Appendix A
Background

In this appendix, we provide some background necessary for the material presented in Part III of the book. Specifically, in Sects. A.1 and A.2 we introduce polytopes and polyhedral operations. In Sect. A.3, we discuss the computation of images and pre-images of polytopes through affine functions, under the assumption that the map is non-singular. We relax this assumption and generalize polytopes to semi-linear sets in Sect. A.4. An overview of discrete-time Lyapunov stability and polyhedral Lyapunov functions, which are used in Chap. 10, is presented in Sect. A.5. The details of the vertex interpolation and contractive sets methods used in Chap. 11 to design polytope-to-polytope controllers are shown in Sect. A.6. Finally, candidate control potential functions, which are defined and used in Chap. 12, are presented in Sect. A.7.

A.1 Polytopes

Definition A.1 (*Convex Set*) A set $C \subset \mathbb{R}^N$ is *convex* if the line segment between any two points in C lies in C. In other words, for all $x_1, x_2 \in C$ and $0 \leq \lambda \leq 1$, we have $\lambda x_1 + (1-\lambda)x_2 \in C$.

Definition A.2 (*Convex Combination*) A point $x = \sum_{i=1}^{n} \lambda_i x_i$, where $\sum_{i=1}^{n} \lambda_i = 1$ and $\lambda_i \geq 0$ for all $i = 1, \ldots, n$ is a *convex combination* of points $x_1, \ldots, x_n$.

Similarly, a point $x = \sum_{i=1}^{n} \lambda_i x_i$, where $\sum_{i=1}^{n} \lambda_i = 1$ is an *affine combination* of points $x_1, \ldots, x_n$. Points $x_1, \ldots, x_n$ are called *affinely independent* if there does not exist an $1 \leq i \leq n$ such that point x_i is an affine combination of points $x_1, \ldots, x_{i-1}, x_{i+1}, \ldots x_n$.

Definition A.3 (*Convex Hull*) The *convex hull* of a set C, denoted as $hull(C)$, is the set of all convex combinations from points in C:

$$hull(C) = \{x \in \mathbb{R}^N \mid x = \sum_{i=1}^{n} \lambda_i x_i,\ \lambda_i \geq 0,\ x_i \in C,\ i = 1, \ldots, n,\ \sum_{i=1}^{n} \lambda_i = 1\}.$$

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The *convex hull* of a set C is the smallest convex set containing C. A *hyperplane* is a set of the form

$$\{x \in \mathbb{R}^N \mid h^\top x = k,\ h \in \mathbb{R}^N,\ h \neq 0,\ k \in \mathbb{R}\}.$$

A hyperplane divides $\mathbb{R}^N$ into two *half-spaces*.

Definition A.4 (*Half-space*) A closed half-space is a set of the form

$$\{x \in \mathbb{R}^N \mid h^\top x \leq k,\ h \in \mathbb{R}^N,\ h \neq 0,\ k \in \mathbb{R}\}.$$

A *supporting hyperplane* of a set C is a hyperplane $\{h^\top x = k\}$, such that C is entirely contained in one of the two closed half-spaces defined by the hyperplane and C has at least one point on the hyperplane.

Definition A.5 (*Polytope*) A closed full dimensional *polytope* $\mathbf{X} \subset \mathbb{R}^N$ is defined as the convex hull of at least $N + 1$ affinely independent points in $\mathbb{R}^N$.

The set of points $v_1, \ldots, v_n \in \mathbb{R}^N$ whose convex hull gives $\mathbf{X}$ and with the property that for all $i = 1, \ldots, n$, point v_i is not contained in the convex hull of $v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n$ is called the set of *vertices* of $\mathbf{X}$ and is denoted by $V(\mathbf{X})$. A polytope is completely described by its set of vertices:

$$\mathbf{X} = hull(V(\mathbf{X})). \tag{A.1}$$

Alternatively, a polytope $\mathbf{X}$ can be described as the intersection of at least $N + 1$ closed half spaces. In other words, there exists a $n \geq N + 1$ and $h_i \in \mathbb{R}^N$, $k_i \in \mathbb{R}$, $i = 1, \ldots, n$ such that

$$\mathbf{X} = \bigcap_{i=1}^{n} \{x \in \mathbb{R}^N \mid h_i^\top x \leq k_i\}. \tag{A.2}$$

Equivalently, by constructing a matrix $H \in \mathbb{R}^{n \times N}$, where $H = [h_1^\top; \ldots; h_n^\top]$ and vector $K \in \mathbb{R}^n$, where $K = [k_1; \ldots; k_n]$: [1]:

$$\mathbf{X} = \{x \in \mathbb{R}^N \mid Hx \leq K\}, \tag{A.3}$$

where the comparison "≤" is interpreted element-wise.

Forms (A.1) and (A.2) (or equivalently (A.3)) are referred to as the V- and H-representations of a polytope, respectively. Given a polytope, there exist algorithms for translation between its V- and H-representations [113, 134]. A *facet* of a polytope $\mathbf{X}$ is the intersection of $\mathbf{X}$ with one of its supporting hyperplanes. A polytope without its facets is called an *open polytope* and we use $int(\mathbf{X})$ to denote $\mathbf{X}$ without its facets (i.e., the interior of $\mathbf{X}$). Given an open polytope $\mathbf{X}$, we use $cl(\mathbf{X})$ to denote its closure (i.e., the union of $\mathbf{X}$ and its facets).

[1] We use standard Matlab notation for constructing concatenations of matrices.

A.2 Operations on Polytopes

Given polytopes $\mathbf{X}_1, \mathbf{X}_2 \subset \mathbb{R}^N$, we define the following operations:

Definition A.6 (*Set Difference*) The *set difference* of $\mathbf{X}_1$ and $\mathbf{X}_2$ is defined as:

$$\mathbf{X}_1 \backslash \mathbf{X}_2 = \{x \in \mathbb{R}^N \mid x \in \mathbf{X}_1, x \notin \mathbf{X}_2\}.$$

Note that convex polytopes are not closed under the set difference operation (i.e., $\mathbf{X}_1 \backslash \mathbf{X}_2$ is not necessarily convex, even if $\mathbf{X}_1$ and $\mathbf{X}_2$ are).

