Appendix A
Mono-jet Cross Sections in the EFT Limit

A.1 Generalities

In this Appendix we show the details of the calculations of the tree-level cross sections for the hard scattering process $f(p_1) + \bar{f}(p_2) \to \chi(p_3) + \chi(p_4) + g(k)$, where f is either a quark (operators D1–D10 and DT1) or a gluon (D11–D14), and the final gluon is emitted from the initial state. For the t-channel operator DT1, we also computed the analytic cross section for the process with (anti-)quark emission, $q(p_1) + g(p_2) \to \chi(p_3) + \chi(p_4) + q(k)$.

The differential cross section is generically given by

$$d\hat{\sigma} = \frac{\sum \overline{|\mathcal{M}|^2}}{4(p_1 \cdot p_2)} \mathrm{d}\Phi_3 \,, \tag{A.1}$$

where the three-body phase space is

$$\mathrm{d}\Phi_3 = (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4 - k) \frac{\mathrm{d}\mathbf{p}_3}{(2\pi)^3 2 p_3^0} \frac{\mathrm{d}\mathbf{p}_4}{(2\pi)^3 2 p_4^0} \frac{\mathrm{d}\mathbf{k}}{(2\pi)^3 2 k^0} \,. \tag{A.2}$$

A.2 Matrix Elements

The matrix elements at the parton level are given by the sum of the Feynman diagrams in Fig. 6.1 for the operators D1–D14 and in Fig. 6.2 for the operator DT1.

In the limit of massless light quarks, they have definite helicity and it makes no difference for the cross sections whether there is q or $\gamma^5 q$ in the operator. Therefore the following identifications between pairs of operators hold:

$$D1' \leftrightarrow D3', \qquad D2' \leftrightarrow D4', \qquad D5 \leftrightarrow D7, \qquad D6 \leftrightarrow D8, \qquad D9 \leftrightarrow D10 \,, \tag{A.3}$$

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while the "primed" and "unprimed" operators are related as in Eq. (6.28). For definiteness, we choose to work with $D1'$, $D4'$, $D5$, $D8$, $D9$ and D11–D14.

The amplitudes are given by

$$\mathcal{M}_{D1'} = -ig_s \frac{1}{\Lambda^2}\epsilon^{*a}_{\mu}(k)\left[\frac{\bar{v}(p_2)(\not{p}_1-\not{k})\gamma^{\mu}T^a u(p_1)}{(p_1-k)^2} - \frac{\bar{v}(p_2)\gamma^{\mu}T^a(\not{p}_2-\not{k})u(p_1)}{(p_2-k)^2}\right]\bar{u}(p_3)v(p_4)\,, \tag{A.4}$$

$$\mathcal{M}_{D4'} = -ig_s \frac{1}{\Lambda^2}\epsilon^{*a}_{\mu}(k)\left[\frac{\bar{v}(p_2)\gamma^5(\not{p}_1-\not{k})\gamma^{\mu}T^a u(p_1)}{(p_1-k)^2} - \frac{\bar{v}(p_2)\gamma^{\mu}T^a(\not{p}_2-\not{k})\gamma^5 u(p_1)}{(p_2-k)^2}\right] \times\bar{u}(p_3)\gamma^5 v(p_4)\,, \tag{A.5}$$

$$\mathcal{M}_{D5} = -ig_s \frac{g_{\nu\rho}}{\Lambda^2}\epsilon^{*a}_{\mu}(k)\left[\frac{\bar{v}(p_2)\gamma^{\nu}(\not{p}_1-\not{k})\gamma^{\mu}T^a u(p_1)}{(p_1-k)^2} - \frac{\bar{v}(p_2)\gamma^{\mu}T^a(\not{p}_2-\not{k})\gamma^{\nu} u(p_1)}{(p_2-k)^2}\right] \times\bar{u}(p_3)\gamma^{\rho} v(p_4)\,, \tag{A.6}$$

$$\mathcal{M}_{D8} = -ig_s \frac{g_{\nu\rho}}{\Lambda^2}\epsilon^{*a}_{\mu}(k)\left[\frac{\bar{v}(p_2)\gamma^{\nu}\gamma^5(\not{p}_1-\not{k})\gamma^{\mu}T^a u(p_1)}{(p_1-k)^2} - \frac{\bar{v}(p_2)\gamma^{\mu}T^a(\not{p}_2-\not{k})\gamma^{\nu}\gamma^5 u(p_1)}{(p_2-k)^2}\right] \times\bar{u}(p_3)\gamma^{\rho}\gamma^5 v(p_4)\,, \tag{A.7}$$

$$\mathcal{M}_{D9} = -i\frac{g_s}{16} \frac{g_{\mu\rho}g_{\nu\sigma}}{\Lambda^2}\epsilon^{*a}_{\alpha}(k)\left[\frac{\bar{v}(p_2)\sigma^{\mu\nu}(\not{p}_1-\not{k})\gamma^{\alpha}T^a u(p_1)}{(p_1-k)^2} - \frac{\bar{v}(p_2)\gamma^{\alpha}T^a(\not{p}_2-\not{k})\sigma^{\mu\nu} u(p_1)}{(p_2-k)^2}\right] \times\bar{u}(p_3)\sigma^{\rho\sigma} v(p_4)\,, \tag{A.8}$$

$$\begin{aligned}\mathcal{M}_{D11} = \frac{g_s^3}{4\pi}\frac{1}{\Lambda^3} f_{abc}\epsilon_{\mu}(p_1)\epsilon_{\nu}(p_2)\epsilon^*_{\rho}(k)\bar{u}(p_3)v(p_4)\\ \Bigg[\frac{(g^{\mu\sigma}(2p_1-k)^{\rho}+g^{\rho\sigma}(2k-p_1)^{\mu}-g^{\mu\rho}(k+p_1)^{\sigma})((p_1-k)^{\nu}p_{2\sigma}-(p_1-k)\cdot p_2 g^{\nu}_{\sigma})}{(p_1-k)^2}\\ -\frac{(g^{\nu\sigma}(2p_2-k)^{\rho}+g^{\rho\sigma}(2k-p_2)^{\nu}-g^{\nu\rho}(k+p_2)^{\sigma})((p_2-k)^{\mu}p_{1\sigma}-(p_2-k)\cdot p_1 g^{\mu}_{\sigma})}{(p_2-k)^2}\\ -\frac{(g^{\mu\nu}(p_1-p_2)^{\sigma}+g^{\nu\sigma}(p_1+2p_2)^{\mu}-g^{\mu\sigma}(2p_1+p_2)^{\nu})((p_1+p_2)^{\rho}k_{\sigma}-k\cdot(p_1+p_2)g^{\rho}_{\sigma})}{(p_1+p_2)^2}\\ +g^{\mu\nu}(p_1-p_2)^{\rho}+g^{\nu\rho}(p_2+k)^{\mu}-g^{\mu\rho}(k+p_1)^{\nu}\Bigg]\,,\end{aligned} \tag{A.9}$$

$$\begin{aligned}\mathcal{M}_{D12} = i\frac{g_s^3}{4\pi}\frac{1}{\Lambda^3} f_{abc}\epsilon_{\mu}(p_1)\epsilon_{\nu}(p_2)\epsilon^*_{\rho}(k)\bar{u}(p_3)\gamma^5 v(p_4)\\ \Bigg[\frac{(g^{\mu\sigma}(2p_1-k)^{\rho}+g^{\rho\sigma}(2k-p_1)^{\mu}-g^{\mu\rho}(k+p_1)^{\sigma})((p_1-k)^{\nu}p_{2\sigma}-(p_1-k)\cdot p_2 g^{\nu}_{\sigma})}{(p_1-k)^2}\\ -\frac{(g^{\nu\sigma}(2p_2-k)^{\rho}+g^{\rho\sigma}(2k-p_2)^{\nu}-g^{\nu\rho}(k+p_2)^{\sigma})((p_2-k)^{\mu}p_{1\sigma}-(p_2-k)\cdot p_1 g^{\mu}_{\sigma})}{(p_2-k)^2}\\ -\frac{(g^{\mu\nu}(p_1-p_2)^{\sigma}+g^{\nu\sigma}(p_1+2p_2)^{\mu}-g^{\mu\sigma}(2p_1+p_2)^{\nu})((p_1+p_2)^{\rho}k_{\sigma}-k\cdot(p_1+p_2)g^{\rho}_{\sigma})}{(p_1+p_2)^2}\\ +g^{\mu\nu}(p_1-p_2)^{\rho}+g^{\nu\rho}(p_2+k)^{\mu}-g^{\mu\rho}(k+p_1)^{\nu}\Bigg]\,,\end{aligned} \tag{A.10}$$

$$\mathcal{M}_{D13} = -\frac{g_s^3}{4\pi}\frac{1}{\Lambda^3} f_{abc}\epsilon_\mu(p_1)\epsilon_\nu(p_2)\epsilon_\rho^*(k)\bar{u}(p_3)v(p_4)$$
$$\Big[\frac{(g_\sigma^\mu(2p_1-k)^\rho + g_\sigma^\rho(2k-p_1)^\mu - g^{\mu\rho}(k+p_1)_\sigma)(\epsilon^{\sigma\nu\eta\chi}p_{2\eta}(p_1-k)_\chi))}{(p_1-k)^2}$$
$$+\frac{(g_\sigma^\nu(2p_2-k)^\rho + g_\sigma^\rho(2k-p_2)^\nu - g^{\nu\rho}(k+p_2)_\sigma)(\epsilon^{\sigma\mu\eta\chi}p_{1\eta}(p_2-k)_\chi)}{(p_2-k)^2}$$
$$+\frac{(g^{\mu\nu}(p_1-p_2)_\sigma + g_\sigma^\nu(p_1+2p_2)^\mu - g_\sigma^\mu(2p_1+p_2)^\nu)(\epsilon^{\rho\eta\sigma\chi}k_\eta(p_1+p_2)_\chi)}{(p_1+p_2)^2}$$
$$-\epsilon^{\mu\nu\rho\sigma}(p_1+p_2-k)_\sigma\Big], \tag{A.11}$$

$$\mathcal{M}_{D14} = -i\frac{g_s^3}{4\pi}\frac{1}{\Lambda^3} f_{abc}\epsilon_\mu(p_1)\epsilon_\nu(p_2)\epsilon_\rho^*(k)\bar{u}(p_3)\gamma^5 v(p_4)$$
$$\Big[\frac{(g_\sigma^\mu(2p_1-k)^\rho + g_\sigma^\rho(2k-p_1)^\mu - g^{\mu\rho}(k+p_1)_\sigma)(\epsilon^{\sigma\nu\eta\chi}p_{2\eta}(p_1-k)_\chi))}{(p_1-k)^2}$$
$$+\frac{(g_\sigma^\nu(2p_2-k)^\rho + g_\sigma^\rho(2k-p_2)^\nu - g^{\nu\rho}(k+p_2)_\sigma)(\epsilon^{\sigma\mu\eta\chi}p_{1\eta}(p_2-k)_\chi)}{(p_2-k)^2}$$
$$+\frac{(g^{\mu\nu}(p_1-p_2)_\sigma + g_\sigma^\nu(p_1+2p_2)^\mu - g_\sigma^\mu(2p_1+p_2)^\nu)(\epsilon^{\rho\eta\sigma\chi}k_\eta(p_1+p_2)_\chi)}{(p_1+p_2)^2}$$
$$-\epsilon^{\mu\nu\rho\sigma}(p_1+p_2-k)_\sigma\Big]. \tag{A.12}$$

where p_1, p_2 are the initial momenta, k the momenta of the gluon, and p_3, p_4 the momenta of the DM particle/antiparticle, g_s is the SU(3) gauge coupling and T^a are the SU(3) generators in the fundamental representation, *i.e.* the standard QCD Gell-Mann matrices. The matrix elements with the operator DT1 are instead

$$\mathcal{M}_{DT1}^g = -i\,\frac{g^2 g_s}{M^2}\epsilon_\mu^* T_{ij}^a \times$$
$$\times\left\{\frac{\bar{u}(p_3)P_L(\not{p}_1-\not{p}_2)\gamma^\mu u(p_1)\bar{v}(p_2)P_R v(p_4)}{(p_1-k)^2} - \frac{\bar{u}(p_3)P_L u(p_1)\bar{v}(p_2)P_R\gamma^\mu(\not{p}_2-\not{k})v(p_4)}{(p_2-k)^2}\right\}$$

$$\mathcal{M}_{DT1}^q = -i\,\frac{g^2 g_s}{M^2}\epsilon_\mu T_{ij}^a \times$$
$$\times\left\{\frac{\bar{u}(k)P_R v(p_3)\bar{u}(p_4)P_L(\not{p}_1+\not{p}_2)\gamma^\mu u(p_1)}{(p_1+p_2)^2} - \frac{\bar{u}(k)\gamma^\mu(\not{p}_2-\not{k})P_R v(p_3)\bar{u}(p_4)P_L u(p_1)}{(p_2-k)^2}\right\}$$

$$\mathcal{M}_{DT1}^{\bar{q}} = -i\,\frac{g^2 g_s}{M^2}\epsilon_\mu T_{ij}^a \times$$
$$\times\left\{\frac{\bar{v}(k)P_L u(p_3)\bar{v}(p_4)P_R(\not{p}_1+\not{p}_2)\gamma^\mu v(p_1)}{(p_1+p_2)^2} - \frac{\bar{v}(k)\gamma^\mu(\not{p}_2-\not{k})P_L u(p_3)\bar{v}(p_4)P_R v(p_1)}{(p_2-k)^2}\right\}$$
$$\tag{A.13}$$

for the gluon, quark and anti-quark emission processes respectively. Here we denote the gluon polarization vector by ϵ_μ and the left and right projectors $(1-\gamma_5)/2$ and $(1+\gamma_5)/2$ with P_L and P_R respectively. The anti-quark matrix element is simply obtained from the quark one by exchanging quarks with anti-quarks and left with right projectors. The parton level cross sections for the two processes are thus the same, so here we only show the explicit derivation of the quark one.

