Appendix

A.1 The Edge of the Wedge Theorem

In the body of the volume, we used several times the edge-of-the-wedge theorem. For the convenience of the reader, we give a statement of this theorem and make some remarks. In its most basic form, the theorem deals with the following situation. $U = (x_1, x_2)$ is an open interval in $\mathbb{R}$, F_1 a function that is holomorphic on the upper half plane of $\mathbb{C}$, F_2 a function holomorphic on the lower half plane, both F_1 and F_2 have the same bounded, continuous limit on U. Then there exists a function F, holomorphic in the cut plane $\mathbb{C}\backslash[(-\infty, x_1] \cup [x_2, \infty)]$, which is a joint extension of F_1, F_2.

A more general version of the theorem applies to analytic functions F_1, F_2 holomorphic on a domain of $\mathbb{C}^n$ of the form $U + iC$ resp. $U - iC$, where $U \subset \mathbb{R}^n$ is an open domain, and where $C \subset \mathbb{R}^n$ is the intersection of some open, convex cone with an open ball. It is assumed that

$$T_1(f) = \lim_{y \in C, y \to 0} \int \mathrm{d}^n x \, F_1(x + iy) f(x), \quad T_2(f) = \lim_{y \in C, y \to 0} \int \mathrm{d}^n x \, F_2(x - iy) f(x) \tag{A.1}$$

define distributions on U such that, actually, $T_1 = T_2$. The edge of the wedge theorem is (see e.g. [1]):

Theorem 19 *There exists a function F which is holomorphic on an open complex neighborhood $N \subset \mathbb{C}^n$ containing U such that F extends both F_1, F_2 where defined.*

One often applies the theorem to the case when a holomorphic function F_1 on $U + iC$ is given with distributional boundary value $T_1 = 0$. Then choosing $F_2 \equiv 0$, one learns that also $F_1 = 0$ where defined.

The edge of the wedge theorem has a straightforward generalization to the case when F_1, F_2 take values in a Banach space, $\mathcal{X}$, which we also use in this volume. A function F valued in $\mathcal{X}$ is called (weakly) holomorphic near z_0 if $\psi(F(z))$ is holomor-

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phic near z_0 for any linear functional ψ in the topological dual $\mathcal{X}^*$. It is easy to see (see e.g. [2]) that a weakly holomorphic function is in fact even strongly holomorphic in the sense that it has a norm-convergent expansion $F(z) = \sum_{n\geq 0} x_n(z-z_0)^n$, $x_n \in \mathcal{X}$ near z_0 (and of course vice versa). By going through the proof of the edge of the wedge-theorem in the $\mathbb{C}$-valued case, one can see as a consequence that an $\mathcal{X}$-valued version holds true, too: if F_i are holomorphic $\mathcal{X}$-valued functions in $U \pm iC$ such that their distributional boundary values (A.1) (limit in the norm topology on $\mathcal{X}$) on U coincide as distributions valued in $\mathcal{X}$, then there is a holomorphic extension F on N. For a related discussion, see also [3].

We also use in this volume the following (related) lemma about $\mathcal{X}$-valued holomorphic functions.

Lemma 14 *Let U be an open domain in $\mathbb{C}$ and let $F : U \to \mathcal{X}$ be a holomorphic function with continuous limit on ∂U. Then $\overline{U} \ni z \mapsto \|F(z)\|_{\mathcal{X}}$ assumes its maximum on the boundary ∂U.*

Proof The norm $u(z) \equiv \|F(z)\|_{\mathcal{X}}$ is continuous on $\overline{U}$ and for each continuous linear map $l : \mathcal{X} \to \mathbb{C}$ the scalar function $l \circ F$ is continuous on $\overline{U}$ and holomorphic on the interior. If $\mathcal{X}_1^*$ denotes the unit ball of the dual space $\mathcal{X}^*$, then

$$\begin{aligned}\max_{z\in\overline{U}} u(z) &= \max_{z\in\overline{U}} \max_{l\in\mathcal{X}_1^*} |l(F(z))| = \max_{l\in\mathcal{X}_1^*} \max_{z\in\overline{U}} |l(F(z))| \\ &= \max_{l\in\mathcal{X}_1^*} \max_{z\in\partial U} |l(F(z))| = \max_{z\in\partial U} \max_{l\in\mathcal{X}_1^*} |l(F(z))| = \max_{z\in\partial U} u(z),\end{aligned} \tag{A.2}$$

where we applied the maximum principle to $l \circ F$ to get to the second line. □

References

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3. A. Strohmaier, R. Verch, M. Wollenberg, Microlocal analysis of quantum fields on curved spacetimes: analytic wavefront sets and Reeh-Schlieder theorems. J. Math. Phys. **43**, 5514 (2002)