Definition A.7 (*Minkowski Sum*) The *Minkowski sum* of $\mathbf{X}_1$ and $\mathbf{X}_2$ is defined as:

$$\mathbf{X}_1 \oplus \mathbf{X}_2 = \{x_1 + x_2 \in \mathbb{R}^N \mid x_1 \in \mathbf{X}_1, x_2 \in \mathbf{X}_2\}.$$

Definition A.8 (*Minkowski Difference*) The *Minkowski difference* of $\mathbf{X}_1$ and $\mathbf{X}_2$ is defined as:

$$\mathbf{X}_1 \ominus \mathbf{X}_2 = \{x_1 - x_2 \in \mathbb{R}^N \mid x_1 \in \mathbf{X}_1, x_2 \in \mathbf{X}_2\}.$$

The Minkowski difference $\mathbf{X}_1 \ominus \mathbf{X}_2$ can also be computed as the Minkowski sum $\mathbf{X}_1 \oplus (-\mathbf{X}_2)$, where $(-\mathbf{X}_2) = \{x \in \mathbb{R}^N \mid -x \in \mathbf{X}_2\}$ is the mirror image of $\mathbf{X}_2$ around the origin. Note that our definition of Minkowski difference follows [118] and is different from the Pontryagin (Minkowski) difference from [78, 112].

Definition A.9 (*Chebyshev Ball*) The *Chebyshev ball* of a polytope $\mathbf{X} \subset \mathbb{R}^N$ is the largest radius ball $B_r(x_c) = \{x \in \mathbb{R}^N \mid ||x - x_c||_2 \leq r\}$ such that $B_r(x_c) \subset \mathbf{X}$. We use $c(\mathbf{X})$ to denote the center and $r(\mathbf{X})$ to denote the radius of the Chebyshev ball of $\mathbf{X}$.

A.3 Affine Functions on Polytopes

Definition A.10 (*Affine function*) A function $f : \mathbb{R}^N \to \mathbb{R}^M$ is called affine if it can be written as $f(x) = Ax + b$, $A \in \mathbb{R}^{M\times N}$, $b \in \mathbb{R}^M$, for all $x \in \mathbb{R}^N$.

If $\mathbf{X}$ is a full dimensional polytope in $\mathbb{R}^N$ with set of vertices $V(\mathbf{X}) = \{v_1, \ldots, v_n\}$ and $f : \mathbb{R}^N \to \mathbb{R}^M$ is an affine function, then

$$f(\mathbf{X}) = hull\{f(v_1), \ldots, f(v_n)\}, \tag{A.4}$$

i.e., the image of a polytope through an affine function is the convex hull of the vertex images through the affine function.

In the particular case $N = M$, if matrix A is nonsingular, then the vertices, facets, and interior of the polytope map through the affine transformation to the vertices, facets, and interior of the image of the polytope, respectively. Therefore

$$f(cl(\mathbf{X})) = cl(f(\mathbf{X})). \tag{A.5}$$

The pre-image of a polytope $\mathbf{X}$ in $\mathbb{R}^M$ through an affine function $f : \mathbb{R}^N \to \mathbb{R}^M$ is a convex set in $\mathbb{R}^N$ and is defined as

$$f^{-1}(\mathbf{X}) = \{x \in \mathbb{R}^N \mid Ax + b \in \mathbf{X}\}. \tag{A.6}$$

For a polytope $\mathbf{X}$ in $\mathbb{R}^M$ represented as the intersection of $m \geq M$ half-spaces as in Eq. (A.2), the pre-image is given by

$$f^{-1}(\mathbf{X}) = \bigcap_{i=1}^{m} \{x \in \mathbb{R}^N \mid h_i^\top Ax \leq k_i - h_i^\top b\}. \tag{A.7}$$

Note that if there are $N + 1$ linearly independent vectors in the set $\{h_1^\top A, \ldots, h_m^\top A\}$, then $f^{-1}(\mathbf{X})$ is a polytope in $\mathbb{R}^N$.

Consider a function $g : \mathbb{R}^N \times \mathbb{R}^L \to \mathbb{R}^M$ defined as $g(x, u) = Ax + Bu + b$, where $A \in \mathbb{R}^{M\times N}$, $B \in \mathbb{R}^{M\times L}$, $b \in \mathbb{R}^M$. The function g can be represented as summation of two affine functions $g_1 : \mathbb{R}^N \to \mathbb{R}^M$ and $g_2 : \mathbb{R}^L \to \mathbb{R}^M$ such that $g_1(x) = Ax + b$ and $g_2(u) = Bu$, i.e., $g(x, u) = g_1(x) + g_2(u)$ for all $x \in \mathbb{R}^N$, $u \in \mathbb{R}^L$. Then, the image of polytopes $\mathbf{X} \subset \mathbb{R}^N$ and $\mathbf{U} \subset \mathbb{R}^L$ through function g can be computed as

$$g(\mathbf{X}, \mathbf{U}) = g_1(\mathbf{X}) \oplus g_2(\mathbf{U}),$$

where $\oplus$ denoted Minkowski summation. Also note that function $g : \mathbb{R}^N \times \mathbb{R}^L \to \mathbb{R}^M$ can equivalently be represented as an affine function $f : \mathbb{R}^{N+L} \to \mathbb{R}^M$ as given in Definition A.10:

$$f(x) = \bar{A}\bar{x} + b, \text{ where } \bar{x} \in \mathbb{R}^{N+L} \text{ and } \bar{A} = [A, B] \in \mathbb{R}^{M\times(N+L)}. \tag{A.8}$$

Then, $g(\mathbf{X}, \mathbf{U}) = f(\mathbf{X} \times \mathbf{U})$. Moreover, the pre-image of the function g over $\mathbb{R}^N$ can be computed through orthogonal projection of $f^{-1}(\mathbf{X} \times \mathbf{U})$ to $\mathbb{R}^N$.