The corresponding squared amplitudes, averaged over initial states (colour and spin) and summed over the final states are

$$\sum \overline{|\mathcal{M}_{D1'}|^2} = \frac{16}{9}\frac{g_s^2}{\Lambda^4}\frac{[(p_3\cdot p_4)-m_{\rm DM}^2]\left[(k\cdot(p_1+p_2))^2-2(p_1\cdot p_2)(k\cdot p_1+k\cdot p_2-p_1\cdot p_2)\right]}{(k\cdot p_1)(k\cdot p_2)}, \tag{A.14}$$

$$\sum \overline{|\mathcal{M}_{D4'}|^2} = \frac{16}{9}\frac{g_s^2}{\Lambda^4}\frac{[(p_3\cdot p_4)+m_{\rm DM}^2]\left[(k\cdot(p_1+p_2))^2-2(p_1\cdot p_2)(k\cdot p_1+k\cdot p_2-p_1\cdot p_2)\right]}{(k\cdot p_1)(k\cdot p_2)}, \tag{A.15}$$

$$\begin{aligned}\sum \overline{|\mathcal{M}_{D5}|^2} = -\frac{32}{9}\frac{g_s^2}{\Lambda^4}\Bigg[&\frac{(k\cdot p_1)\left[(k\cdot p_1)+(k\cdot p_2)-3(p_1\cdot p_2)-m_{\rm DM}^2\right]}{(k\cdot p_2)}\\ &+\frac{(k\cdot p_2)\left[(k\cdot p_1)+(k\cdot p_2)-3(p_1\cdot p_2)-m_{\rm DM}^2\right]}{(k\cdot p_1)}-4(p_1\cdot p_2)\\ &-2\frac{(p_1\cdot p_2)}{(k\cdot p_1)(k\cdot p_2)}\Big[(k\cdot p_3)\left((p_1\cdot p_3)+(p_2\cdot p_3)\right)+(p_1\cdot p_2)\left(m_{\rm DM}^2+(p_1\cdot p_2)\right)\\ &\quad-2(p_1\cdot p_3)(p_2\cdot p_3)\Big]\\ &+2\frac{(k\cdot p_3)(p_1\cdot p_3)-(p_2\cdot p_3)(p_1\cdot p_3)+(p_2\cdot p_3)^2+2(p_1\cdot p_2)^2+m_{\rm DM}^2(p_1\cdot p_2)}{(k\cdot p_2)}\\ &+2\frac{(k\cdot p_3)(p_2\cdot p_3)-(p_1\cdot p_3)(p_2\cdot p_3)+(p_1\cdot p_3)^2+2(p_1\cdot p_2)^2+m_{\rm DM}^2(p_1\cdot p_2)}{(k\cdot p_1)}\Bigg],\end{aligned} \tag{A.16}$$

$$\begin{aligned}\sum \overline{|\mathcal{M}_{D8}|^2} = \frac{32}{9}\frac{g_s^2}{\Lambda^4}\Bigg[&\frac{(k\cdot p_1)\left[(k\cdot p_1)+(k\cdot p_2)-3(p_1\cdot p_2)+m_{\rm DM}^2+2(p_3\cdot p_4)\right]}{(k\cdot p_2)}\\ &+\frac{(k\cdot p_2)\left[(k\cdot p_1)+(k\cdot p_2)-3(p_1\cdot p_2)+m_{\rm DM}^2+2(p_3\cdot p_4)\right]}{(k\cdot p_1)}-4(p_1\cdot p_2)\\ &+2\frac{(p_1\cdot p_2)}{(k\cdot p_1)(k\cdot p_2)}\Big[(p_1\cdot p_2)\left(2(p_3\cdot p_4)+m_{\rm DM}^2\right)+(k\cdot p_3)\left((p_1\cdot p_3)+(p_2\cdot p_3)\right)\\ &\quad+2(p_1\cdot p_3)(p_2\cdot p_3)-(p_1\cdot p_2)^2\Big]\\ &+2\frac{(p_1\cdot p_3)\left[-(k\cdot p_3)+(p_2\cdot p_3)\right]-(p_2\cdot p_3)^2+(p_1\cdot p_2)\left[2(p_1\cdot p_2)-m_{\rm DM}^2-2(p_3\cdot p_4)\right]}{(k\cdot p_2)}\\ &+2\frac{(p_2\cdot p_3)\left[-(k\cdot p_3)+(p_1\cdot p_3)\right]-(p_1\cdot p_3)^2+(p_1\cdot p_2)\left[2(p_1\cdot p_2)-m_{\rm DM}^2-2(p_3\cdot p_4)\right]}{(k\cdot p_1)}\Bigg],\end{aligned} \tag{A.17}$$

$$\begin{aligned}\sum \overline{|\mathcal{M}_{D9}|^2} = \frac{128}{9}\frac{g_s^2}{\Lambda^4}\Bigg[&-2[m_{\rm DM}^2-(k\cdot p_3)]+\frac{(k\cdot p_1)\left[-(k\cdot p_3)+(p_1\cdot p_3)-(p_2\cdot p_3)+m_{\rm DM}^2\right]}{(k\cdot p_2)}\\
&-2\frac{(p_1\cdot p_2)\left[-2(k\cdot p_3)+(p_1\cdot p_3)+(p_2\cdot p_3)+m_{\rm DM}^2\right]}{(k\cdot p_2)}\\
&-4\frac{[(k\cdot p_3)-(p_2\cdot p_3)]\,[(p_1\cdot p_3)-(p_2\cdot p_3)]}{(k\cdot p_2)}\\
&+\frac{(k\cdot p_2)\left[-(k\cdot p_3)+(p_2\cdot p_3)-(p_1\cdot p_3)+m_{\rm DM}^2\right]}{(k\cdot p_1)}\\
&-2\frac{(p_1\cdot p_2)\left[-2(k\cdot p_3)+(p_1\cdot p_3)+(p_2\cdot p_3)+m_{\rm DM}^2\right]}{(k\cdot p_1)}\\
&-4\frac{[(k\cdot p_3)-(p_1\cdot p_3)]\,[(p_2\cdot p_3)-(p_1\cdot p_3)]}{(k\cdot p_1)}\\
&-2\frac{(p_1\cdot p_2)\,[(k\cdot p_3)-(p_1\cdot p_3)-(p_2\cdot p_3)]\,[2(k\cdot p_3)+(p_1\cdot p_2)]}{(k\cdot p_1)(k\cdot p_2)}\\
&+2\frac{(p_1\cdot p_2)\left[-4(p_1\cdot p_3)(p_2\cdot p_3)+m_{\rm DM}^2(p_1\cdot p_2)\right]}{(k\cdot p_1)(k\cdot p_2)}\Bigg], \end{aligned} \tag{A.18}$$

$$\begin{aligned}\sum \overline{|\mathcal{M}_{D11}|^2} = \frac{3}{32\pi^2}\frac{g_s^6}{\Lambda^6}\left[(p_3\cdot p_4)-m_{\rm DM}^2\right]\Bigg\{&\frac{(k\cdot p_1)^3}{(k\cdot p_2)(p_1\cdot p_2)}+\frac{(k\cdot p_2)^3}{(k\cdot p_1)(p_1\cdot p_2)}+\frac{(p_1\cdot p_2)^3}{(k\cdot p_1)(k\cdot p_2)}\\
&+3\frac{(k\cdot p_1)(k\cdot p_2)}{(p_1\cdot p_2)}+\frac{(k\cdot p_1)(p_1\cdot p_2)-(k\cdot p_1)^2}{(k\cdot p_2)}+\frac{(k\cdot p_2)(p_1\cdot p_2)-(k\cdot p_2)^2}{(k\cdot p_1)}\\
&-\frac{(k_-\cdot p_1)(k\cdot p_2)^3}{(k\cdot k_-)(k\cdot p_1)(p_1\cdot p_2)}-\frac{(k_-\cdot p_2)(k\cdot p_1)^3}{(k\cdot k_-)(k\cdot p_2)(p_1\cdot p_2)}\\
&+\frac{(k_-\cdot p_1)}{(k\cdot k_-)(p_1\cdot p_2)}[(k\cdot p_1)^2+(k\cdot p_1)(k\cdot p_2)-(k\cdot p_2)^2]\\
&+\frac{(k_-\cdot p_2)}{(k\cdot k_-)(p_1\cdot p_2)}[(k\cdot p_2)^2+(k\cdot p_1)(k\cdot p_2)-(k\cdot p_1)^2]\\
&+2\frac{(k_-\cdot p_1)}{(k\cdot k_-)(k\cdot p_1)}[(k\cdot p_2)^2-(p_1\cdot p_2)(k\cdot p_2)]\\
&+2\frac{(k_-\cdot p_2)}{(k\cdot k_-)(k\cdot p_2)}[(k\cdot p_1)^2-(p_1\cdot p_2)(k\cdot p_1)]\\
&+2\frac{(k_-\cdot p_1)}{(k\cdot k_-)}[(p_1\cdot p_2)+(k\cdot p_1)-2(k\cdot p_2)]\\
&+2\frac{(k_-\cdot p_2)}{(k\cdot k_-)}[(p_1\cdot p_2)+(k\cdot p_2)-2(k\cdot p_1)]\\
&+(k\cdot p_1)+(k\cdot p_2)+6(p_1\cdot p_2)\Bigg\}, \end{aligned} \tag{A.19}$$

$$\begin{aligned}\sum \overline{|\mathcal{M}_{D12}|^2} = \frac{3}{32\pi^2}\frac{g_s^6}{\Lambda^6}\left[(p_3\cdot p_4)+m_{\rm DM}^2\right]\Bigg\{&\frac{(k\cdot p_1)^3}{(k\cdot p_2)(p_1\cdot p_2)}+\frac{(k\cdot p_2)^3}{(k\cdot p_1)(p_1\cdot p_2)}+\frac{(p_1\cdot p_2)^3}{(k\cdot p_1)(k\cdot p_2)}\\
&+3\frac{(k\cdot p_1)(k\cdot p_2)}{(p_1\cdot p_2)}+\frac{(k\cdot p_1)(p_1\cdot p_2)-(k\cdot p_1)^2}{(k\cdot p_2)}+\frac{(k\cdot p_2)(p_1\cdot p_2)-(k\cdot p_2)^2}{(k\cdot p_1)}\\
&-\frac{(k_-\cdot p_1)(k\cdot p_2)^3}{(k\cdot k_-)(k\cdot p_1)(p_1\cdot p_2)}-\frac{(k_-\cdot p_2)(k\cdot p_1)^3}{(k\cdot k_-)(k\cdot p_2)(p_1\cdot p_2)}\\
&+\frac{(k_-\cdot p_1)}{(k\cdot k_-)(p_1\cdot p_2)}[(k\cdot p_1)^2+(k\cdot p_1)(k\cdot p_2)-(k\cdot p_2)^2]\end{aligned}$$

$$+\frac{(k_- \cdot p_2)}{(k \cdot k_-)(p_1 \cdot p_2)}[(k \cdot p_2)^2 + (k \cdot p_1)(k \cdot p_2) - (k \cdot p_1)^2]$$
$$+2\frac{(k_- \cdot p_1)}{(k \cdot k_-)(k \cdot p_1)}[(k \cdot p_2)^2 - (p_1 \cdot p_2)(k \cdot p_2)]$$
$$+2\frac{(k_- \cdot p_2)}{(k \cdot k_-)(k \cdot p_2)}[(k \cdot p_1)^2 - (p_1 \cdot p_2)(k \cdot p_1)]$$
$$+2\frac{(k_- \cdot p_1)}{(k \cdot k_-)}[(p_1 \cdot p_2) + (k \cdot p_1) - 2(k \cdot p_2)]$$
$$+2\frac{(k_- \cdot p_2)}{(k \cdot k_-)}[(p_1 \cdot p_2) + (k \cdot p_2) - 2(k \cdot p_1)]$$
$$+(k \cdot p_1) + (k \cdot p_2) + 6(p_1 \cdot p_2)\Big\}\ , \tag{A.20}$$

