Appendix A
Algebra Techniques

A.1 Partial Fractions

The integration of a rational function—that is, a function of the form P/Q, where P and Q are polynomials—becomes much easier if the function can be written as a sum of simpler expressions, commonly referred to as *partial fractions*. This is done by means of a procedure that consists of the following steps:

(i) Using long division, if necessary, we isolate the fractional part of the given function, for which the degree of P is strictly less than the degree of Q.

(ii) We split Q into a product of linear factors (first-degree polynomials) and irreducible quadratic factors (second-degree polynomials with complex roots). Some of these factors may be repeated.

(iii) For each single linear factor $ax + b$, we write a fraction of the form $C/(ax + b)$. For each repeated linear factor $(ax + b)^n$, we write a sum of fractions of the form

$$\frac{C_1}{ax+b} + \frac{C_2}{(ax+b)^2} + \cdots + \frac{C_n}{(ax+b)^n}.$$

(iv) For each single irreducible quadratic factor $ax^2 + bx + c$, we write a fraction of the form $(C_1x+C_2)/(ax^2+bx+c)$. For each repeated irreducible quadratic factor $(ax^2 + bx + c)^n$, we write a sum of fractions of the form

$$\frac{C_{11}x + C_{12}}{ax^2+bx+c} + \frac{C_{21}x + C_{22}}{(ax^2+bx+c)^2} + \cdots + \frac{C_{n1}x + C_{n2}}{(ax^2+bx+c)^n}.$$

(v) The fractional part of the given function is equal to the sum of all the partial fractions and sums of partial fractions constructed in steps (iii) and (iv). The unknown coefficients C, C_i, and C_{ij} are determined (uniquely) from this equality.

A.1 Example. The denominator of the fraction $(x - 5)/(x^2 - 4x + 3)$ is a quadratic polynomial, but it is not irreducible. Its roots are 1 and 3, so we write

$$\frac{x-5}{x^2-4x+3} = \frac{x-5}{(x-1)(x-3)} = \frac{A}{x-1} + \frac{B}{x-3}.$$

If we eliminate the denominators, we arrive at the equality

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$$x - 5 = A(x-3) + B(x-1) = (A+B)x - 3A - B,$$

which, in fact, is an identity, meaning that it must hold for all admissible values (in this case, all real values) of x. Then, matching the coefficients of x and the constant terms on both sides, we obtain the system

$$A + B = 1, \quad 3A + B = 5,$$

with solution $A = 2$ and $B = -1$. Hence,

$$\frac{x-5}{x^2 - 4x + 3} = \frac{2}{x-1} - \frac{1}{x-3}. \ \blacksquare$$

A.2 Example. Similarly, we have

$$\frac{x^2 + 7x + 4}{x^3 + 4x^2 + 4x} = \frac{x^2 + 7x + 4}{x(x+2)^2} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2},$$

or

$$x^2 + 7x + 4 = A(x+2)^2 + Bx(x+2) + Cx.$$

Equating the coefficients of x^2 and x and the constant terms on both sides would yield a system of 3 equations in 3 unknowns, whose solution would require a little time and effort. This can be avoided if we recall that the above equality, which—we must emphasize—has been set up correctly, holds for all real values of x. Thus, for $x = 0$ we obtain $4 = 4A$, so $A = 1$, and for $x = -2$ we get $-6 = -2C$, so $C = 3$. We chose these particular values of x because they made some of the terms on the right-hand side vanish. To find B, we can take x to be any other number, for example, 1, and replace A and C by their already determined values. Then $12 = 9A + 3B + C = 12 + 3B$, from which $B = 0$; hence,

$$\frac{x^2 + 7x + 4}{x^3 + 4x^2 + 4x} = \frac{1}{x} + \frac{3}{(x+2)^2}. \ \blacksquare$$

