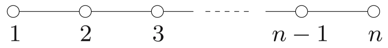
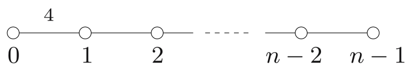
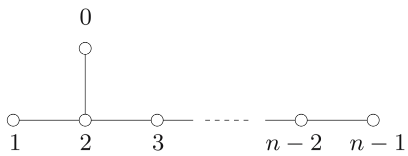
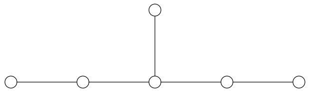
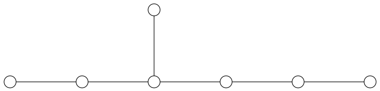
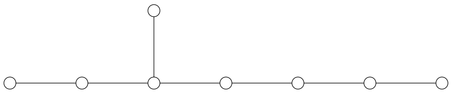
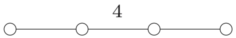
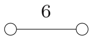
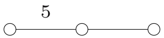
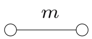
Appendix A1

Classification of finite and affine Coxeter groups

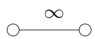
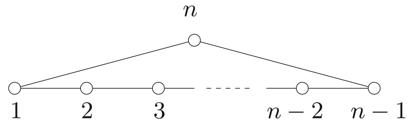
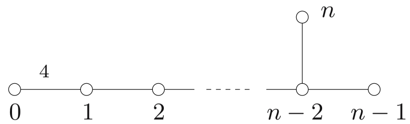
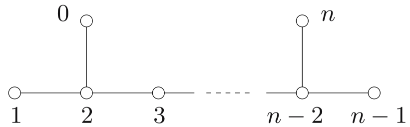
Table I. The finite irreducible Coxeter systems

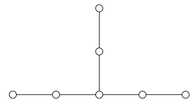
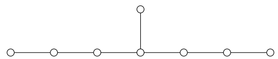
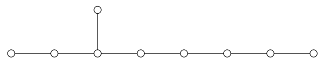
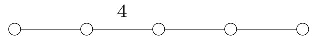
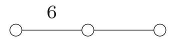
Name	Diagram	Order	$\lvert \mathbf{T} \rvert$	Exponents
A_n $(n \geq 1)$		$(n+1)!$	$\binom{n+1}{2}$	$1, 2, \ldots, n$
B_n $(n \geq 2)$		$2^n n!$	n^2	$1, 3, \ldots, 2n-1$
D_n $(n \geq 4)$		$2^{n-1} n!$	$n^2 - n$	$1, 3, \ldots, 2n-3, n-1$

E_6		$2^7\,3^4\,5$	36	1, 4, 5, 7, 8, 11
E_7		$2^{10}\,3^4\,5\,7$	63	1, 5, 7, 9, 11, 13, 17
E_8		$2^{14}\,3^5\,5^2\,7$	120	1, 7, 11, 13, 17, 19, 23, 29
F_4		1152	24	1, 5, 7, 11
G_2		12	6	1, 5
H_3		120	15	1, 5, 9
H_4		14400	60	1, 11, 19, 29
$I_2(m)$ $(m \geq 3)$		$2m$	m	$1,\ m-1$

The underlying groups are pairwise nonisomorphic, except that $I_2(3) = A_2$, $I_2(4) = B_2$ and $I_2(6) = G_2$.

Table II. The affine irreducible Coxeter systems

Name	Diagram
$\widetilde{A}_1$ $= I_2(\infty)$	
$\widetilde{A}_{n-1}$ $(n \geq 3)$	
$\widetilde{C}_n$ $(n \geq 2)$	
$\widetilde{B}_n$ $(n \geq 3)$	
$\widetilde{D}_n$ $(n \geq 4)$	

Name	Diagram
$\widetilde{E}_6$	
$\widetilde{E}_7$	
$\widetilde{E}_8$	
$\widetilde{F}_4$	
$\widetilde{G}_2$	

Appendix A2

Graphs, posets, and complexes

Graphs, posets, and simplicial complexes are, together with permutations and tableaux, the basic combinatorial notions used. They play an important role throughout the book.

In this appendix, we define and recall some terminology, notation, and results. More details, proofs and references for the first two sections can be found, for example, in [497], and for the last three, for example, in [64, Section 4.7].

A2.1 Graphs and Digraphs

By a *graph* we mean a pair $G = (V, E)$, where V is a set and $E \subseteq \binom{V}{2}$. We call V the set of *nodes* or *vertices* of G, and E the set of *edges* of G. A *path* in G is a sequence $\Gamma = (x_0, \ldots, x_k) \in V^{k+1}$ such that $\{x_i, x_{i+1}\} \in E$ for all $i = 0, \ldots, k-1$. If $x_0 = x_k$ and $k \geq 1$, then we call Γ a *cycle*. We also say that the path Γ *connects* x_0 and x_k. A graph is *connected* if for all $x, y \in V$ there is a path Γ that connects x and y.

A *rooted graph* is a pair (G, x), where G is a graph and x is a vertex of G, called the *root*. A *tree* is a connected graph with no cycles. A vertex v of a tree is a *leaf* if $|\{x \in V : \ \{x, v\} \in E\}| = 1$. Note that if (G, x) is a rooted tree, then for every $v \in V$ there is a unique path $\Gamma(v)$ connecting x and v. Given two vertices $u, v \in V$, we then say that u is a *descendant* of v if $v \in \Gamma(u)$.

By a *directed graph* (or *digraph*, for short) we mean a pair $D = (V, A)$, where V is a set and $A \subseteq V^2$. We call V the set of *nodes* or *vertices* of D and A the set of *directed edges* of D. We write $x \to y$ to mean that $(x, y) \in A$. An edge $x \to x$ is called a *loop*. A *directed path* (or, *path*, for short) in D is a sequence $\Gamma = (x_0, x_1, \ldots, x_k) \in V^{k+1}$ such that $x_0 \to x_1 \to \cdots \to x_k$. We say that Γ goes *from* x_0 *to* x_k, and we call k the *length* of Γ. If $x_0 = x_k$, we call Γ a *directed cycle*. The edges of a directed cycle of length $k = 2$ are sometimes referred to as a pair of *antiparallel edges*. If $S \subseteq V$, then $(S, A \cap S^2)$ is also a digraph called the *induced directed subgraph*, induced by D on S.

It is sometimes convenient to allow multiple edges in graphs and digraphs. This means that E is a multiset of elements from $\binom{V}{2}$ (resp., A is a multiset of elements from V^2). Such graphs with multiple edges appear a few times in the book. We do not distinguish this more general case terminologically or notationally.

A2.2 Posets

The word *poset* is an abbreviation of *partially ordered set*. Thus, a poset $P = (P, \leq)$ consists of a set P together with a partial order relation $\leq$. The relation is suppressed from the notation when it is clear from context. If $Q \subseteq P$, we may refer to Q also as a poset, having in mind the *induced subposet* $(Q, \leq)$, whose order relation is the restriction of P's. Two elements $x, y \in P$ are said to be *comparable* if either $x \leq y$ or $y \leq x$, and *incomparable* otherwise.

A sequence $(x_0, x_1, \ldots, x_h)$ of elements of P is called a *chain* (respectively *multichain*) if $x_0 < x_1 < \cdots < x_h$ (respectively, $x_0 \leq x_1 \leq \cdots \leq x_h$). We then also say that the chain (respectively, multichain) goes from x_0 to x_h. The integer h is called the *length* of the chain (respectively, multichain). The supremum of this number over all chains of P is the *rank* (or *length*) of P. A chain is *maximal* if its elements are not a proper subset of those of any other chain. If all maximal chains are of the same finite length, then P is *pure*. An element $x \in P$ is *maximal* if there is no element $y \in P$ such that $x < y$.

Suppose that P is pure of length k. Define the *rank* $r(x)$ of $x \in P$ to be the length of the subposet $\{y \in P : y \leq x\}$. The rank function $r : P \to [0, k]$ restricts to a bijection on each maximal chain, and decomposes P into *rank levels* $P_i = \{x \in P : r(x) = i\}$, $i \in [0, k]$.