$$\sum \overline{|\mathcal{M}_{D13}|^2} = \frac{3}{32\pi^2}\frac{g_s^6}{\Lambda^6}\Big[(p_3 \cdot p_4) - m_{\rm DM}^2\Big]\Big\{\frac{(k \cdot p_1)^3}{(k \cdot p_2)(p_1 \cdot p_2)} + \frac{(k \cdot p_2)^3}{(k \cdot p_1)(p_1 \cdot p_2)} + \frac{(p_1 \cdot p_2)^3}{(k \cdot p_1)(k \cdot p_2)}$$
$$+3\frac{(k \cdot p_1)(k \cdot p_2)}{(p_1 \cdot p_2)} + \frac{(k \cdot p_1)(p_1 \cdot p_2) - (k \cdot p_1)^2}{(k \cdot p_2)} + \frac{(k \cdot p_2)(p_1 \cdot p_2) - (k \cdot p_2)^2}{(k \cdot p_1)}$$
$$-\frac{(k_- \cdot p_1)(k \cdot p_2)^3}{(k \cdot k_-)(k \cdot p_1)(p_1 \cdot p_2)} - \frac{(k_- \cdot p_2)(k \cdot p_1)^3}{(k \cdot k_-)(k \cdot p_2)(p_1 \cdot p_2)}$$
$$+\frac{(k_- \cdot p_1)}{(k \cdot k_-)(p_1 \cdot p_2)}[(k \cdot p_1)^2 - 3(k \cdot p_1)(k \cdot p_2) + 3(k \cdot p_2)^2]$$
$$+\frac{(k_- \cdot p_2)}{(k \cdot k_-)(p_1 \cdot p_2)}[(k \cdot p_2)^2 - 3(k \cdot p_1)(k \cdot p_2) + 3(k \cdot p_1)^2]$$
$$+2\frac{(k_- \cdot p_1)}{(k \cdot k_-)(k \cdot p_1)}[(k \cdot p_2)^2 - (p_1 \cdot p_2)(k \cdot p_2)]$$
$$+2\frac{(k_- \cdot p_2)}{(k \cdot k_-)(k \cdot p_2)}[(k \cdot p_1)^2 - (p_1 \cdot p_2)(k \cdot p_1)]$$
$$+2\frac{(k_- \cdot p_1)}{(k \cdot k_-)}[(p_1 \cdot p_2) + (k \cdot p_1) - 2(k \cdot p_2)]$$
$$+2\frac{(k_- \cdot p_2)}{(k \cdot k_-)}[(p_1 \cdot p_2) + (k \cdot p_2) - 2(k \cdot p_1)]$$
$$-3(k \cdot p_1) - 3(k \cdot p_2) + 2(p_1 \cdot p_2)\Big\}\ , \tag{A.21}$$

$$\sum \overline{|\mathcal{M}_{D14}|^2} = \frac{3}{32\pi^2}\frac{g_s^6}{\Lambda^6}\Big[(p_3 \cdot p_4) + m_{\rm DM}^2\Big]\Big\{\frac{(k \cdot p_1)^3}{(k \cdot p_2)(p_1 \cdot p_2)} + \frac{(k \cdot p_2)^3}{(k \cdot p_1)(p_1 \cdot p_2)} + \frac{(p_1 \cdot p_2)^3}{(k \cdot p_1)(k \cdot p_2)}$$
$$+3\frac{(k \cdot p_1)(k \cdot p_2)}{(p_1 \cdot p_2)} + \frac{(k \cdot p_1)(p_1 \cdot p_2) - (k \cdot p_1)^2}{(k \cdot p_2)} + \frac{(k \cdot p_2)(p_1 \cdot p_2) - (k \cdot p_2)^2}{(k \cdot p_1)}$$
$$-\frac{(k_- \cdot p_1)(k \cdot p_2)^3}{(k \cdot k_-)(k \cdot p_1)(p_1 \cdot p_2)} - \frac{(k_- \cdot p_2)(k \cdot p_1)^3}{(k \cdot k_-)(k \cdot p_2)(p_1 \cdot p_2)}$$
$$+\frac{(k_- \cdot p_1)}{(k \cdot k_-)(p_1 \cdot p_2)}[(k \cdot p_1)^2 - 3(k \cdot p_1)(k \cdot p_2) + 3(k \cdot p_2)^2]$$
$$+\frac{(k_- \cdot p_2)}{(k \cdot k_-)(p_1 \cdot p_2)}[(k \cdot p_2)^2 - 3(k \cdot p_1)(k \cdot p_2) + 3(k \cdot p_1)^2]$$
$$+2\frac{(k_- \cdot p_1)}{(k \cdot k_-)(k \cdot p_1)}[(k \cdot p_2)^2 - (p_1 \cdot p_2)(k \cdot p_2)]$$
$$+2\frac{(k_- \cdot p_2)}{(k \cdot k_-)(k \cdot p_2)}[(k \cdot p_1)^2 - (p_1 \cdot p_2)(k \cdot p_1)]$$

$$+2\frac{(k_{-}\cdot p_1)}{(k\cdot k_{-})}[(p_1\cdot p_2)+(k\cdot p_1)-2(k\cdot p_2)]$$
$$+2\frac{(k_{-}\cdot p_2)}{(k\cdot k_{-})}[(p_1\cdot p_2)+(k\cdot p_2)-2(k\cdot p_1)]$$
$$-3(k\cdot p_1)-3(k\cdot p_2)+2(p_1\cdot p_2)\}\ . \tag{A.22}$$

$$\begin{aligned}\sum\overline{|\mathcal{M}^g_{DT1}|^2} = {} & \frac{1}{9}\frac{g_s^2}{\Lambda^4}\frac{1}{(k\cdot p_1)(k\cdot p_2)}\times \\ & \Big\{p_1\cdot p_3\Big[(k\cdot p_4)(k\cdot p_1)-(k\cdot p_4)(p_1\cdot p_2)-(k\cdot p_2)(p_1\cdot p_4)\Big]+ \\ & +p_2\cdot p_4\Big[(k\cdot p_3)(k\cdot p_2)-(k\cdot p_3)(p_1\cdot p_2)-(k\cdot p_1)(p_2\cdot p_3)\Big]+ \\ & +(p_1\cdot p_3)(p_2\cdot p_4)\Big[2p_1\cdot p_2-k\cdot p_1-k\cdot p_2\Big]\Big\}\end{aligned} \tag{A.23}$$

$$\begin{aligned}\sum\overline{|\mathcal{M}^q_{DT1}|^2} = {} & \frac{1}{6}\frac{g_s^2}{\Lambda^4}\frac{1}{(k\cdot p_1)(k\cdot p_2)}\times \\ & \Big\{p_1\cdot p_4\Big[(k\cdot p_2)(p_1\cdot p_3)-(k\cdot p_1)(p_2\cdot p_3)+ \\ & \quad +(k\cdot p_2)(k\cdot p_3)+(k\cdot p_1)(k\cdot p_3)-(p_1\cdot p_2)(k\cdot p_3)\Big]+ \\ & +p_2\cdot p_2\Big[(p_1\cdot p_4)(p_2\cdot p_3)-(k\cdot p_4)(k\cdot p_3)\Big]+ \\ & +(k\cdot p_3)\Big[(k\cdot p_1)(p_1\cdot p_4)+(k\cdot p_1)(p_2\cdot p_4)+(k\cdot p_2)(p_2\cdot p_4)\Big]\Big\}\end{aligned} \tag{A.24}$$

where the polarization 4-vector in Eqs. (A.19)–(A.22) is defined as $k_{-} \equiv P(k^{\nu})/\sqrt{k^{\mu}\cdot P(k_{\mu})}$, where P is the parity operation.

A.3 Cross Sections

Now, the next step is to compute the cross sections in the lab frame. To this end we proceed by first evaluating the matrix elements and the phase space density in the centre-of-mass frame and then boosting the result to the lab frame. In the centre-of-mass (c.o.m) frame, let us parametrize the four-momenta involved in the process as

$$p_1 = x\frac{\sqrt{s}}{2}(1,0,0,1)\,,\qquad p_2 = x\frac{\sqrt{s}}{2}(1,0,0,-1)\,,\qquad k = x\frac{\sqrt{s}}{2}(z_0, z_0\hat{k})\,, \tag{A.25}$$
$$p_3 = x\frac{\sqrt{s}}{2}(1-y_0, \sqrt{(1-y_0)^2-a^2}\,\hat{p}_3)\,,\quad p_4 = x\frac{\sqrt{s}}{2}(1+y_0-z_0, \sqrt{(1+y_0-z_0)^2-a^2}\,\hat{p}_4)\,,$$

where the two colliding partons carry equal momentum fractions $x_1 = x_2 \equiv x$ of the incoming protons, $a \equiv 2m_{\rm DM}/(x\sqrt{s}) < 1$, $\hat{k} = (0, \sin\theta_0, \cos\theta_0)$, and θ_0 is the polar angle of $\hat{k}$ with respect to the beam line, in the c.o.m. frame. With the subscript $_0$ we will refer to quantities evaluated in the c.o.m. frame. The polarization 4-vector k_- in the c.o.m. frame simply reads $k_- = (1/\sqrt{2})(1, 0, -\sin\theta_0, -\cos\theta_0)$.

The conservation of three-momentum sets the angle $\theta_{0\,3j}$ between $\hat{p}_3$ and $\hat{k}$ as: $\cos\theta_{0\,3j} = (\mathbf{p}_4^2 - \mathbf{k}^2 - \mathbf{p}_3^2)/2|\mathbf{k}||\mathbf{p}_3|$. Integration over the azimuthal angle ϕ_0 of the outgoing jet simply gives a factor of 2π, while the matrix element does depend in the t-channel case on the azimuthal angle of the three-momentum $\mathbf{p}_3$ with respect to $\mathbf{k}$, $\phi_{0\,3\rm j}$, and so it can not be integrated over at this stage, contrary to the s-channel case. Taking all of this into account, the phase space density simplifies to

$$d\Phi_3 = \frac{1}{(4\pi)^3} dE_3\, dk\, d\cos\theta_0 = \frac{1}{(4\pi)^3}\frac{x^2 s}{4} dz_0\, dy_0\, d\cos\theta_0\,. \tag{A.26}$$

$$d\Phi_3 = \frac{1}{8(2\pi)^4}\, dE_3\, d|\mathbf{k}|\, d\cos\theta_0\, d\phi_{0\,3\rm j} = \frac{x^2 s}{32(2\pi)^4}\, dy_0\, dz_0\, d\cos\theta_0\, d\phi_{0\,3\rm j}. \tag{A.27}$$

in the s and t-channel respectively. The kinematical domains of y_0, z_0 and $\phi_{0\,3\rm j}$ are

$$\frac{z_0}{2}\left(1 - \sqrt{\frac{1 - z_0 - a^2}{1 - z_0}}\right) \le\; y_0 \;\le \frac{z_0}{2}\left(1 + \sqrt{\frac{1 - z_0 - a^2}{1 - z_0}}\right) \tag{A.28}$$

$$0 \le\; z_0 \;\le 1 - a^2 \tag{A.29}$$

$$0 \le \phi_{0\,3\rm j} \le 2\pi \tag{A.30}$$

The variables y_0 and $\phi_{0\,3\rm j}$ refer to the momentum $\mathbf{p}_3$ of an invisible DM particle; they are therefore not measurable, and we integrate over them. In the t-channel case, finding the total integrated cross section is useless, since these variables enter our definition of the momentum transfer $Q_{\rm tr}$, and the condition $Q_{\rm tr} < \Lambda$ which we used to define the ratio R_Λ.

For the doubly-differential cross sections with respect to the energy and angle of the emitted gluon, in the c.o.m. frame, we obtain

$$\left.\frac{d^2\hat{\sigma}}{dz_0 d\cos\theta_0}\right|_{D1'} = \frac{\alpha_s}{36\pi^2}\frac{x^2 s}{\Lambda^4}\frac{\left[1 - z_0 - \frac{4m_{\rm DM}^2}{x^2 s}\right]^{3/2}}{\sqrt{1 - z_0}}\frac{[1 + (1 - z_0)^2]}{z_0 \sin^2\theta_0}\,, \tag{A.31}$$

$$\left.\frac{d^2\hat{\sigma}}{dz_0 d\cos\theta_0}\right|_{D4'} = \frac{\alpha_s}{36\pi^2}\frac{x^2 s}{\Lambda^4}\frac{\left[1 - z_0 - \frac{4m_{\rm DM}^2}{x^2 s}\right]^{1/2}}{\sqrt{1 - z_0}}\frac{[1 + (1 - z_0)^2]}{z_0 \sin^2\theta_0}\,, \tag{A.32}$$

$$\left.\frac{d^2\hat{\sigma}}{dz_0 d\cos\theta_0}\right|_{D5} = \frac{\alpha_s}{108\pi^2}\frac{x^2 s}{\Lambda^4}\frac{\sqrt{1 - z_0 - \frac{4m_{\rm DM}^2}{x^2 s}}}{\sqrt{1 - z_0}}$$

$$\times \frac{(1 - z_0 + \frac{2m_{\rm DM}^2}{x^2 s})(8 - 8z_0 + (3 + \cos 2\theta_0)z_0^2)}{z_0 \sin^2\theta_0}\,, \tag{A.33}$$

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}z_0 \mathrm{d}\cos\theta_0}\right|_{D8} = \frac{\alpha_s}{108\pi^2}\frac{x^2 s}{\Lambda^4}\frac{[1 - z_0 - \frac{4m_{\rm DM}^2}{x^2 s}]^{3/2}}{\sqrt{1 - z_0}}\frac{8 - 8z_0 + (3 + \cos 2\theta_0)z_0^2}{z_0 \sin^2\theta_0}\,, \tag{A.34}$$

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}z_0 \mathrm{d}\cos\theta_0}\right|_{D9} = \frac{\alpha_s}{27\pi^2}\frac{x^2 s}{\Lambda^4}\frac{\sqrt{1 - z_0 - \frac{4m_{\rm DM}^2}{x^2 s}}}{[1 - z_0]^{3/2}}$$
$$\times \frac{(1 - z_0 + \frac{2m_{\rm DM}^2}{x^2 s})(4 - 8z_0 + 6z_0^2 - (1 + \cos 2\theta_0)z_0^3)}{z_0 \sin^2\theta_0}\,, \tag{A.35}$$

