Appendix A
FORTRAN Variables

The following Tables A.1–A.5 contains a list of all variables used in the FORTRAN codes:

Table A.1 List of deformation related variables used in the FORTRAN codes

Deformation				
Type	Variable	Symbol	Unit	Description
Real*8	eps	ε_{n+1}^{j+1}	[−]	Strain at current increment and iteration
Real*8	epsinc	ε_n	[−]	Strain at previous increment
Real*8	epsit	ε_{n+1}^{j}	[−]	Strain at current increment and previous iteration
Real*8	deps	$\Delta\varepsilon_n^{j+1}$	[−]	Strain increment at current iteration
Real*8	epspl	κ_{n+1}^{j+1}	[−]	Effective plastic strain at current increment and iteration
Real*8	epsplinc	κ_n	[−]	Effective plastic strain at previous increment
Real*8	epsplt	$\kappa_{n+1}^{\mathrm{t}}$	[−]	Trial effective plastic strain at current increment
Real*8	dl	$\Delta\lambda$	[−]	Consistency parameter
Real*8	utot	u_{tot}	[−]	Total user prescribed displacement
Real*8	du	Δu_n^{j+1}	[−]	Displacement increment at current iteration
Real*8	u	u_{n+1}^{j+1}	[−]	Displacement at current increment and iteration
Real*8	uinc	u_n	[−]	Displacement at previous increment
Real*8	uit	u_{n+1}^{j}	[−]	Displacement at current increment and previous iteration

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Table A.2 List of stress and force related variables used in the FORTRAN codes

Stress and force				
Type	Variable	Symbol	Unit	Description
Real*8	ds	$\Delta\sigma_n^{j+1}$	[MPa]	Stress increment at current iteration
Real*8	s	σ_{n+1}^{j+1}	[MPa]	Stress at current increment and iteration
Real*8	sdev	σ'^{j+1}_{n+1}	[MPa]	Deviatoric stress at current increment and iteration
Real*8	sinc	σ_n	[MPa]	Stress at previous iteration
Real*8	st	σ_{n+1}^t	[MPa]	Trial stress at current increment
Real*8	funF	$F(\sigma, k(\kappa))$	[MPa]	Yield function
Real*8	dFds	$\frac{\partial F}{\partial \boldsymbol{\sigma}}$	[−]	Derivative of the yield function with respect to the stress
Real*8	dFdJ2dev	$\frac{\partial F}{\partial J_2'}$	$\left[\frac{1}{\text{MPa}}\right]$	Derivative of the yield function
Real*8	d2Fds2	$\frac{\partial^2 F}{\partial \boldsymbol{\sigma}^2}$	$\left[\frac{1}{\text{MPa}}\right]$	Second derivative of the yield function
Real*8	d2FdJ2dev2	$\frac{\partial^2 F}{\partial \left(J_2'\right)^2}$	$\left[\frac{1}{\text{MPa}^2}\right]$	Second derivative of the yield function
Real*8	r	r	[−]	Function of flow direction
Real*8	seff	σ_{eff}	[−]	Effective stress
Real*8	J2dev	J_2'	[−]	Second deviatoric invariant
Real*8	v	$\mathbf{v}_{n+1}^{j+1}$	[MPa, −]	Solution vector at current increment and iteration
Real*8	v0	$\mathbf{v}_{n+1}^{j=0}$	[MPa, −]	Solution vector at the beginning of the increment
Real*8	vt	$\mathbf{v}_{n+1}^{t}$	[MPa, −]	Solution vector at the trial state
Real*8	vold	$\mathbf{v}_{n+1}^{j}$	[MPa, −]	Solution vector at previous iteration
Real*8	m	$\mathbf{m}_{n+1}^{j+1}$	[MPa, −]	Residual vector at current increment and iteration
Real*8	dmdv	$\left(\frac{\partial \mathbf{m}}{\partial \mathbf{v}}\right)_{n+1}^{j+1}$	$\left[\frac{1}{\text{MPa}}, -\right]$	Residual matrix of derivatives with respect to the solution vector
Real*8	dmdvinv	$\left(\left(\frac{\partial \mathbf{m}}{\partial \mathbf{v}}\right)_{n+1}^{j+1}\right)^{-1}$	[MPa, −]	Residual matrix inverse (3D)
Real*8	mtilinv	$\left(\left(\frac{\partial \mathbf{m}}{\partial \mathbf{v}}\right)_{n+1}^{j+1}\right)^{-1}$	[MPa, −]	Residual matrix inverse (1D)
Real*8	Ftot	F_{tot}	[N]	Total user prescribed force
Real*8	F	F_{n+1}^{j+1}	[N]	Force at current increment and iteration
Real*8	dF	ΔF_n^{j+1}	[N]	Force increment at current iteration
Real*8	r	r_{n+1}^{j+1}	[N]	Force residual at the current increment and iteration

