to the Hamiltonian function of all variables if the latter is strictly convex, see Clarke and Ekeland [CE].

In Sect. 4.2, the direct method is a general principle in the calculus of variations. We introduce a few interesting examples showing how the principle works.

Harmonic maps were introduced by Eells and Sampson [ES] in representing homotopy classes of mappings between two manifolds. Here we only touch on the existence of a weak solution. For $m = 2$ the harmonic map with minimal energy is smooth, see Morrey [Mo 2]; for $m > 2$, Schoen and Uhlenbeck [ScU 1], [ScU 2] proved that the singularity has at most a codimension 3 finite Hausdorff measure. A nonsmooth minimal energy harmonic map was given by Lin [Lin]. As to nonminimal energy harmonic maps, the smoothness for $m = 2$ was proved by Helein [Hel 1], and the existence of a nowhere continuous harmonic for $m = 3$ constructed by Riviere [Ri].

The example on constant mean curvature surface is taken from Hildebrandt [Hi 1]; a systematic introduction can be found in Struwe [St 2].

The prescribing scalar curvature problem can be found in Kazdan [Ka], Kazdan and Warner [KW 1], [KW 2]. The result for $\chi(M) = 0$ is due to Berger [Ber 2], that for real projective space P^2 is due to Moser [Mos 4]; however, we present an easy proof with the aid of Aubin’s inequality [Au 3].

Section 4.3 is from Morrey [Mo 1] [Mo 2], Dacorogna [Dac], Acerbi and Fusco [AF], Marcellini [Mar], Muller [Mul 1] [Mul 2] Sverak [Sv 1] [Sv 2], Zhang [Zh 1] and Ball [Bal 1].

The relaxation method is from Buttazzo [Bu] and Dacorogna [Dac]. For the two-well problem, Zhang [Zh 2] has obtained an explicit expression of the quasi-convex envelope for the square distance function.

The Young measure was introduced by Young [Yo]; the presentation here is due to Ball [Bal 2], see also [Mul 1], Kinderlehrer and Pedregal [KP 1, KP 2]. In Chipot [Chi], there are examples on the computation of Young measures.

There are few books in dealing with the BV space, see for instance, Giusti [Gi], Evans and Gariepy [EG], Zimer [Zi], Ambrosio [Amb] etc. Section 4.5.1 is adapted from these books.

The Hardy space and BMO space are important parts in harmonic analysis, see Stein [Ste 1, Ste 2] and Stein–Weiss [SW]. They are applied to PDEs in compensated compactness (as seen in Sect. 4.5.2) and in regularity (for harmonic maps see Helein [Hel 2] Evans [Ev 2]. For the background material we recommend Brezis and Nirenberg [BN 3], and Semmes [Se] as references. The connection between Hardy space and the compensated compactness is revealed by Coifman, Lions, Meyer and Semmes [CLMS]. In the applications to the calculus of variations, the biting theorem due to Chacon is required, see Brooks and Chacon [BC]. Compensated compactness is also an important

tool in nonlinear analysis; we refer the reader to Tartar [Tar 1, Tar 2], DiPerna [Di] and Evans [Ev 2].

We briefly introduce the Γ -convergence in Sect. 4.6 by a result of Modica and Mortola [MM], see also Alberti [Al]. Readers who want to know more can read Dal Maso [D].

The Mumford–Shah model [MS] in the segmentation of the image processing is an interesting subject; the existence proof was given by Ambrosio [Amb], De Giorgi and [DG 2], and De Giorgi and Ambrosio [GA]. More details can be found in Ambrosio Fusco and Pallara [AFP], Dal Maso, Morel and Solimini [DMS] and Morel and Solimini [MS]. As to the regularity of the edge curve, see Morel [Mor]. However, all of these results require deep knowledge on BV functions. A much simplified model presented here is from Nordström [No].

Concentration compactness [LP] in combination with blowing up analysis [SU] is one of the most important techniques in nonlinear problems without compactness. Since interesting concrete problems require more background knowledge, they are out of the scope of this book. We present in Sect. 4.7 just an introduction of the idea: The concentration phenomenon and the role of the bubble (or best constant). Two examples on semilinear elliptic equations are studied: The subcritical exponent on R^n , see Coti Zelati [CZ], and the critical exponent on bounded domain, see Brezis and Nirenberg [BN 1].

Section 4.8 is the preliminary of the minimax method. The Palais–Smale condition [PS] and the mountain pass lemma due to Ambrosetti and Rabinowitz [AR] are introduced in accordance with the Ekeland minimizing principle. The proof was given by Shi [Shi] and Aubin and Ekeland [AE]. Examples are taken from Mawhin [Maw 2] and Ambrosetti and Rabinowitz [AR].

Chapter 5

Section 5.1: The most recommended book in introducing Morse theory on compact manifolds is Milnor [Mi 1]. The classics are Morse [Mo] and Morse and Cairns [MC]. The extension to infinite-dimensional space can be found in Chang [Ch 4], [Ch 5]. See also Mawhin and Willem [MW].

The background material in this section can be found in Palais [Pa 1], Palais and Smale [PS], Rothe [Ro 1], [Ro 2], [Ro 3], Marino and Prodi [MP 1], [MP 2], Chang [Ch 7], [Ch 8], Wang [Wa 1], [Wa 2], Castro and Lazer [CaL].

The critical point theory has been extended to non-differentiable functionals, see Chang [Ch 6] for locally Lipschitzian functionals on Banach space, and Kartiel [Ka], Ioffe, Schwartzmann [IS], and Corvellec, De Giovanni, Marzocchi [CGM] for continuous functions on metric spaces.

Section 5.2: There are several books on minimax methods, see for instance Rabinowitz [Ra 4], Struwe [St 1], Ghoussoub [GH], and Willem [Wi] etc.

The background material can be found in Rabinowitz [Ra 4], [Ra 5], Palais [Pa 2], [Pa 3], Ni [Ni], Chang [Ch 5], Tian [Ti], Liu [Liu 1], [Liu 2], Viterbo [Vi 2], Solimini [So], Chang, Long and Zehnder [CYZ], Fournier and Willem [FoW].

The notion of category was discovered by Ljusternik and Schnirelmenn [LjS 1]. The genus was introduced by Krasnosel'ski in [Kr 1], while for general index theory see Fadell and Rabinowitz [Fr]. The geometric index for S^1 is taken from Benci [Be 1] and Nirenberg [Ni 3]; other extensions of the index theory can be found in Benci and Rabinowitz [BR 2].

Section 5.3: There are many papers on periodic solutions for Hamiltonian systems via variational methods. The results related to this section are Rabinowitz [Ra 3], Weinstein [We 1], [We 2], Viterbo [Vi 1], Hofer and Zehnder [HZ 2], Struwe [St 1]. Further development on the Weinstein conjecture can be found in Hofer and Viterbo [HV], Hofer [Ho 2] and Liu and Tian [LT 1].

The following books are recommended: Hofer and Zehnder [HZ 1], Ekeland [Ek 2], Mawhin and Willem [MW].

Other important topics on the periodic solutions include:

- For Arnold conjecture on the symplectic fixed points and Lagrangian intersections, see Conley and Zehnder [CZ 1], Floer [Fl 1], [Fl 2], [Fl 3], Liu and Tian, [LT 2], Fukaya Ono [FO].
- The number of the periodic orbits on compact convex hypersurface has been estimated by Long and Zhu [LZ].
- For the N -body problem, see Bahri and Rabinowitz [Br].

Section 5.4: The main result of this section is taken from Chang and Yang [CY 1], [CY 2], and Chang and Liu [CL 2]. See also Han [Han], Chen and Li [ChL 1], [ChL 2]. The related prescribing scalar curvature problem on S^n has received extensive attention. For $n = 3$ see Bahri and Coron [BC], and for high dimension, see Schoen and Zhang [SZ], Bahri [Bar 1], [Bar 2], Li [Li 1], and Chen and Lin [ChL].

Section 5.5: Conley's index theory was introduced by Conley [Co], in which isolating neighborhoods for isolated invariant sets are compact. The theory is completed by Conley and Zehnder [CZ 2], Salamon [Sal], Salamon and Zehnder [SZ]. There are many ways to extend the theory to infinite-dimensional spaces, see Rybakowski [Ry], Rybakowski and Zehnder [RZ], Benci [Be 2] etc. The relationship between the Conley theory and the Morse theory can be found in Chang and Ghoussoub [CG]. For further reading see Michaiikov [Mic] and Smoller [Smo].