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}z_0 \mathrm{d}\cos\theta_0}\right|_{D11} = \frac{3\alpha_s^3 x^4 s^2}{32768\pi^2\Lambda^6}\frac{\left[1 - z_0 - \frac{4m_{\rm DM}^2}{x^2 s}\right]^{3/2}}{z_0\sqrt{1 - z_0}\sin^2\theta_0}\,[128 - 128(1 + \cos 2\theta_0)z_0$$
$$+(304 + 64\cos 2\theta_0 + 16\cos 4\theta_0)z_0^2 - 128(1 + \cos 2\theta_0)z_0^3$$
$$+(79 + 44\cos 2\theta_0 + 5\cos 4\theta_0)z_0^4\big]\,, \tag{A.36}$$

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}z_0 \mathrm{d}\cos\theta_0}\right|_{D12} = \frac{3\alpha_s^3 x^4 s^2}{32768\pi^2\Lambda^6}$$
$$\times \frac{\sqrt{1 - z_0 - \frac{4m_{\rm DM}^2}{x^2 s}}\sqrt{1 - z_0}}{z_0 \sin^2\theta_0}\,[128 - 128(1 + \cos 2\theta_0)z_0$$
$$+(304 + 64\cos 2\theta_0 + 16\cos 4\theta_0)z_0^2 - 128(1 + \cos 2\theta_0)z_0^3$$
$$+(79 + 44\cos 2\theta_0 + 5\cos 4\theta_0)z_0^4\big]\,, \tag{A.37}$$

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}z_0 \mathrm{d}\cos\theta_0}\right|_{D13} = \frac{3\alpha_s^3 x^4 s^2}{32768\pi^2\Lambda^6}\frac{\left[1 - z_0 - \frac{4m_{\rm DM}^2}{x^2 s}\right]^{3/2}}{z_0\sqrt{1 - z_0}\sin^2\theta_0}\,[128 - 128(1 + \cos 2\theta_0)z_0$$
$$+(240 + 128\cos 2\theta_0 + 16\cos 4\theta_0)z_0^2$$
$$-16(11 + 4\cos 2\theta_0 + \cos 4\theta_0)z_0^3$$
$$+(79 + 44\cos 2\theta_0 + 5\cos 4\theta_0)z_0^4\big]\,, \tag{A.38}$$

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}z_0 \mathrm{d}\cos\theta_0}\right|_{D14} = \frac{3\alpha_s^3 x^4 s^2}{32768\pi^2\Lambda^6}\frac{\sqrt{1 - z_0 - \frac{4m_{\rm DM}^2}{x^2 s}}\sqrt{1 - z_0}}{z_0 \sin^2\theta_0}$$
$$[128 - 128(1 + \cos 2\theta_0)z_0\ (240 + 128\cos 2\theta_0 + 16\cos 4\theta_0)z_0^2$$
$$-16(11 + 4\cos 2\theta_0 + \cos 4\theta_0)z_0^3$$
$$+(79 + 44\cos 2\theta_0 + 5\cos 4\theta_0)z_0^4\big]\,. \tag{A.39}$$

$$\frac{d^4\hat{\sigma}}{dz_0\, d\cos\theta_0\, dy_0\, d\phi_{0\,3j}}\Bigg|_{DT1}^{g} = \frac{1}{4608\pi^4}\frac{g_s^2}{\Lambda^4}\frac{1-z_0}{z_0^4}$$
$$\Big\{4x(2-z_0)\csc\theta_0\cos\phi_{0\,3j}(\cos\theta_0(z_0-2y_0)+z_0)$$
$$\sqrt{s\left(sx^2y_0(z_0-1)(y_0-z_0)-m_{\rm DM}^2 z_0^2\right)}$$
$$-8m_{\rm DM}^2 z_0^2\cos^2\phi_{0\,3j}+sx^2((z_0-2)z_0+2)\left(\sec^2(\theta_0/2)\,y_0^2\right.$$
$$\left.+\csc^2(\theta_0/2)\,(y_0-z_0)^2\right)$$
$$-2sx^2y_0^2((z_0-6)z_0+6)+4sx^2y_0(z_0-1)(y_0-z_0)\cos(2\phi_{0\,3j})$$
$$+2sx^2y_0((z_0-6)z_0+6)z_0-sx^2z_0^2((z_0-2)z_0+2)\Big\}, \qquad (A.40)$$

$$\frac{d^4\hat{\sigma}}{dz_0\, d\cos\theta_0\, dy_0\, d\phi_{0\,3j}}\Bigg|_{DT1}^{q} = \frac{1}{98304\pi^4}\frac{g_s^2}{\Lambda^4}\frac{1-z_0}{z_0^3\cos^2\frac{\theta_0}{2}}$$
$$\Big\{8x\sqrt{s}\left[z_0(z_0-y_0-1)-\left(z_0^2-(1+y_0)z_0+2y_0\right)\right.$$
$$\left.\times\cos\theta_0\right]\cos\phi_{0\,3j}\sin\theta_0\times\sqrt{sx^2y_0(z_0-y_0)(1-z_0)-m_{\rm DM}^2z_0^2}$$
$$-2(1-\cos(2\theta_0))m_{\rm DM}^2z_0^2$$
$$+4\Big[sx^2y_0(z_0-y_0)(1-z_0)-m_{\rm DM}^2z_0^2\Big]\cos(2\phi_{0\,3j})\sin^2\theta_0$$
$$+sx^2\Big[11z_0^4-(6+22y_0)z_0^3+(11y_0^2+8y_0+3)z_0^2$$
$$-2y_0(1+y_0)z_0+2y_0^2\Big]$$
$$+sx^2\Big[z_0^4-2(1+y_0)z_0^3+(y_0^2+8y_0+1)z_0^2$$
$$-6y_0(1+y_0)z_0+6y_0^2\Big]\cos(2\theta_0)$$
$$-4sx^2z_0\Big[z_0^3-2(1+y_0)z_0^2+(y_0^2+4y_0+1)z_0-2y_0(1+y_0)\Big]\cos\theta_0\Big\}. \qquad (A.41)$$

Equation (A.31)–(A.34) agree with the findings in Refs. [1, 2], up to the factor of 1/9, as we are considering coloured colliding particles.

To get the cross sections in the lab frame we perform a boost in the $\hat{z}$ axis, accounting for the generic parton momentum fractions x_1, x_2. The velocity of the c.o.m. of the colliding particles with respect to the lab frame is given by

$$\beta_{\rm c.o.m.} = \frac{x_1-x_2}{x_1+x_2}, \qquad (A.42)$$

so that the relations between the quantities z_0, θ_0 and the analogous ones z, θ in the lab frame are

$$z_0 = \frac{(x_1+x_2)^2 + (x_2^2 - x_1^2)\cos\theta}{4x_1x_2} z$$
$$\sin^2\theta_0 = \frac{4x_1x_2}{[(x_1+x_2)+(x_2-x_1)\cos\theta]^2}\sin^2\theta. \tag{A.43}$$

The Jacobian factor to transform $\mathrm{d}z_0\,\mathrm{d}\cos\theta_0 \to \mathrm{d}z\,\mathrm{d}\cos\theta$ is simply obtained using Eq. (A.43); the cross section in the lab frame is then

$$\frac{\mathrm{d}^4\hat{\sigma}}{\mathrm{d}z\,\mathrm{d}\cos\theta\,\mathrm{d}y_0\,\mathrm{d}\phi_{0\,3\mathrm{j}}} = \frac{x_1+x_2}{x_1+x_2+(x_1-x_2)\cos\theta} \times \left.\frac{\mathrm{d}^4\hat{\sigma}}{\mathrm{d}z_0\,\mathrm{d}\cos\theta_0\,\mathrm{d}y_0\,\mathrm{d}\phi_{0\,3\mathrm{j}}}\right|_{\substack{z_0\to z_0(z)\\ \theta_0\to\theta_0(\theta)}}. \tag{A.44}$$

Expressing the energy of the emitted gluon or (anti-)quark in terms of the transverse momentum and rapidity, $k^0 = p_\mathrm{T}\cosh\eta$, one finds

$$z = \frac{4p_\mathrm{T}\cosh\eta}{(x_1+x_2)\sqrt{s}}, \qquad \cos\theta = \tanh\eta \tag{A.45}$$

which allows us to express the differential cross sections with respect to the transverse momentum and pseudo-rapidity of the emitted jet:

$$\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}p_\mathrm{T}\,\mathrm{d}\eta} = \frac{4}{(x_1+x_2)\sqrt{s}\cosh\eta}\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}z\,\mathrm{d}\cos\theta\,\mathrm{d}y_0}\right|_{\substack{z\to z(p_\mathrm{T},\eta)\\ \theta\to\theta(p_\mathrm{T},\eta)}}. \tag{A.46}$$

$$\frac{\mathrm{d}^4\hat{\sigma}}{\mathrm{d}p_\mathrm{T}\,\mathrm{d}\eta\,\mathrm{d}y_0\,\mathrm{d}\phi_{0\,3\mathrm{j}}} = \frac{4}{(x_1+x_2)\sqrt{s}\cosh\eta}\left.\frac{\mathrm{d}^4\hat{\sigma}}{\mathrm{d}z\,\mathrm{d}\cos\theta\,\mathrm{d}y_0\,\mathrm{d}\phi_{0\,3\mathrm{j}}}\right|_{\substack{z\to z(p_\mathrm{T},\eta)\\ \theta\to\theta(p_\mathrm{T},\eta)}}. \tag{A.47}$$

This way we get the translation of Eqs. (A.31)–(A.35) into the lab frame:

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}p_\mathrm{T}\mathrm{d}\eta}\right|_{D1'} = \frac{\alpha_s}{36\pi^2}\frac{x_1x_2s}{\Lambda^4}\frac{1}{p_\mathrm{T}}\frac{\left[1-f-\frac{4m_\mathrm{DM}^2}{x_1x_2s}\right]^{3/2}\left[1+(1-f)^2\right]}{\sqrt{1-f}}, \tag{A.48}$$

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}p_\mathrm{T}\mathrm{d}\eta}\right|_{D4'} = \frac{\alpha_s}{36\pi^2}\frac{x_1x_2s}{\Lambda^4}\frac{\sqrt{1-f}}{p_\mathrm{T}}\left[1-f-\frac{4m_\mathrm{DM}^2}{x_1x_2s}\right]^{1/2}\left[1+(1-f)^2\right], \tag{A.49}$$

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}p_\mathrm{T}\mathrm{d}\eta}\right|_{D5} = \frac{\alpha_s}{27\pi^2}\frac{x_1x_2s}{\Lambda^4}\frac{\sqrt{1-f-\frac{4m_\mathrm{DM}^2}{x_1x_2s}}}{\sqrt{1-f}} \times \frac{\left[1-f+\frac{2m_\mathrm{DM}^2}{x_1x_2s}\right]\left[1+(1-f)^2-2\frac{p_\mathrm{T}^2}{x_1x_2s}\right]}{p_\mathrm{T}}, \tag{A.50}$$

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}p_\mathrm{T}\mathrm{d}\eta}\right|_{D8} = \frac{\alpha_s}{27\pi^2}\frac{x_1x_2s}{\Lambda^4}\frac{[1-f-\frac{4m_\mathrm{DM}^2}{x_1x_2s}]^{3/2}}{\sqrt{1-f}}\frac{1+(1-f)^2-2\frac{p_\mathrm{T}^2}{x_1x_2s}}{p_\mathrm{T}}, \tag{A.51}$$

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}p_\mathrm{T}\mathrm{d}\eta}\right|_{D9} = \frac{2\alpha_s}{27\pi^2}\frac{x_1x_2s}{\Lambda^4}\frac{\sqrt{1-f-\frac{4m_\mathrm{DM}^2}{sx_1x_2}}}{[1-f]^{3/2}} \times \frac{(1-f+\frac{2m_\mathrm{DM}^2}{x_1x_2s})\left[(1-f)(1+(1-f)^2)+f\frac{4p_\mathrm{T}^2}{x_1x_2s}\right]}{p_\mathrm{T}}, \tag{A.52}$$

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}p_\mathrm{T}\mathrm{d}\eta}\right|_{D11} = \frac{3\alpha_s^3x_1^2x_2^2s^2}{256\pi^2\Lambda^6}\frac{(1-f-\frac{4m_\mathrm{DM}^2}{sx_1x_2})^{3/2}}{p_\mathrm{T}f^2\sqrt{1-f}} \times \left[16\frac{p_\mathrm{T}^4}{x_1^2x_2^2s^2}+8\frac{p_\mathrm{T}^2}{x_1x_2s}f+(1-8\frac{p_\mathrm{T}^2}{x_1x_2s}+5\frac{p_\mathrm{T}^4}{x_1^2x_2^2s^2})f^2 + (-2+8\frac{p_\mathrm{T}^2}{x_1x_2s})f^3+(3-4\frac{p_\mathrm{T}^2}{x_1x_2s})f^4-2f^5+f^6\right], \tag{A.53}$$