A.3 Example. Since $x^2 + 1$ is an irreducible quadratic polynomial, we have

$$\frac{2 + x - x^2}{x^3 + x} = \frac{2 + x - x^2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1},$$

or

$$2 + x - x^2 = A(x^2 + 1) + (Bx + C)x.$$

Using either the method in Example A.1 or that in Example A.2, we find that $A = 2$, $B = -3$, and $C = 1$, so

$$\frac{2 + x - x^2}{x^3 + x} = \frac{2}{x} + \frac{1 - 3x}{x^2 + 1}. \ \blacksquare$$

A.4 Example. Without giving computational details but mentioning the use of long division, we have

$$\begin{aligned}
\frac{x^5 + 2x^2 - 3x + 2}{x^4 - x^3 + x^2} &= x + 1 + \frac{x^2 - 3x + 2}{x^4 - x^3 + x^2} = x + 1 + \frac{x^2 - 3x + 2}{x^2(x^2 - x + 1)} \\
&= x + 1 + \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 - x + 1} \\
&= x + 1 - \frac{1}{x} + \frac{2}{x^2} + \frac{x - 2}{x^2 - x + 1}. \ \blacksquare
\end{aligned}$$

A.2 Synthetic Division

Consider a polynomial equation of the form

$$a_0 r^n + a_1 r^{n-1} + \cdots + a_{n-1} r + a_n = 0, \tag{A.1}$$

where all the coefficients $a_0, \ldots, a_n$ are integers. If a root of this equation is an integer, then that root is a divisor of the constant term a_n and can be determined by means of a simple algorithm.

(i) Let r_0 be a divisor of a_n. We line up the coefficients of the equation in decreasing order of the powers of r (writing 0 if a term is missing) as the first row in Table A.1.

Table A.1

a_0	a_1	a_2	$\cdots$	a_{n-1}	a_n	

(ii) Next, we complete the second row in Table A.1 with numbers $b_0, \ldots, b_n$ computed as shown in Table A.2.

Table A.2

a_0	a_1	a_2	$\cdots$	a_n	
$b_0 = a_0$	$b_1 = b_0 r_0 + a_1$	$b_2 = b_1 r_0 + a_2$	$\cdots$	$b_n = b_{n-1} r_0 + a_n$	r_0

(iii) If $b_n \neq 0$, then r_0 is not a root of the equation, and we repeat the procedure with the next candidate—that is, the next divisor of the constant term. If $b_n = 0$, then r_0 is a root; furthermore, b_0, b_1, $\ldots$, b_{n-1} are the coefficients of the polynomial equation of degree $n-1$ which yields the remaining roots.

(iv) When this new equation is not easily solvable, we may try applying the procedure again, starting with the new set of coefficients b_i.

A.5 Example. For the equation

$$r^4 - 2r^3 - 3r^2 + 4r + 4 = 0,$$

the divisors of the constant term 4 are ± 1, ± 2, and ± 4. To check whether, say, 1 is a root, we apply the algorithm described above and arrive at Table A.3.

Table A.3

1	−2	−3	4	4	
1	−1	−4	0	4	1

The number 4 in the last place before the vertical bar in the second row means that 1 is not a root, so we try the next candidate, −1. This time we get Table A.4.

Table A.4

1	−2	−3	4	4	
1	−3	0	4	0	−1

Since the last number in the second row before the bar is 0, we conclude that −1 is a root. We can try −1 again, starting the operation directly from the second row, as shown in Table A.5.

Table A.5

1	−2	−3	4	4	
1	−3	0	4	0	−1
1	−4	4	0		−1

Hence, −1 is a double root, and the coefficients 1, −4, and 4 in the third row tell us that the remaining two roots of the characteristic equation are given by the equation $r^2 - 4r + 4 = 0$, which yields another double root, namely 2. ■

We can extend the above algorithm to determine whether an equation of the form (A.1) has rational roots; that is, roots of the form a/b, where $b \neq 0$ and a and b are integers with no common factor greater than 1. If this is the case, then the numerator a is a divisor of the constant term a_n and the denominator b is a divisor of the leading coefficient a_0.