References

- [AF] Acerbi E., Fusco N., Semicontinuity problems in the calculus of variations, *Arch. Rat. Mech. Anal.* 86 (1984), 125–145.
- [Ad] Adams R. A., *Sobolev spaces*, Acad. Press (1975).
- [Al] Alberti G., *Variational models for phase transition, an approach via Γ -convergence calculus of variations and partial differential equations*. Topics on geometrical evolution problems and degree theory (ed. by Ambrossio, Dancer), Springer-Verlag, (2000) 95–114.
- [Am 1] Amann H., *Order structures and fixed points*, Ruhr-Universität, Bochum (1977).
- [Am 2] Amann H., Fixed point equations and elliptic eigenvalue problems in ordered Banach spaces, *SIAM Rev.* 18 (1976), 620–709.
- [Am 3] Amann H., On the number of solutions of nonlinear equations in ordered Banach spaces, *J. Funct. Anal.* 14 (1973), 346–384.
- [Am 4] Amann H., Saddle points and multiple solutions of differential equations, *Math. Zeit.* 169 (1979), 122–166.
- [AC] Amann H., Crandall M., On some existence theorems for semi-linear elliptic equations, *Indiana Univ. Math. J.* 27 (1978), 779–790.
- [AR] Ambrosetti A., Rabinowitz P., Dual variational methods in the critical point theory and applications, *J. Funct. Anal.* 14 (1973), 349–381.
- [Amb] Ambrosio L., Variational problems in SBV and image segmentation, *Acta Appl. Math.* 17 (1989), 1–40.
- [AFP] Ambrosio L., Fusco N., Pallara D., *Functions of bounded variation and free discontinuity problem*, Oxford Sci Publ. (2000).
- [Ar 1] Arnold V. I., *Mathematical methods of classical mechanics*, Springer-Verlag (1978).
- [Ar 2] Arnold V. I., Proof of a theorem of A. N. Kolmogorov on the conservation of quasi-periodic motions under a small change of the Hamiltonian function, *Uspekhi Mat. Nauk* 18:5 (1963), 9–36.
- [AE] Aubin J. P., Ekeland I., *Applied nonlinear analysis*, John Wiley and Sons (1984).
- [Au 1] Aubin Th., *Nonlinear analysis on manifolds, Monge Ampere equations*, Grundlehren 252 (1982), Springer-Verlag.
- [Au 2] Aubin Th., Equations différentielles nonlinéaires et problème de Yamabe concernant la courbure scalaire, *J. Math. Pure Appl.* 55 (1976), 269–293.

- [Au 3] Aubin Th., Meilleures constants dans le théorème d'inclusion de Sobolev et un théorème de Fredholm nonlinéaire pour la transformation conforme de la courbure scalaire, *J. Funct. Anal.* 32 (1979), 148–174.
- [Ba 1] Bahri A., Critical points at infinity in some variational problems, Pitman Research Notes in Math. V. 182, Longman (1989).
- [Ba 2] Bahri A., The scalar curvature problem on sphere of dimension larger or equal to 7, Preprint (1994).
- [BC] Bahri A., Coron J. M., The scalar curvature problem on the standard three dimensional sphere, *J. Funct. Anal.* 95 (1991), 106–172.
- [Bal 1] Ball J., Convexity conditions and existence theorems in nonlinear elasticity, *Arch. Rat. Mech. Anal.* 63 (1977), 337–403.
- [Bal 2] Ball J., A version of fundamental theorem for Young measures, PDE's and continuum models of phase transitions (Rasclé M. Serre D. Slemrod M. eds.), Lecture Notes in Physics, 344, Springer-Verlag (1989), 207–215.
- [Bm] Ballmann W., Der Satz von Lyusternik und Schnirelmann, *Bonn. Math. Schr. Nr. 102* (1978).
- [B] Bauschke H. H., The approximation of fixed points of compositions of nonexpansive mappings in Hilbert space, *JMAA* 202 (1996) 150–159.
- [BB] Bauschke H. H., Borwein J. M., On projection algorithms for solving convex feasible problems, *SIAM Rev.* 38 (1996), 367–426.
- [Be 1] Benci V., A geometric index for the group S^1 and some applications to the study of periodic solutions of ordinary differential equations. *Comm. Pure Appl. Math.* 274 (1981) 393–432.
- [Be 2] Benci V., A new approach to the Morse Conley theory and some applications. *Ann. Mat. Pura Appl. (IV)* 158 (1991), 231–305.
- [BR] Benci V., Rabinowitz P., Critical point theorems for indefinite functionals, *Invent. Math.* 52 (1979), 241–273.
- [Ber 1] Berger M. S., Nonlinearity and functional analysis, Acad. Press (1977).
- [Ber 2] Berger M. S., On Riemannian structures of prescribing Gaussian curvature for compact 2-manifolds, *J. Diff. Geom.* 5 (1971), 325–332.
- [Bi] Birkhoff G. D., Dynamical systems with two degrees of freedom, *Trans. AMS* 18 (1917), 199–300.
- [BZS] Borisovich Y. G., Zvyagin V. G., Sapronov Y. I., Nonlinear Fredholm maps and the Leray Schauder degree theory, *Russian Math. Surveys* 324 (1977) 1–54.
- [Bo 1] Bott R., Nondegenerate critical manifolds, *Ann. Math.* 60 (1954), 248–261.
- [Bo 2] Bott R., Lectures on Morse theory, old and new, *Bull. AMS* 7 (1982), 331–358.
- [BT] Bott R., Tu L. W., *Differential forms in algebraic topology*, Springer-Verlag (1982).
- [Bo] Bourbaki N., *Topologie générale*, Hermann, Paris (1940).
- [Bou] Bourgain J., Global solutions of nonlinear Schrödinger equations, *AMS Colloquium Publ.* Vol. 46 (1999).
- [Br 1] Brezis H., Opérateurs maximaux monotones, Lecture Notes, Vol. 5, North Holland Amsterdam (1973).
- [BrC] Brezis H., Coron J. M., Multiple solutions of H-systems and Rellich's conjecture, *Comm. Pure Appl. Math.* 37 (1984), 149–187.
- [BL] Brezis H., Lieb E. H., A relation between pointwise convergence of functions and convergence of functionals, *Proc. AMS* 88 (1983), 486–490.

- [BN 1] Brezis H., Nirenberg L., Positive solutions of nonlinear elliptic equations involving critical Sobolev exponents, *Comm. Pure Appl. Math.* 36 (1983), 437–477.
- [BN 2] Brezis H., Nirenberg L., Remarks on finding critical points, *Comm. Pure Appl. Math.* 64 (1991), 939–963.
- [BN 3] Brezis H., Nirenberg L., Degree theory and BMO, *Selecta Math.* 1 (1995), 197–263.
- [BNS] Brezis H., Nirenberg L., Stampacchia G., A remark on Ky Fan’s minimax principle, *Boll. Un. Mat. Ital.* (4) 6, (1973), 293–300.
- [BC] Brooks J. K., Chacon R. V., Continuity and compactness of measures, *Adv. Math.* 37 (1980), 16–26.
- [Br] Brouwer L. E. J., Über Abbildung von Mannigfaltigkeiten, *Math. Ann.* 71 (1912), 97–115.
- [Bd 1] Browder F., Covering spaces, fibre spaces and local homeomorphisms, *Duke Math. J.* 21 (1974), 329–336.
- [Bd 2] Browder F., Non-expansive nonlinear operators in a Banach space, *Proc. Nat. Acad. Sci. USA* 54 (1965), 1041–1044.
- [Bd 3] Browder F., On the unification of the calculus of variations and the theory of monotone nonlinear operators in Banach space, *Proc. Nat. Acad. USA* 56 (1966), 419–425.
- [Bd 4] Browder F., Nonlinear elliptic boundary value problems, *Bull. AMS* 69 (1963), 862–874.
- [Bd 5] Browder F., The fixed point theory of multi-valued mappings in topological vector spaces, *Math. Ann.* 117 (1968), 283–301.
- [Bu] Buttazzo G., Semicontinuity, relaxation and integral representation in the calculus of variations, *Pitman Res. Notes Math. Ser.*, 207, Longman (1989).
- [Cac 1] Caccioppoli R., Sugli elementi uniti delle trasformazioni funzionali, *Rend. Sem. Mat. Padova* 3 (1932), 1–15.
- [Cac 2] Caccioppoli R., Sulle corrispondenze funzionali inverse diramate, teoria generale e applicazioni ad alcune equazioni funzionali nonlineari al problema di Plateau, I, II, *Rend. Acad. Naz. Lincei* (6) 24 (1936), 258–263, 416–421.
- [CGS] Caffarelli L., Gidas B., Spruck J., Asymptotic symmetry and local behavior of semilinear elliptic equations with critical Sobolev growth, *Comm. Pure Appl. Math.* 42 (1989), 271–297.
- [Ca] Caristi J., Fixed point theorems for mappings satisfying inwardness conditions, *Tran. AMS* 215 (1976), 241–251.
- [CC] Carleson L., Chang A. S. Y., On the existence of an extremal function for an inequality of J. Moser, *Bull. Sc. Math.* 2 110 (1986), 113–127.
- [CaL] Castro A., Lazer A. C., Critical point theory and the number of solutions of a nonlinear Dirichlet problem, *Ann. Pura Appl.* 70 (1979), 113–137.
- [Ce] Cellina A., Approximation of set-valued functions and fixed point theorems, *Ann. Pura Appl. Mat.* 4 (1969), 17–24.
- [CY 1] Chang A. S. Y., Yang P., Prescribing Gaussian curvature on S^2 , *Acta Math.* 159 (1987), 215–259.
- [CY 2] Chang A. S. Y., Yang P., Conformal deformation of metric on S^2 , *J. Diff. Geom.* 23 (1988), 259–296.
- [Ch 1] Chang K. C., The obstacle problem and partial differential equations with discontinuous nonlinear terms, *Comm. Pure Appl. Math.*, 33 (1980), 117–146.