If $x \leq y$ in P, we define the *closed interval* (or *interval*, for short) $[x, y] = \{z \in P : x \leq z \leq y\}$, the *open interval* $(x, y) = \{z \in P : x < z < y\}$, and the *half-open interval* $(x, y] = \{z \in P : x < z \leq y\}$. A *bottom element* $\widehat{0}$ (resp. a *top element* $\widehat{1}$) is an element satisfying $\widehat{0} \leq x$ (resp. $x \leq \widehat{1}$) for all $x \in P$. If P has a bottom element $\widehat{0}$ and every interval $[\widehat{0}, x]$ is pure, then

P is *graded.* The *rank function* $r : P \to \mathbb{N}$ is defined for a graded poset P as for a pure one. If $|P_i| < \infty$ for all $i \geq 0$, then we call the formal power series $\sum_{i \geq 0} |P_i| q^i$ the *rank generating function* of P.

Suppose from now on that all intervals in P are finite (only such posets appear in this book). A pair (x, y) such that $x < y$ and no $z \in P$ satisfies $x < z < y$ is called a *covering* and is denoted by $x \lhd y$ (or $y \rhd x$). Let $\mathrm{Cov}(P)$ be the set of all coverings in P. This set of ordered pairs implies all other order relations by transitivity, and $\mathrm{Cov}(P)$ is clearly minimal with this property. A chain is *saturated* if all successive relations are coverings: $x_0 \lhd x_1 \lhd \cdots \lhd x_h$. If P has a $\widehat{0}$ (respectively, a $\widehat{1}$), then an element $x \in P$ is an *atom* (respectively, *coatom*) of P if $\widehat{0} \lhd x$ (respectively, $x \lhd \widehat{1}$).

The standard way of depicting a poset P is to draw a digraph with the elements of P as nodes and the elements of $\mathrm{Cov}(P)$ as upward-directed edges. This graph is called the *diagram* of P (sometimes the *Hasse diagram*). For instance, Figure 2.10 depicts a graded poset of length 3, with the rank levels P_1 and P_2 both of cardinality k.

A map $f : P \to Q$ of posets is *order-preserving* if $x \leq y$ implies $f(x) \leq f(y)$, for all $x, y \in P$. If, instead, $x \leq y$ implies $f(x) \geq f(y)$, the map is *order-reversing.* Two posets P and Q are *isomorphic* if there exists an order-preserving bijection $f : P \to Q$ such that f^{-1} is also order-preserving. A poset P is a *Boolean algebra* if there is a set X such that P is isomorphic to the set of all subsets of X, partially ordered by inclusion. A bijection $f : P \to P$ is an *automorphism* if f and f^{-1} are order-preserving, and an *antiautomorphism* if f and f^{-1} are order-reversing.

A poset P is a *lattice* if for all $x, y \in P$, the subposet $\{z \in P : z \leq x,\ z \leq y\}$ has a top element, the *meet* $x \wedge y$, and — dually — the subposet $\{z \in P : z \geq x,\ z \geq y\}$ has a bottom element, the *join* $x \vee y$. If only the meet $x \wedge y$ is guaranteed to exist, P is a *meet-semilattice.* See Section 3.2 for a few more definitions pertaining to (semi)lattices.

The *Möbius function* of P assigns to each ordered pair $x \leq y$ an integer $\mu(x, y)$ according to the following recursion:

$$\mu(x,y) = \begin{cases} 1, & \text{if } x = y, \\ -\sum_{x \leq z < y} \mu(x,z), & \text{if } x < y. \end{cases} \tag{A2.1}$$

Let $\mathrm{Int}(P) \stackrel{\mathrm{def}}{=} \{(x, y) \in P^2 :\ x \leq y\}$. Given a commutative ring R, the *incidence algebra* of P with coefficients in R, denoted $I(P; R)$, is the set of all functions $f : \mathrm{Int}(P) \to R$ with sum and product defined by

$$(f+g)(x,y) \stackrel{\mathrm{def}}{=} f(x,y) + g(x,y)$$

and

$$(fg)(x,y) \stackrel{\mathrm{def}}{=} \sum_{x \leq z \leq y} f(x,z)\, g(z,y), \tag{A2.2}$$

for all $f, g \in I(P; R)$ and $(x, y) \in \mathrm{Int}(P)$. For instance, $\mu \in I(P; \mathbb{Z})$.

It is well known (see, e.g., [497, Section 3.6]) that $I(P;R)$ is an associative algebra having δ as identity element (where $\delta(x,y) \stackrel{\text{def}}{=} 1$ if $x=y$, and $\stackrel{\text{def}}{=} 0$ otherwise) and that an element $f \in I(P;R)$ is invertible if and only if $f(x,x)$ is invertible for all $x \in P$. If f is invertible, then we denote by f^{-1} its (two-sided) inverse.

See [497, Chapter 3] for more about posets and the Möbius function.

A2.3 Simplicial complexes

By an *(abstract simplicial) complex* on vertex set V is meant a nonempty collection Δ of finite subsets of V, called *faces*, which is closed under containment: $F \subseteq F' \in \Delta$ implies $F \in \Delta$. Since we assume that $\Delta \neq \emptyset$, it follows that $\emptyset \in \Delta$. If $F \subseteq F' \in \Delta$, let $[F, F'] = \{G \in \Delta : F \subseteq G \subseteq F'\}$. The *dimension* of a face is defined by $\dim F = |F| - 1$, and the dimension of Δ by $\dim\Delta = \sup_{F\in\Delta} \dim F$ (which can be equal to ∞). So, for example, $\dim\{\emptyset\} = -1$.

A complex Δ is *pure d-dimensional* if every face is contained in some d-dimensional face. Almost all complexes dealt with in this book are pure. In this case, the d-dimensional faces are called *facets* and the $(d-1)$-dimensional faces are called *panels*. The collection of all facets is denoted by $\mathcal{F}(\Delta)$. Two facets C and C' are *adjacent* if $\dim(C \cap C') = d - 1$.

Let $\Delta \subseteq \Delta'$ be complexes and assume that x is a vertex of Δ' but not of Δ. Then, Δ' is said to be a *cone* over Δ with *cone point* x if

$$C \in \mathcal{F}(\Delta) \quad \Leftrightarrow \quad C \cup \{x\} \in \mathcal{F}(\Delta').$$

For any complex Δ, we let $\|\Delta\|$ denote its topological space, or *geometric realization*. For this construction, as well as such notions as Euler characteristic, simplicial homology and homotopy type, and their connections, see any textbook on algebraic topology (e.g. [401]).

Let Δ be a finite d-dimensional simplicial complex, and let f_i be the number of i-dimensional faces of Δ. The sequence $f = (f_0, \ldots, f_d)$ is called the *f-vector* of Δ. We put $f_{-1} = 1$. The *h-vector* $h = (h_0, \ldots, h_{d+1})$ of Δ is defined by the equation

$$\sum_{i=0}^{d+1} f_{i-1} x^{d+1-i} = \sum_{i=0}^{d+1} h_i (x+1)^{d+1-i}. \tag{A2.3}$$

Note that $h_0 = 1$, $h_1 = n - d - 1$, and

$$h_{d+1} = f_d - f_{d-1} + \cdots + (-1)^d f_0 + (-1)^{d+1} = (-1)^d \widetilde{\chi}(\Delta),$$

where $\widetilde{\chi}(\Delta)$ is the reduced Euler characteristic of Δ. In particular,

$$h_{d+1} = \begin{cases} 1, & \text{if } \|\Delta\| \text{ is homeomorphic to a sphere,} \\ 0, & \text{if } \|\Delta\| \text{ is homeomorphic to a ball.} \end{cases}$$

An important way in which complexes arise in combinatorics is from posets. If P is a poset, let $\Delta(P)$ be the collection of all finite chains $x_0 < x_1 < \cdots < x_k$ in P. A subset of a chain is again a chain, so this is a simplicial complex, called the *order complex* of P.

We make use of the following two facts. Let $x < y$ in P. Then, the Möbius function $\mu(x, y)$ is equal to the reduced Euler characteristic of the order complex of the open interval (x, y):

Fact A2.3.1 $\quad \mu(x, y) = \widetilde{\chi}(\Delta((x, y)))$.

See [497, Proposition 3.8.6] for a proof.