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}p_\mathrm{T}\mathrm{d}\eta}\right|_{D12} = \frac{3\alpha_s^3x_1^2x_2^2s^2}{256\pi^2\Lambda^6}\frac{\sqrt{1-f-\frac{4m_\mathrm{DM}^2}{sx_1x_2}}\sqrt{1-f}}{p_\mathrm{T}f^2}\left[16\frac{p_\mathrm{T}^4}{x_1^2x_2^2s^2}+8\frac{p_\mathrm{T}^2}{x_1x_2s}f +(1-8\frac{p_\mathrm{T}^2}{x_1x_2s}+5\frac{p_\mathrm{T}^4}{x_1^2x_2^2s^2})f^2+(-2+8\frac{p_\mathrm{T}^2}{x_1x_2s})f^3 +(3-4\frac{p_\mathrm{T}^2}{x_1x_2s})f^4-2f^5+f^6\right], \tag{A.54}$$

$$\left.\frac{\mathrm{d}^2\hat{\sigma}}{\mathrm{d}p_\mathrm{T}\mathrm{d}\eta}\right|_{D13} = \frac{3\alpha_s^3x_1^2x_2^2s^2}{256\pi^2\Lambda^6}\frac{(1-f-\frac{4m_\mathrm{DM}^2}{sx_1x_2})^{3/2}}{p_\mathrm{T}f^2\sqrt{1-f}}\left[16\frac{p_\mathrm{T}^4}{x_1^2x_2^2s^2}+8(\frac{p_\mathrm{T}^2}{x_1x_2s}-2\frac{p_\mathrm{T}^4}{x_1^2x_2^2s^2})f + (1-12\frac{p_\mathrm{T}^2}{x_1x_2s}+5\frac{p_\mathrm{T}^4}{x_1^2x_2^2s^2})f^2+(-2+8\frac{p_\mathrm{T}^2}{x_1x_2s})f^3 +(3-4\frac{p_\mathrm{T}^2}{x_1x_2s})f^4-2f^5+f^6\right], \tag{A.55}$$

$$\left.\frac{d^2\hat{\sigma}}{dp_T d\eta}\right|_{D14} = \frac{3\alpha_s^3 x_1^2 x_2^2 s^2}{256\pi^2\Lambda^6} \frac{\sqrt{1-f-\frac{4m_{DM}^2}{sx_1x_2}}\sqrt{1-f}}{p_T f^2}$$
$$\times\left[16\frac{p_T^4}{x_1^2x_2^2s^2} + 8(\frac{p_T^2}{x_1x_2s} - 2\frac{p_T^4}{x_1^2x_2^2s^2})f\right.$$
$$+ (1 - 12\frac{p_T^2}{x_1x_2s} + 5\frac{p_T^4}{x_1^2x_2^2s^2})f^2 + (-2 + 8\frac{p_T^2}{x_1x_2s})f^3$$
$$\left.+(3 - 4\frac{p_T^2}{x_1x_2s})f^4 - 2f^5 + f^6\right], \tag{A.56}$$

where we have defined

$$f(p_T, \eta, x_1, x_2) \equiv \frac{p_T(x_1e^{-\eta} + x_2e^{\eta})}{x_1x_2\sqrt{s}}. \tag{A.57}$$

For the emission of a photon, rather than a gluon, from a quark with charge Q_q one simply replaces $(4/3)\,\alpha_s \rightarrow Q_q^2\alpha$ in Eqs. (A.48)–(A.52). The cross section for the corresponding simplified models are simply obtained by replacing

$$\Lambda^4 = \frac{\left(Q_{tr}^2 - M_{med}^2\right)^2 + \Gamma^2 M_{med}^2}{g_q^2 g_\chi^2}, \tag{A.58}$$

From these expressions one reproduces the results reported in Eqs. (6.17)–(6.25). We do not report the explicit expressions in the t-channel because they are too cumbersome and of limited interest.

References

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Appendix B
Annihilation Cross Section with a Pure Vector/Axial Mediator (as in Chap. 8)

In this Appendix we collect the results of cross sections calculations for the process of DM annihilation into SM fermions

$$\chi\bar{\chi} \to f\bar{f} \tag{B.1}$$

We performed the calculation at zero temperature in the lab frame where χ is at rest, and the centre of mass energy $s = 2m_{\rm DM}^2\left(\frac{1}{\sqrt{1-v^2}} + 1\right)$. This is equivalent to performing the calculation in the Moeller frame, and is the correct frame for the relic density calculations [1].

$$\begin{aligned}(\sigma v)_V = &\frac{N_C(g_f^V)^2(g_{\rm DM}^V)^2}{2\pi}\frac{\sqrt{1-m_f^2/m_{\rm DM}^2}}{(M^2-4m_{\rm DM}^2)^2+\Gamma^2M^2}\Big[(m_f^2+2m_{\rm DM}^2)+\\ &v^2\Big(\frac{11m_f^4+2m_f^2m_{\rm DM}^2-4m_{\rm DM}^4}{24m_{\rm DM}^2(1-\frac{m_f^2}{m_{\rm DM}^2})}+2\frac{m_{\rm DM}^2(m_f^2+2m_{\rm DM}^2)(M^2-4m_{\rm DM}^2)}{(M^2-4m_{\rm DM}^2)^2+\Gamma^2M^2}\Big)\Big],\end{aligned} \tag{B.2}$$

$$\begin{aligned}(\sigma v)_A = &\frac{N_C(g_f^A)^2(g_{\rm DM}^A)^2}{2\pi}\frac{\sqrt{1-m_f^2/m_{\rm DM}^2}}{(M^2-4m_{\rm DM}^2)^2+\Gamma^2M^2}\Big[m_f^2\\ &+v^2\Big(\frac{23m_f^4-28m_f^2m_{\rm DM}^2+8m_{\rm DM}^4}{24m_{\rm DM}^2(1-\frac{m_f^2}{m_{\rm DM}^2})}+2\frac{m_f^2m_{\rm DM}^2(M^2-4m_{\rm DM}^2)}{(M^2-4m_{\rm DM}^2)^2+\Gamma^2M^2}\Big)\Big].\end{aligned} \tag{B.3}$$

E. Morgante, *Aspects of WIMP Dark Matter Searches at Colliders and Other Probes*, Springer Theses, https://doi.org/10.1007/978-3-319-67606-7

In the limit $m_f \to 0$ these expressions become

$$(\sigma v)_V = \frac{N_C (g_f^V)^2 (g_{\rm DM}^V)^2}{\pi} \frac{m_{\rm DM}^2}{(M^2 - 4m_{\rm DM}^2)^2 + \Gamma^2 M^2} \times \left[1 + v^2 \left(-\frac{1}{12} + \frac{2m_{\rm DM}^2 (M^2 - 4m_{\rm DM}^2)}{(M^2 - 4m_{\rm DM}^2)^2 + \Gamma^2 M^2}\right)\right], \quad \text{(B.4)}$$

$$(\sigma v)_A = \frac{N_C (g_f^A)^2 (g_{\rm DM}^A)^2}{6\pi} \frac{m_{\rm DM}^2}{(M^2 - 4m_{\rm DM}^2)^2 + \Gamma^2 M^2} v^2. \quad \text{(B.5)}$$

In the limit $m_{\rm DM} \ll M$ the effective operator approximation holds:

$$(\sigma v)_V = \frac{N_C m_{\rm DM}^2}{2\pi \Lambda^4} \sqrt{1 - \frac{m_f^2}{m_{\rm DM}^2}} \left[\left(\frac{m_f^2}{m_{\rm DM}^2} + 2\right) + v^2 \frac{11 m_f^4/m_{\rm DM}^4 + 2m_f^2/m_{\rm DM}^2 - 4}{24(1 - m_f^2/m_{\rm DM}^2)}\right], \quad \text{(B.6)}$$

$$(\sigma v)_A = \frac{N_C}{2\pi \Lambda^4} \sqrt{1 - \frac{m_f^2}{m_{\rm DM}^2}} \left[m_f^2 + v^2 \frac{23 m_f^4/m_{\rm DM}^2 - 28 m_f^2 + 8 m_{\rm DM}^2}{24(1 - m_f^2/m_{\rm DM}^2)}\right]. \quad \text{(B.7)}$$

The widths for the vector mediator decay to a pair of fermions are given by

$$\Gamma_V = \frac{N_C (g_f^V)^2 (M^2 + 2m_f^2) \sqrt{1 - 4m_f^2/M^2}}{12\pi M}, \quad \text{(B.8)}$$

$$\Gamma_A = \frac{N_C (g_f^A)^2 M (1 - 4m_f^2/M^2)^{3/2}}{12\pi}. \quad \text{(B.9)}$$

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Appendix C
Details of the Annihilation Rate Calculation in the Z' Model

In this appendix we present the calculation of the annihilation cross sections of the DM into the SM in detail. We also go in detail over the one-loop order annihilation into the WW. The results of this calculations are summarized on Fig. 9.6

The Feynman diagrams for the possible annihilation channels of $\chi\chi$ into the SM particles are shown on Fig. C.1. For each channel, we consider the leading order (tree level or one-loop), moreover we always restrict the calculation to the leading order in the mixing angle between Z and Z', ψ.

At the tree level the DM annihilates into $f\bar{f}$, and if $\theta \neq \pi/2$ also to W^+W^- and Zh. Equations (C.1)–(C.6) summarize the annihilation cross sections into all these channels at the tree level, distinguishing between the polarization of the vector bosons in the final states through a superscript (T) or (L) for transverse or longitudinal polarization, respectively.

$$\sigma(\chi\chi \to f\bar{f}) = \frac{g_\chi^2 g_{Z'}^4 \cos^4\psi\, N_c^f}{3\pi s\left((s-m_{Z'}^2)^2 + \Gamma_{Z'}^2 m_{Z'}^2\right)} \times \sqrt{\frac{s-4m_f^2}{s-4m_\chi^2}}\Bigg((g_f^V)^2(s-4m_\chi^2)(s+2m_f^2) + (g_f^A)^2\Bigg(s(s-4m_\chi^2) + 4m_f^2\Bigg(m_\chi^2\Bigg(7-6\frac{s}{m_{Z'}^2}+3\frac{s^2}{m_{Z'}^2}\Bigg)-s\Bigg)\Bigg)\Bigg), \tag{C.1}$$

$$\sigma(\chi\chi \to Z^{(L)}h) = g_\chi^2 g_{Z'}^4 \cos^2\theta\cos^4\psi \frac{\sqrt{(s-(m_h^2+m_Z^2))^2 - 4m_h^2 m_Z^2}}{((s-m_{Z'}^2)^2) + \Gamma_{Z'}^2 m_{Z'}^2)} \frac{1}{48\pi s^{5/2}} \cdot \Big(\Big(m_h^4(2m_\chi^2+s) - 2m_h^2(s+2m_\chi^2)(s+m_Z^2) + 2m_\chi^2(s^2-10sm_Z^2+m_Z^4)$$

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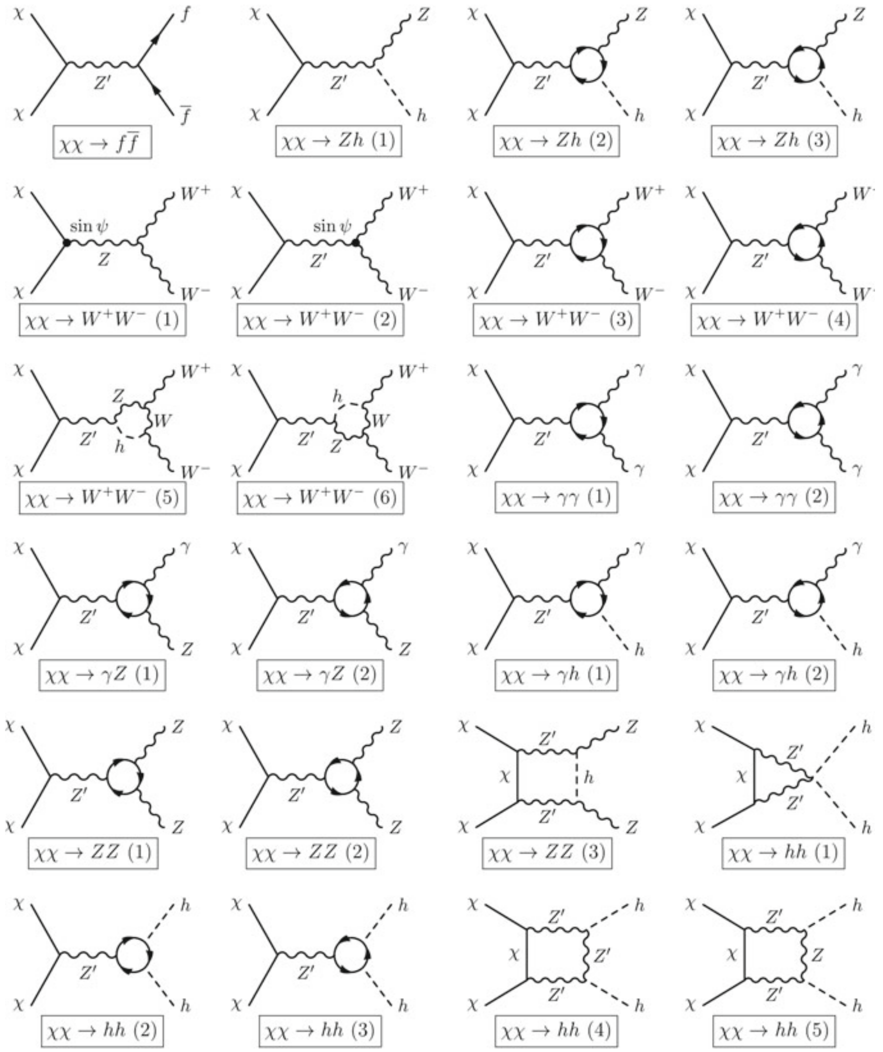