A.4 Semi-linear Sets and Affine Functions

Definition A.11 (*Semi-linear set*) A *semi-linear set* $\mathbf{S}$ in $\mathbb{R}^N$ is defined as unions, intersections, and complements of sets $\{x \in \mathbb{R}^N \mid a^\top x \sim b, \sim \in \{=, <\}\}$, for some $a \in \mathbb{R}^N$ and $b \in \mathbb{R}$.

A semi-linear set is also called a *polyhedron*. A closed convex bounded semi-linear set is a polytope, and a convex bounded semi-linear set is a polytope with some of its facets removed.

Note that the basic set operation definitions, such as Minkowski sum and difference, and set difference defined above, also apply to semi-linear sets. Furthermore, the image (or pre-image) of a semi-linear set through an affine function can be computed by performing the computation on a finite number of polytopes since a semi-linear set can be represented as a union of a finite number of convex sets. In particular, a semi-linear set is either a polytope, or a union of polytopes, or a convex and bounded set, or a general non-convex set. Therefore, the computation of the image (or pre-image) of a semi-linear set $\mathbf{S}$ through an affine function f falls into one of the following case:

i. If $\mathbf{S}$ is a closed polytope, then $f(\mathbf{S})$ and $f^{-1}(\mathbf{S})$ are computed as explained in Sect. A.3. Note that the computation applies to a polytope of any dimension.
ii. If $\mathbf{S}$ is a union of polytopes, one can use a standard convex decomposition method to decompose $\mathbf{S}$ into a set of polytopes $\{\mathbf{P}_i\}_{i\in I}$ (see, e.g., [79]), i.e.,

$$\mathbf{S} = \bigcup_{i\in I} \mathbf{P}_i, \text{ where for any } i \neq j, \mathbf{P}_i \cap \mathbf{P}_j = \emptyset,$$

and compute the image as

$$f(\mathbf{S}) = \bigcup_{i\in I} f(\mathbf{P}_i)$$

using case Sect. A.4 (similarly $f^{-1}(\mathbf{S}) = \bigcup_{i\in I} f^{-1}(\mathbf{P}_i)$).
iii. If $\mathbf{S}$ is a convex and bounded semi-linear set, then, in general,

$$\mathbf{S} = \mathbf{P} \setminus \bigcup_{i\in I} \mathbf{F}_i,$$

for some polytope $\mathbf{P}$ and a subset of its facets $\{\mathbf{F}_i\}_{i\in I}$. The image is computed as

$$f(\mathbf{S}) = f(\mathbf{P}) \setminus \left(\bigcup_{i\in I} f(\mathbf{F}_i) \setminus f(\mathbf{P}) \right),$$

where the right hand side can be computed as described in case Sect. A.4. Similarly, the pre-image can be computed as

$$f^{-1}(\mathbf{S}) = f^{-1}(\mathbf{P}) \setminus \left(\bigcup_{i\in I} f^{-1}(\mathbf{F}_i) \setminus f^{-1}(\mathbf{P}) \right).$$

iv. If $\mathbf{S}$ is a general (non-convex) bounded semi-linear set, then again it can be decomposed into convex and bounded semi-linear sets $\mathbf{S} = \bigcup_{i\in I} \mathbf{S}_i$. Then,

$$f(\mathbf{S}) = \bigcup_{i\in I} f(\mathbf{S}_i),$$

and each $f(\mathbf{S})$ can be computed as described in case Sect. A.4.

As summarized above, the image and the pre-image of a semi-linear set through an affine function can always be implemented by convex decompositions and repeated applications of Eqs. (A.4) and (A.7), respectively.

A.5 Lyapunov Theory

An autonomous discrete-time system is defined by

$$x(k+1) = \Phi(x(k)), \quad k \in \mathbb{N}, \tag{A.9}$$

where $x(k) \in \mathbb{R}^N$ is the state at time k and $\Phi : \mathbb{R}^N \to \mathbb{R}^N$ is an arbitrary map with $\Phi(0) = 0$. Given a state $x \in \mathbb{R}^N$, $x' := \Phi(x)$ is called a *successor* state of x.

Definition A.12 (*Contractive set and positively invariant set*) Let $\lambda \in [0, 1]$. A set $\mathbf{P} \subseteq \mathbb{R}^N$ is called λ-*contractive* (shortly, contractive) for system (A.9) if for all $x \in \mathbf{P}$ it holds that $\Phi(x) \in \lambda\mathbf{P}$. For $\lambda = 1$, $\mathbf{P}$ is called a *positively invariant* set.

Definition A.13 (*Class $\mathcal{K}_\infty$ function*) A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ belongs to class $\mathcal{K}_\infty$ if it is continuous, strictly increasing, $\alpha(0) = 0$, and $\lim_{s\to\infty} \alpha(s) = \infty$.

Theorem A.1 *Let $\mathbf{P}$ be a positively invariant set for (A.9) with $0 \in int(\mathbf{P})$. Furthermore, let $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$, $\rho \in (0, 1)$ and $V : \mathbb{R}^N \mapsto \mathbb{R}_+$ such that:*

$$\alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|), \forall x \in \mathbf{P}, \tag{A.10}$$

$$V(\Phi(x)) \leq \rho V(x), \forall x \in \mathbf{P}. \tag{A.11}$$

Then (the origin of) system (A.9) is asymptotically stable in $\mathbf{P}$ [93, 119].