Fact A2.3.2 *Let $f : P \to P$ be an order-preserving map such that $x \geq f(x) = f^2(x)$ for all $x \in P$. Then, the order complexes of P and of $f(P)$ are homotopy equivalent.*

It is not hard to give a direct proof that $\Delta(f(P))$ is a strong deformation retract of $\Delta(P)$ in this situation. For another simple proof, see [60, Corollary 10.12].

A2.4 Shellability

Throughout this section, let Δ be a pure d-dimensional complex of at most countable cardinality. We will be considering linear orderings $C_1, C_2, C_3, \ldots$ of $\mathcal{F}(\Delta)$. Given such an ordering, let $\Delta_k = [\emptyset, C_1] \cup [\emptyset, C_2] \cup \cdots \cup [\emptyset, C_k]$, for $k \geq 1$. Thus, Δ_k is the subcomplex generated by the k first facets.

Definition A2.4.1 *The complex Δ is said to be* shellable *if its facets can be arranged in linear order $C_1, C_2, C_3, \ldots$ in such a way that $\Delta_{k-1} \cap [\emptyset, C_k]$ is pure $(d-1)$-dimensional, for $k = 2, 3, \ldots$. Such an ordering of $\mathcal{F}(\Delta)$ is called a* shelling.

In other words, a linear order $C_1, C_2, C_3, \ldots$ is a shelling if and only if whenever $i < k$, there exists some $j < k$ such that $C_i \cap C_k \subseteq C_j \cap C_k$, and $|C_j \cap C_k| = |C_k| - 1$. Note that

if Δ' is a cone over Δ, then Δ' is shellable if and only if Δ is. (A2.4)

Given a shelling, define the *restriction* of facet C_k by

$$\mathcal{R}(C_k) = \{x \in C_k : C_k \setminus \{x\} \in \Delta_{k-1}\}. \tag{A2.5}$$

(Here, and whenever else needed, we let $\Delta_0 = \emptyset$.) Shellings and their restriction maps have several useful characterizations, of which we mention the following.

Fact A2.4.2 *Given an ordering $C_1, C_2, C_3, \ldots$ of $\mathcal{F}(\Delta)$ and a map $\mathcal{R} : \mathcal{F}(\Delta) \to \Delta$, the following are equivalent:*

(i) $C_1, C_2, C_3, \ldots$ *is a shelling and* $\mathcal{R}$ *its restriction map.*

(ii) $\Delta_k = \biguplus_{i=1}^{k} [\,\mathcal{R}(C_i), C_i\,]$, *for all* $k \geq 1$.

A pure complex Δ is said to be *thin* if every panel is contained in exactly two facets. It is called *subthin* if every panel is contained in at most two facets and it is not thin. It is *locally finite* if every vertex is contained in only finitely many facets.

Let $\mathbb{B}^d$ and $\mathbb{S}^d$ denote the standard PL d-ball and d-sphere (i.e., a geometric d-simplex and the boundary of a geometric $(d+1)$-simplex, respectively). Using some simple facts from PL (piecewise linear) topology, one derives the following.

Fact A2.4.3 *Let Δ be a shellable pure d-dimensional simplicial complex.*

(i) *If Δ is finite and subthin, then $\|\Delta\|$ is PL homeomorphic to $\mathbb{B}^d$.*

(ii) *If Δ is finite and thin, then $\|\Delta\|$ is PL homeomorphic to $\mathbb{S}^d$.*

(iii) *If Δ is infinite and thin, then $\|\Delta\|$ is contractible.*

(iv) *If Δ is infinite, thin, and locally finite, then $\|\Delta\|$ is PL homeomorphic to $\mathbb{R}^d$.*

If Δ is finite and shellable, then the h-vector has the following interpretation in terms of the restriction map:

$$h_i = \operatorname{card}\{C \in \mathcal{F}(\Delta) : |\mathcal{R}(C)| = i\}. \qquad \text{(A2.6)}$$

The definition of the h-vector can be extended to infinite shellable complexes via equation (A2.6).

Fact A2.4.4 *Let Δ be a shellable pure d-dimensional complex and let $h \stackrel{\text{def}}{=} h_{d+1}$. Then, $\|\Delta\|$ has the homotopy type of a wedge of h copies of the d-sphere. Consequently,*

$$\widetilde{H}_i(\Delta;\mathbb{Z}) = \begin{cases} \mathbb{Z}^h, & \text{if } i = d, \\ 0, & \text{if } i \neq d. \end{cases} \qquad \text{(A2.7)}$$

Here, $\widetilde{H}_i(\Delta;\mathbb{Z})$ denotes reduced simplicial homology with integer coefficients.

The *link* of a face $F \in \Delta$ (including $F = \emptyset$) is the subcomplex $lk_\Delta(F) \stackrel{\text{def}}{=} \{G \in \Delta : G \cup F \in \Delta \text{ and } G \cap F = \emptyset\}$. The complex Δ is said to be *Cohen-Macaulay* if

$$\widetilde{H}_i(lk_\Delta(F);\mathbb{Z}) = 0, \text{ for all } F \in \Delta \text{ and } i < \dim lk_\Delta(F). \qquad \text{(A2.8)}$$

By a theorem of Reisner, this property is (in the finite case) equivalent to the Cohen-Macaulayness (in the sense of commutative algebra) of a certain ring $\mathbf{k}[\Delta]$, for every coefficient field $\mathbf{k}$. The ring $\mathbf{k}[\Delta]$ is the quotient of the polynomial ring $\mathbf{k}[x_1, \ldots, x_n]$, whose indeterminates are the vertices x_i of Δ, modulo the ideal generated by the square-free monomials $x_{i_1} x_{i_2} \cdots x_{i_k}$

corresponding to nonfaces $\{x_{i_1}, x_{i_2}, \ldots, x_{i_k}\} \notin \Delta$. See [496] for a detailed discussion of this connection to ring theory.

It is easy to see that shellability of Δ is inherited by all links $lk_\Delta(F)$. Hence, from equation (A2.7) we get the following:

Fact A2.4.5 *If Δ is shellable, then Δ is Cohen-Macaulay.*

A *colored* complex Δ, on vertex set V and with color set S, is by definition a pure d-dimensional complex Δ with a partition $V = \biguplus_{s \in S} V_s$ such that $|C \cap V_s| = 1$ for all $C \in \mathcal{F}(\Delta)$ and all $s \in S$. It is convenient to think of S as a set of colors, the condition being that every facet has exactly one vertex of each color. Clearly, $|S| = d+1$. Examples of colored complexes are provided by order complexes of pure posets P of length d, where $S = [0, d]$ and the color classes V_s are the rank levels P_i.

Let Δ be a colored complex as above. Define the *type* of a face $F \in \Delta$ as its set of colors: $\tau(F) = \{s \in S : F \cap V_s \neq \emptyset\}$. Then, for $E \subseteq S$, let $\Delta_E = \{F \in \Delta : \tau(F) \subseteq E\}$. The *type-selected subcomplex* Δ_E is pure $(|E| - 1)$-dimensional.

Fact A2.4.6 *Suppose that Δ is colored and shellable. Fix $E \subseteq S$. Then, Δ_E is shellable and*

$$h_{|E|}(\Delta_E) = \operatorname{card}\{C \in \mathcal{F}(\Delta) : \tau(\mathcal{R}(C)) = E\}.$$

A2.5 Regular CW complexes

By a *ball* in a topological space T we mean a subspace $\sigma \subseteq T$ that is homeomorphic to the ball $\mathbb{B}^d$, for some $d \geq 0$. The (relative) interior $\overset{\circ}{\sigma}$ and boundary $\partial\sigma = \sigma \setminus \overset{\circ}{\sigma}$ are defined via transfer from $\mathbb{B}^d$. If $\dim \sigma = 0$, then $\overset{\circ}{\sigma} = \sigma = \{\text{point}\}$.

Definition A2.5.1 *A* regular CW complex Γ *is a collection of balls in a Hausdorff space $\|\Gamma\| = \cup_{\sigma \in \Gamma} \sigma$ such that the following hold:*

(i) The interiors $\overset{\circ}{\sigma}$ partition $\|\Gamma\|$.

(ii) The boundary $\partial\sigma$ is a union of some members of Γ, for all $\sigma \in \Gamma$ of positive dimension.