A.6 Example. For the equation

$$2r^3 - 9r^2 + 14r - 5 = 0,$$

the divisors of the constant term −5 are ±1 and ±5, but none of them satisfies the equation. Since the divisors of the leading coefficient 2 are ±1 and ±2, it follows that any possible nonintegral rational roots need to be sought amongst the numbers ±1/2 and ±5/2. Trying the first one, 1/2, with the procedure described above, we construct Table A.6.

Table A.6

2	−9	14	−5	
2	−8	10	0	1/2

This shows that $r_1 = 1/2$ is a root and that the other two roots are given by the quadratic equation $r^2 - 4r + 5 = 0$ (for convenience, we have divided the coefficients 2, −8, and 10 in the second row by 2); they are $r_2 = 2 + i$ and $r_3 = 2 - i$. ■

Appendix B
Calculus Techniques

B.1 Sign of a Function

If $f = f(x)$ is a continuous function on an interval J and a and b are two points in J such that $f(a)$ and $f(b)$ have opposite signs, then, by the intermediate value theorem, there is at least one point c between a and b such that $f(c) = 0$. In other words, a continuous function cannot change sign unless it goes through the value 0. This observation offers a simple way to find where on J the function f has positive values and where it has negative values.

B.1 Example. To determine the sign of

$$f(x) = (x+3)(x-1)(5-x)$$

at all points x on the real line, we note that the roots of the equation $f(x) = 0$ are $x_1 = -3$, $x_2 = 1$, and $x_3 = 5$. Since the sign of f does not change on any of the subintervals $x < -3$, $-3 < x < 1$, $1 < x < 5$, and $x > 5$, we can test it by computing the value of f at an arbitrarily chosen point in each of these subintervals; here we picked the points -4, 0, 2, and 6. The conclusions are listed in Table B.1.

Table B.1

x		(-4)		-3		(0)		1		(2)		5		(6)	
$f(x)$	$+$	$+$	$+$	0	$-$	$-$	$-$	0	$+$	$+$	$+$	0	$-$	$-$	$-$

This makes it unnecessary to study and combine the sign of every factor in the expression of $f(x)$. ■

B.2 Integration by Parts

If u and v are continuously differentiable functions of a variable x, then

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx,$$

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or, in abbreviated form,

$$\int u\,dv = uv - \int v\,du.$$

B.2 Example. Let

$$I_1 = \int e^{ax}\cos(bx)\,dx, \quad I_2 = \int e^{ax}\sin(bx)\,dx.$$

Then

$$I_1 = \frac{1}{a}\,e^{ax}\cos(bx) - \int \frac{1}{a}\,e^{ax}(-\sin(bx))b\,dx = \frac{1}{a}\,e^{ax}\cos(bx) + \frac{b}{a}\,I_2,$$

$$I_2 = \frac{1}{a}\,e^{ax}\sin(bx) - \int \frac{1}{a}\,e^{ax}\cos(bx)b\,dx = \frac{1}{a}\,e^{ax}\sin(bx) - \frac{b}{a}\,I_1.$$

This is a simple algebraic system for I_1 and I_2, which yields

$$I_1 = \int e^{ax}\cos(bx)\,dx = \frac{1}{a^2+b^2}\,e^{ax}[a\cos(bx) + b\sin(bx)] + C, \tag{B.1}$$

$$I_2 = \int e^{ax}\sin(bx)\,dx = \frac{1}{a^2+b^2}\,e^{ax}[a\sin(bx) - b\cos(bx)] + C. \ \blacksquare \tag{B.2}$$