- [Ch 2] Chang K. C., Free boundary problems and set valued mappings, *J. Diff. Eq.* 49 (1981), 1–28.
- [Ch 3] Chang K. C., Remarks on some free boundary problems for equilibrium equation of plasmas, *Comm. PDE* 5 (1980), 741–751
- [Ch 4] Chang K. C., Infinite dimensional Morse theory and its applications, les presses de l' univ. de Montreal SMS 97 (1985).
- [Ch 5] Chang K. C., Infinite dimensional Morse theory and multiple solution problems, Birkhauser (1993).
- [Ch 6] Chang K. C., Variational methods for non-differentiable functionals and their applications to PDE, *J. Math. Anal. Appl.* 80 (1981), 102–129.
- [Ch 7] Chang K. C., Solutions of asymptotic linear operator equations via Morse theory, *Comm. Pure Appl. Math.* 34 (1981), 693–712.
- [Ch 8] Chang K. C., H^1 versus C^1 isolated critical points, *C. R. Acad. Sci. Paris* 319 (1994), 441–446.
- [CG] Chang K. C., Ghoussoub N., The Conley index and the critical groups via an extension of Gromoll-Meyer theory, *Topol. Methods in Nonlinear Analysis* 7 (1996), 77–93.
- [CL 1] Chang K. C., Liu J. Q., Morse theory under general boundary conditions, *J. System Sci. and Math. Sci.* 4 (1991), 78–83.
- [CL 2] Chang K. C., Liu J. Q., On Nirenberg's problem, *International J. Math.* 4 (1993) 35–58.
- [CYZ] Chang K. C., Long Y., Zehnder E., Forced oscillations for the triple pendulum, *Analysis et cetera* (P. Rabinowitz, E. Zehnder eds.) Academic Press, (1990).
- [ChL] Chen C. C., Lin C. S., Blowing up with infinite energy of conformal metric on S^n , *Comm. PDE* 24 (1999), 785–799.
- [CD] Chen W. X., Ding W. Y., Scalar curvatures on S^2 . *Tran. Amer. Math. Soc.* 303 (1987), 365–382.
- [ChL 1] Chen W., Li C., A priori estimates for prescribing scalar curvature equations, *Ann. Math.* 145 (1997), 547–564.
- [ChL 2] Chen W., Li C., A necessary and sufficient condition for the Nirenberg problem, *Comm. Pure Appl. Math.* 48 (1995), 657–667.
- [Chi] Chipot M., *Elements in nonlinear analysis*, Birkhauser, (2000).
- [CH] Chow S. N., Hale J., *Methods of bifurcation theory*, Springer-Verlag, (1982).
- [Cl] Clarke F. H., *Optimization and nonsmooth analysis*, Wiley Interscience (1983).
- [CE] Clarke F. H., Ekeland I., Hamiltonian trajectories having prescribed minimal period, *Comm. Pure Appl. Math.* 33 (1980), 103–116.
- [CLMS] Coifman R., Lions, P. L., Meyer Y., Semmes S., Compensated compactness and Hardy spaces, *J. Math. Pure Appl.* (9), 72 (1993), 247–286.
- [Co] Conley, C., Isolated invariant sets and the Morse index. *CBMS* 38 (1978), AMS.
- [CZ 1] Conley C., Zehnder E., The Birkhoff-Lewis fixed point theorem and a conjecture of V. I. Arnold, *Inv. Math.* 73 (1983) 33–49.
- [CZ 2] Conley C., Zehnder E., A Morse type index theory for flows and periodic solutions to Hamiltonian systems, *Comm. Pure Appl. Math.* 37 (1984), 207–253.

- [CGM] Corvellec J. N., De Giovanni M., Marzocchi M., Deformation properties for continuous functions and critical point theory, preprint Univ. di Pisa (1992).
- [CZ] Coti Zelati V., Critical point theory and applications to elliptic equations in R^n , *Nonlinear Functional Analysis and Appl. to Diff. Eqs.* (A. Ambrosetti, K. C. Chang, I. Ekeland eds.) World Sci. (1998), 102–121.
- [CR] Coti Zelati V., Rabinowitz P., Homoclinic type solutions for a semilinear elliptic partial differential equation on R^n , *Comm. Pure Appl. Math.* 45, (1992), 1217–1269.
- [CLi] Crandall M., Liggett T., Generations of semigroups of nonlinear transformations on general Banach spaces, *Amer. J. Math.* 93 (1971), 265–298.
- [CR 1] Crandall M., Rabinowitz P., Bifurcation, perturbations of simple eigenvalue and linearized stability, *Arch. Rat. Mech. Anal.* 52 (1971), 161–180.
- [CR 2] Crandall M., Rabinowitz P., The Hopf bifurcation theorem, *Arch. Rat. Math. Anal.* 67 (1977), 53–72.
- [Dac] Dacorogna B., *Direct methods in the calculus of variations*, Springer-Verlag (1989).
- [DM] Dacorogna B., Marcellini, P. A counter example in the vectorial calculus of variations, *Material Instabilities in Continuum Mechanics*, Proc. (ed. J. Ball), Oxford Sci. Publ. (1988), 77–83.
- [D] Dal Maso G., *An introduction to Γ -convergence*, Birkhauser (1993).
- [DMS] Dal Maso G., Morel J. M., Solimini S., A variational method in image segmentation: existence and approximation results, *Acta Math.* 168 (1992), 89–151.
- [Dan 1] Dancer N., Global solution branches for positive maps, *Arch. Rat. Math. Anal.* 55 (1974), 207–213.
- [Dan 2] Dancer N., The effect of domain shape on the number of positive solutions of certain nonlinear equations, *J. Diff. Equs.* 87 (1990), 316–339.
- [Dar] Darbo G., Punti uniti in trasformazioni a codominio non compatto, *Rend. Sem. Univ. Padua* 24 (1955), 84–92.
- [dF] de Figueiredo D. G., Lectures on Ekeland variational principle with applications and detours, *Lect. Notes, College on variational problems in analysis*, ICTP Trieste (1988).
- [FLN] de Figueiredo D. G., Lions P. L., Nussbaum R., J. A priori estimates and existence of positive solutions of semilinear elliptic equations, *Math. Pures et Appl.* 61 (1982), 41–63.
- [DG 1] De Giorgi E., Frontiere orientate di misura minima, *Sem. Mat. Scuola Norm. Sup. Pisa* (1961).
- [DG 2] De Giorgi E., Free discontinuity problems in the calculus of variations, *Frontiers in pure and applied Mathematics, a collection of papers dedicated to J. L. Lions* (Dautray R., ed.) North Holland (1991), 55–62.
- [DA] De Giorgi E., Ambrosio L., Un nuovo funzionale del calcolo delle variazioni, *Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. Rend. Lincei* (8), Mat. Appl. 82 (1988), 199–210.
- [De] Deimling K., *Nonlinear functional analysis*, Springer-Verlag (1985).
- [DT] DeTurck D., Existence of metrics with prescribing Ricci curvatures: local theory, *Invent. Math.* 65 (1981), 179–208.
- [DiT] Ding W. Y., Tian G., Energy identity for a class of approximate harmonic maps from surfaces, *Comm Anal. Geom.* 3 (1995), 543–544.