Figure C.1 Feynman diagrams for the annihilation of $\chi\chi$ into pairs of SM particles that have been considered in this work. In the fermion loops, the amplitude is summed over all the SM fermions

$$+ s(s + m_Z^2)^2\Big) + \frac{1}{m_{Z'}^2}\Big(- 6m_\chi^2 s(m_h^4 - 2m_h^2(m_Z^2 + s) + (m_Z^2 - s)^2)\Big)$$
$$+ \frac{1}{m_{Z'}^4}\Big(3m_\chi^2 s^2(m_h^4 - 2m_h^2(s + m_Z^2) + (s - m_Z^2)^2)\Big)\Bigg), \qquad \text{(C.2)}$$

$$\sigma(\chi\chi \to Z^{(T)}h) = g_\chi^2 g_{Z'}^4 \cos^2\theta \cos^4\psi \frac{\sqrt{(s-(m_h^2+m_Z^2))^2 - 4m_h^2 m_Z^2}}{((s-m_{Z'}^2)^2) + \Gamma_{Z'}^2 m_{Z'}^2)} \frac{m_Z^2\left(s-4m_\chi^2\right)^{1/2}}{6\pi s^{3/2}}, \tag{C.3}$$

$$\sigma(\chi\chi \to W^{+(L)}W^{-(L)}) = g_\chi^2 g_{Z'}^4 \alpha_W \cos^2\theta_W \cos^2\psi \sin^2\psi\left(s-4m_W^2\right)^{3/2}\left(s-4m_\chi^2\right)^{1/2} \cdot \cdot \frac{(m_{Z'}^2-m_Z^2)^2 + (\Gamma_{Z'}m_{Z'} - \Gamma_Z m_Z)^2}{((s-m_Z^2)^2 - \Gamma_Z^2 m_Z^2)((s-m_{Z'}^2)^2 - \Gamma_{Z'}^2 m_{Z'}^2)} \frac{(2m_W^2+s)^2}{12 m_W^4 s}, \tag{C.4}$$

$$\sigma(\chi\chi \to W^{\pm(T)}W^{\mp(L)}) = g_\chi^2 g_{Z'}^4 \alpha_W \cos^2\theta_W \cos^2\psi \sin^2\psi\left(s-4m_W^2\right)^{3/2}\left(s-4m_\chi^2\right)^{1/2} \cdot \cdot \frac{(m_{Z'}^2-m_Z^2)^2 + (\Gamma_{Z'}m_{Z'} - \Gamma_Z m_Z)^2}{((s-m_Z^2)^2 - \Gamma_Z^2 m_Z^2)((s-m_{Z'}^2)^2 - \Gamma_{Z'}^2 m_{Z'}^2)} \frac{4}{3\, m_W^2}, \tag{C.5}$$

$$\sigma(\chi\chi \to W^{+(T)}W^{-(T)}) = g_\chi^2 g_{Z'}^4 \alpha_W \cos^2\theta_W \cos^2\psi \sin^2\psi\left(s-4m_W^2\right)^{3/2}\left(s-4m_\chi^2\right)^{1/2} \cdot \cdot \frac{(m_{Z'}^2-m_Z^2)^2 + (\Gamma_{Z'}m_{Z'} - \Gamma_Z m_Z)^2}{((s-m_Z^2)^2 - \Gamma_Z^2 m_Z^2)((s-m_{Z'}^2)^2 - \Gamma_{Z'}^2 m_{Z'}^2)} \frac{2}{3s}. \tag{C.6}$$

A few clarifications are in order about the annihilation cross sections of the DM at tree level.

$f\overline{f}$: We denote the number of colours of the fermion f by N_c^f, and its vector and axial vector couplings to Z' by g_f^V, g_f^A respectively. The values of g_f^V, g_f^A are given in Table 9.2 In the zero velocity limit, corresponding to $s = 4m_\chi^2$, the cross section is proportional to m_f^2, because of the helicity suppression for pairs of annihilating fermions. In that limit, $\sigma \propto (g_f^A)^2 \propto \cos^2\theta$, i.e. the a coefficient in the low velocity expansion $\sigma v \simeq a + bv^2$ comes from the $U(1)_Y$ component of the $U(1)'$ extension.

Zh: The diagram on Fig. C.1 contains the tree level vertex $Z'Zh$ of Eq. (9.9). In the zero velocity limit, the only contribution comes from the production of a longitudinally polarized Z, since in Eq. (C.3) the factor $(s-4m_\chi^2)^{1/2}$ vanishes.

WW: We denote $\alpha_W = g_W^2/(4\pi)$, where g_W is the weak coupling constant. The amplitude is the sum of the two diagrams on Fig. C.1: the annihilation occurs via the mixing of Z and Z' and the SM trilinear gauge vertex ZWW, see Eq. (9.10). For each of the final polarization states, the cross section is proportional to $(s-4m_\chi^2)^{1/2}$.

Given that $\sigma(\chi\chi \to WW)v_{\rm DM}$ (where $v_{\rm DM}$ is the DM velocity) is suppressed by $\sin^2\psi$ and by $v_{\rm DM}^2$ at tree level it is worth checking whether contributions arising at one loop can become important. The contribution to the amplitude of diagrams 5 and 6 on Fig. C.1 is velocity suppressed because of the same argument reported at the end of Sec. 9.3.6. Therefore we only considered the contribution to the cross section coming from the sum of diagrams 3 and 4, also ignoring the interference terms with the other diagrams.

We also computed the cross sections for the annihilations into ZZ, $\gamma\gamma$, γh, γZ and hh at one loop. It is worth computing these corrections because of the velocity suppression of some tree level channels, and because in the pure $U(1)_{B-L}$ case the tree level annihilations into WW and Zh disappear. As for the Zh channel, we computed the one loop cross section only in the pure $U(1)_{B-L}$ case ($\theta = \pi/2$), in which the tree level amplitude vanishes, and the only remaining contribution comes from diagrams 2 and 3 in Fig. C.1. The results of the loop calculation are:

$$\begin{aligned}
\sigma(\chi\chi \to W^{+(T)}W^{-(T)}) = &\frac{g_{Z'}^4 g_\chi^2 N_c^i \left(s-4m_W^2\right)^{3/2} \alpha_W^2}{768\pi^3 \left(\Gamma_{Z'}^2 m_{Z'}^2 + \left(m_{Z'}^2 - s\right)^2\right) s\sqrt{s-4m_\chi^2}} \cdot \\
&\cdot \left\{ \frac{48 m_\chi^2 \left(s-m_{Z'}^2\right)^2}{m_{Z'}^4 (s-4m_W^2)^2} \left| \sum_i N_c^i \left\{ (g_{d^i}^L - g_{d^i}^R) B_0(s, m_{d^i}^2, m_{d^i}^2) m_{d^i}^2 - \right.\right.\right. \\
&- (g_{d^i}^L - g_{d^i}^R)(m_{d^i}^2 - m_{u^i}^2 - m_W^2) C_0(m_W^2, m_W^2, s, m_{d^i}^2, m_{u^i}^2, m_{d^i}^2) m_{d^i}^2 - \frac{1}{2}(g_{d^i}^L + g_{u^i}^L)(s-4m_W^2) + \\
&+ \left((g_{d^i}^R - g_{d^i}^L) m_{d^i}^2 + (g_{u^i}^R - g_{u^i}^L) m_{u^i}^2\right) B_0(m_W^2, m_{d^i}^2, m_{u^i}^2) + (g_{u^i}^L - g_{u^i}^R) m_{u^i}^2 B_0(s, m_{u^i}^2, m_{u^i}^2) + \\
&\left.\left. + (g_{u^i}^L - g_{u^i}^R) m_{u^i}^2 (m_{d^i}^2 - m_{u^i}^2 + m_W^2) C_0(m_W^2, m_W^2, s, m_{u^i}^2, m_{d^i}^2, m_{u^i}^2) \right\} \right|^2 \\
&+ \frac{32}{81} \frac{s-4m_\chi^2}{m_W^4 (s-4m_W^2)^4} \left| \sum_i N_c^i \left\{ \frac{1}{2}(s-4m_W^2)\left((3m_{d^i}^2 - 3m_{u^i}^2 + 7m_W^2 - 5s/2) g_{d^i}^L + \right.\right.\right. \\
&+ (3m_{d^i}^2 - 3m_{u^i}^2 - 7m_W^2 + 5s/2) g_{u^i}^L\big) m_W^2 + \frac{3}{2}\Big[3 g_{d^i}^R (s-4m_W^2) m_{d^i}^2 + g_{d^i}^L \big(-6m_{d^i}^4 + 2(6m_{u^i}^2 + 2m_W^2 + s) m_{d^i}^2 - \\
&- 6m_{u^i}^4 + 6m_W^4 + s^2 - 3m_{u^i}^2 s - 7m_W^2 s\big)\Big] B_0(s, m_{d^i}^2, m_{d^i}^2) m_W^2 - \frac{3}{2}\Big[3 g_{u^i}^R (s-4m_W^2) m_{u^i}^2 + \\
&+ g_{u^i}^L \big(-6m_{d^i}^4 - 3(s-4m_{u^i}^2) m_{d^i}^2 - 6m_{u^i}^4 + 6m_W^4 + s^2 - 7m_W^2 s + 2m_{u^i}^2 (s+2m_W^2)\big)\Big] B_0(s, m_{u^i}^2, m_{u^i}^2) m_W^2 - \\
&- \frac{9}{2}(m_W^2 - m_{d^i}^2 + m_{u^i}^2)\Big[-g_{d^i}^R (s-4m_W^2) m_{d^i}^2 + g_{d^i}^L \Big(2m_{d^i}^4 - (s+4m_{u^i}^2) m_{d^i}^2 + \\
&+ 2\big(m_{u^i}^4 + (s-2m_W^2) m_{u^i}^2 + m_W^4\big)\Big)\Big] C_0(m_W^2, m_W^2, s, m_{d^i}^2, m_{u^i}^2, m_{d^i}^2) m_W^2 + \\
&+ 9\Big[-\frac{1}{2}(s-4m_W^2) g_{u^i}^R m_{u^i}^2 + \big(m_{d^i}^4 + (s-2m_{u^i}^2 - 2m_W^2) m_{d^i}^2 + m_{u^i}^4 + m_W^4 - m_{u^i}^2 s/2\big) g_{u^i}^L \Big] \cdot \\
&\cdot (m_{d^i}^2 - m_{u^i}^2 + m_W^2) C_0(m_W^2, m_W^2, s, m_{u^i}^2, m_{d^i}^2, m_{u^i}^2) m_W^2 + \frac{3}{2}(m_{d^i}^2 - m_{u^i}^2)\big(g_{d^i}^L (m_{d^i}^2 - m_{u^i}^2 - m_W^2) - \\
&- g_{u^i}^L (m_{d^i}^2 - m_{u^i}^2 + m_W^2)\big)(s-4m_W^2) B_0(0, m_{d^i}^2, m_{u^i}^2) + 3\Big[-\frac{3}{2}(s-4m_W^2)(g_{d^i}^R m_{d^i}^2 - g_{u^i}^R m_{u^i}^2) m_W^2 + \\
&+ g_{u^i}^L \Big(\frac{1}{2}\big(m_{d^i}^4 - 2(m_{u^i}^2 + m_W^2) m_{d^i}^2 + m_{u^i}^4 + m_W^4 + m_{u^i}^2 m_W^2\big)s - m_W^2\big(5m_{d^i}^4 + 2(m_W^2 - 5m_{u^i}^2) m_{d^i}^2 + 5m_{u^i}^4 + \\
&+ 5m_W^4 - 4m_{u^i}^2 m_W^2\big)\Big) + g_{d^i}^L \Big(m_W^2\big(5m_{d^i}^4 - 2(5m_{u^i}^2 + 2m_W^2) m_{d^i}^2 + 5m_{u^i}^4 + 5m_W^4 + 2m_{u^i}^2 m_W^2\big) - \\
&\left.\left.\left. - \frac{1}{2}\big(m_{d^i}^4 + (m_W^2 - 2m_{u^i}^2) m_{d^i}^2 + (m_{u^i}^2 - m_W^2)^2\big)s\Big)\Big] B_0(m_W^2, m_{d^i}^2, m_{u^i}^2) \right\} \right|^2 \right\}
\end{aligned} \tag{C.7}$$

We do not report the formula for $\sigma(\chi\chi \to W^{\pm(T)}W^{\mp(L)})$ because this channel turns out to have $a = 0$ (therefore it is irrelevant for the annihilation process in the Sun) and the corresponding formula is too cumbersome to be reported here.