Table A.3 List of geometry parameters used in the FORTRAN codes

Geometry parameters				
Type	Variable	Symbol	Unit	Description
Real*8	A	A	$[\mathrm{m}^2]$	Cross-sectional area of specimen
Real*8	L	L	[m]	Length of specimen
Integer	ele	ele	[−]	Total number of elements
Integer	m	m	[−]	Element counter
Integer	nod	nod	[−]	Total number of nodes
Integer	i	i	[−]	Node counter

Table A.4 List of material parameters used in the FORTRAN codes

Material parameters				
Type	Variable	Symbol	Unit	Description
Real*8	nu	ν	[−]	POISSON ratio
Real*8	Etil	$\tilde{E}_n^{j+1}$	[MPa]	(Approximated) tangent modulus at current iteration
Real*8	E	E	[MPa]	YOUNG's modulus
Real*8	Epl	E^{pl}	[MPa]	Plastic modulus
Real*8	kinit	k^{init}	[MPa]	Initial yield stress
Real*8	Eelpl	E^{elpl}	[MPa]	Elasto-plastic modulus
Real*8	K	K_n^{j+1}	$\left[\frac{\mathrm{kg}}{\mathrm{s}^2}\right]$	(Approximated) tangent stiffness at current iteration
Real*8	Km	$\mathbf{K}_n^{j+1}$	$\left[\frac{\mathrm{kg}}{\mathrm{s}^2}\right]$	Tangent stiffness matrix at current iteration
Real*8	Kmred	$\mathbf{K}_{\mathrm{red},n}^{j+1}$	$\left[\frac{\mathrm{kg}}{\mathrm{s}^2}\right]$	Reduced tangent stiffness matrix (to enable matrix inversion)
Real*8	Kminv	$\left(\mathbf{K}_n^{j+1}\right)^{-1}$	$\left[\frac{\mathrm{kg}}{\mathrm{s}^2}\right]$	Tangent stiffness matrix inverse
Real*8	Kserial	$\sum_m (K^m)$	$\left[\frac{\mathrm{kg}}{\mathrm{s}^2}\right]$	Sum of elemental stiffness inverses
Real*8	funK	$k(\kappa)$	[MPa]	Yield stress
Real*8	h	h	[−]	Hardening function
Real*8	dhds	$\frac{\mathrm{d}h}{\mathrm{d}\sigma}$	$\left[(\mathrm{MPa})^{-1}\right]$	Derivative of the hardening function with respect to the stress

Table A.5 List of miscellaneous parameters used in the FORTRAN codes

Miscellaneous				
Type	Variable	Symbol	Unit	Description
Real*8	tolILoop	tol	[−]	Tolerance of the NEWTON- RAPHSON algorithm which iterates $\Delta\varepsilon_n^{j+1}$
Real*8	junk	–	[−]	Placeholder variable
Real*8	tol	tol	[−]	Tolerance at current iteration
Real*8	tolCPP	tol	[−]	Tolerance of the CPP algorithm
Integer	inc	inc	[−]	Total number of increments
Integer	n	n	[−]	Increment counter
Integer	j	j	[−]	Iteration counter (reset)
Integer	z	z	[−]	Iteration counter (not reset)
Integer	it	j	[−]	Iteration counter, used when j is already assigned
Integer	g	–	[−]	Row counter
Integer	p	–	[−]	Row counter
Integer	h	–	[−]	Column counter
Integer	q	–	[−]	Column counter
Integer	Input	–	[−]	Input file label
Integer	Output	–	[−]	Output file label
Logical	tolexceeded	–	[−]	Boolean for tolerance check
Logical	CPPtolexceeded	–	[−]	Boolean for tolerance check
Logical	tolex	–	[−]	Boolean for tolerance check
Logical	linCPP	–	[−]	Flag, T: linear hardening function, F: non-linear hardening function
Logical	Fconrolled	–	[−]	Flag, T: force-controlled setup, F: displacement-controlled setup
Logical	EqEle	–	[−]	Flag, T: element parameters are equal, F: individual parameters

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