This definition of a regular CW complex is not the standard one in the topological literature, where an approach via attaching maps (applicable also to general "non-regular" CW complexes) is more common. In that setting, regularity means that the attaching map of each cell should be a homeomorphism on *the whole* cell that is being attached, not only on its interior. For detailed topological treatments of regular cell complexes, see [158] or [363]. For a discussion of regular CW complexes from a combinatorial point of view, including motivation for the equality of the two definitions, see [64, Section 4.7].

The balls $\sigma \in \Gamma$ are the *closed cells* of Γ; their interiors $\overset{\circ}{\sigma}$ are the *open cells*. If $\|\Gamma\| \cong T$, then Γ is said to provide (via the homeomorphism) a *regular CW decomposition* of the space T. The geometric realizations of abstract simplicial complexes are examples of regular CW complexes whose cells are the geometric simplices representing the abstract faces.

The *cell poset* $\mathcal{C}(\Gamma)$ is the set of closed cells of Γ ordered by containment. Now, it turns out that the order complex of $\mathcal{C}(\Gamma)$ is homeomorphic to $\|\Gamma\|$, which has the following consequence.

Fact A2.5.2 *The cell poset determines the topology of* $\|\Gamma\|$ *and its cellular structure up to cell-preserving homeomorphism.*

For any CW complex Γ, there exists an algebraic chain complex, the *cellular chain complex*,

$$\cdots \longrightarrow C_{i+1} \xrightarrow{d_{i+1}} C_i \xrightarrow{d_i} C_{i-1} \longrightarrow \cdots$$

with the following properties:

(i) C_i is a free Abelian group with a basis indexed by the i-dimensional cells of Γ.

(ii) $H_i\left(\|\Gamma\|; \mathbb{Z}\right) \cong \operatorname{Ker} d_i \;/\; \operatorname{Im} d_{i+1}$.

Furthermore, if Γ is regular, there exists a mapping from pairs of cells (σ, τ) such that $\sigma \subset \tau$ and $\dim \sigma + 1 = \dim \tau$ (or, in other words, from coverings $\sigma \lhd \tau$ in $\mathcal{C}(\Gamma)$) to numbers $[\sigma : \tau] \in \{+1, -1\}$ (called *incidence numbers*) such that the boundary maps are given by

$$d_i\left(\tau\right) = \sum_{\sigma \lhd \tau} [\sigma : \tau]\, \sigma, \tag{A2.9}$$

where we identify cells with the corresponding basis elements.

Appendix A3

Permutations and tableaux

Permutations play a central role throughout the book. They have close connections with the combinatorics of tableaux, which is of importance in Chapters 6 and 7.

Here, we first review the basic definitions and establish notation for permutations and tableaux. Then, in Sections A3.3 – A3.9 we summarize those properties of the Robinson-Schensted correspondence that are needed in Chapter 6. The final section concerns properties of dual equivalence of skew tableaux, needed in Chapter 7.

For a detailed treatment with proofs of this material, see [450]. Much of the material can also be found in [248], [328], and [498].

A3.1 Permutations

Fix a set E, finite or infinite. Bijections $x : E \to E$ are called *permutations* of E. They form a group under composition that we denote by $S(E)$. Subgroups of $S(E)$ are called *permutation groups.* A *permutation representation* of a group W is a homomorphism $f : W \to S(E)$ for some set E.

If $G \subseteq S(E)$ is a permutation group and $A \subseteq E$, let

$$\mathrm{Stab}\,(A) \stackrel{\mathrm{def}}{=} \{x \in G : x(A) = A\}.$$

The notation means that x maps A onto A *as a set* (not necessarily fixing each of its elements). This defines a subgroup of G called the *stabilizer* of A.

The finite groups $S_n \stackrel{\text{def}}{=} S([n])$ are called the *symmetric groups*. Suppose that E is a finite subset of $\mathbb{Z}$, such as $[n]$ or $[\pm n]$. Then, permutations $x \in S(E)$ can be denoted by listing all the values $x(i)$ left to right in order of increasing argument i. For instance, 74185236 denotes the permutation $1 \mapsto 7$, $2 \mapsto 4$, $3 \mapsto 1$, etc., an element of S_8. We call this the *complete notation*[1] for x. To keep notation simple, we omit commas in the complete notation wherever, as in the given example, confusion cannot arise. So, writing $x = x_1 x_2 \ldots x_n$ for a permutation x of a finite set $E \subset \mathbb{Z}$, this means that $x_i = x(e_i)$ for all i, where e_i is the i-th element of E in increasing order.

At times, we also write permutations in *disjoint cycle form*, omitting to write the 1-cycles. For instance, we have that

$$74185236 = (1, 7, 3)(2, 4, 8, 6),$$

where the left-hand side uses complete notation and the right hand side disjoint cycle form. Permutations of the form (i, j) are called *transpositions*.

Our convention for multiplying permutations is to read the product right to left as composition of mappings. For instance, with permutations of $[5]$ expressed in complete notation, we have

$$31524 \cdot 15243 = 34125.$$

This has the consequence for S_n that multiplying $x = x_1 x_2 \ldots x_n$ (complete notation) on the right by a transposition $t_{i,j} = (i, j)$ has the effect of transposing the values in *positions* i and j, whereas multiplying on the left transposes the *values* i and j. For example,

$$24531 \cdot t_{2,5} = 21534 \quad \text{and} \quad t_{2,5} \cdot 24531 = 54231.$$

Given a sequence $(x_1, x_2, \ldots, x_n) \in \mathbb{Z}^n$, define

$$\begin{aligned}
\operatorname{inv}(x_1, x_2, \ldots, x_n) &\stackrel{\text{def}}{=} |\{(i,j) :\ 1 \le i < j \le n \text{ and } x_i > x_j\}|, \\
\operatorname{neg}(x_1, x_2, \ldots, x_n) &\stackrel{\text{def}}{=} |\{i \in [n] :\ x_i < 0\}|, \\
\operatorname{nsp}(x_1, x_2, \ldots, x_n) &\stackrel{\text{def}}{=} \left|\left\{\{i,j\} \in \binom{[n]}{2} :\ x_i + x_j < 0\right\}\right|, \\
D(x_1, x_2, \ldots, x_n) &\stackrel{\text{def}}{=} \{i \in [n-1] :\ x_i > x_{i+1}\}.
\end{aligned}$$

These definitions apply, in particular, to permutations $x = x_1 x_2 \ldots x_n \in S(E)$, where $E \subseteq \mathbb{Z}$, $|E| = n$.

[1] Of course, listing only the first *seven* values (i.e., the images of $1, 2, \ldots, 7$, in this order) uniquely identifies a permutation of S_8. Thus, the last entry of the complete notation of a permutation is redundant and could be omitted. Although it would make no sense to use this shorter notation for the elements of the symmetric group, such "window notation" is extremely convenient for other permutation groups, including all those discussed in Chapter 8.

The functions "neg" and "nsp" (short for "number of negative entries" and "number of negative sum pairs") appear only in Chapter 8. For more about the "descent set" $D = D_R$, see Section A3.4.

A pair $(i, j) \in [n]^2$ is an *inversion* of a permutation $x = x_1 x_2 \dots x_n$ (or of a sequence $(x_1, \dots, x_n)$) if $i < j$ and $x_i > x_j$. The *inversion table* of x (or of $(x_1, \dots, x_n)$) is the sequence

$$(I_1(x), \dots, I_n(x)),$$

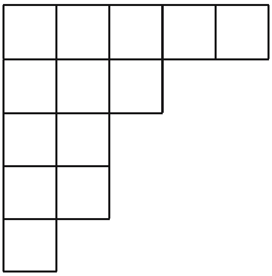
where

$$I_i(x) \stackrel{\text{def}}{=} |\{j \in [n] : \ i < j,\ x_i > x_j\}|.$$

In the rest of this appendix, all permutations will be elements of S_n.