Definition A.14 (*Lyapunov function*) A function $V : \mathbb{R}^N \mapsto \mathbb{R}_+$ is called a *Lyapunov function* (LF) in $\mathbf{P}$ if it satisfies (A.10) and (A.11). If $\mathbf{P} = \mathbb{R}^N$, then V is called a *global Lyapunov function*.

The parameter ρ is called the *contraction rate* of V. For any $\Gamma > 0$,

$$\mathbf{P}_\Gamma := \{x \in \mathbb{R}^N \mid V(x) \leq \Gamma\}$$

is called a *sublevel set* of V.

A difference inclusion system is defined as

$$x(k+1) \in \Phi(x(k)), \quad k \in \mathbb{N}, \tag{A.12}$$

where $x(k) \in \mathbb{R}^N$ is the state at time k and $\Phi : \mathbb{R}^N \to 2^{\mathbb{R}^N}$ is an arbitrary map with $\Phi(0) = 0$. Note that the system is non-deterministic, and the successor of state x takes values from a set $\Phi(x)$.

The invariant set, contractive set, and Lyapunov function definitions apply directly to difference inclusions in the absolute sense, i.e., given x, the corresponding conditions must hold for all $x' \in \Phi(x)$. For example, a set $\mathbf{P}$ is positively invariant if $\Phi(x) \subseteq \mathbf{P}$ for all $x \in \mathbf{P}$.

An infinity norm Lyapunov function is in the form

$$V(x) = \|Lx\|_\infty, \quad L \in \mathbb{R}^{l \times N}, l \geq N, l \in \mathbb{N}, \tag{A.13}$$

where L has full-column rank. Infinity norm Lyapunov functions are particular types of polyhedral Lyapunov functions.

If $L \in \mathbb{R}^{l \times N}$ has full-column rank and V as defined in (A.13) is a global Lyapunov function for system (A.9) with contraction rate $\rho \in (0, 1)$, then for all $\Gamma > 0$, the induced sublevel set $\mathbf{P}_\Gamma$ is a polytope and $0 \in int(\mathbf{P}_\Gamma)$. Moreover, if $\Phi(x) = Ax$ for some $A \in \mathbb{R}^{N \times N}$, then for all $\Gamma > 0$, the induced sublevel set $\mathbf{P}_\Gamma$ is a ρ-contractive polytope for (A.9) [32, 120].

A.6 Reach Control Problems on Polytopes

Consider an affine control system of the form

$$x(k+1) = Ax(k) + Bu(k) + c, x(k) \in \mathbb{R}^N, \ u(k) \in \mathbf{U}, \tag{A.14}$$

where $A \in \mathbb{R}^{N \times N}$, $B \in \mathbb{R}^{N \times M}$ and $c \in \mathbb{R}^N$ describe the system dynamics, at each time step $k = 0, 1, \ldots, x(k) \in \mathbb{R}^N$ is the state of the system and $u(k)$ is the control restricted to a polytopic set $\mathbf{U} \subset \mathbb{R}^M$.

Given two polytopes $\mathbf{S}$ and $\mathbf{T}$ in $\mathbb{R}^N$ with $\mathbf{T} \subseteq \mathbf{S}$, and a control system (A.14), the reach control problem concerns synthesis of a feedback control law $g : \mathbb{R}^N \to \mathbb{R}^M$ such that for all $x \in \mathbf{S}$ there exists a $k_x \in \mathbb{N}$, $k_x < \infty$ and:

$$\begin{aligned}
&x(0) = x, \\
&x(k+1) = Ax(k) + Bg(x(k)) + c, && k = 0, \ldots, k_x - 1, \\
&x(k) \in \mathbf{S} && k = 0, \ldots, k_x, \\
&x(k_x) \in \mathbf{T}.
\end{aligned}$$

The reach control problem might not be feasible, i.e., such a control law g might not exists. In the remainder of this section, several approaches for synthesis of a feedback control law solving the reach control problem are provided. For convenience of notation, a H-representation of a polytope $\mathbf{P}$ is denoted by matrices $H_\mathbf{P}$,$K_\mathbf{P}$, where $\mathbf{P} = \{x \mid H_\mathbf{P} x \leq K_\mathbf{P}\}$.

A.6.1 Iterative Pre-computation

The set of states of system (A.14) that can reach $\mathbf{T}$ in one-step can be computed via Pre operator and orthogonal projection as explained in Sect. A.3:

$$Pre(\mathbf{T} \times \mathbf{U})|_N,$$

where, for a set $\mathbf{P}$, $\mathbf{P}|_N$ denotes the orthogonal projection of $\mathbf{P}$ to $\mathbb{R}^N$. Simply $Pre(\mathbf{T} \times \mathbf{U})|_N$ is denoted by $Pre(\mathbf{T})$. Then, the set of states $\mathbf{T}^1 \subseteq \mathbf{S}$ that can reach $\mathbf{T}$ in one-step is the intersection of $Pre(\mathbf{T})$ with $\mathbf{S}$, i.e.,

$$\mathbf{T}^1 = \mathbf{S} \cap Pre(\mathbf{T}). \tag{A.15}$$

By iteratively applying (A.15), one can compute the set of states $\mathbf{T}^k \subseteq \mathbf{S}$ that can reach $\mathbf{T}$ *exactly* at step $k \in \mathbf{N}_+$, $k \geq 1$:

$$\mathbf{T}^k = \mathbf{S} \cap Pre(\mathbf{T}^{k-1}), \qquad \text{where } \mathbf{T}^0 = \mathbf{T}. \tag{A.16}$$

The set of states $\mathbf{T}^{\leq k} \subseteq \mathbf{S}$ that can reach $\mathbf{T}$ *within* $k \in \mathbb{N}$ steps is simply the union of $\mathbf{T}^0, \dots, \mathbf{T}^k$:

$$\mathbf{T}^{\leq k} = \bigcup_{i=0,\dots,k} \mathbf{T}^k. \tag{A.17}$$

Finally, the maximal set of states $\mathbf{T}^{\leq} \subseteq \mathbf{S}$ that can reach $\mathbf{T}$ in a finite number of steps is defined as

$$\mathbf{T}^{\leq} = \mathbf{T}^{\leq k} \text{ where } \mathbf{T}^{\leq k} = \mathbf{T}^{\leq k-1}. \tag{A.18}$$

Note that if $\mathbf{T}^{\leq} \neq \mathbf{S}$, then the reach-control problem is infeasible.