- [Di] DiPerna R. J., Compensated compactness and general systems of conservation laws, *Trans. AMS* 292 (1985), 383–420.
- [DM] DiPerna R. J., Majda A. J. Oscillations and concentration in weak solutions of the incompressible fluid equations, *Comm. Math. Phys.* 108, (1987), 667–689.
- [Du 1] Dugundji J., An extension of Tietze’s theorem, *Pac. J. Math.* 1 (1951), 353–367.
- [Du 2] Dugundji J., *Topology*, Allyn and Bacon (1966).
- [DG] Dugundji J., Granas A., Fixed point theory, *Monografie Matematyczne* 16, Polish Sci. Publ. (1982).
- [DS] Dunford N., Schwartz, J. T., *Linear operators*, Vol. I., Interscience (1958).
- [DL] Duvaut G., Lions J. L., *Les inéquations en mécanique et en physique*, Dunod (1972).
- [ES] Eells J., Sampson J. H., Harmonic mappings of Riemannian manifolds, *J. AMS* 86 (1964), 109–160.
- [Ek 1] Ekeland I., Nonconvex minimization problems, *Bull. AMS* 1 (1979), 443–474.
- [Ek 2] Ekeland I., Convexity methods in Hamiltonian mechanics, *Ergebnisse d. Math. (Ser. III)* 19, Springer-Verlag (1990).
- [ET] Ekeland I., Temam R., *Convex analysis and variational problems*, North Holland (1976).
- [EIT 1] Elworthy K. D., Tromba A. J., Degree theory on Banach manifolds, *Non-linear Functional Analysis (F. Browder ed.) Proc. Symp. Pure Math. Part I* 18 (1970), 86–94.
- [EIT 2] Elworthy K. D., Tromba A. J., Differential structures and Fredholm maps on Banach manifolds, *Global analysis (S. S. Chern and S. Smale eds.) Proc. Symp. Pure Math.* 15 (1970), 45–94.
- [Ev 1] Evans L. C., *Weak convergence methods for nonlinear partial differential equations*, AMS, Providence (1990).
- [Ev 2] Evans L. C., Quasi-convexity and partial regularity in the calculus of variations, *Arch. Rat. Mech. Anal.* 95 (1986), 227–252.
- [EG] Evans L. C., Gariepy R., *Measure theory and fine properties of functions*, CRC Press (1992).
- [EM] Evans L. C., Muller S., Hardy spaces and the two dimensional Euler equations with nonnegative vorticity, *J. AMS* (1994), 199–219.
- [Fa] Faber C., Beweis, dass unter allen homogenen Membrane von gleicher Fläche und gleicher Spannung die Kriesformige die tiefsten Grundton gibt, *Sitzungsber-Bayer Acad. Wiss. Math. Phys. Munich* (1923), 169–172.
- [FR] Fadell E. R., Rabinowitz P., Generalized cohomological index theories for Lie group actions with applications to bifurcation questions for Hamiltonian systems, *Invent. Math.* 45 (1978), 139–174.
- [FK 1] Fan K., Fixed point and minimax theorems in locally convex topological spaces, *Proc. Nat. Acad. Sci. USA* 38 (1952), 121–126.
- [FK 2] Fan K., A generalization of Tychnoff’s fixed point theorem, *J. Math. Anal. Appl.* 142 (1961), 305–310.
- [FK 3] Fan K., A minimax inequality and applications, *Inequalities-III (Shisha O. ed.) Acad. Press* (1972).
- [Fe] Federer H., *Geometric measure theory*, Springer-verlag (1969).
- [FP] Fitzpatrick P. M., Pejsachowicz J., Parity and generalized multiplicity, *TAMS* 326 (1991), 281–305.

- [FPR] Fitzpatrick P. M., Pejsachowicz J., Rabier P. J., Orientability of Fredholm families and topological degree for orientable Fredholm mappings, *J. Funct. Anal.* 124 (1994), 1–39.
- [Fl 1] Floer A., Morse theory for fixed points of symplectic diffeomorphisms, *Bull. AMS* 16 (1987), 279–281.
- [Fl 2] Floer A., Witten complex and infinite dimensional Morse theory, *J. Diff. Geom.* 18 (1988), 207–221.
- [Fl 3] Floer A., Symplectic fixed points and holomorphic spheres, *Comm. Math. Phys.* 120 (1989), 576–611.
- [FW] Floer A., Weinstein A., Nonspreading wave packets for cubic Schrodinger equations with a bounded potential, *J. Funct. Anal.* 69 (1986), 397–408.
- [FoW] Fournier G., Willem M., Relative category and the calculus of variations, Preprint.
- [Fr] Friedman A., Variational principle and free boundary problems, John Wiley and Sons (1982).
- [FO] Fukaya K., Ono K., Arnold conjecture and Gromov-Witten invariants for general symplectic manifolds, *The Arnoldfest*, Fields Inst. Commun. 24, (173–190).
- [Gh] Ghoussoub N., Duality and perturbation method in critical point theory, Cambridge Univ. Press (1993).
- [GMS] Giaquinta M., Modica G., Soucek J., Cartesian currents in the calculus of variations, Part I, Cartesian currents, Part II, Variational integrals, Springer-Verlag (1998).
- [GNN] Gidas B., Ni W. M., Nirenberg L., *Comm. Math. Phys.* 68 (1979), 209.
- [GS] Gidas B., Spruck J., A priori bounds for positive solutions of nonlinear elliptic equations, *Comm. PDE* 6 (1981), 883–901.
- [GT] Gilbarg D., Trudinger N. S., Elliptic partial differential equations of second order, 2nd edition, *Grundlehren der Mathematik*, 224, Springer-Verlag (1983).
- [Gi] Giusti E., Minimal surfaces and functions of bounded variations, *Monographs in Mathematics* 80, Birkhauser (1984).
- [GK] Goebel K., Kirk W., Iteration process for nonexpansive mappings, *Comtemp. Math.* 21 (1983), 115–123.
- [GH] Greenberg M. J., Harper J. R., Algebraic topology, A first course, The Benjamin/Cummings Publ. Co. (1981).
- [GM] Gromoll D., Meyer W., On differential functions with isolated critical points, *Topology* 8 (1969), 361–369.
- [Gu] Gui C., Existence of multi-bump solutions for nonlinear Schrodinger equations via variational method, *Comm. Part. Diff. Eq.* 21 (1996), 787–820.
- [GP] Guillemin V., Pollack A., Differential topology, Prentice-Hall (1974).
- [Ha] Hadamard J., Sur les transformations ponctuelles, *Bull. Soc. Math. France*, 34, (1906), 7–84.
- [Ham] Hamilton, R., The inverse function theorem of Nash and Moser, *Bull. AMS* 7 (1982), 65–222.
- [Han] Han Z. C. Prescribing Gaussian curvature on S^2 , *Duke Math. J.* 61 (1990), 679–703.
- [Hann] Hanner, Some theorems on absolute neighborhood retracts, *Arkiv Math.* 1 (1951), 389–408.
- [HS] Hartman P., Stampacchia G., On some nonlinear elliptic differential equations, *Acta Math.* 115 (1966), 271–310.