A3.2 Tableaux

A *partition* $\lambda = (\lambda_1, \dots, \lambda_k)$ of the integer n (written $\lambda \vdash n$ or $|\lambda| = n$) is a weakly decreasing sequence of positive integers $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_k$ such that $\lambda_1 + \cdots + \lambda_k = n$. The integers $\lambda_1, \dots, \lambda_k$ are called the *parts* of λ, and k is called the *length* of λ. If $k = 0$, then λ is the *empty partition*. A partition is geometrically represented by its *(Ferrers) diagram*, a left-justified arrangement of boxes (also called *cells*) having λ_i boxes in row i. For instance, the following is the diagram of $(5, 3, 2, 2, 1)$:



A partititon $\lambda = (\lambda_1, \dots, \lambda_k)$ is a *hook* if $\lambda_2 = \lambda_3 = \cdots = \lambda_k = 1$, a *rectangle* if $\lambda_1 = \lambda_2 = \cdots = \lambda_k$, a *square* if $\lambda_1 = \cdots = \lambda_k = k$, and a *staircase* if $(\lambda_1, \dots, \lambda_k) = (k, k-1, \dots, 2, 1)$. We let $\delta_n \stackrel{\text{def}}{=} (n, n-1, \dots, 2, 1)$ for all $n \in \mathbb{P}$.

Given two partitions $\mu = (\mu_1, \dots, \mu_r)$, $\lambda = (\lambda_1, \dots, \lambda_k)$, we write $\mu \subseteq \lambda$ to mean that $r \leq k$ and $\mu_i \leq \lambda_i$ for $i = 1, \dots, r$. In this case, we call $\lambda \setminus \mu$ a *skew partition*. Skew partitions are also represented geometrically as diagrams. A skew partition of the form $\lambda \setminus \delta_n$, where λ is a square of length n, is called an *antistaircase*.

Given two skew partitions θ and ρ, we say that θ is an *extension* of (or *extends*) ρ if there exist three partitions λ, μ, and ν, with $\nu \subseteq \mu \subseteq \lambda$ such that $\rho = \mu \setminus \nu$ and $\theta = \lambda \setminus \mu$. We then write $\lambda \setminus \nu = \rho \cup \theta$. We say that

θ is a *final segment* (respectively, *initial segment*) of ρ if there exists three partitions λ, μ, ν with $\nu \subseteq \mu \subseteq \lambda$ such that $\rho = \lambda \setminus \nu$ and $\lambda \setminus \mu = \theta$ (respectively, $\mu \setminus \nu = \theta$). Flipping the diagram of a skew partition along the main diagonal yields the diagram of another skew partition, called its *conjugate.* For example, the conjugate of $(5, 3, 2, 2, 1) \setminus (2, 2, 1)$ is $(5, 4, 2, 1, 1) \setminus (3, 2)$. A skew partition is called *self-conjugate* (or *symmetric*) if it coincides with its conjugate. For example, $(4, 2, 1, 1) \setminus (1)$ is self-conjugate.

By a *connected* skew partition we mean a skew partition whose diagram is rookwise connected. The diagram of a partition (and hence of a skew partition) can be naturally identified with a subset of $\mathbb{N}^2$. Giving $\mathbb{N}^2$ its natural partial order ($(a, b) \leq (c, d)$ if and only if $a \leq c$ and $b \leq d$) then gives a partial order on the cells of the diagram. For this reason, we often identify a diagram with its corresponding poset in this way.

A *tableau* is a filling of the boxes of a diagram by distinct integers so that each row and each column is strictly increasing when read left to right and top to bottom. We call these integers the *entries* of the tableau. A tableau is called *standard* (or a *standard Young tableau*) if its entries are the numbers $1, 2, \ldots, n$, for some n. For instance,

1	2	5	8	11
3	6	10		
4	9			
7	13			
12				

is a standard Young tableau.

If we reflect a tableau T across the main diagonal, then we get another tableau, which we call the *transpose* of T, and denote by T'. Given a tableau T and $i \in \mathbb{Z}$, we let $T_{|+i}$ be the tableau obtained by adding i to each entry of T. If T is standard, we sometimes abuse terminology and call $T_{|+i}$ also a standard tableau. Given a tableau T, we denote by $T_{a,b}$ its b-th entry (from the left) in its a-th row (from the top).

The (possibly skew) partition associated with the diagram of a tableau T is called its *shape* , denoted $\mathrm{sh}(T)$, so, for example, the shape of the preceding tableau is $(5, 3, 2, 2, 1)$. We say that a tableau T has *normal shape* if $\mathrm{sh}((T)$ is a partition. If we wish to emphasize that there are no restrictions on the shape of T, then we say that T is a *skew tableau* (or that it has *skew shape*).

We let SYT_n denote the set of all standard Young tableaux with n boxes, SYT_λ the subset of those having shape λ, and $f^\lambda \stackrel{\text{def}}{=} |SYT_\lambda|$. The word "tableau" in this book means (unless otherwise explicitly stated) standard Young tableau.

A3.3 The Robinson-Schensted correspondence

With each permutation $x \in S_n$ is associated a pair $(P(x), Q(x))$ of tableaux of the same normal shape according to the following rule.

Let $x = x_1 x_2 \ldots x_n$. Starting with the pair of empty tableaux $(\emptyset, \emptyset)$, iterate the following procedure n times. Assume that (P_i, Q_i) has been constructed after i steps. Now, if x_{i+1} is greater than all entries in the first row of P_i, then place it at the end of that row (adding a new box). Otherwise, if, say, $p_{1,j} < x_{i+1} < p_{1,j+1}$, then replace (or "bump") $p_{1,j+1}$ by x_{i+1}. Then, repeat the same operation on the second row with $p_{1,j+1}$ playing the role of x_{i+1}. This bumping process will continue row by row until either a new box is created at the end of some existing row or a new one-box row is created. With this algorithm, a new box is created somewhere and the left tableau P_i grows to P_{i+1}. Let the right tableau Q_i grow to Q_{i+1} by placing $i+1$ in the correspondingly located new box. Due to this formation algorithm, the left tableau $P(x)$ is often called the *insertion tableau* and the right tableau $Q(x)$ is called the *recording tableau.*

The whole procedure is best explained by an example. Let $x = 35214$. The various steps in the formation of P and Q are:

	P_i	Q_i
Step1 :	$\begin{array}{\|c\|}\hline 3 \\ \hline\end{array}$	$\begin{array}{\|c\|}\hline 1 \\ \hline\end{array}$
Step2 :	$\begin{array}{\|c\|c\|}\hline 3 & 5 \\ \hline\end{array}$	$\begin{array}{\|c\|c\|}\hline 1 & 2 \\ \hline\end{array}$
Step3 :	$\begin{array}{\|c\|c\|}\hline 2 & 5 \\ \hline 3 \\ \cline{1-1}\end{array}$	$\begin{array}{\|c\|c\|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array}$
Step4 :	$\begin{array}{\|c\|c\|}\hline 1 & 5 \\ \hline 2 \\ \cline{1-1} 3 \\ \cline{1-1}\end{array}$	$\begin{array}{\|c\|c\|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array}$
Step5 :	$\begin{array}{\|c\|c\|}\hline 1 & 4 \\ \hline 2 & 5 \\ \hline 3 \\ \cline{1-1}\end{array}$	$\begin{array}{\|c\|c\|}\hline 1 & 2 \\ \hline 3 & 5 \\ \hline 4 \\ \cline{1-1}\end{array}$

The last pair of tableaux is $(P(x), Q(x))$.

Fact A3.3.1 *The mapping* $x \mapsto (P(x), Q(x))$ *is a bijection between permutations* $x \in S_n$ *and pairs of tableaux* $(P, Q) \in \bigcup_{\lambda \vdash n} SYT_\lambda^2$.

This is called the *Robinson-Schensted correspondence*, and much of this appendix is concerned with summarizing its key properties.

A3.4 Descent sets

The *right* and *left descent sets* of $x \in S_n$ are, by definition,

$$D_R(x) = \{i \in [n-1] \,:\, x(i) > x(i+1)\}$$

and $D_L(x) = D_R(x^{-1})$. For example,

$$D_R(41253) = \{1, 4\}\ ,\ D_L(41253) = \{3\}.$$

Note that the left descent set of a permutation $x \in S_n$ consists of those $i \in [n-1]$ such that $i+1$ appears to the left of i in the complete notation of x.

The *descent set* of a tableau T is the set $D(T)$ consisting of those entries i such that $i+1$ appears in a strictly lower row. This is related to the previous definition via the Robinson-Schensted correspondence as follows.