The above explained steps allows us to compute the maximal set $\mathbf{T}^{\leq}$. This method also induces a control strategy such that the trajectories originating from $\mathbf{T}^{\leq k}$ reach $\mathbf{S}$ with in k steps.

Consider the sets $\mathbf{T}^k$ and $\mathbf{T}^{k-1}$ defined above, and let $V(\mathbf{T}^k) = \{v^1, \dots, v^n\}$. Consider the following set of linear inequalities in the variables $u^1, \dots, u^n$:

$$\begin{aligned} H_{\mathbf{T}^{k-1}}(Av^i + Bu^i + c) &\leq K_{\mathbf{T}^{k-1}}, \\ H_{\mathbf{U}} u^i &\leq K_{\mathbf{U}}. \end{aligned} \tag{A.19}$$

A solution to (A.19) can be obtained by solving a feasibility linear program (LP). Note that the LP is always feasible since $\mathbf{T}^{\leq k} \subseteq Pre(\mathbf{T}^{\leq k-1})$. The control law

$$g_k(x) := \sum_{i=1}^{n} \lambda^i u^i, \tag{A.20}$$

where $\lambda^i \in \mathbb{R}$, $0 \leq \lambda^i \leq 1$, are such that $x = \sum_{i=1}^{n} \lambda^i v^i$, guarantees that the trajectories originating from $\mathbf{T}^k$ reach $\mathbf{T}^{k-1}$ in the next time instant. The evaluation of the control law (A.20) requires calculation of the coefficients $\lambda^1, \ldots, \lambda^n$, which amounts to solving a system of linear equations and can also be formulated as a feasibility LP.

Alternatively, an explicit PWA form of g_k can be obtained by a simplicial partition of $\mathbf{T}^k$. Then, the evaluation of g_k requires solving a point location problem [169], which consists of checking a finite number of linear inequalities. Although efficient ways to solve point location problems exist, depending on the complexity of the partition (number of simplices), the point location problem may be more computationally expensive than calculating the coefficients $\lambda^1, \ldots, \lambda^n$.

For a given state $x(0) \in \mathbf{S}$, the control strategy solving the reach control problem (under the assumption that the problem is feasible, i.e., $\mathbf{T}^{\leq} = \mathbf{S}$) amounts to finding $\mathbf{T}^k$ such that $x(0) \in \mathbf{T}^k$, and applying the control sequence $g_k(x(0)), g_{k-1}(x(1)), \ldots, g_1(x(k-1))$, where $x(i+1) = Ax(i) + Bg_{k-i}(x(i)) + c$, $i = 0, \ldots, k-1$.

Note that this method is guaranteed to find the solution to the reach-control problem if one exists. However, it is computationally expensive. Next, computationally more efficient, but conservative methods are presented.

A.6.2 Vertex Interpolation

Let $V(\mathbf{S}) = \{v^1, \ldots, v^n\}$ be the vertices of $\mathbf{S}$, and let $\{\mathbf{u}^1, \ldots, \mathbf{u}^n\}$ denote a corresponding set of finite sequences of control actions, where $\mathbf{u}^i := u_0^i, \ldots, u_{S-1}^i$ for all $i = 1, \ldots, n$, and $S \geq 1$. For each v^i define the following set of linear equality and inequality constraints in the variables $u_0^i, \ldots, u_{S-1}^i$:

$$\begin{aligned} x^i(0) &:= v^i, \\ x^i(k+1) &= Ax^i(k) + Bu^i(k) + c, \quad \forall k = 0, \ldots, S-1, \\ H_{\mathbf{S}} x^i(k) &\leq K_{\mathbf{S}}, \quad \forall k = 0, \ldots, S-1, \\ H_{\mathbf{U}} u^i(k) &\leq K_{\mathbf{U}}, \quad \forall k = 0, \ldots, S-1, \\ H_{\mathbf{U}} x^i(S) &\leq K_{\mathbf{U}}. \end{aligned} \tag{A.21}$$

A solution to the set of problems (A.21) can be searched by solving repeatedly a corresponding set of feasibility LPs starting with $S = 1$, for all $i = 1, \ldots, n$, and increasing S until a feasible solution is obtained for all LPs and the same value of S. Let $S^* \geq 1$ denote the minimal S for which a feasible solution was found. Then, it is straightforward to establish that for any $x \in \mathbf{S}$, the control law

$$g(x(k)) := \sum_{i=1}^{n} \lambda^i u^i(k), \quad k = 0, \ldots, S^* - 1, \tag{A.22}$$

where $x(0) = x$ and $\lambda^i \in \mathbb{R}$, $0 \leq \lambda^i \leq 1$, are such that $x = \sum_{i=1}^{n} \lambda^i v^i$, solves the reach control problem for $\mathbf{S}$ and $\mathbf{T}$, and yields that closed-loop trajectories that reach $\mathbf{T}$ in at most S^* discrete-time instants.

Evaluation of the control law g of (A.22) at time $k = 0$ requires calculation of the coefficients $\lambda^1, \ldots, \lambda^n$, which is an LP, while at every $k = 1, \ldots, S^* - 1$ the analytic expression of g is implemented. However, a faster convergence to $\mathbf{T}$ can be obtained by taking $\lambda^i \in \mathbb{R}$, $0 \leq \lambda^i \leq 1$, such that $x = \sum_{i=1}^{n} \lambda^i x^i(j^*)$, where

$$j^* := \arg\max\{j \in \{0, \ldots, S^*\} \mid x \in hull(x^1(j), \ldots, x^n(j)).$$

Then, the resulting closed-loop trajectories will reach $\mathbf{T}$ in at most $S^* - j^*$ discrete-time instants.