B.3 Integration by Substitution

Let $f = f(x)$ be a continuous function on an interval $[a, b]$, and let $x = x(t)$ be a continuously differentiable function on an interval $[c, d]$, which takes values in $[a, b]$ and is such that $x(c) = a$ and $x(d) = b$. Then

$$\int_a^b f(x)\,dx = \int_c^d f(x(t))x'(t)\,dt.$$

B.3 Example. If $g = g(t)$, then, in the absence of prescribed limits,

$$\int \frac{g'(t)}{g(t)}\,dt = \int \frac{1}{g(t)}\,g'(t)\,dt = \int \frac{dg}{g} = \ln|g| + C. \ \blacksquare$$

B.4 Overview of the Hyperbolic Functions

The basic hyperbolic functions are defined for all (real) values of x by

$$\sinh x = \tfrac{1}{2}\left(e^x - e^{-x}\right), \quad \cosh x = \tfrac{1}{2}\left(e^x + e^{-x}\right),$$

$$\tanh x = \frac{\sinh x}{\cosh x}, \quad \coth x = \frac{\cosh x}{\sinh x}\,,$$

$$\operatorname{sech} x = \frac{1}{\cosh x}, \quad \operatorname{csch} x = \frac{1}{\sinh x}\,.$$

The names of these functions are the same as those of the corresponding trigonometric (circular) functions but with the word 'hyperbolic' preceding them; for example, the first one above is read 'hyperbolic sine of x'. These names are chosen because many of the properties of hyperbolic functions are fully analogous (though not identical) to those of their trigonometric counterparts. Here is a succinct list of some of the major properties—easily proved directly from their definitions—of the hyperbolic sine and hyperbolic cosine, which are of interest to us.

Algebraic relations:

$$\sinh(-x) = -\sinh x, \quad \cosh(-x) = \cosh x,$$
$$\cosh^2 x - \sinh^2 x = 1,$$
$$\sinh(x+y) = (\sinh x)(\cosh y) + (\cosh x)(\sinh y),$$
$$\cosh(x+y) = (\cosh x)(\cosh y) + (\sinh x)(\sinh y),$$
$$2\sinh^2 x = \cosh(2x) - 1, \quad 2\cosh^2 x = \cosh(2x) + 1.$$

Derivatives and integrals:

$$(\sinh x)' = \cosh x, \quad (\cosh x)' = \sinh x,$$
$$(\sinh(a(x-b)))' = a\cosh(a(x-b)), \quad (\cosh(a(x-b)))' = a\sinh(a(x-b)),$$
$$\int \sinh x\, dx = \cosh x + C, \quad \int \cosh x\, dx = \sinh x + C.$$

Taylor series expansions:

$$\sinh x = \frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!},$$
$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}.$$

The analogy between the trigonometric and hyperbolic sines and cosines breaks down when it comes to their graphical representations. The latter have some simple symmetry but no periodicity or boundedness, as the former do. This is clearly seen from Figs. B.1 and B.2.

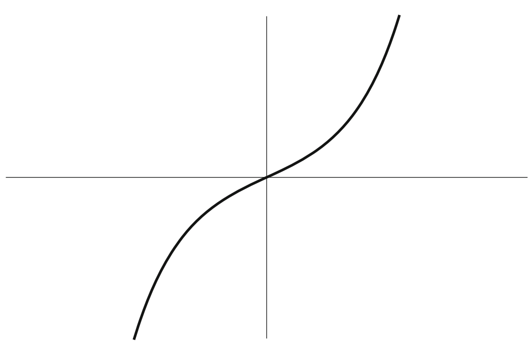

Fig. B.1 Graph of $\sinh x$.

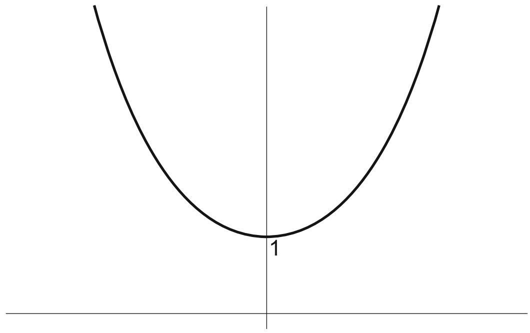

Fig. B.2 Graph of $\cosh x$.

