Appendix A

Comment on Equations (2.48), (2.51), and (2.52)

Consider a periodic array (2.45), (2.46) of segments distributed along the ζ-axis.

The segments have lengths $\ell_1 = m_1\delta, \quad \ell_2 = m_2\delta$, and are occupied, respectively, by materials 1 and 2 immovable in the laboratory frame (z, t).

A general solution to the system (2.44) is given by

$$\left.\begin{array}{l} \bar{u} = Ae^{s\frac{\zeta}{V-a_1}} + Be^{s\frac{\zeta}{V+a_1}}, \\ \bar{v} = -\theta_1^{-1/2}\left(Ae^{s\frac{\zeta}{V-a_1}} - Be^{s\frac{\zeta}{V+a_1}}\right), \end{array}\right\} - \ell_1 \le \zeta \le 0, \tag{A.1}$$

$$\left.\begin{array}{l} \bar{u} = Ce^{s\frac{\zeta}{V-a_2}} + De^{s\frac{\zeta}{V+a_2}}, \\ \bar{v} = -\theta_2^{-1/2}\left(Ce^{s\frac{\zeta}{Va_2}} - De^{s\frac{\zeta}{V+a_2}}\right), \end{array}\right\} 0 \le \zeta \le \ell_2. \tag{A.2}$$

Here $\theta_i = (k_i\rho_i)^{-1}, \quad i = 1, 2$.

By Floquet theory,

$$\bar{u}(\zeta) = e^{\mu\delta}\bar{u}(\zeta - \delta), \quad \bar{v}(\zeta) = e^{\mu\delta}\bar{v}(\zeta - \delta), \tag{A.3}$$

where μ is the characteristic exponent. Given (A.1) and (A.3), we represent a solution in the interval $\ell_2 \le \zeta \le \ell_1 + \ell_2$ as

$$\begin{array}{l} \bar{u} = e^{\mu\delta}\left(Ae^{s\frac{\zeta-\delta}{V-a_1}} + Be^{s\frac{\zeta-\delta}{V+a_1}}\right), \\ \bar{v} = -\theta_1^{-1/2}e^{\mu\delta}\left(Ae^{s\frac{\zeta-\delta}{V-a_1}} - Be^{s\frac{\zeta-\delta}{V+a_1}}\right). \end{array}$$

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The compatibility conditions

$$[u]_{\zeta=0^-}^{\zeta=0^+} = [v]_{\zeta=0^-}^{\zeta=0^+} = [u]_{\zeta=\ell_2-0}^{\zeta=\ell_2+0} = [v]_{\zeta=\ell_2-0}^{\zeta=\ell_2+0} = 0$$

produce a linear system

$$\begin{aligned} A+B &= C+D, \\ 1/\sqrt{\theta_1}(A-B) &= 1/\sqrt{\theta_2}(C-D), \end{aligned} \tag{A.4}$$

$$e^{\mu\delta}\left(Ae^{-s\frac{\ell_1}{V-a_1}} + Be^{-s\frac{\ell_1}{V+a_1}}\right) = Ce^{s\frac{\ell_2}{V-a_2}} + De^{s\frac{\ell_2}{V+a_2}},$$

$$-1\sqrt{\theta_1}\, e^{\mu\delta}\left(Ae^{-s\frac{\ell_1}{V-a_1}} - Be^{-s\frac{\ell_1}{V+a_1}}\right) = -1/\sqrt{\theta_2}\left(Ce^{s\frac{\ell_2}{V-a_2}} - De^{s\frac{\ell_2}{V+a_2}}\right),$$

with determinant

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ -1/\sqrt{\theta_1} & 1/\sqrt{\theta_1} & -1/\sqrt{\theta_2} & 1/\sqrt{\theta_2} \\ Ye^{-s\frac{\ell}{V-a_1}} & Ye^{-s\frac{\ell_1}{V+a_1}} & e^{s\frac{\ell_2}{V-a_2}} & e^{s\frac{\ell_2}{V+a_2}} \\ -1/\sqrt{\theta_1}Ye^{-s\frac{\ell_1}{V-a_1}} & 1/\sqrt{\theta_1}Ye^{-s\frac{\ell_1}{V+a_1}} & -1/\sqrt{\theta_2}e^{s\frac{\ell_2}{V-a_2}} & 1/\sqrt{\theta_2}e^{s\frac{\ell_2}{V+a_2}} \end{vmatrix}, \tag{A.5}$$

where

$$Y = e^{\mu\delta}. \tag{A.6}$$

By setting the determinant (A.5) equal to zero, we obtain, after some calculation,

$$Y^2 e^{-2s\frac{V\ell_1}{V^2-a_1^2}} - 2Y[c_1c_2 + \sigma s_1 s_2] + e^{2s\frac{V\ell_2}{V^2-a_2^2}} = 0\,. \tag{A.7}$$