- [Hel 1] Helein F., Régularité des applications faiblement harmoniques entre une surface et une variété riemannienne, *CR Acad. Sci. Paris* 312 (1991), 591–596.
- [Hel 2] Helein F., Constant mean curvature surfaces, harmonic maps and integrable systems, Birkhauser (2001).
- [Hi 1] Hildebrandt S., Nonlinear elliptic systems and harmonic mappings, *Proc. Beijing Symp. Diff Geom. and Diff Eqs.*, Gordon and Breach (1983), 481–615.
- [Hi 2] Hildebrandt S., Calculus of variations, I, II. *Grundlehrer Mathematik Wiss.* 310 311, Springer-Verlag (1996).
- [Ho 1] Hofer H., A note on the topological degree at a critical point of mountain pass type, *Proc. AMS* 90 (1984), 309–315.
- [Ho 2] Hofer H., Pseudo holomorphic curves in symplectisations with applications to the Weinstein conjecture in dimension three, *Invent. Math.* 114 (1993), 515–563.
- [HV] Hofer H., Viterbo C., The Weinstein conjecture in the presence of holomorphic curves, *Comm. Pure Appl. Math.* 45 (1992), 583–622.
- [HZ 1] Hofer H., Zehnder E., Periodic solutions on hyper-surfaces and a result by C. Viterbo, *Invent. Math.* 90 (1987), 1–9.
- [HZ 2] Hofer H., Zehnder E., Symplectic invariants and Hamiltonian dynamics, Birkhauser (1994).
- [Hon 1] Hong C. W., A best Constant and the Gaussian curvature, *Proc. AMS* 97 (1986), 737–747.
- [Hon 2] Hong C. W., A note on prescribed Gaussian curvature on S^n , *Partial Diff. Eqs.* (in Chinese), 1 (1987), 13–20.
- [Hor 1] Hormander L., The analysis of linear partial differential operators, Vol. III, *Grundlehren der Math. Wiss.* 274, Springer-Verlag (1984).
- [Hor 2] Hormander L., The boundary value problems of physical geodesy, *Arch. Rat. Math. Anal.* 62 (1976), 1–52.
- [IS] Ioffe A., Schwartzman E., Metric critical point theory I, Morse regularity and homology stability of a minimum, *J. Math. Pure Appl.* 75 (1996), 125–153.
- [Iz] Ize J., Bifurcation theory for Fredholm operators, *Memoirs, AMS*, 174, Providence (1976).
- [JLS] Jeanjean H., Lucia M., Stuart C. A., Branches of solutions to semilinear elliptic equations on R^n , *Math. Z.* 230 (1999), 79–105.
- [Kak] Kakutani S., A generalization of Brouwer’s fixed point theorem, *Duke Math. J.* 8 (1941), 457–459.
- [Kar] Kartiel G., Mountain pass theorems and global homeomorphism theorems, *Analyse Nonlineaire* 11 (1994), 189–209.
- [Ka] Kazdan J. L., Prescribing the curvature of a Riemannian manifold, *CBMS* no. 57 (1987).
- [KW 1] Kazdan J., Warner F., Curvature functions for compact 2-manifolds, *Ann. of Math.* 99 (1974), 14–47.
- [KW 2] Kazdan J., Warner F., Existence and conformal deformation of metrics with prescribing Gaussian and scalar curvatures, *Ann. of Math.* 101 (1975), 317–331.
- [KP 1] Kinderlehrer D., Pedregal P., Characterization of Young measure generated by gradients, *Arch. Rat. Mech. Anal.* 115 (1991), 329–365.

- [KP 2] Kinderlehrer D., Pedregal P., Gradient Young measure generated by sequences in Sobolev spaces, *J. Geometric Analysis* 4 (1994), 59–90.
- [KS] Kinderlehrer D., Stampacchia G., An introduction of variational inequalities and their applications, Acad. Press (1980).
- [Kl] Klingenberg W., Lecture on closed geodesics, *Grundlehren der Math.* 230, Springer-Verlag (1978).
- [Ko] Kolmogorov A. N., On the conservation of conditionally periodic motions for a small change in Hamilton's function (Russian), *Dokl. Acad. Nauk SSSR* 98 (1954), 525–530.
- [KKM] Knaster B., Kuratowski K., Mazurkiewicz S., Ein Beweis des Fixpunktsatzes für n -dimensionale Simplexe, *Fundamenta Mathematica* 14 (1929), 132–137.
- [Kn] Kneser H., Eine direkte Ableitung des Zornsche Lemmas aus dem Auswahlaxiom, *Math. Zeit.* 53 (1950), 110–113.
- [Krh] Krahn E., Über eine von Rayleigh formulierte Minimaleigenschaft des Kreises, *Math. Ann.* 94 (1925), 97–100.
- [Kr 1] Krasnosel'ski M. A., Topological methods in the theory of nonlinear integral equations, MacMillan (1964).
- [Kr 2] Krasnosel'ski M. A., Positive solutions of operator equations, Groningen, Noordhoff (1964).
- [Kui] Kuiper N. H., The homotopy type of the unitary group of Hilbert space, *Topology* 3 (1965) 19–30.
- [Kuk] Kuksin S. B., Nearly integrable infinite dimensional Hamiltonian systems, *Lecture Notes in Math.* 1556, Springer-Verlag (1993).
- [LU] Ladyzhenskaya O. A., Ural'ceva N. N., Linear and quasilinear elliptic equations, Acad. Press (1968).
- [La] Larsen R., Functional analysis, Marcel Dekker (1973).
- [Le] Leray J., La théorie des points fixes et ses applications en analyse, *Proc. ICM*, Cambridge 2 (1950), 202–208.
- [LL] Leray J., Lions J. L., Quelques résultats de Vishik sur les problèmes elliptiques non linéaires par les méthodes de Minty Browder, *Bull. Soc. Math. France* 93 (1965), 97–107.
- [LS] Leray J., Schauder J., Topologie et équations fonctionnelles, *Ann. Sci. Ecole Norm. Sup.* 51 (1934), 45–78.
- [Li 1] Li Y. Y. Liouville-type theorems and Harnack-type inequalities for semi-linear elliptic equations, *J. d'analyse mathématique* 90 (2003), 27–87.
- [Li 2] Li Y. Y. Prescribing scalar curvature on S^n and related problems, Part I, *J. Diff. Eqs.* 120, (1995), 319–410, Part II, *Comm. Pure Appl. Math.* 49 (1996), 541–597.
- [Lieb] Lieb E. H. Sharp constants in the Hardy-Littlewood-Sobolev and related inequalities, *Ann. of Math.* 118 (1983), 349–374.
- [Lie] Lieberman G. M., 2nd order parabolic differential equations, World Sci. (1996).
- [Lin] Lin F. H., A remark on the map $\frac{x}{|x|}$, *C. R. Sci. Paris* 305 (1987), 529–531.
- [Lid] Lindqvist P., On the equation $div(|\nabla u|^{p-2}\nabla u) + \lambda|u|^{p-2}u = 0$, *Proc. AMS* 109 (1992), 157–164.
- [LJ 1] Lions J. L., Quelques méthodes de résolution des problèmes aux limites non linéaires, Dunod (1969).
- [LS] Lions J. L., Stampacchia G., Variational inequality, *Comm. Pure Appl. Math.* 20 (1967), 493–519.