Fact A3.4.1 *$D_L(x) = D(P(x))$ and $D_R(x) = D(Q(x))$.*

For instance, for

$Q(35214) =$

1	2
3	5
4	

computed earlier, both x and $Q(x)$ have (right) descent set $= \{2, 3\}$.

A3.5 Special tableaux

A tableau is *row superstandard* if when reading its rows from left to right and from top to bottom, we get the integers $1, 2, \ldots, n$ in their natural order. For instance, the following is the row superstandard tableau of shape $(5, 3, 1)$:

1	2	3	4	5
6	7	8		
9				

By symmetry there is a corresponding notion of *column superstandard* tableaux.

The *reading word* $\rho(T)$ of a tableau T is obtained by reading the rows of T from left to right and from bottom to top. For instance, let T be the row superstandard tableau of shape $(5, 3, 1)$. Then, $\rho(T) = 967812345$.We will sometimes consider $\rho(T)$ as an element of S_n if T has n boxes (and is standard). For a given partition $\lambda = (\lambda_1, \ldots, \lambda_k)$, there is a unique tableau T_λ having descent set $\{\lambda_k, \lambda_k + \lambda_{k-1}, \ldots, \lambda_k + \cdots + \lambda_2\}$. It is called the *reading tableau* of shape λ, because of the bijection

$$\rho(T) \longleftrightarrow (T, T_\lambda),$$

which holds under Robinson-Schensted for all $T \in SYT_\lambda$. For example, the reading tableau of shape $(5, 3, 1)$ is

1	3	4	8	9
2	6	7		
5				

A3.6 Knuth equivalence

Let $x, y \in S_n$. We write $x \underset{K}{\approx} y$ if there exist $1 < i < n$ such that $x_1 x_2 \dots x_n$ and $y_1 y_2 \dots y_n$ differ only on the substrings $x_{i-1} x_i x_{i+1}$ and $y_{i-1} y_i y_{i+1}$, and these substrings are related to each other in either of the following two ways:

$$bca \leftrightarrow bac \quad \text{or} \quad cab \leftrightarrow acb,$$

where $a < b < c$. This is called *elementary Knuth equivalence*. It means that a commutation $ac \leftrightarrow ca$ is allowed if and only if the commuted pair has a neighbor b of intermediate value placed immediately to the left or immediately to the right.

Let $x \underset{K}{\sim} y$ be the equivalence relation (called *Knuth equivalence*) generated by $x \underset{K}{\approx} y$. For instance,

$$2\,1\,5\,\underline{4}\,\underline{3}\,\underline{6} \underset{K}{\approx} 2\,1\,\underline{5}\,\underline{4}\,\underline{6}\,3 \underset{K}{\approx} \underline{2}\,\underline{1}\,\underline{5}\,6\,4\,3 \underset{K}{\approx} 2\,\underline{5}\,\underline{1}\,\underline{6}\,4\,3 \underset{K}{\approx} 2\,5\,6\,1\,4\,3$$

shows that $215436 \underset{K}{\sim} 256143$.

The equivalence classes with respect to $\underset{K}{\sim}$ are called *Knuth classes*. This turns out to characterize the relation of having the same insertion tableau.

Fact A3.6.1 *$P(x) = P(y)$ if and only if $x \underset{K}{\sim} y$, for all $x, y \in S_n$.*

This result has a dual form characterizing equality of recording tableaux. The dual form can be easily deduced using Fact A3.9.1 below. Because of its importance in Chapter 6, we nevertheless give the explicit statement.

For $x, y \in S_n$, we write $x \underset{dK}{\approx} y$ if x and y differ by transposition of two values i and $i+1$, and either $i-1$ or $i+2$ occurs in a position between those of i and $i+1$. This defines *elementary dual Knuth equivalence*, and *dual Knuth equivalence* (written $x \underset{dK}{\sim} y$) is the transitive closure. For instance,

$$\underline{2}\,\underline{1}\,5\,4\,\underline{3}\,6 \underset{dK}{\approx} \underline{3}\,1\,\underline{5}\,\underline{4}\,2\,6 \underset{dK}{\approx} 4\,\underline{1}\,5\,\underline{3}\,\underline{2}\,6 \underset{dK}{\approx} 4\,2\,5\,3\,1\,6$$

shows that $215436 \underset{dK}{\sim} 425316$.

Fact A3.6.2 *$Q(x) = Q(y)$ if and only if $x \underset{dK}{\sim} y$, for all $x, y \in S_n$.*

The equivalence classes under "$\underset{dK}{\sim}$" are called *dual Knuth classes.*

A3.7 Jeu de taquin slides

Let T be a skew tableau and $x \in \mathbb{N}^2$ be such that $\mathrm{sh}(T)$ extends $\{x\}$. We then define the *backward jeu de taquin slide* (or *backward slide*, for short) of T into x to be the skew tableau, denoted $j^x(T)$, obtained as follows. We first fill cell x by "sliding" into it the smaller (or only one) of the entries of T that occupy the cells immediately to the right and immediately below cell x. This will vacate a new cell x', which we now fill by the same sliding rule, and so on until we have vacated a cell y for which neither the cell directly below it nor the one directly to its right are in $\mathrm{sh}(T)$.

For example, if

$$T = \begin{array}{cccc} \bullet & 1 & 4 & 8 \\ 2 & 5 & 6 & \square \\ 3 & 7 & & \end{array} \tag{A3.1}$$

and x is the cell marked by a $\bullet$, then we obtain

$$\begin{array}{cccc} \bullet & 1 & 4 & 8 \\ 2 & 5 & 6 & \\ 3 & 7 & & \end{array} \to \begin{array}{cccc} 1 & \bullet & 4 & 8 \\ 2 & 5 & 6 & \\ 3 & 7 & & \end{array} \to \begin{array}{cccc} 1 & 4 & \bullet & 8 \\ 2 & 5 & 6 & \\ 3 & 7 & & \end{array} \to \begin{array}{cccc} 1 & 4 & 6 & 8 \\ 2 & 5 & \bullet & \\ 3 & 7 & & \end{array}$$

and so

$$j^x(T) = \begin{array}{cccc} 1 & 4 & 6 & 8 \\ 2 & 5 & & \\ 3 & 7 & & \end{array}$$

Similarly, if $x \in \mathbb{N}^2$ is such that $\{x\}$ extends $\mathrm{sh}(T)$, then we define a *forward jeu de taquin slide* (or *forward slide*, for short) of T into x to be the skew tableau, denoted $j_x(T)$, defined as follows. We first fill cell x by "sliding" into it the largest (or only one) of the entries of T that occupy the cells immediately to the left and immediately above cell x. This vacates a new cell x', which we fill by the same rule until we have vacated a cell y for which neither the cell immediately to its left nor the one immediately above it are in $\mathrm{sh}(T)$.

For example, if T is the tableau in (A3.1) and x is the cell marked by a $\square$, then we obtain

$$\begin{array}{cccc} & 1 & 4 & 8 \\ 2 & 5 & 6 & \square \\ 3 & 7 & & \end{array} \to \begin{array}{cccc} & 1 & 4 & \square \\ 2 & 5 & 6 & 8 \\ 3 & 7 & & \end{array} \to \begin{array}{cccc} & 1 & \square & 4 \\ 2 & 5 & 6 & 8 \\ 3 & 7 & & \end{array} \to \begin{array}{cccc} & \square & 1 & 4 \\ 2 & 5 & 6 & 8 \\ 3 & 7 & & \end{array}$$

so

$$j_x(T) = \begin{array}{cccc} & & 1 & 4 \\ 2 & 5 & 6 & 8 \\ 3 & 7 & & \end{array}$$

A *slide sequence* for T is a sequence of cells $(x_1, \dots, x_r)$ such that it is meaningful to form the tableaux $T_r, \dots, T_1$, where, for each $i = 1, \dots, r$, either $T_i = j_{x_i}(T_{i-1})$ or $T_i = j^{x_i}(T_{i-1})$ (and where $T_0 \stackrel{\text{def}}{=} T$).

A3.8 Evacuation and antievacuation

We describe two operations called *evacuation* and *antievacuation* that transform a tableau T to other tableaux $e(T)$, $e^*(T)$ of the same shape. These operations turn out to be involutions.