Here we introduced notation

$$\begin{aligned} c_1 &= \frac{1}{2}\left(e^{-\frac{s\ell_1}{V-a_1}} + e^{-\frac{s\ell_1}{V+a_1}}\right), \\ c_2 &= \frac{1}{2}\left(e^{\frac{s\ell_1}{V-a_2}} + e^{\frac{s\ell_2}{V+a_2}}\right), \\ s_1 &= \frac{1}{2}\left(e^{-s\frac{\ell_1}{V-a_1}} - e^{-s\frac{\ell_1}{V+a_1}}\right), \\ s_2 &= \frac{1}{2}\left(e^{s\frac{\ell_2}{V+a_2}} - e^{s\frac{\ell_2}{V-a_2}}\right); \end{aligned}$$

parameter σ is defined by (2.49).

We now check by direct inspection that

$$e^{2s\frac{V\ell_1}{V^2-a_1^2}}[c_1c_2 + \sigma s_1 s_2] = e^{V\left(\frac{\theta_1}{a_1}+\frac{\theta_2}{a_2}\right)}(\cosh\theta_1\cosh\theta_2 + \sigma\sinh\theta_1\sinh\theta_2),$$

and

$$e^{2sV\left(\frac{\ell_1}{V^2-a_1^2}+\frac{\ell_2}{V^2-a_2^2}\right)} = e^{2V\left(\frac{\theta_1}{a_1}+\frac{\theta_2}{a_2}\right)},$$

with symbols ϑ_1, ϑ_2 defined by (2.49). Equation (A.7) now takes on the form

$$Y^2-2Ye^{V\left(\frac{\vartheta_1}{a_1}+\frac{\vartheta_2}{a_2}\right)}(\cosh\vartheta_1\cosh\vartheta_2+\sigma\sinh\vartheta_1\sinh\vartheta_2)+e^{2V\left(\frac{\vartheta_1}{a_1}+\frac{\vartheta_2}{a_2}\right)} = 0. \quad \text{(A.8)}$$

We look for the roots of this equation presented as

$$Y_{1,2} = e^{V\left(\frac{\vartheta_1}{a_1}+\frac{\vartheta_2}{a_2}\right)\pm\chi}. \quad \text{(A.9)}$$

The sum of the roots equals

$$2e^{V\left(\frac{\vartheta_1}{a_1}+\frac{\vartheta_2}{a_2}\right)}\cosh\chi;$$

this becomes consistent with (A.8) if parameter χ is defined by the equation

$$\cosh\chi = \cosh\vartheta_1\cosh\vartheta_2 + \sigma\sinh\vartheta_1\sinh\vartheta_2,$$

introduced in (2.49). By (A.6) and (A.9) we conclude that the characteristic exponents $\mu_{1,2}$ are specified as

$$\mu_{1,2}\delta = V\left(\frac{\vartheta_1}{a_1}+\frac{\vartheta_2}{a_2}\right) \pm \chi. \quad \text{(A.10)}$$

By using (2.49), we, after some calculation, rewrite equation (2.50) as

$$\chi = s\delta\frac{a_1a_2}{\Delta_1\Delta_2}\sqrt{(V^2\tilde{\rho}-\tilde{k})\left(V^2\left(\tilde{\frac{1}{k}}\right)-\left(\tilde{\frac{1}{\rho}}\right)\right)}; \quad \text{(A.11)}$$

here Δ_i is defined by (2.10) with a replaced by a_i

With reference to (2.49) and (A.11), we rewrite (A.10) as

$$\mu_{1,2}\delta = \frac{s\delta}{\Delta_1\Delta_2}\left[V(V^2-\widetilde{a^2}) \pm a_1a_2\sqrt{(V^2\tilde{\rho}-\tilde{k})\left(V^2\left(\tilde{\frac{1}{k}}\right)-\left(\tilde{\frac{1}{\rho}}\right)\right)}\right]. \quad \text{(A.12)}$$

The system (2.51) for E, G, H now follows from the formulae (2.39) for P, Q along with equations (2.47) and (A.4). As to eqn. (2.52) for $v_{1,2} = V - \frac{s}{\mu_{1,2}}$, it follows from (A.12) after some algebraic work.