- [LP 1] Lions P. L., The concentration compactness principle in the calculus of variations, Part I and II, *Analyse Nonlinéaire* 1 (1984), 109–145, 223–283.
- [LT 1] Liu G., Tian G., Weinstein conjecture and GW-invariants,
- [LT 2] Liu G., Tian G., Floer homology and the Arnold conjecture, *J. Diff. Geom.* 49 (1998), 1–74.
- [Liu 1] Liu J. Q., A Morse index of saddle points, *System Sci. and Math. Sci.* 2 (1989), 32–39.
- [Liu 2] Liu J. Q., A generalized saddle point theorem, *J. Diff. Eqs.* (1989) 372–385.
- [Lj] Ljusternik L., The topology of function spaces and the calculus of variations in the large, *Trudy Mat. Inst. Steklov* 19 (1947).
- [LjS 1] Ljusternik L., Schnirelmann L., *Méthodes topologiques dans les problèmes variationnelles*, *Actualités Sci. Industr.* 188 (1934).
- [LjS 2] Ljusternik L., Schnirelmann L., Sur le problème de trois géodesiques fermées sur les surfaces de genre 0, *C. R. Acad. Sci Paris* 189 (1929), 269–271.
- [Ll] Lloyd N. G., *Degree theory*, Cambridge Univ. Press (1977).
- [Lo] Long Y. M., *Index theory for symplectic paths with applications*, *Progress in Math.* 207, Birkhauser (2002).
- [LZ] Long Y. M., Zhu C. Closed characteristics on compact hypersurfaces in R^{2n} , *Ann. of Math.* 155 (2002), 317–368.
- [Ma] Ma T. W., *Topological degrees of set-valued compact fields in locally convex spaces*, *Dissertationes Mathematicae*, 92 Warszawa (1972).
- [Man] Mann W. R., Mean value methods in iterations, *Proc. AMS* 4 (1953), 506–510.
- [Mar] Marcellini P., Approximation of quasiconvex functions and lower semi-continuity of multiple integrals, *Manuscripta Math.* 51 (1985), 1–28.
- [MP 1] Marino A., Prodi G., La teoria di Morse per spazi di Hilbert, *Rend. Sem. Mat. Univ. Padova*, 41 (1968), 43–68.
- [MP 2] Marino A., Prodi G., Metodi perturbativi nella teoria di Morse, *Boll. Un. Mat. Ital. Suppl.* 3 (1975), 1–32.
- [Maw 1] Mawhin J., *Topological degree methods in nonlinear boundary value problems*, AMS (1979).
- [Maw 2] Mawhin J., *Problèmes de Dirichlet, variationnels nonlinéaires*, Univ. de Montréal 104 (1987)
- [Maw 3] Mawhin J. *Functional analysis and BVP*, *Studies in Math.* 14 (J. Hale ed.) (1977), 128–168.
- [MW] Mawhin J., Willem M., *Critical point theory and Hamiltonian systems*, *Appl. Math. Sci.* 74, Springer-Verlag (1989).
- [Mic] Michaikov K., *Conley index theory*, *Lect. Notes*, 1609 Springer-Verlag (1996), 119–207.
- [Mi 1] Milnor J., *Morse theory*, Princeton Univ. Press (1963).
- [Mi 2] Milnor J., *Topology from the differential viewpoint*, Univ. Press of Virginia, Charlottesville (1969).
- [Mi 3] Milnor J., *Lecture on the h-cobordism theorem*, Princeton Univ. Press (1965).
- [Min] Minty G., On a monotonicity method for the solutions of nonlinear equations in Banach spaces, *Proc. Nat. Acad. Sci. USA*, 50 (1963), 1038–1041.
- [MM] Modica L., Mortola S., Un esempio di Γ -convergenza, *Boll. U. M. I.* 14-B (1977), 285–299.

- [Mor] Morel J. M., The Mumford Shah conjecture in image processing, *Asterisque*, 241, Expose 813 (1997), 221–242.
- [MS] Morel J. M., Solimini S., *Variational models in image segmentation*, Birkhauser (1994).
- [Mo 1] Morrey C. B., Quasi-convexity and the lower semicontinuity of multiple integrals, *Pacific J. Math.* 2 (1952), 25–53.
- [Mo 2] Morrey C. B., *Multiple integrals in the calculus of variations*, Springer (1966).
- [Mo] Morse M., *The calculus of variations in the large*, AMS Coll. Publ. 18 (1934).
- [MC] Morse M., Cairns S. S., *Critical point theory in global analysis and differential topology*, Acad. Press (1969).
- [MT] Morse M., Tompkins C. B., The existence of minimal surfaces of general critical types, *Ann. of Math.* 40 (1939), 443–472.
- [Mos 1] Moser J., A new technique for the construction of solutions of nonlinear differential equations, *Proc. Nat. Acad. Sci. USA* 47 (1961), 1824–1831.
- [Mos 2] Moser J., A rapidly convergent iteration method and nonlinear partial differential equations, I, II, *Ann. Scuola Norm. Sup. Pisa* 20 (1966), 226–315, 449–535.
- [Mos 3] Moser J., Convergent series expansions for quasi-periodic motions, *Math. Ann.* 169 (1967), 136–176.
- [Mos 4] Moser J., On a nonlinear problem in differential geometry, *Dynamical systems*, Acad. Press (1973).
- [Mos 5] Moser J., A sharp form of an inequality by N. Trudinger, *Indiana U. Math. J.* 20 (1971), 1077–1092.
- [Mul 1] Muller S., *Variational models for microstructure and phase transitions*, *Lecture Notes in Math*, 1713 *Calculus of variations and geometric evolution problems* (Hildebrandt S., Struwe M., eds.) Springer-Verlag (1999), 85–210.
- [Mul 2] Muller S., A sharp version of Zhang’s theorem on truncating sequences of gradients, *Trans. AMS*,
- [MS] Mumford D., Shah J., *Boundary detection by minimizing functionals*, *IEEE Conference on computer vision and pattern recognition*, San Francisco (1985).
- [Na 1] Nash J., The embedding problem for Riemann manifolds, *Ann. of Math.* 63 (1956), 20–63.
- [Na 2] Nash J., Continuity of solutions of parabolic and elliptic equations, *Amer. J. Math.* 80 (1958), 931–954.
- [Na 3] Nash J., Non-cooperative games, *Ann. of Math.* 54 (1951), 286–295.
- [NN] Newlander A., Nirenberg L., Complex coordinates in almost complex manifolds, *Ann. Math.* 65 (1957), 391–404.
- [Ni] Ni W. M., Some minimax principles and their applications in nonlinear elliptic equations, *J. d’analyse Math.* 37 (1980), 248–275.
- [NT] Ni W. M., Takagi I., On the shape of least energy solutions to a semilinear Neumann problem, *Comm. Pure Appl. Math.* 45 (1991), 819–851.
- [Ni 1] Nirenberg L., *Topics in nonlinear functional analysis*, *Courant Institute Lecture Notes*, New York (1974).
- [Ni 2] Nirenberg L., *Linear partial differential equations*, *CBMS*, no.17 (1972).
- [Ni 3] Nirenberg L., *Comments on nonlinear problems*, *Le Mathematiscche* 16 (1981).

- [Ni 4] Nirenberg L., Variational and topological methods in nonlinear problems, *Bull. AMS* 3 (1981), 267–302.
- [No] Nordstrom N., Minimization of energy functional with curve-represented edges (J. J. Koenderink ed.), 155–168.
- [Nu] Nussbaum R. D., The fixed point index for local condensing maps, *Ann. Mat. Pura Appl.* 37 (1972), 741–766.
- [Ob] Obata M., The conjectures on conformal transformations of Riemannian manifolds, *J. Diff. Geom.* 6 (1971), 247–258.
- [Oh] Oh Y. G., On positive multi-lump bound states of nonlinear Schrödinger equations under multiple well potential, *Comm Math. Phys.* 131 (1990), 223–253.
- [Pa 1] Palais R. S., Morse theory on Hilbert manifolds, *Topology* (1963), 299–340.
- [Pa 2] Palais R. S., Ljusternik Schnirelmann theory on Banach manifolds, *Topology* 5 (1966), 115–132.
- [Pa 3] Palais R. S., Homotopy theory of infinite dimensional manifolds, *Topology* 5 (1966), 1–16.
- [PS] Palais R. S., Smale S., A generalized Morse theory, *Bull. AMS* 70 (1964), 165–171.
- [PR 1] Pejsachowicz J., Rabier P. J., Degree theory for C^1 Fredholm mappings of index 0, *J'Analyse mathématique* 76 (1998), 289–319.
- [PR 2] Pejsachowicz J., Rabier P. J., A substitute for the Sard-Smale theorem in the C^1 case, *ibid.*, 76 (1998), 265–288.
- [Pl] Plastock R., Homeomorphisms between Banach spaces, *TAMS* 200 (1974), 169–183.
- [Po] Poschel J., Small divisors with spatial structure in infinite dimensional Hamiltonian systems, *Comm. Math. Phys.* 127 (1990), 351–393.
- [Pw] Protter M. H., Weinberger H. F., *Maximum principles in differential equations*, Prentice-Hall, (1967).
- [QS] Quinn F., Sard A., Hausdorff conullity of critical images of Fredholm maps, *Amer. J. Math.* 94 (1972), 1101–1110.
- [Ra 1] Rabinowitz P., A global theorem for nonlinear eigenvalue problems and applications, *Contributions to Nonlinear Functional Analysis*, Academic Press (1971), 11–30.
- [Ra 2] Rabinowitz P., Théorie du degré topologique et applications á des problèmes aux limites non linéaires, *Univ. laborat. anal. num. Paris* (1975).
- [Ra 3] Rabinowitz P., Periodic solutions for Hamiltonian systems, *Comm. Pure Appl. Math.* 31 (1978), 157–184.
- [Ra 4] Rabinowitz P., Minimax methods in critical point theory with applications to differential equations, *CBMS Regional Conference Series Math.* 65 (1986), AMS Providence.
- [Ra 5] Rabinowitz P., Variational method for nonlinear eigenvalue problems (ed. L.G. Prodi), *Cremonese, Roma* (1974), 141–195.
- [Re 1] Reshetnyak Yu. G., On the stability of conformal mappings in multidimensional spaces, *Sib. Math. J.* 8 (1967), 69–85.
- [Re 2] Reshetnyak Yu. G., Space mapping with bounded distortion, *AMS* (1989).
- [Ri] Riviere T., Everywhere discontinuous harmonic maps into spheres, *Acta Math.* 175 (1995), 197–226.
- [Ro 1] Rothe E., Critical points theory in Hilbert space under regular boundary conditions, *J. Math. Anal. Appl.* 36 (1971), 377–431.