Similar to the case of the control law from Eq. (A.20), simplicial decompositions of $\mathbf{S}$ can be employed to obtain an explicit PWA form of the control law $g(x(k))$, $k = 0, \ldots, S^* - 1$, both for its standard and faster variants presented above.

A.6.3 Contractive Sets

In this section, first a Lyapunov function based solution to the reach control problem is presented, which is referred as "polyhedral LFs" method. In this method, the synthesized feedback controller guarantees that a point $x^s \in int(\mathbf{T})$ is an equilibrium point of the closed loop system and $\mathbf{P}$ is a sublevel set of a Lyapunov function. The contractive property of the sublevel set guarantees that the trajectories reach the set $\mathbf{T}$ in finite time. Then, these conditions are relaxed, and a less conservative solution, referred to as "contractive sets", is presented.

The set of equilibrium points in the interior of $\mathbf{T}$ is defined as

$$\mathbf{E_T} := \{x^s \in int(\mathbf{T}) \mid \exists u^s \in \mathbf{U} \,:\, x^s = Ax^s + Bu^s + c\}.$$

Polyhedral LFs

If $\mathbf{E_T} \neq \emptyset$, an explicit PWA solution to the reach control problem can be obtained via polyhedral LFs, see, e.g., [33, 120], as follows. Let

$$\mathscr{W}(x) := \max_{i=1,\ldots,w} W_i^\top (x - x^s) \tag{A.23}$$

denote a function induced by the polytope $\mathbf{S}$ and a point $x^s \in \mathbf{E_T}$, where $w \geq N + 1$ is the number of lines of the matrix $W = [W_1^\top; \ldots; W_w^\top]$, which is such that $\mathbf{S} = \{x \in \mathbb{R}^N \mid \mathscr{W}(x) \leq 1\}$. Next, consider the conic polytopic partition $\mathbf{C}_1, \ldots, \mathbf{C}_w$ of $\mathbf{S}$ induced by x^s, which is constructed as follows:

$$\mathbf{C}_i := \{x \in \mathbf{S} | (W_i^\top - W_j^\top)(x - x^s) \geq 0,\ j = 1, \ldots, w\} \cup \{x^s\}. \tag{A.24}$$

Notice that $\cup_{i=0,\dots,w}\mathbf{C}_i = \mathbf{S}$ and $int(\mathbf{C}_i) \cap int(\mathbf{C}_j) = \emptyset$ for all $i \neq j$. Let $\rho \in \mathbb{R}$ with $0 \leq \rho < 1$ denote a desired convergence rate. Consider the PWA control law

$$g(x) := F_i x + f_i \quad \text{if} \quad x \in \mathbf{C}_i \tag{A.25}$$

and the following feasibility LP in the variables $(F_1, f_1), \dots, (F_w, f_w)$:

$$\begin{aligned}
&\rho W_i^\top (x - x^s) - W_j^\top (Ax + Bg(x) + c - x^s) \geq 0, \\
&\qquad\qquad\qquad\qquad \forall x \in V(\mathbf{C}_i), \forall j = 1, \dots, w, \\
&F_i x + f_i \in \mathbf{U}, \quad \forall i = 1, \dots, w, \\
&(A + BF_i)x^s + Ba_i + c = x^s, \quad \forall i = 1, \dots, w.
\end{aligned} \tag{A.26}$$

Note that ρ can be minimized to obtain an optimal convergence rate and a different ρ_i can be assigned to each cone $\mathbf{C}_i$, while (A.26) remains an LP.

Proposition A.1 *Suppose that the LP* (A.26) *is feasible. Then the function* $\mathscr{W}$ *is a Lyapunov function and* **S** *is a* ρ*-contractive set for system* (A.14) *in closed-loop with the PWA control law* (A.25)*, with respect to the equilibrium* $x^s \in int(\mathbf{T})$.

The proof of Proposition A.1 follows from Theorem III.6 from [120], i.e., the constraints given in the LP (A.26) are the necessary conditions for constructing a Lyapunov function in Theorem III.6 from [120].

Let $k^* := \arg\min\{k \geq 1 \mid \rho^k(\mathbf{S} \oplus \{-x^s\}) \subseteq (\mathbf{T} \oplus \{-x^s\})\}$. All trajectories of system (A.14) in closed-loop with (A.25) that start in **S** reach **T** in at most k^* discrete-time instants. Thus, the PWA control law (A.25) solves the reach control problem for **S** and **T**. The evaluation of (A.25) reduces to a point location problem that can be solved in logarithmic time due to the specific conic partition.

Next, we present a less conservative solution by relaxing the constraints of (A.26).

Contractive Sets

Pick any $\bar{x} \in int(\mathbf{T})$ (not necessarily an equilibrium point) and let $\mathscr{W}(x)$ denote the function induced by **S** and the point $\bar{x}$ as defined in (A.23). Moreover, let

$$\alpha^\star_{\bar{x}} = \max_{\alpha > 0}\{\alpha(\mathbf{S} \oplus \{-\bar{x}\}) \subseteq (\mathbf{T} \oplus \{-\bar{x}\})\}.$$