Appendix B

Comment on Equations (3.47)

If a plane electromagnetic wave travels along the z-axis, then its electromagnetic field is characterized by the electromagnetic tensors F and f specified by (3.21). The material tensor s participating in the constitutive relation (3.33) is given for an immovable material by the formula

$$s = -\frac{1}{\mu c} a_{13} a_{13} - \epsilon c a_{14} a_{14}.$$

If the dielectric is brought into motion with a uniform speed v along the x_3-axis, then the relevant expression for s becomes

$$s = -\frac{1}{\mu c} a'_{13} a'_{13} - \epsilon c a'_{14} a'_{14}, \tag{B.1}$$

with the "primed" tensors a'_{13}, a'_{14} given by (c.f. (3.22))

$$a'_{13} = a_{13}\cosh\phi + i a_{14}\sinh\phi, \quad a'_{14} = a_{14}\cosh\phi - i a_{13}\sinh\phi,$$

and the angle ϕ defined by $\tanh\phi = v/c$. By referring to (3.21), (3.6), and (B.1), we reduce the material relation (3.33) to the system of two equations

$$Q u_{x_3} + i T u_{x_4} = i v_{x_4}, \quad -T u_{x_3} - i R u_{x_4} = v_{x_3}, \tag{B.2}$$

with parameters Q, T, R defined by (3.48). Consider now two dielectric media moving with different speeds v_1 and v_2 along the x_3-axis, and let these media be separated by a point moving with velocity $V < c$ along the same axis. This point of separation will trace the line L with the slope ψ, $\tanh\psi = V/c$, in the (z, t)-plane.

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The derivative u_τ of u along this line equals

$$u_\tau = iu_{x_3}\tanh\psi - u_{x_4}; \tag{B.3}$$

this derivative should be continuous across L along with a similar derivative of v. Bearing this in mind, we eliminate u_{x_4}, v_{x_4} from (B.2), and arrive at the system

$$u_{x_3} = iu_\tau \frac{R\tanh\psi - T}{W} + iv_\tau \frac{1}{W},$$

$$v_{x_3} = iu_\tau \frac{T^2 - QR}{W} + iv_\tau \frac{R\tanh\psi - T}{W},$$

with W defined by (3.48).

We now take average values of both sides of either equation bearing in mind the continuity of u_τ, v_τ. Returning to notation (B.3), we arrive, after some calculation, at the system

$$\begin{aligned} \alpha c u_z + \beta u_t &= V v_z + v_t, \\ V u_z + u_t &= \theta(\alpha c v_z + \beta v_t), \end{aligned} \tag{B.4}$$

with α, β, θ defined by (3.47). A simple algebra reduces (B.4) to a standard form (2.13).

Appendix C

A Mechanical Implementation of a Discontinuous Velocity Pattern Along an Elastic Bar

A discontinuous velocity distribution along the bar may be produced through the following arrangements suggested by B. P. Lavrov (B.P. Lavrov, private communication, 2003).

First Version

Consider a thin elastic band stretched by a tensile force. With respect to longitudinal vibrations, the band performs as an elastic bar, with material displacements occurring about the static equilibrium.

The band is split into many independent sections, each section fabricated as a closed loop mounted on four supporting rolls (see Figure C.1). One of the rolls serves as a carrier bringing the whole loop into motion, another bridle roll maintains the tension of the band. The upper rolls are suspended to the ceiling by the rods connected through hinges, so the entire section, being rectangular in statics, preserves the freedom of horizontal motion. Through such a motion, it becomes distorted and takes the shape of a parallelogram shown in Figure C.2.