- [Ro 2] Rothe E., Morse theory in Hilbert space, *Rocky Mountain J. Math.* 3 (1973), 251–274.
- [Ro 3] Rothe E., On the connection between critical point theory and Leray Schauder degree, *J. Math. Anal. Appl.* 88 (1982), 265–269.
- [Ry] Rybakowski K. P., *The homotopy index and partial differential equations*, Springer-Verlag (1987).
- [RZ] Rybakowski K. P., Zehnder E., On the Morse equation in Conley's index theory for semiflows on metric spaces, *Ergodic Theory Dyn. Syst.* 5 (1985), 123–143.
- [SU] Sacks J., Uhlenbeck K., The existence of minimal immersions of 2 spheres, *Ann. Math.* 113 (1981), 1–24.
- [Sa] Sadovskii B., Limit compact and condensing operators (Russian), *Uspehi Mat. Nauk* 27(1) (1972), 81–146.
- [Sal] Salamon D., Connected simple systems and the Conley index of isolated invariant sets, *Trans. Amer. Math. Soc.* 291 (1985), 1–41.
- [SZ] Salamon D., Zehnder, E., Morse theory for periodic solutions of Hamiltonian systems and the Morse index, *Comm. Pure Appl. Math.* 45 (1992), 1303–1360.
- [Sca] Schaefer H., Über die Methode der a-priori Schranken, *Math. Ann.* 129 (1955), 415–416.
- [Sc] Schauder J., Der Fixpunktsatz in Funktionalräumen, *Studia Math.* 2 (1930), 171–180.
- [Sch] Schiffman M., The Plateau problem for non-relative minima, *Ann. of Math.* 40 (1939), 834–854.
- [Sco] Schoen R., Conformal deformation of a Riemannian metric to a constant scalar curvature, *J. Diff. Geom.* 20 (1984), 479–495.
- [ScU 1] Schoen R., Uhlenbeck K., A regularity for harmonic maps, *J. Diff. Geom.* 17 (1982), 307–335.
- [ScU 2] Schoen R., Uhlenbeck K., Boundary regularity and the Dirichlet problem for harmonic maps, *J. Diff. Geom.* 18 (1983), 253–268.
- [SZ] Schoen R., Zhang D., Prescribing scalar curvature on the n-sphere, *Calc. Var. PDE* 4 (1996), 1–25.
- [Scw] Schwartz J. T., *Nonlinear functional analysis*, Golden Beach Publ. (1969).
- [Se] Semmes S., A primer on Hardy spaces and some remarks on a theorem of Evans and Muller, *Comm. PDE*, 19 (1994), 277–319.
- [Se] Séré E., Existence of infinitely many homoclinic orbits in Hamiltonian systems, *Math. Zeit.* 209 (1992), 27–42.
- [Shi] Shi S.Z., Ekeland's variational principle and the mountain pass lemma, *Acta Math. Sinica (NS)* 1 (1985) 348–355.
- [Si] Sion M., On general minimax theorems, *Pacific J. Math.* 8 (1958), 171–176.
- [Sm 1] Smale S., Generalized Poincaré's conjecture in dimension great than four, *Ann. of Math.* 74 (1961), 391–406.
- [Sm 2] Smale S., Morse theory and a nonlinear generalization of the Dirichlet problem, *Ann. of Math.* 80 (1964), 382–396.
- [Sm 3] Smale S., An infinite dimensional version of Sard's theorem, *Amer. J. Math.* 87 (1965), 861–867.
- [Sma] Smart D. R., *Fixed point theorems*, Cambridge Univ. Press (1974).
- [Smo] Smoller J., *Shock waves and reaction diffusion equations*, *Grundlehren der Mathematischen Wissenschaften* 258, Springer-Verlag (1982).

- [So] Solimini S., Morse index estimates in Min-Max theorems, *Manusc. Math.* 32 (1989), 421–454.
- [Sp] Spanier E. H., *Algebraic topology*, Springer-Verlag (1966).
- [Ste 1] Stein E., *Singular integrals and differentiability properties of functions*, Princeton Univ. Press (1970).
- [Ste 2] Stein E., *Harmonic analysis: real variable methods, orthogonality and oscillatory integrals*, Princeton Univ. Press (1993).
- [SW] Stein E., Weiss G., *Introduction to Fourier analysis on Euclidean spaces*, Princeton Univ. Press (1971).
- [St 1] Struwe M., *Variational Methods, Applications to nonlinear partial differential equations and Hamiltonian systems*, 2nd ed., Springer-Verlag (1996).
- [St 2] Struwe M., *Plateau problem and the calculus of variations*, Princeton Univ. Press (1988).
- [ST] Stuart C. A., Toland J. F., The fixed point index of a linear k -set contraction, *J. London Math. Soc.* 6 (1973), 317–320.
- [Sv 1] Šverák V., Rank one convexity does not imply quasiconvexity, *Proc. Royal Soc. Edinburgh* 120 (1992), 185–189.
- [Sv 2] Šverák V., Lower semicontinuity of variational integrals and compensated compactness, *Proc. ICM 1994, Vol 2*, Birkhauser (1995), 1153–1158.
- [Tal] Talenti G., Best constant in Sobolev inequality, *Ann. Mat. Pure Appl.* 110 (1976), 353–372.
- [Tar 1] Tartar L., The compensated compactness method applied to systems of conservation laws, *Systems of Nonlinear Partial Differential Equations* (Ball J., ed.), NATO ASI series, Vol CIII, Reidel (1983), 263–285.
- [Tar 2] Tartar L., H -measures, a new approach for studying homogenization, oscillations and concentration effects in partial differential equations, *Proc. Roy. Soc. Edinburgh A* 115 (1990), 193–230,
- [Ta] Tarski A., A lattice theoretical fixed point theorem and its applications, *Pac. J. Math.* 5 (1955), 285–309.
- [Tau] Taubes C. H. Self-dual connections on non-self-dual 4-manifolds, *JDG* 17 (1982), 139–170.
- [Ti] Tian G., On the mountain pass lemma, *Kexue Tongbao* 14 (1983), 833–835.
- [Tr] Trudinger N. S., On imbeddings into Orlicz spaces and some applications, *J. Math. Mech.* 17 (1967), 473–483.
- [Tu] Turner R., Positive solutions of nonlinear eigenvalue problems (ed. LG. Prodi) *Eigenvalues of Nonlinear Problems*, Crmenese, Roma (1974), 213–239.
- [Uh] Uhlenbeck K., Generic properties of eigenfunctions, *Amer. J. Math.* 98 (1976), 1059–1078.
- [Va] Vainberg M., *Variational method for the study of nonlinear operators* (Russian), Gostehizdt (1956).
- [Vi 1] Viterbo C., A proof of the Weinstein conjecture on R^{2n} , *Ann. Inst. H. Poincaré, Analyse non lineaire* 4 (1987), 337–356.
- [Vi 2] Viterbo C., Indices de Morse des points critiques obtenus par minimax, *Ann. Inst. H. Poincaré, Analyse nonlinéaire* 5 (1988), 221–225.
- [VN] Von Neumann J., Zur Theorie der Gesellschaftsspiele, *Math. Ann.* 100 (1927), 295–320.
- [Wa 1] Wang Z. Q., A note on the deformation theorem, *Acta Math. Sinica* 30 (1987), 106–110.