Let T be a tableau with n cells. Delete entry "n" from T and perform a forward slide into the cell that contained it. This will vacate a cell of T. Now do the same for entry "$n-1$," then for entry "$n-2$," etc., and finally for entry "1." You will now have the empty tableau. Then, the *antievacuation tableau* of T, denoted $e^*(T)$, is the tableau whose entries record the order in which the cells of T have been vacated.

For example, if

$$T = \begin{array}{cccc} & 1 & 3 & 6 \\ 2 & 4 & 7 & \\ 5 & & & \end{array}$$

then we obtain from it the following sequence of tableaux:

$$T \;\rightarrow\; \begin{array}{cccc} & 1 & 3 & 6 \\ \square & 2 & 4 & \\ 5 & & & \end{array} \;\rightarrow\; \begin{array}{cccc} & \square & 1 & 3 \\ \square & 2 & 4 & \\ 5 & & & \end{array} \;\rightarrow\; \begin{array}{cccc} & \square & 1 & 3 \\ \square & 2 & 4 & \\ \square & & & \end{array}$$

$$\rightarrow\; \begin{array}{cccc} & \square & 1 & 3 \\ \square & \square & 2 & \\ \square & & & \end{array} \;\rightarrow\; \begin{array}{cccc} & \square & \square & 1 \\ \square & \square & 2 & \\ \square & & & \end{array} \;\rightarrow\; \begin{array}{cccc} & \square & \square & 1 \\ \square & \square & \square & \\ \square & & & \end{array}$$

and, therefore,

$$e^*(T) = \begin{array}{cccc} & 2 & 5 & 7 \\ 1 & 4 & 6 & \\ 3 & & & \end{array}$$

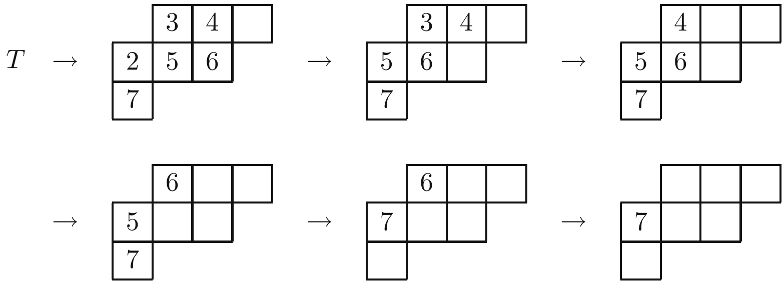
Note that $e^*(T)$ has the same shape as T. It is a fact that the mapping e^* is an involution on the set of tableaux of any given shape.

Next, starting from T, we first delete the entry "1" and perform a backward slide on the cell that contained it; then we do the same for the entry "2," etc.... Then, the tableau that records (in reverse) the order in which the cells of T have been vacated in this process is called the *evacuation* of T and is denoted by $e(T)$.

For example, if

$$T = \begin{array}{cccc} & \boxed{1} & \boxed{3} & \boxed{4} \\ \boxed{2} & \boxed{5} & \boxed{6} & \\ \boxed{7} & & & \end{array}$$

then we obtain the following sequence of tableaux :



and, hence,

$$e(T) = \begin{array}{cccc} & \boxed{2} & \boxed{5} & \boxed{7} \\ \boxed{1} & \boxed{4} & \boxed{6} & \\ \boxed{3} & & & \end{array}$$

Again, $e(T)$ has the same shape as T, and the mapping e is an involution.

A3.9 Symmetries of the R-S correspondence

Let w_0 be the reverse permutation $w_0 = n \ldots 3\ 2\ 1$. We summarize the remarkable effects on tableaux that multiplication with w_0 and the operation $x \mapsto x^{-1}$ have under the Robinson-Schensted correspondence.

With our convention for multiplying permutations, we get the following combinatorial meanings of the algebraic operations:

$$\begin{array}{rcl} x^{-1} & \longleftrightarrow & \text{switch places and values,} \\ xw_0 & \longleftrightarrow & \text{reverse the places,} \\ w_0x & \longleftrightarrow & \text{reverse the values,} \\ w_0xw_0 & \longleftrightarrow & \text{reverse both.} \end{array}$$

For instance, if $x = 24135$, then $x^{-1} = 31425$, $xw_0 = 53142$, $w_0x = 42531$, and $w_0xw_0 = 13524$.

Recall that for a tableau P, we let P' denote the transposed tableau (i.e., P mirrored in its main diagonal). This clearly commutes with evacuation: $e(P') = e(P)'$.

Fact A3.9.1 *If $x \leftrightarrow (P, Q)$ are matched under the Robinson-Schensted correspondence, then so are*

$$\begin{array}{rcl} x^{-1} & \longleftrightarrow & (Q, P), \\ xw_0 & \longleftrightarrow & (P', e(Q)'), \\ w_0 x & \longleftrightarrow & (e(P)', Q'), \\ w_0 x w_0 & \longleftrightarrow & (e(P), e(Q)). \end{array}$$

Note that the last relation implies that evacuation is an involution on SYT_λ.

We leave it to the reader to exemplify these relations; for instance, starting from the pair

$$24135 \quad \longleftrightarrow \quad \left(\begin{array}{|c|c|c|} \hline 1 & 3 & 5 \\ \hline 2 & 4 \\ \cline{1-2} \end{array} \; , \; \begin{array}{|c|c|c|} \hline 1 & 2 & 5 \\ \hline 3 & 4 \\ \cline{1-2} \end{array} \right).$$

A3.10 Dual equivalence

In this section, we summarize the properties of an equivalence relation for skew tableaux that is closely connected to the dual Knuth equivalence of permutations, as discussed in Section A3.6.

Let P and Q be skew tableaux. We say that P is *dual equivalent* to Q, denoted $P \approx Q$, if whenever a slide sequence can be applied to both P and Q, then the resulting tableaux are of the same shape.

Note that the sequence in the definition can be empty. Thus, two dual equivalent tableaux necessarily have the same shape. The converse, however, is not true in general. For example, if

$$S = \begin{array}{ccc} \cline{2-3} & \multicolumn{1}{|c|}{2} & \multicolumn{1}{c|}{3} \\ \hline \multicolumn{1}{|c|}{1} \\ \cline{1-1} \end{array} \quad \text{and} \quad T = \begin{array}{ccc} \cline{2-3} & \multicolumn{1}{|c|}{1} & \multicolumn{1}{c|}{3} \\ \hline \multicolumn{1}{|c|}{2} \\ \cline{1-1} \end{array}$$

then S and T are not dual equivalent. In fact, performing a backward slide into the cell $(1,1)$ yields

$$j^{(1,1)}(S) = \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline \end{array} \quad \text{and} \quad j^{(1,1)}(T) = \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 \\ \cline{1-1} \end{array}$$

which do not have the same shape. We do have, however, the following remarkable result.

Fact A3.10.1 *Let U and V be two tableaux of the same normal shape. Then, $U \approx V$.*

Although the definition of dual equivalence is a global one, this concept can be characterized locally, and this is one of the most important of its many properties.

Fact A3.10.2 *Let X, S, T, and Y be four tableaux such that* $\mathrm{sh}(Y)$ *extends* $\mathrm{sh}(T)$, $\mathrm{sh}(T)$ *extends* $\mathrm{sh}(X)$, *and* $S \approx T$. *Then,* $X \cup S \cup Y \approx X \cup T \cup Y$.

Note that in this lemma we are writing, for simplicity, $X \cup S \cup Y$ instead of $X \cup S_{|+|X|} \cup Y_{|+|X|+|S|}$, etc., thereby tacitly using the convention stated in Section A3.2. We will do this routinely.

A skew partition $\lambda \setminus \mu$ is said to be *miniature* if $|\lambda \setminus \mu| = 3$. A tableau T is said to be *miniature* if $\mathrm{sh}(T)$ is miniature.