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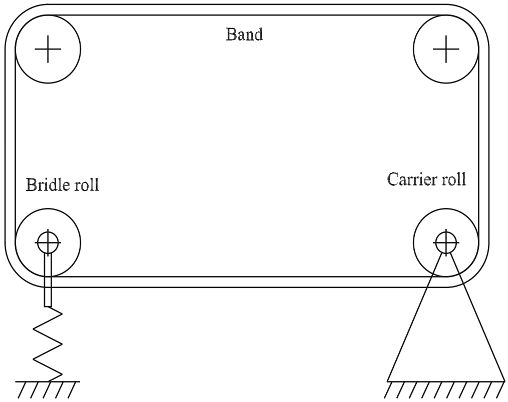

Fig. C.1. A section of the elastic bar

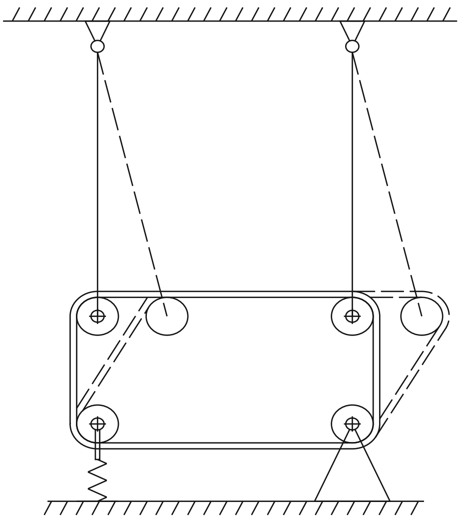

Fig. C.2. A suspended section of the bar

Two rolls out of four on each section have the axes common with rolls belonging to the adjacent sections (such rolls occupy the upper row in Figure C.3). All of the rolls rotate freely, without friction, about their axes. The neighboring sections occupy alternating positions along the common rotation axes, so their respective bands come onto the rolls as shown in Figure C.3. The lower rolls in the alternating sections are placed at different horizontal levels to secure the access necessary for mounting the independent carrier and bridle rolls in order to maintain the required velocity and tension of the band.

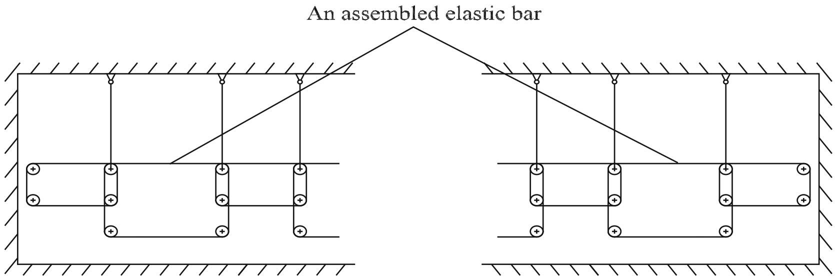

Fig. C.3. An elastic bar as an assembly of sections

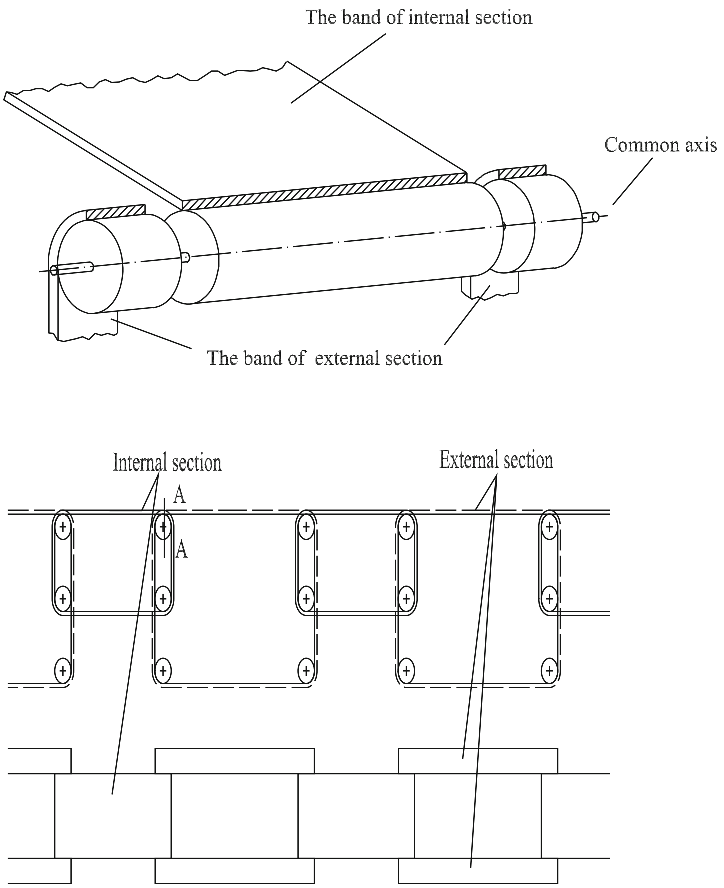

Fig. C.4. Rolls from two adjacent sections mounted on the common axis

The ultimate arrangement of an elastic bar is shown in Figure C.4. It is combined of the top horizontal parts of each section. The extreme left and right rolls of the arrangement are either attached to the walls or connected to the devices that generate longitudinal vibrations or pulses.