Note that $\alpha^\star_{\bar{x}} < 1$ whenever $\mathbf{T} \subseteq \mathbf{S}$ and $\mathbf{T} \neq \mathbf{S}$; pick any $\alpha_{\bar{x}}$ such that $0 < \alpha_{\bar{x}} < \alpha^\star_{\bar{x}}$. Let $\{\mathbf{C}_i\}_{i=1,\dots,w}$ be the conic partition of **S** induced by $\bar{x}$ as defined in (A.24), and let $\rho \in \mathbb{R}$ with $0 \leq \rho < 1$ denote a desired convergence rate. Consider the PWA control law (A.25), and the following feasibility LP in the variables $(F_1, f_1), \dots, (F_w, f_w)$:

$$\begin{aligned}
&\rho(W_i^\top (x - \bar{x}) - \alpha_{\bar{x}}) - (W_j^\top (Ax + Bg(x) + c - \bar{x}) - \alpha_{\bar{x}}) \geq 0, \\
&\qquad\qquad\qquad \forall x \in V(\mathbf{C}_i), \forall j = 1, \dots, w, \forall i = 1, \dots, w, \\
&F_i x + f_i \in \mathbf{U}, \quad \forall x \in V(\mathbf{C}_i), \forall i = 1, \dots, w.
\end{aligned} \tag{A.27}$$

Similar to the polyhedral LFs method, ρ can be minimized to obtain an optimal convergence rate and a different ρ_i can be assigned to each cone $\mathbf{C}_i$, while (A.27) remains an LP. If the LP (A.27) is feasible, then by construction and the definition of the function $\mathscr{W}$, for all $x \in \mathbf{S}$ it holds that

$$\rho\mathscr{W}(x) - \mathscr{W}(Ax + Bg(x) + c) \geq \alpha_{\bar{x}}(\rho - 1). \tag{A.28}$$

The recursive application of (A.28) implies that for a closed-loop trajectory $x(0), x(1), \ldots$, the following inequality holds for all $k \geq 0$:

$$\mathscr{W}(x(k)) \leq \rho^k \mathscr{W}(x(0)) - \alpha_{\bar{x}}\rho^k + \alpha_{\bar{x}}. \tag{A.29}$$

The above property can be exploited to establish the following result.

Lemma A.1 *Suppose that the LP (A.27) is feasible and let*

$$k^\star := \left\lceil \frac{\ln\left(\frac{\alpha_{\bar{x}}^\star - \alpha_{\bar{x}}}{1-\alpha_{\bar{x}}}\right)}{\ln(\rho)} \right\rceil. \tag{A.30}$$

Then for all trajectories $x(0), x(1), \ldots$ with $x(0) \in \mathbf{S}$ of system (A.14) in closed-loop with (A.25) there exists a $k \geq 0$ such that $x(k) \in \mathbf{T}$ and, moreover, $k \leq k^\star$.

Proof Let $x(0) \in \mathbf{S}$. To prove the claim consider two cases, i.e., $\mathscr{W}(x(0)) \leq \alpha_{\bar{x}}^\star$ and $\mathscr{W}(x(0)) > \alpha_{\bar{x}}^\star$. Notice that for any $x \in \mathbf{S}$, $\mathscr{W}(x) \leq \alpha_{\bar{x}}^\star$ implies that $x \in (\alpha_{\bar{x}}^\star(\mathbf{S} \oplus \{-\bar{x}\}) \oplus \{\bar{x}\})$ and thus, it implies that $x \in \mathbf{T}$. As such, in the case when $\mathscr{W}(x(0)) \leq \alpha_{\bar{x}}^\star$, the claim holds by the definition of $k^\star$ (A.30).

Next, consider the case when $\mathscr{W}(x(0)) > \alpha_{\bar{x}}^\star$ and suppose that the inequality

$$\alpha_{\bar{x}}^\star < \rho^k \mathscr{W}(x(0)) - \alpha_{\bar{x}}\rho^k + \alpha_{\bar{x}} \tag{A.31}$$

holds for all $k \geq 0$. By taking the limit when k tends to infinity in the above inequality yields that $\alpha_{\bar{x}}^\star < \alpha_{\bar{x}}$ and thus, a contradiction was reached. As such, there must exist a $k \geq 0$ such that

$$\rho^k \mathscr{W}(x(0)) - \alpha_{\bar{x}}\rho^k + \alpha_{\bar{x}} \leq \alpha_{\bar{x}}^\star \tag{A.32}$$

holds. Equations (A.29) and (A.32) imply that $\mathscr{W}(x(k)) \leq \alpha_{\bar{x}}^\star$ and hence, $x(k) \in \mathbf{T}$. Furthermore, reordering the terms and taking the logarithm of both sides in (A.32) yields

$$k \leq \frac{\ln\left(\frac{\alpha_{\bar{x}}^\star - \alpha_{\bar{x}}}{\mathscr{W}(x(0)) - \alpha_{\bar{x}}}\right)}{\ln(\rho)}. \tag{A.33}$$

Noticing that $\mathscr{W}(x) \leq 1$ for all $x \in \mathbf{S}$, the fact that $k \leq k^\star$ follows directly from the definition of $k^\star$ (A.30). As the point $x(0) \in \mathbf{S}$ was chosen arbitrarily, the claim is established. ∎

If the LP (A.27) is feasible, the PWA control law (A.25) is an admissible solution to the reach control problem as shown in Lemma A.1. As in polyhedral LFs solution, the evaluation of (A.25) reduces to a point location problem that can be solved in logarithmic time due to the specific conic partition.

The contractive sets method is a relaxation of the Lyapunov function based method. First, $\bar{x}$ is no longer required to be an equilibrium point. Second, the contraction condition of (A.27) is a relaxation of the contraction condition of (A.26). In particular, if $\bar{x}$ is an equilibrium point, then by augmenting the constraints of the LP (A.27) with the equilibrium point constraints, i.e., $(A + BF_i)\bar{x} + Bf_i + c = \bar{x}$, for all $i = 1, \ldots, w$, and setting $\alpha_{\bar{x}}$ to 0, the polyhedral LFs solution is recovered and the function $\mathscr{W}$ becomes a standard Lyapunov function with respect to the equilibrium $\bar{x}$.