Fact A3.10.3 *Each dual equivalence class of miniature tableaux consists of at most two tableaux. Furthermore, a miniature tableau T is in a two-element dual equivalence class if and only if its reading word is either* 132, 231, 213, *or* 312. *In each case, the unique tableau S dual equivalent to T is equal to T except that its reading word is the reverse of that of T.*

Two tableaux U and V are *elementary dual equivalent* if there exist four tableaux X, S, T, and Y as in the hypotheses of Fact A3.10.2 such that S and T are miniature, $S \approx T$, $U = X \cup S \cup Y$, and $V = X \cup T \cup Y$. So, for example,

	1	2	3
	4	5	
6			

and

	1	2	3
	4	6	
5			

are elementary dual equivalent, but

	1	2	3
	4	5	
6			

and

	1	2	6
	4	5	
3			

are not.

It is clear from this definition and Fact A3.10.2 that two tableaux that are related by a chain of elementary dual equivalences are dual equivalent. Remarkably, the converse is also true.

Fact A3.10.4 *Let U and V be two tableaux. Then, $U \approx V$ if and only if U can be obtained from V by a sequence of elementary dual equivalences.*

Appendix A4

Enumeration and symmetric functions

In this appendix, we review some results, notation and terminology regarding formal power series and symmetric functions. These are needed in connection with the enumerative theory of Coxeter groups. Further details, including proofs, can be found in [450], [497], [498], and [258].

A4.1 Formal power series

Let $\mathbf{x} = (x_1, x_2, \ldots)$ be a sequence of independent variables and R be a commutative ring with identity. We denote by $R[[\mathbf{x}]]$ the ring of formal power series in $x_1, x_2, \ldots$. Given $(a_1, a_2, \ldots, a_p) \in \mathbb{N}^p$ and $F \in R[[\mathbf{x}]]$, we denote by $[x_1^{a_1} \cdots x_p^{a_p}](F)$ the coefficient of the monomial $x_1^{a_1} \cdots x_p^{a_p}$ in F, and we also write $F(0) \stackrel{\text{def}}{=} [x_1^0 x_2^0 \cdots](F)$. If $F \in R[[\mathbf{x}]]$ is invertible, we write $G = F^{-1}$ (or $G = 1/F$) to mean that $FG = GF = 1$. An element $F \in R[[\mathbf{x}]]$ is *rational* if there exist polynomials $P, Q \in R[x_1, x_2, \ldots]$ such that $Q(0)$ is invertible in R and

$$F = \frac{P}{Q}.$$

Recall that there is a notion of convergence in $R[[\mathbf{x}]]$. Namely, if $F, F_1, F_2, \ldots \in R[[\mathbf{x}]]$, then we write

$$\lim_{n\to\infty} F_n = F$$

if for each monomial $x_1^{a_1}x_2^{a_2}\cdots$, there is an integer N (depending on $a_1, a_2, \ldots$) such that $[x_1^{a_1}x_2^{a_2}\cdots](F_n) = [x_1^{a_1}x_2^{a_2}\cdots](F)$ for all $n \geq N$. We then say that the sequence $\{F_n\}_{n=1,2,\ldots}$ *converges* to F. In particular, we write

$$\prod_{n\geq 1} F_n = F$$

to mean that

$$\lim_{n\to\infty} \prod_{i=1}^{n} F_i = F.$$

Given $F \in R[[\mathbf{x}]]$ such that $F(0) = 1$, we define

$$\log(F) \stackrel{\text{def}}{=} \sum_{n\geq 1} (-1)^{n-1} \frac{(F-1)^n}{n}.$$

Let now z, x, and q be three independent variables. The following result is usually known as the q-Binomial Theorem, see, e.g., [258, Appendix II.3].

Fact A4.1.1 *We have that*

$$\sum_{n\geq 0} \prod_{i=0}^{n-1} \left(\frac{1-zq^i}{1-q^{i+1}}\right) x^n = \prod_{i\geq 0} \frac{(1-zxq^i)}{(1-xq^i)}$$

in $\mathbb{Q}[[z, x, q]]$.

Let D be a directed graph on vertex set $[n]$. The *adjacency matrix* of D is the matrix $Z \stackrel{\text{def}}{=} (Z_{u,v})_{u,v\in[n]}$ defined by

$$Z_{u,v} = \begin{cases} 1, & \text{if } u \to v, \\ 0, & \text{otherwise.} \end{cases}$$

For $u, v \in [n]$, define a formal power series $F_{u,v}(q) \in \mathbb{Z}[[q]]$ by

$$F_{u,v}(q) \stackrel{\text{def}}{=} \sum_{n\geq 0} F_{u,v}(n)\, q^n,$$

where $F_{u,v}(n)$ equals the number of paths of length n from u to v (so $F_{u,v}(1) = Z_{u,v}$, $F_{u,v}(0) = \delta_{u,v}$).

The following basic result is sometimes known as the Transfer Matrix Method. See Theorem 4.7.2 in [497] for a detailed discussion

Fact A4.1.2 *Let* $u, v \in [n]$. *Then,*

$$F_{u,v}(q) = \frac{(-1)^{u+v}\, det(I - qZ; v, u)}{det(I - qZ)},$$

where $(I - qZ; v, u)$ *denotes the matrix obtained from* $I - qZ$ *by deleting its* v*-th row and* u*-th column, and* I *is the* $n \times n$ *identity matrix.*

Corollary A4.1.3 *The series* $F_{u,v}(q)$ *is rational.*

A4.2 Symmetric functions

An element $F \in R[[\mathbf{x}]]$ is said to be *symmetric* if

$$F(x_1, x_2, \ldots) = F(x_{u(1)}, x_{u(2)}, \ldots)$$

for all bijections $u : \mathbb{P} \to \mathbb{P}$, and it is said to be *bounded* if there is a constant M such that all of the monomials appearing in F have degree $\leq M$. F is called a *symmetric function* if it is both symmetric and bounded. For example, $\prod_{i\geq 1}(1 + x_i)$ is symmetric but not a symmetric function.

Let $\lambda = (\lambda_1, \ldots, \lambda_k)$ be a partition. A *column strict plane partition* T of shape λ is a filling of the boxes of the diagram of λ with positive integers so that each row is weakly decreasing when read from left to right and each column is strictly decreasing when read from top to bottom. The *content* of T is the vector

$$m(T) = (m_1(T), m_2(T), \ldots),$$

where $m_i(T)$ equals the number of entries of T that are equal to i ($i \in \mathbb{P}$).

The *Schur function* associated to λ is defined by

$$s_\lambda(\mathbf{x}) \stackrel{\text{def}}{=} \sum_T x_1^{m_1(T)} x_2^{m_2(T)} \cdots ,$$

where T runs over all the column strict plane partitions of shape λ. It is a remarkable fact that $s_\lambda(\mathbf{x})$ is always a symmetric function.

Fact A4.2.1 *Let λ be a partition; then, $s_\lambda(\mathbf{x})$ is a symmetric function, homogeneous of degree $|\lambda|$.*

In fact, much more is true. It is clear that the symmetric functions that are homogeneous of a given degree n form a vector space, and it turns out that the set $\{s_\lambda(\mathbf{x})\}_{\lambda\vdash n}$ is a basis for it.

Let $p \in \mathbb{P}$ and $S \subseteq [p-1]$. A sequence $(a_1, \ldots, a_p) \in \mathbb{P}^p$ is *compatible* with S if the following hold:

(i) $a_1 \leq a_2 \leq \cdots \leq a_p$.

(ii) $a_i < a_{i+1}$ if $i \in S$.

Denote by C_S the set of all the sequences compatible with S. The *fundamental quasi-symmetric function* $Q_{S,p}(\mathbf{x})$ is

$$Q_{S,p}(\mathbf{x}) \stackrel{\text{def}}{=} \sum_{(a_1,\ldots,a_p)\in C_S} x_{a_1} \cdots x_{a_p}.$$

We then have the following result.

Fact A4.2.2 *Let λ be a partition. Then,*

$$\sum_{T\in \mathrm{SYT}_\lambda} Q_{D(T),|\lambda|}(\mathbf{x}) = s_\lambda(\mathbf{x}),$$

where $D(T)$ is the descent set of T (see Section A3.4).

For example, if $\lambda = (2,1)$, then there are two tableaux of shape λ, namely

$$\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array} \quad \text{and} \quad \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1}\end{array}$$

so

$$s_{(2,1)}(\mathbf{x}) = Q_{\{2\},3}(\mathbf{x}) + Q_{\{1\},3}(\mathbf{x}).$$

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Index of notation

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