The velocity of the band in each section may be independently sustained, both in magnitude and direction, by the use of the relevant single drive. The tension is, however, common to all sections; it is maintained by a tensile force generated by bridle rolls. To secure a reliable performance of a build up, the stress in the band should not exceed the yield force of the material.

Second Version

A bar is imitated by a gas (air) column. A segment of a pipeline is assembled of sections separated from each other by toroidal chambers (see Figure C.5). By manipulating compressions and rarefactions in the chambers, it is possible to produce, within each section, the velocity pattern variable in magnitude and direction.

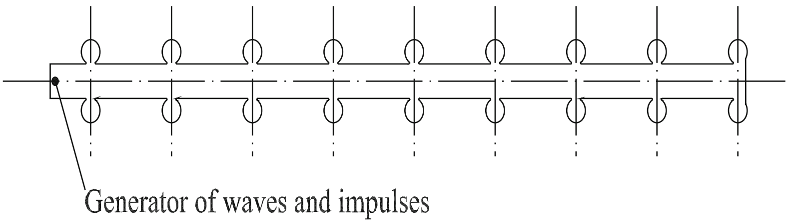

Fig. C.5. A pipeline assembled of sections

In particular, one may generate a standing wave and register variations of its frequency caused by the variable velocity distribution. The pressure may be adjusted individually for each chamber, by virtue of reducing valves. The air should be able to leave some of the chambers, also through such valves.

A base pressure level in a system may be maintained by a common compressor; control of the pressure in various chambers may be carried out through individual reduction gears. The velocity of sound is affected by pressure, and may be accordingly controlled by pressure variations. There must be a way for the air to leave the system, also through the reduction gears.

Appendix D

Comment on Equations (6.139), (6.140)

We reproduce below a standard homogenization procedure for the wave equation

$$\text{div}(\mu^{-1}\text{grad}u) - (\epsilon u_t)_t = 0 \tag{D.1}$$

in two spatial variables x, z, with the pattern of (ϵ, μ) defined as an activated laminate depending on the fast variable $(\xi x + \eta z - Vt)/\delta$, with period 1.

The asymptotics of solution of (D.1) is sought for in the form

$$u(x,z,t) = u_0\left(x,z,t,\frac{\xi x + \eta z - Vt}{\delta}\right) + \delta u_1\left(x,z,t,\frac{\xi x + \eta z - Vt}{\delta}\right) + \delta^2 u_2\left(x,z,t,\frac{\xi x + \eta z - Vt}{\delta}\right). \tag{D.2}$$

where $u_i(x,z,t,\zeta)$ is 1-periodic function of ζ, i.e., $u_i(x,z,t,\zeta+1) = u_i(x,z,t,\zeta)$, and $\xi = \cos\psi, \eta = \sin\psi$. By substituting (D.2) into (A.1) we obtain

$$\begin{aligned}
&- \delta^{-2}(\xi^2 L_{\zeta\zeta}u_0 + \eta^2 L_{\zeta\zeta}u_0 - V^2 M_{\zeta\zeta}u_0)\\
&- \delta^{-1}(\xi L_{x\zeta}u_0 + \eta L_{z\zeta}u_0 + \xi L_{\zeta x}u_0 + \eta L_{\zeta z}u_0 + \xi^2 L_{\zeta\zeta}u_1\eta^2 L_{\zeta\zeta}u_1 + VM_{\zeta t}u_0 +\\
&+ VM_{t\zeta}u_0 - V^2 M_{\zeta\zeta}u_1) - \delta^0(L_{xx}u_0 + L_{zz}u_0 + \xi L_{x\zeta}u_1 + \eta L_{z\zeta}u_1\\
&+ \xi L_{\zeta x}u_1 + \eta L_{\zeta z}u_1 + \xi^2 L_{\zeta\zeta}u_2 + \eta^2 L_{\zeta\zeta}u_2 - M_{tt}u_0 + VM_{t\zeta}u_1 + VM_{\zeta t}u_1\\
&+ VM_{\zeta t}u_1 - V^2 M_{\zeta\zeta}u_2) + \delta r(x,z,t,\delta) = 0.
\end{aligned} \tag{D.3}$$