- [Wa 2] Wang Z. Q., On a superlinear elliptic equation, *Analyse Nonlineaire* 8 (1991), 43–58.
- [Way] Wayne C. E., Periodic and quasi-periodic solutions of nonlinear wave equations via KAM theory, *Comm. Math. Phys.* 127 (1990), 479–528.
- [Wei 1] Weinstein A., Periodic orbits for convex Hamiltonian systems, *Ann Math.* 108 (1978), 507–518.
- [Wei 2] Weinstein A., On the hypotheses of Rabinowitz's periodic orbit theorem, *J. Diff. Eq.* 33 (1979), 353–358.
- [Wi] Willem M., *Minimax methods*, Birkhauser (1994).
- [Ya] Yamabe H., On the deformation of Riemannian structures on compact manifolds, *Osaka Math. J.* 12 (1960), 21–37.
- [Yo] Young L. C., *Lectures on the calculus of variations and optimal control theory*, Saunders (1969).
- [Ze] Zehnder E., Generalized implicit function theorem with applications to some small divisor problems, I, II, *Comm. Pure Appl. Math.* 28 (1975), 91–140 (1976), 49–111.
- [Zei] Zeidler, E., *Nonlinear functional analysis and its applications I-IV*, Springer-Verlag, New York (1988).
- [Zh 1] Zhang K. W., A construction of quasiconvex function with linear growth at infinity, *Ann. S. N. S. Pisa* 19 (1992), 313–326.
- [Zh 2] Zhang K. W., Two well structure and intrinsic mountain pass points, preprint.
- [Zi] Ziemer W. P., *Weakly differentiable functions*, Springer-Verlag (1989).

Springer Monographs in Mathematics

This series publishes advanced monographs giving well-written presentations of the “state-of-the-art” in fields of mathematical research that have acquired the maturity needed for such a treatment. They are sufficiently self-contained to be accessible to more than just the intimate specialists of the subject, and sufficiently comprehensive to remain valuable references for many years. Besides the current state of knowledge in its field, an SMM volume should also describe its relevance to and interaction with neighbouring fields of mathematics, and give pointers to future directions of research.

Abhyankar, S.S. **Resolution of Singularities of Embedded Algebraic Surfaces** 2nd enlarged ed. 1998

Alexandrov, A.D. **Convex Polyhedra** 2005

Andrievskii, V.V.; Blatt, H.-P. **Discrepancy of Signed Measures and Polynomial Approximation** 2002

Angell, T. S.; Kirsch, A. **Optimization Methods in Electromagnetic Radiation** 2004

Ara, P.; Mathieu, M. **Local Multipliers of C^* -Algebras** 2003

Armitage, D.H.; Gardiner, S.J. **Classical Potential Theory** 2001

Arnold, L. **Random Dynamical Systems** corr. 2nd printing 2003 (1st ed. 1998)

Arveson, W. **Noncommutative Dynamics and E-Semigroups** 2003

Aubin, T. **Some Nonlinear Problems in Riemannian Geometry** 1998

Auslender, A.; Teboulle M. **Asymptotic Cones and Functions in Optimization and Variational Inequalities** 2003

Bang-Jensen, J.; Gutin, G. **Digraphs** 2001

Baues, H.-J. **Combinatorial Foundation of Homology and Homotopy** 1999

Brown, K.S. **Buildings** 3rd printing 2000 (1st ed. 1998)

Chang, K.-C. **Methods in Nonlinear Analysis** 2005

Cherry, W.; Ye, Z. **Nevanlinna’s Theory of Value Distribution** 2001

Ching, W.K. **Iterative Methods for Queuing and Manufacturing Systems** 2001

Crabb, M.C.; James, I.M. **Fibrewise Homotopy Theory** 1998

Chudinovich, I. **Variational and Potential Methods for a Class of Linear Hyperbolic Evolutionary Processes** 2005

Dineen, S. **Complex Analysis on Infinite Dimensional Spaces** 1999

Dugundji, J.; Granas, A. **Fixed Point Theory** 2003

Elstrodt, J.; Grunewald, F. Mennicke, J. **Groups Acting on Hyperbolic Space** 1998

Edmunds, D.E.; Evans, W.D. **Hardy Operators, Function Spaces and Embeddings** 2004

Engler, A.J.; Prestel, A. **Valued Fields** 2005

Fadell, E.R.; Husseini, S. Y. **Geometry and Topology of Configuration Spaces** 2001

Fedorov, Y.N.; Kozlov, V.V. **A Memoir on Integrable Systems** 2001

Flenner, H.; O’Carroll, L. Vogel, W. **Joins and Intersections** 1999

Gelfand, S.I.; Manin, Y.I. **Methods of Homological Algebra** 2nd ed. 2003

Griess, R.L. Jr. **Twelve Sporadic Groups** 1998

Gras, G. **Class Field Theory** corr. 2nd printing 2005

Hida, H. **p -Adic Automorphic Forms on Shimura Varieties** 2004

Ischebeck, F.; Rao, R.A. **Ideals and Reality** 2005

Ivrii, V. **Microlocal Analysis and Precise Spectral Asymptotics** 1998

Jech, T. **Set Theory** (3rd revised edition) 2002

Jorgenson, J.; Lang, S. **Spherical Inversion on $SL_n(\mathbb{R})$** 2001

Kanamori, A. **The Higher Infinite** corr. 2nd printing 2005 (2nd ed. 2003)

Kanovei, V. **Nonstandard Analysis, Axiomatically** 2005

Khoshnevisan, D. **Multiparameter Processes** 2002

Koch, H. **Galois Theory of p -Extensions** 2002

Komornik, V. **Fourier Series in Control Theory** 2005
 Kozlov, V.; Maz'ya, V. **Differential Equations with Operator Coefficients** 1999
 Landsman, N.P. **Mathematical Topics between Classical & Quantum Mechanics** 1998
 Leach, J.A.; Needham, D.J. **Matched Asymptotic Expansions in Reaction-Diffusion Theory** 2004
 Lebedev, L.P.; Vorovich, I.I. **Functional Analysis in Mechanics** 2002
 Lemmermeyer, F. **Reciprocity Laws: From Euler to Eisenstein** 2000
 Malle, G.; Matzat, B.H. **Inverse Galois Theory** 1999
 Mardesic, S. **Strong Shape and Homology** 2000
 Margulis, G.A. **On Some Aspects of the Theory of Anosov Systems** 2004
 Murdock, J. **Normal Forms and Unfoldings for Local Dynamical Systems** 2002
 Narkiewicz, W. **Elementary and Analytic Theory of Algebraic Numbers** 3rd ed. 2004
 Narkiewicz, W. **The Development of Prime Number Theory** 2000
 Parker, C.; Rowley, P. **Symplectic Amalgams** 2002
 Peller, V. (Ed.) **Hankel Operators and Their Applications** 2003
 Prestel, A.; Delzell, C.N. **Positive Polynomials** 2001
 Puig, L. **Blocks of Finite Groups** 2002
 Ranicki, A. **High-dimensional Knot Theory** 1998
 Ribenboim, P. **The Theory of Classical Valuations** 1999
 Rowe, E.G.P. **Geometrical Physics in Minkowski Spacetime** 2001
 Rudyak, Y.B. **On Thom Spectra, Orientability and Cobordism** 1998
 Ryan, R.A. **Introduction to Tensor Products of Banach Spaces** 2002
 Saranen, J.; Vainikko, G. **Periodic Integral and Pseudodifferential Equations with Numerical Approximation** 2002
 Schneider, P. **Nonarchimedean Functional Analysis** 2002
 Serre, J-P. **Complex Semisimple Lie Algebras** 2001 (reprint of first ed. 1987)
 Serre, J-P. **Galois Cohomology** corr. 2nd printing 2002 (1st ed. 1997)
 Serre, J-P. **Local Algebra** 2000
 Serre, J-P. **Trees** corr. 2nd printing 2003 (1st ed. 1980)
 Smirnov, E. **Hausdorff Spectra in Functional Analysis** 2002
 Springer, T.A. Veldkamp, F.D. **Octonions, Jordan Algebras, and Exceptional Groups** 2000
 Sznitman, A.-S. **Brownian Motion, Obstacles and Random Media** 1998
 Taira, K. **Semigroups, Boundary Value Problems and Markov Processes** 2003
 Talagrand, M. **The Generic Chaining** 2005
 Tauvel, P.; Yu, R.W.T. **Lie Algebras and Algebraic Groups** 2005
 Tits, J.; Weiss, R.M. **Moufang Polygons** 2002
 Uchiyama, A. **Hardy Spaces on the Euclidean Space** 2001
 Üstünel, A.-S.; Zakai, M. **Transformation of Measure on Wiener Space** 2000
 Vasconcelos, W. **Integral Closure. Rees Algebras, Multiplicities, Algorithms** 2005
 Yang, Y. **Solitons in Field Theory and Nonlinear Analysis** 2001
 Zieschang P.-H. **Theory of Association Schemes** 2005