$$\sigma(\chi\chi \to W^{+(L)}W^{-(L)}) = \frac{g_{Z'}^4 g_\chi^2 \alpha_W^2 \sqrt{s - 4mx^2}}{62208\pi^3 m_W^8 [(s - m_{Z'}^2)^2 + \Gamma_{Z'}^2 m_{Z'}^2] s (s - 4m_W^2)^{5/2}} \cdot$$

$$\cdot \Bigg| \sum_i N_c^i \Bigg\{ (4m_W^2 - s) m_W^2 \Big(g_{u^i}^L (-4m_W^4 + 12m_{u^i}^2 m_W^2 + 2sm_W^2 - s^2 - 6m_{d^i}^2 (s - 2m_W^2))$$

$$- g_{d^i}^L (-4m_W^4 + 12m_{d^i}^2 m_W^2 + 2sm_W^2 - s^2 - 6m_{u^i}^2 (s - 2m_W^2)) \Big)$$

$$+ 6 \Big(A_0(m_{d^i}^2) g_{d^i}^L - A_0(m_{u^i}^2) g_{u^i}^L \Big) (s - 4m_W^2)^2 m_W^2$$

$$- 3B_0(0, m_{d^i}^2, m_{u^i}^2)(m_{d^i}^2 - m_{u^i}^2)(4m_W^2 - s) \Big(g_{u^i}^L (4m_W^4 - 2sm_W^2 + m_{u^i}^2 (8m_W^2 - s) + m_{d^i}^2 (s - 8m_W^2)) +$$

$$g_{d^i}^L (4m_W^4 - 2sm_W^2 + m_{d^i}^2 (8m_W^2 - s) + m_{u^i}^2 (s - 8m_W^2)) \Big)$$

$$+ 3B_0(m_W^2, m_{d^i}^2, m_{u^i}^2) \Big(6(g_{d^i}^R m_{d^i}^2 - g_{u^i}^R m_{u^i}^2)(4m_W^2 - s) m_W^4 + g_{d^i}^L [8m_W^8 + 12sm_W^6 - 2s^2 m_W^4 +$$

$$(m_{d^i}^2 - m_{u^i}^2)^2 (32m_W^4 - 6sm_W^2 + s^2) + m_{u^i}^2 (8m_W^4 + 6sm_W^2 + s^2) m_W^2 +$$

$$m_{d^i}^2 (-16m_W^6 - 12sm_W^4 + s^2 m_W^2)] - g_{u^i}^L [8m_W^8 + 12sm_W^6 - 2s^2 m_W^4 +$$

$$(m_{d^i}^2 - m_{u^i}^2)^2 (32m_W^4 - 6sm_W^2 + s^2) + m_{u^i}^2 (-16m_W^4 - 12sm_W^2 + s^2) m_W^2 + m_{d^i}^2 (m_W^2 (8m_W^4 + 6sm_W^2 + s^2))] \Big)$$

$$+ 3B_0(s, m_{u^i}^2, m_{u^i}^2) m_W^2 \Big(6g_{u^i}^R m_{u^i}^2 (4m_W^2 - s) m_W^2 + g_{u^i}^L [-24m_W^6 + 12sm_W^4 + 4s(s - 4m_{u^i}^2) m_W^2 + 6m_{d^i}^4 s$$

$$+ s(6m_{u^i}^4 + sm_{u^i}^2 - s^2) + 3m_{d^i}^2 (8m_W^4 - 2sm_W^2 + s(s - 4m_{u^i}^2))] \Big)$$

$$- 3B_0(s, m_{d^i}^2, m_{d^i}^2) m_W^2 \Big(6g_{d^i}^R m_{d^i}^2 (4m_W^2 - s) m_W^2 + g_{d^i}^L [-24m_W^6 + 12sm_W^4 + 4s(s - 4m_{d^i}^2) m_W^2 + 6m_{u^i}^4 s$$

$$+ s(6m_{d^i}^4 + sm_{d^i}^2 - s^2) + 3m_{u^i}^2 (8m_W^4 - 2sm_W^2 + s(s - 4m_{d^i}^2))] \Big)$$

$$+ 18C_0(m_W^2, m_W^2, s, m_{u^i}^2, m_{d^i}^2, m_{u^i}^2) m_W^2 \Big(g_{u^i}^R m_{u^i}^2 (m_{d^i}^2 - m_{u^i}^2 + m_W^2)(4m_W^2 - s) m_W^2$$

$$+ g_{u^i}^L [sm_{d^i}^6 + (4m_W^4 - 2sm_W^2 + s(s - 3m_{u^i}^2)) m_{d^i}^4 + (-8m_W^6 + 5sm_W^4 + 3m_{u^i}^4 s$$

$$- m_{u^i}^2 (4m_W^4 + sm_W^2 + s^2)) m_{d^i}^2 - (m_{u^i}^2 - m_W^2)(4m_W^6 - 2m_{u^i}^2 sm_W^2 + m_{u^i}^4 s)] \Big)$$

$$+ 18C_0(m_W^2, m_W^2, s, m_{d^i}^2, m_{u^i}^2, m_{d^i}^2) m_W^2 \Big(g_{d^i}^R m_{d^i}^2 (m_{d^i}^2 - m_{u^i}^2 - m_W^2)(4m_W^2 - s) m_W^2$$

$$+ g_{d^i}^L [-4m_W^8 + m_{d^i}^6 s - m_{u^i}^6 s - 3m_{d^i}^4 (m_{u^i}^2 + m_W^2) s + m_{u^i}^2 (8m_W^6 - 5m_W^4 s)$$

$$- m_{u^i}^4 (4m_W^4 - 2sm_W^2 + s^2) + m_{d^i}^2 (3sm_{u^i}^4 + (4m_W^4 + sm_W^2 + s^2) m_{u^i}^2 + 2m_W^4 (2m_W^2 + s))] \Big) \Bigg\} \Bigg|^2, \tag{C.8}$$

$$\sigma(\chi\chi \to Z^{(T)}Z^{(T)}) = \frac{4g_\chi^2 g_{Z'}^4 m_\chi^2 m_Z^4 (m_{Z'}^2 - s)^2}{\pi^5 m_{Z'}^4 s v^4 \sqrt{s - 4m_\chi^2} \sqrt{s - 4m_Z^2} \left(\Gamma_{Z'}^2 m_{Z'}^2 + \left(m_{Z'}^2 - s \right)^2 \right)} \cdot$$

$$\cdot \Bigg| \sum_f N_c^f \Bigg\{ 4g_f^A m_Z^2 (c_f^L)^2 - 4g_f^V m_Z^2 (c_f^L)^2 - g_f^A s (c_f^L)^2 + g_f^V s (c_f^L)^2 + 4g_f^A (c_f^R)^2 m_Z^2 + 4(c_f^R)^2 g_f^V m_Z^2$$

$$- g_f^A (c_f^R)^2 s - (c_f^R)^2 g_f^V s - \Big[g_f^A \Big((4m_f^2 - 4m_Z^2 + s)(c_f^L)^2 - 8c_f^R m_f^2 c_f^L + (c_f^R)^2 (4m_f^2 - 4m_Z^2 + s) \Big)$$

$$+ \Big((c_f^L)^2 - (c_f^R)^2 \Big) g_f^V (4m_Z^2 - s) \Big] B_0(m_Z^2, m_f^2, m_f^2) + \Big[\Big((c_f^L)^2 - (c_f^R)^2 \Big) g_f^V (4m_Z^2 - s)$$

$$+ g_f^A \Big((4m_f^2 - 4m_Z^2 + s)(c_f^L)^2 - 8c_f^R m_f^2 c_f^L + (c_f^R)^2 (4m_f^2 - 4m_Z^2 + s) \Big) \Big] B_0(s, m_f^2, m_f^2)$$

$$+ 4g_f^A m_f^2 \Big((c_f^L)^2 m_Z^2 + (c_f^R)^2 m_Z^2 + c_f^L c_f^R (2m_Z^2 - s) \Big) C_0(s, m_Z^2, m_Z^2, m_f^2, m_f^2, m_f^2) \Bigg\} \Bigg|^2, \tag{C.9}$$

$$\sigma(\chi\chi \to Z^{(T)}Z^{(L)}) = \frac{4g_\chi^2 g_{Z'}^4 m_Z^2\sqrt{s-4m_\chi^2}}{3\pi^5 s v^4\left(s-4m_Z^2\right)^{3/2}\left(\Gamma_{Z'}^2 m_{Z'}^2+\left(m_{Z'}^2-s\right)^2\right)}\cdot$$
$$\cdot\Bigg|\sum_f N_c^f\Bigg\{-4g_f^A(c_f^L)^2m_Z^4-4g_f^A(c_f^R)^2m_Z^4+4(c_f^L)^2g_f^Vm_Z^4-4(c_f^R)^2g_f^Vm_Z^4$$
$$+g_f^A(c_f^L)^2sm_Z^2+g_f^A(c_f^R)^2sm_Z^2-(c_f^L)^2g_f^Vsm_Z^2+(c_f^R)^2g_f^Vsm_Z^2$$
$$+\frac{1}{2}\Bigg[g_f^A\Bigg(-\big(20m_Z^4-6sm_Z^2+s^2+4m_f^2(s-4m_Z^2)\big)(c_f^L)^2+8c_f^Rm_f^2(s-4m_Z^2)c_f^L$$
$$-(c_f^R)^2\big(20m_Z^4-6sm_Z^2+s^2+4m_f^2(s-4m_Z^2)\big)\Bigg)$$
$$+\big((c_f^L)^2-(c_f^R)^2\big)g_f^V\big(20m_Z^4-6sm_Z^2+s^2\big)\Bigg]B_0(m_Z^2,m_f^2,m_f^2)$$
$$-\frac{1}{2}\Bigg[g_f^A\Bigg(-\big(20m_Z^4-6sm_Z^2+s^2+4m_f^2(s-4m_Z^2)\big)(c_f^L)^2+8c_f^Rm_f^2(s-4m_Z^2)c_f^L$$
$$-(c_f^R)^2\big(20m_Z^4-6sm_Z^2+s^2+4m_f^2(s-4m_Z^2)\big)\Bigg)$$
$$+\big((c_f^L)^2-(c_f^R)^2\big)g_f^V\big(20m_Z^4-6sm_Z^2+s^2\big)\Bigg]B_0(s,m_f^2,m_f^2)$$
$$+2\Bigg[-\frac{1}{2}\big((c_f^L)^2-(c_f^R)^2\big)g_f^V\Big(-2m_Z^6+2sm_Z^4+m_f^2\big(8m_Z^4-6sm_Z^2+s^2\big)\Big)$$
$$-\frac{1}{2}g_f^A\Bigg(\big(2m_Z^6-2sm_Z^4+4m_f^2sm_Z^2-m_f^2s^2\big)(c_f^L)^2+2c_f^Rm_f^2\big(8m_Z^4-6sm_Z^2+s^2\big)c_f^L$$
$$+(c_f^R)^2\Big(2m_Z^6-2sm_Z^4+4m_f^2sm_Z^2-m_f^2s^2\Big)\Bigg)\Bigg]C_0(s,m_Z^2,m_Z^2,m_f^2,m_f^2,m_f^2)\Bigg\}\Bigg|^2, \tag{C.10}$$

$$\sigma(\chi\chi \to Z^{(L)}Z^{(L)}) = 0, \tag{C.11}$$

$$\sigma(\chi\chi \to Z^{(T)}h)\big|_{\theta=\pi/2} = \frac{\alpha_W g_\chi^2 g_{Z'}^4}{24\pi^4\left(m_{Z'}^4+\Gamma_{Z'}^2m_{Z'}^2-2sm_{Z'}^2+s^2\right)}\cdot$$
$$\cdot\frac{\sqrt{s-4m_\chi^2}\sqrt{(m_H^2-m_Z^2)^2+s^2-2s(m_H^2+m_Z^2)}}{s^{3/2}v^2\left((m_H-m_Z)^2-s\right)^2\left((m_H+m_Z)^2-s\right)^2}\Bigg|\sum_f g_f^Vm_f^2N_c^f(c_f^L+c_f^R)\cdot$$
$$\cdot\Bigg\{C_0(m_H^2,m_Z^2,s,m_f^2,m_f^2,m_f^2)s^3+2(-m_H^2-m_Z^2+s)B_0(s,m_f^2,m_f^2)s$$
$$-\Big[2B_0(m_H^2,m_f^2,m_f^2)+(-4m_f^2+3m_H^2+m_Z^2)C_0(m_H^2,m_Z^2,s,m_f^2,m_f^2,m_f^2)-2\Big]s^2$$
$$+\Big[-2B_0(m_Z^2,m_f^2,m_f^2)m_Z^2+2(m_H^2+2m_Z^2)B_0(m_H^2,m_f^2,m_f^2)$$
$$-\Big((8m_f^2-3m_H^2+m_Z^2)C_0(m_H^2,m_Z^2,s,m_f^2,m_f^2,m_f^2)+4\Big)(m_H^2+m_Z^2)\Big]s$$

$$
\begin{aligned}
&- (m_H^2 - m_Z^2)\Big[- 2B_0(m_H^2, m_f^2, m_f^2)m_Z^2 + 2B_0(m_Z^2, m_f^2, m_f^2)m_Z^2 \\
&+ \Big((-4m_f^2 + m_H^2 - m_Z^2)C_0(m_H^2, m_Z^2, s, m_f^2, m_f^2, m_f^2) - 2\Big)(m_H^2 - m_Z^2)\Big]\Big\}\Big|^2 , \qquad \text{(C.12)}
\end{aligned}
$$

$$
\begin{aligned}
\sigma(\chi\chi \to Z^{(L)}h)\big|_{\theta=\pi/2} = &\frac{\alpha_W g_\chi^2 g_{Z'}^4}{3\pi^4\left(\Gamma_{Z'}^2 m_{Z'}^2 + m_{Z'}^4 - 2m_{Z'}^2 s + s^2\right)} \cdot \\
&\cdot \frac{m_Z^2\sqrt{s-4m_\chi^2}}{\sqrt{s}v^2\left(-2s\left(m_H^2+m_Z^2\right)+\left(m_H^2-m_Z^2\right)^2+s^2\right)^{3/2}}\Big|\sum_f g_f^V m_f^2 N_c^f (c_f^L + c_f^R)\cdot \\
&\cdot \Big\{m_H^2\Big[(-m_H^2 + m_Z^2 + s)C_0(m_H^2, m_Z^2, s, m_f^2, m_f^2, m_f^2) - 2B_0(m_H^2, m_f^2, m_f^2)\Big] \\
&\quad + (m_H^2 + m_Z^2 - s)B_0(m_Z^2, m_f^2, m_f^2) + (m_H^2 - m_Z^2 + s)B_0(s, m_f^2, m_f^2)\Big\}\Big|^2 . \qquad \text{(C.13)}
\end{aligned}
$$

$$
\begin{aligned}
\sigma(\chi\chi \to \gamma\gamma) = &\frac{16\alpha_{\rm em}^2 g_\chi^2 g_{Z'}^4}{\pi^3\left(\Gamma_{Z'}^2 m_{Z'}^2 + \left(m_{Z'}^2 - s\right)^2\right)} \frac{m_\chi^2\sqrt{s}(s-m_{Z'}^2)^2}{m_{Z'}^4\sqrt{s-4m_\chi^2}} \cdot \\
&\cdot \Big|\sum_f g_f^A N_c^f Q_f^2\Big[2m_f^2 C_0(0,0,s,m_f^2,m_f^2,m_f^2)+1\Big]\Big|^2 , \qquad \text{(C.14)}
\end{aligned}
$$