Here

$$L_{\alpha\beta}u_i(x,z,t,\zeta) = \frac{\partial}{\partial\alpha}\left(\mu^{-1}(\zeta)\frac{\partial}{\partial\beta}u_i(x,z,t,\zeta)\right),$$

$$M_{\alpha\beta}u_i(x,z,t,\zeta) = \frac{\partial}{\partial\alpha}\left(\epsilon(\zeta)\frac{\partial}{\partial\beta}u_i(x,z,t,\zeta)\right),$$

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$$r(x,z,t,\delta) = r_0(x,z,t,\delta) + \delta r_1(x,z,t,\delta),$$

$$r_0(x,z,t,\delta) = -L_{xx}u_1 - L_{zz}u_1 - \xi L_{x\zeta}u_2 - \eta L_{z\zeta}u_2 - \xi L_{\zeta x}u_2 \\ - \eta L_{\zeta z}u_2 + M_{tt}u_2 - VM_{\zeta t}u_2,$$

$$r_1(x,z,t,\delta) = -L_{xx}u_2 - L_{zz}u_2 + M_{tt}u_0.$$

Let us require that the terms of orders $\delta^{-2}, \delta^{-1}, \delta^0$ vanish. Then

$$\xi^2 L_{\zeta\zeta}u_0 + \eta^2 L_{\zeta\zeta}u_0 - V^2 M_{\zeta\zeta}u_0 = 0, \tag{D.4}$$

$$\xi L_{x\zeta}u_0 + \eta L_{z\zeta}u_0 + \xi L_{\zeta x}u_0 + \eta L_{\zeta z}u_0 + \xi^2 L_{\zeta\zeta}u_1 + \eta^2 L_{\zeta\zeta}u_1 \\ + VM_{\zeta t}u_0 + VM_{t\zeta}u_0 - V^2 M_{\zeta\zeta}u_1 = 0, \tag{D.5}$$

$$L_{xx}u_0 + L_{zz}u_0 + \xi L_{x\zeta}u_1 + \eta L_{z\zeta}u_1 + \xi L_{\zeta x}u_1 + \eta L_{\zeta z}u_1 + \xi^2 L_{\zeta\zeta}u_2 \\ + \eta^2 L_{\zeta\zeta}u_2 - M_{tt}u_0 + VM_{t\zeta}u_1 + VM_{\zeta t}u_1 - V^2 M_{\zeta\zeta}u_2 = 0 \tag{D.6}$$

It follows from (D.4) that $\mu^{-1}(\zeta)\partial u_0(x,z,t,\zeta)/\partial\zeta - V^2\epsilon(\zeta)\partial u_0(x,z,t,\zeta)/\partial\zeta$ is independent of ζ, i.e., $\mu^{-1}(\zeta)\partial u_0/\partial\zeta - V^2\epsilon(\zeta)\partial u_0/\partial\zeta = C(x,z,t)$; therefore,

$$\frac{\partial u_0(x,z,t,\zeta)}{\partial\zeta} = \frac{C(x,z,t)}{\mu^{-1}(\zeta) - V^2\epsilon(\zeta)}. \tag{D.7}$$

We adopt the following notation for the *mean over the period* in both the one-dimensional and many-dimensional case,

$$\langle f(x_1,\ldots,x_s,t,\zeta_1,\ldots,\zeta_s)\rangle = \int_0^1 \ldots \int_0^1 f(x_1,\ldots,x_s,t,\zeta_1,\ldots,\zeta_s)d\zeta_1\ldots d\zeta_s,$$

with the variables x and ζ considered independent in the last integral.