A.7 Control Potential Functions

Control potential functions are formally defined in Definition 12.2 in Chap. 12. In this appendix, candidate control potential functions for a transition system $T = (X, \Sigma, \delta, O, o)$ and a dual automaton $A_D = (S_D, S_{D_0}, O_{\mathscr{W}}, \delta_D, \tau_D, F_D)$ with $R_s \subseteq X, \forall s \in S_D$ and transition weight function $\mathbf{W} : S_D \times S_D \to \mathbb{N}_+$ are presented.

A.7.1 Control Potential Function Based on One Step Controllable Sets

Consider control potential function $V_{con,CS} : \bigcup_{\{(s,s')|s' \in \delta_D(s), s \neq s'\}} \{\{(s, s')\} \times R_s\} \longrightarrow \mathbb{N}_+$:

$$V_{con,CS}((s, s'), x) = \arg\min\{k \in \{1, \ldots, \mathbf{W}(s, s')\} \mid x \in B_{ss'}^{\leq k-1}\}, \tag{A.34}$$

where $B_{ss'}^{k-1}$ denotes the set of states in R_s that can reach the beacon $B_{ss'}$ of (s, s') in $k - 1$ steps, hence $R_{s'}$ in k steps. The set $B_{ss'}^{\leq k-1}$ is computed shown in Sect. A.6.1. Therefore, for $V_{con,CS}$, the constraint set $\mathscr{R}_{V_{con}}^{k,ss'}$ (see (12.4)) is defined as

$$\mathscr{R}_{V_{con,CS}}^{k,ss'} = B_{ss'}^{\leq k-1}.$$

A.7.2 Control Potential Function Based on Feedback Controllers

In this section, a control potential function is presented for each of the control synthesis methods given in Sect. A.6 for solving reach control problem.

A.7.2.1 Vertex Interpolation

Let $g : R_s \rightarrow \mathbf{U}$ be a feedback control law synthesized by using the vertex interpolation method to solve the reach control problem from R_s to $B_{ss'}$, and $\mathbf{W}(s, s')$ be the corresponding time bound. Let $x^i(0), \ldots, x^i(\mathbf{W}(s, s') - 1)$ be the trajectory generated by the feedback control g from the vertex $x^i(0) \in V(R_s)$. Note that $x^i(\mathbf{W}(s, s') - 1) \in B_{ss'}$ for each $x^i(0) \in V(R_s)$.

Consider control potential function $V_{con,VI} : \bigcup_{s \in S_D, s' \in \delta_D(s)} \left\{\{(s, s')\} \times R_s\right\} \longrightarrow \mathbb{N}_+$:

$$V_{con,VI}((s, s'), x) = \mathbf{W}(s, s') - \arg\max\{j \in \{0, \ldots, \mathbf{W}(s, s') - 1\} \mid x \in hull(\{x^i(j)\}_{i=1,\ldots,|V(R_s)|})\}. \quad \text{(A.35)}$$

From (12.4) and (A.35), the constraint set for $V_{con,VI}$ is defined as

$$\mathscr{R}^{k,ss'}_{V_{con,VI}} = \bigcup_{j=1,\ldots,k} hull(\{x^i(\mathbf{W}(s, s') - j)\}_{i=1,\ldots,|V(R_s)|}) \quad \text{(A.36)}$$

A.7.2.2 Polyhedral LFs

Let a feedback control law be synthesized by using the polyhedral LFs method to solve the reach control problem from R_s to $B_{ss'}$, and let ρ and x^s be the corresponding contraction rate and equilibrium point, respectively.

Consider control potential function $V_{con,PL} : \bigcup_{s \in S_D, s' \in \delta_D(s)} \left\{\{(s, s')\} \times R_s\right\} \longrightarrow \mathbb{N}_+$:

$$V_{con,PL}((s, s'), x) = \mathbf{W}(s, s') - \arg\max\{j \in \{0, \ldots, \mathbf{W}(s, s') - 1\} \mid x \in (\rho^k(R_s \oplus \{-x^s\}) \oplus x^s)\}. \quad \text{(A.37)}$$

For $V_{con,PL}$, the constraint set is defined as

$$\mathscr{R}^{k,ss'}_{V_{con,PL}} = \rho^{\mathbf{W}(s,s')-k}(R_s \oplus \{-x^s\}) \oplus x^s \cup B_{ss'}. \quad \text{(A.38)}$$

A.7.2.3 Contractive Sets

Let a feedback control law be synthesized by using the contractive sets method to solve the reach control problem from R_s to $B_{ss'}$, and let $\bar{x}$, ρ, $\alpha_{\bar{x}}$, $\alpha^\star_{\bar{x}}$ and the function $\mathscr{W}$ be defined as in (A.28) with respect to $\bar{x}$. For the contractive sets method, a control potential function $V_{con,C} : \bigcup_{s \in S_D, s' \in \delta_D(s)} \left\{\{(s, s')\} \times R_s\right\} \longrightarrow \mathbb{N}_+$ is defined as:

$$V_{con,C}((s,s'),x) = \left\lceil \frac{\ln\left(\frac{\alpha^{\star}_{\bar{x}} - \alpha_{\bar{x}}}{\mathscr{W}(x_0) - \alpha_{\bar{x}}}\right)}{\ln(\rho)} \right\rceil, \tag{A.39}$$

where $\lceil \cdot \rceil$ denotes the ceiling operation. For $V_{con,C}$, the constraint set is defined as

$$\mathscr{R}^{k,ss'}_{V_{con,C}} = \frac{\alpha^{\star}_{\bar{x}} - \alpha_{\bar{x}} + \alpha_{\bar{x}}\rho^{\mathbf{W}(s,s')-k}}{\rho^{\mathbf{W}(s,s')-k}}(R_s \oplus \{-\bar{x}\}) \oplus \bar{x} \cup B_{ss'}. \tag{A.40}$$

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