$$
\begin{aligned}
\sigma(\chi\chi \to \gamma h) = &\frac{\alpha_{\rm em} g_\chi^2 g_{Z'}^4}{6\pi^4\left(\Gamma_{Z'}^2 m_{Z'}^2 + m_{Z'}^4 - 2m_{Z'}^2 s + s^2\right)} \frac{\sqrt{s-4m_\chi^2}}{s^{3/2}v^2\left(s-m_H^2\right)} \cdot \\
&\cdot \Big|\sum_f g_f^V m_f^2 N_c^f Q_f\Big\{ - 2sB_0(m_H^2, m_f^2, m_f^2) + 2sB_0(s, m_f^2, m_f^2) \\
&\quad + (s - m_H^2)\Big[(4m_f^2 - m_H^2 + s)C_0(m_H^2, 0, s, m_f^2, m_f^2, m_f^2) + 2\Big]\Big\}\Big|^2 , \qquad \text{(C.15)}
\end{aligned}
$$

$$
\begin{aligned}
\sigma(\chi\chi \to \gamma Z^{(T)}) = &\frac{\alpha_{\rm em}\alpha_W g_\chi^2 g_{Z'}^4}{3\pi^3\left([(s-m_{Z'}^2)^2 + \Gamma_{Z'}^2 m_{Z'}^2\right] m_{Z'}^4 s^{5/2}\left(s-m_Z^2\right)\sqrt{s-4m_\chi^2}} \cdot \\
&\cdot \Big\{\frac{m_{Z'}^4}{2}(s-4m_\chi^2)\Big|\sum_f N_c^f Q_f\Big\{m_Z^2 s\big(g_f^A(c_f^L + c_f^R) + g_f^V(c_f^R - c_f^L)\big)\big(B_0(m_Z^2, m_f^2, m_f^2) - B_0(s, m_f^2, m_f^2)\big) \\
&- (s - m_Z^2)\Big[2m_f^2 C_0(m_Z^2, 0, s, m_f^2, m_f^2, m_f^2)\big(g_f^A c_f^L m_Z^2 + g_f^A c_f^R m_Z^2 - c_f^L g_f^V s + c_f^R g_f^V s\big)
\end{aligned}
$$

$$+ m_Z^2\big(g_f^A(c_f^L + c_f^R) + g_f^V(c_f^R - c_f^L)\big)\Big]\Big\}\Big|^2 + 3m_\chi^2(s-m_Z^2)^4(s-m_{Z'}^2)^2\cdot$$
$$\cdot\Big|\sum_f N_c^f Q_f\Big\{2g_f^A m_f^2(c_f^L + c_f^R)C_0(m_Z^2,0,s,m_f^2,m_f^2,m_f^2) + g_f^A(c_f^L + c_f^R) + g_f^V(c_f^R - c_f^L)\Big\}\Big|^2, \Big\} \tag{C.16}$$

$$\sigma(\chi\chi\to\gamma Z^{(L)}) = \frac{\alpha_{\rm em}\alpha_W g_\chi^2 g_{Z'}^4}{6\pi^3\left([(s-m_{Z'}^2)^2+\Gamma_{Z'}^2 m_{Z'}^2\right]}\frac{\sqrt{s-4m_\chi^2}}{m_Z^2 s^{3/2}\left(s-m_Z^2\right)}\cdot$$
$$\cdot\Big|\sum_f N_C^f Q_f\Big\{m_Z^2 s\big(g_f^A(c_f^L+c_f^R)+g_f^V(c_f^R-c_f^L)\big)\big(B_0(m_Z^2,m_f^2,m_f^2)-B_0(s,m_f^2,m_f^2)\big)$$
$$-(s-m_Z^2)\Big[2m_f^2 C_0(m_Z^2,0,s,m_f^2,m_f^2,m_f^2)\big(g_f^A c_f^L m_Z^2 + g_f^A c_f^R m_Z^2 - c_f^L g_f^V s + c_f^R g_f^V s\big)$$
$$+ m_Z^2\big(g_f^A(c_f^L+c_f^R)+g_f^V(c_f^R-c_f^L)\big)\Big]\Big\}\Big|^2, \tag{C.17}$$

$$\sigma(\chi\chi\to hh) = \frac{g_\chi^4 g_{Z'}^8 m_\chi^2\cos^4\theta}{2048\pi^5}\frac{\sqrt{s-4m_h^2}}{s\big(s-4m_\chi^2\big)^{3/2}}\cdot$$
$$\cdot\Big|B_0(m_\chi^2,m_\chi^2,m_{Z'}^2) - B_0(s,m_{Z'}^2,m_{Z'}^2) + (2m_\chi^2 + m_{Z'}^2 - s)C_0(m_\chi^2,m_\chi^2,s,m_{Z'}^2,m_\chi^2,m_{Z'}^2)\Big|^2. \tag{C.18}$$

In the cross sections involving a fermionic loop, we denoted by $\sum_f$ a sum over all the SM fermion species. In the WW cross section, instead, we denoted by $\sum_i$ a sum over the six fermion families (three families of quarks and three of leptons), with m_{u^i}, m_{d^i} the upper and lower component of the doublet, respectively, and with $g_{u^i}^L$, $g_{u^i}^R$, $g_{d^i}^L$, $g_{d^i}^R$ the combinations

$$g_{u^i}^L = g_{u^i}^V - g_{u^i}^A,\qquad g_{u^i}^R = g_{u^i}^V + g_{u^i}^A,\qquad g_{d^i}^L = g_{d^i}^V - g_{d^i}^A,\qquad g_{d^i}^R = g_{d^i}^V + g_{d^i}^A. \tag{C.19}$$

The functions A_0, B_0 and C_0 are the standard Passarino-Veltman one loop one-, two- and three-points scalar integrals [1]:

$$A_0\left(m_0^2\right) = \frac{\mu^{4-D}}{i\pi^{D/2}\gamma_\Gamma}\int\frac{d^Dk}{k^2-m_0^2}, \tag{C.20}$$

$$B_0\left(p^2,m_1^2,m_2^2\right) = \frac{\mu^{4-D}}{i\pi^{D/2}\gamma_\Gamma}\int\frac{d^Dk}{\left(k^2-m_1^2\right)\left((k+p)^2-m_2^2\right)}, \tag{C.21}$$

Table C.1 Coupling of the Z boson to SM fermions

SM fermion f	c_f^L	c_f^R
Leptons	$-\frac{1}{2}+\sin^2\theta_W$	$\sin^2\theta_W$
Neutrinos	$\frac{1}{2}$	0
Up quarks	$\frac{1}{2}-\frac{2}{3}\sin^2\theta_W$	$-\frac{2}{3}\sin^2\theta_W$
Down quarks	$-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W$	$\frac{1}{3}\sin^2\theta_W$

$$C_0\left(p_1^2, p_2^2, (p_1+p_2)^2, m_1^2, m_2^2, m_3^2\right) = \frac{\mu^{4-D}}{i\pi^{D/2}\gamma_\Gamma}\int \frac{d^Dk}{\left(k^2-m_1^2\right)\left((k+p_1)^2-m_2^2\right)\left((k+p_1+p_2)^2-m_3^2\right)}, \tag{C.22}$$

with

$$\gamma_\Gamma = \frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}, \quad D = 4-2\epsilon, \tag{C.23}$$

where γ_Γ approaches 1 in the limit $\epsilon \to 0$. In these equations, we denoted by $v \simeq 246$ GeV the vacuum expectation value of the Higgs field, and by c_f^L, c_f^R the SM coupling of the Z boson to left- and right-handed fermions respectively (see Table C.1).

Let us briefly comment on the one loop cross sections:

WW: The bosonic loop diagrams 5 and 6 in Fig. C.1 are velocity suppressed as expected.

ZZ: The box diagram (number 3 on Fig. C.1) is suppressed at low energies by the two heavy propagators in the loop, and gives only a minor effect. Therefore we ignored it in our calculations.

$\gamma\gamma$: The cross section for annihilation into $\gamma\gamma$ vanishes on resonance, due to the factor of $(s-m_{Z'}^2)^2$ in the numerator. This is a consequence of the Landau-Yang theorem [2, 3] that states that a spin-1 particle can not decay into two photons, and is a reassuring cross-check of our results.
Also notice that the $\gamma\gamma$ cross section is proportional to g_f^A, the axial coupling of the fermions to the Z', and vanishes in the limit of pure $B-L$. This is due to the Dirac structure of the fermion loop in the very same way in which the cross section for annihilation into γh is proportional to the vectorial coupling g_f^V, and can be seen as a realization of the Furry theorem [4], which states that any physical amplitude involving an odd number of photons vanishes (in our case one of the photons is replaced by the vectorial part of the Z').

hh: The two diagrams with a fermionic loop (diagrams 2 and 3 in Fig. C.1) sum to zero, while the two box diagrams (numbers 4 and 5) give a contribution at most comparable to that of the triangular diagram (number 1). Since the cross section for annihilation into hh including only the triangular diagram is subdominant by several orders of magnitude, we can safely ignore the contribution of the two box diagrams.

Results of our calculations show that in the low kinetic energy regime that is relevant for DM annihilation in the Sun loop channels are usually subdominant. Some of the cross sections receive a velocity suppression ($\sigma v \simeq b v^2$) and precisely vanish in the zero velocity limit. Those are $W^{(T)}W^{(T)}$, $Z^{(T)}Z^{(L)}$, $\gamma Z^{(L)}$, γh, hh, $Z^{(T)}h$ and, for $\theta = \pi/2$, $Z^{(L)}h$. We do not explicitly show the analytical expansion around $v = 0$ because the velocity appears as an argument of the Passarino-Veltman functions. The only process, which acquires a relevant contribution at the one-loop level is $W^{(L)}W^{(L)}$.

All the cross sections we computed, except for hh that has no s-channel exchange of a Z' boson, vanish around $m_{\rm DM} = m_{Z'}$ in the $v_{\rm DM} \to 0$ limit. Again, this is a cross-check of the correctness of our calculations. Indeed, close to the resonance the cross section for $\chi\chi \to XX$ is proportional to the product $\Gamma(Z' \to \chi\chi)\cdot\Gamma(Z' \to XX)$, but $\Gamma(Z' \to \chi\chi)$ vanishes if $m_{\rm DM} = m_{Z'}/2$, which is implied by a resonant production of Z' with $v_{\rm DM} = 0$.

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Appendix D
General Formalism for the Calculation of the Relic Density

Our technique to compute the abundance and notation follow Refs. [1] and [2]. First we find the freezeout temperature by solving

$$e^{x_F} = \frac{\sqrt{\frac{45}{8}} g_{\text{DoF}} m_{\text{DM}} M_{\text{Pl}} c(c+2)\langle\sigma v\rangle}{2\pi^3 g_\star^{1/2}\sqrt{x_F}}, \tag{D.1}$$

where $x = m_{\text{DM}}/T$ and subscript F denotes the value at freezeout, $g_{\text{DoF}} = 2$ is the number of degrees of freedom of the DM particle, c is a matching constant usually taken to be $1/2$, $g_\star$ is the number of relativistic degrees of freedom, $M_{\text{Pl}} = 1/\sqrt{G_N}$ is the Planck mass. Usually, it is safe to expand in powers of the velocity and use the approximation

$$\langle\sigma v\rangle = a + b\langle v^2\rangle + \mathcal{O}(\langle v^4\rangle) \simeq a + 6b/x_F. \tag{D.2}$$

However, when the mediator width is small, this approximation can down near the s-channel resonance in the annihilation rate at $M \simeq 2m_{\text{DM}}$ [1, 3, 4] if the width is small. Around this point it becomes more accurate to use the full expression,

$$\langle\sigma v\rangle = \frac{x}{8m_{\text{DM}}^5 K_2^2[x]} \int_{4m_{\text{DM}}^2}^{\infty} \mathrm{d}s\, \sigma(s - 4m_{\text{DM}}^2)\sqrt{s} K_1[\sqrt{s}\, x/m_{\text{DM}}]. \tag{D.3}$$

With this information, one can calculate the relic abundance,

$$\Omega_{\text{DM}} h^2 = \Omega_\chi h^2 + \Omega_{\bar{\chi}} h^2 = \frac{2 \times 1.04 \times 10^9\,\text{GeV}^{-1} m_{\text{DM}}}{M_{\text{Pl}} \int_{T_0}^{T_F} g_\star^{1/2}\langle\sigma v\rangle \mathrm{d}T}, \tag{D.4}$$

where the factor of 2 is for Dirac DM. When the non-relativistic approximation to the annihilation rate holds, this simplifies to

$$\Omega_{\text{DM}} h^2 = \frac{2 \times 1.04 \times 10^9\,\text{GeV}^{-1} x_F}{\overline{g_\star}^{1/2} M_{\text{Pl}}\,(a + 3b/x_F)} \tag{D.5}$$

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where $\overline{g_\star}^{1/2}$ is a typical value of $g_\star^{1/2}(T)$ in the range $T_0 \leq T \leq T_F$. We have tested the validity of this approximation and find that there is a negligible difference to the full relativistic calculation, since the widths we consider are relatively large. If the physical widths are used, then care should be taken that this approximation still holds when the width becomes small, especially when the annihilation rate has a larger p-wave component.

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