By applying the operator $\langle\cdot\rangle$ to the equality (D.7), we see from the periodicity of $u_0(x,z,\zeta)$ in ζ that

$$\left\langle \frac{\partial u_0(x,z,t,\zeta)}{\partial\zeta} \right\rangle = \int_0^1 \frac{\partial u_0(x,z,t,\zeta)}{\partial\zeta} d\zeta = 0.$$

Thus $0 = C(x,z,t)\langle(\mu^{-1}(\zeta) - V^2\epsilon(\zeta))^{-1}\rangle$, and consequently, $C(x,z,t) = 0, \partial u_0/\partial\zeta = 0$, and $u_0(x,z,t,\zeta)$ is independent of ζ, i.e.,

$$u_0(x,z,t,\zeta) = u_0(x,z,t). \tag{D.8}$$

Referring to (D.8), we rewrite (D.5) as

$$\frac{\partial}{\partial\zeta}\left(\xi\mu^{-1}\frac{\partial u_0}{\partial x} + \eta\mu^{-1}\frac{\partial u_0}{\partial z} + \mu^{-1}\frac{\partial u_1}{\partial\zeta} + V\epsilon\frac{\partial u_0}{\partial t} - V^2\epsilon\frac{\partial u_1}{\partial\zeta}\right) = 0.$$

This implies that

$$\xi\mu^{-1}\frac{\partial u_0}{\partial x} + \eta\mu^{-1}\frac{\partial u_0}{\partial z} + \mu^{-1}\frac{\partial u_1}{\partial \zeta} + V\epsilon\frac{\partial u_0}{\partial t} - V^2\epsilon\frac{\partial u_1}{\partial \zeta} = C_1(x,z,t),$$

$$\begin{aligned}\frac{\partial u_1}{\partial \zeta} &= C_1\frac{1}{\mu^{-1}(\zeta) - V^2\epsilon(\zeta)} - \xi\frac{\mu^{-1}(\zeta)}{\mu^{-1}(\zeta) - V^2\epsilon(\zeta)}\frac{\partial u_0}{\partial x}\\ &\quad - \eta\frac{\mu^{-1}(\zeta)}{\mu^{-1}(\zeta) - V^2\epsilon(\zeta)}\frac{\partial u_0}{\partial z} - V\frac{\epsilon(\zeta)}{\mu^{-1}(\zeta) - V^2\epsilon(\zeta)}\frac{\partial u_0}{\partial t}.\end{aligned} \tag{D.9}$$

By applying the operator $\langle\cdot\rangle$, we get

$$\left\langle \frac{\partial u_1}{\partial \zeta}\right\rangle = -C_1C + \xi A\frac{\partial u_0}{\partial x} + \eta A\frac{\partial u_0}{\partial z} + VB\frac{\partial u_0}{\partial t},$$

where A, B, C are given by (2.12) and (2.15), with a standard substitution (3.8). Hence,

$$C_1(x,z) = \xi\frac{A}{C}u_{0_x} + \eta\frac{A}{C}u_{0_z} + V\frac{B}{C}u_{0_t},$$

and

$$\begin{aligned}\frac{\partial u_1}{\partial \zeta} &= u_{0_x}\frac{\xi}{\mu^{-1} - V^2\epsilon}\left(\frac{A}{C} - \mu^{-1}\right) + u_{0_z}\frac{\eta}{\mu^{-1} - V^2\epsilon}\left(\frac{A}{C} - \mu^{-1}\right)\\ &\quad + u_{0_t}\frac{v}{\mu^{-1} - V^2\epsilon}\left(\frac{B}{C} - \epsilon\right).\end{aligned}$$

Taking into account this expression for $\frac{\partial u_1}{\partial \zeta}$, we integrate (D.6) with respect to ζ over $[0,1]$ and use the periodicity of $u_1(\zeta), \mu^{-1}(\zeta)$ and $\epsilon(\zeta)$. This yields

$$\begin{aligned}&- u_{0_{tt}}\left(D - V^2\frac{B^2}{C}\right) + u_{0_{xx}}\left(\eta^2E - V^2D + \xi^2\frac{A^2}{C}\right)\\ &+ u_{0_{zz}}\left(\xi^2E - V^2D + \eta^2\frac{A^2}{C}\right)\\ &+ u_{0_{xz}}2\xi\eta\left(\frac{A^2}{C} - E\right) + u_{0_{xt}}2\xi V\left(\frac{AB}{C} - D\right)\\ &+ u_{0_{zt}}2\eta V\left(\frac{AB}{C} - D\right) = 0,\end{aligned} \tag{D.10}$$

where D is defined by (2.12) and (3.8), and E specified by

$$E = \left\langle \frac{1}{\mu}\frac{a^2}{V^2 - a^2}\right\rangle.$$

Equation (D.10) represents the required averaged equation. When $\xi = 0, \eta = 1$, it reduces to (6.140).

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