It should be noted that of these two functions, only $\sinh x$ takes the value 0, and that happens only at $x = 0$. Also, we have

$$\lim_{x\to-\infty} \sinh x = -\infty, \quad \lim_{x\to\infty} \sinh x = \infty,$$

$$\lim_{x\to-\infty} \cosh x = \infty, \quad \lim_{x\to\infty} \cosh x = \infty.$$

Appendix C
Table of Laplace Transforms

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1	$f^{(n)}(t) \quad (n\text{th derivative})$	$s^n F(s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$
2	$f(t-a)H(t-a)$	$e^{-as}F(s)$
3	$e^{at}f(t)$	$F(s-a)$
4	$(f * g)(t)$	$F(s)G(s)$
5	1	$\dfrac{1}{s} \quad (s > 0)$
6	$t^n \quad (n \text{ positive integer})$	$\dfrac{n!}{s^{n+1}} \quad (s > 0)$
7	e^{at}	$\dfrac{1}{s-a} \quad (s > a)$
8	$\sin(at)$	$\dfrac{a}{s^2 + a^2} \quad (s > 0)$
9	$\cos(at)$	$\dfrac{s}{s^2 + a^2} \quad (s > 0)$
10	$\sinh(at)$	$\dfrac{a}{s^2 - a^2} \quad (s > \lvert a\rvert)$
11	$\cosh(at)$	$\dfrac{s}{s^2 - a^2} \quad (s > \lvert a\rvert)$
12	$\delta(t-a) \quad (a \geq 0)$	e^{-as}
13	$\delta(c(t-a)) \quad (c,\, a > 0)$	$\dfrac{1}{c}\, e^{-as}$
14	$f(t)\delta(t-a) \quad (a \geq 0)$	$f(a)e^{-as}$
15	$\int_0^t f(\tau)\, d\tau$	$\dfrac{1}{s}\, F(s)$
16	$t^n f(t) \quad (n \text{ positive integer})$	$(-1)^n F^{(n)}(s)$

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Appendix D
The Greek Alphabet

Below is a table of the Greek letters most used by mathematicians. The recommended pronunciation of these letters as symbols in an academic context, listed in the third column, is that of classical, not modern, Greek.

Letter	**Name**	**Greeks say...**
α	alpha	*ahl*-fah
β	beta	*beh*-tah
γ, $\mathit{\Gamma}$	gamma	*gahm*-mah
δ, $\mathit{\Delta}$	delta	*del*-tah
ε, ϵ	epsilon	*ep*-sih-lohn
ζ	zeta	*zeh*-tah
η	eta	*eh*-tah
θ, $\mathit{\Theta}$	theta	*theh*-tah
ι	iota	*yoh*-tah (as in 'York')
κ	kappa	*kahp*-pah
λ, $\mathit{\Lambda}$	lambda	*lahmb*-dah
μ	mu	mu
ν	nu	nu
ξ, $\mathit{\Xi}$	xi	xih
π, $\mathit{\Pi}$	pi	pih
ρ	rho	roh
σ, $\mathit{\Sigma}$	sigma	*sig*-mah
τ	tau	tau (as in 'how')
υ, $\mathit{\Upsilon}$	upsilon	*ewe*-psih-lohn
ϕ, φ, $\mathit{\Phi}$	phi	fih
χ	chi	khih
ψ, $\mathit{\Psi}$	psi	psih
ω, $\mathit{\Omega}$	omega	oh-*meh*-gah

One allowed exception: π (and its upper case version $\mathit{\Pi}$) can be mispronounced 'pie', to avoid the objectionable connotation of the original sound.

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Further Reading

The following is a short list, by no means exhaustive, of textbooks on ordinary differential equations that contain additional material, technical details, and information on the subject going beyond the scope of this book.

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Answers to Odd-Numbered Exercises

Chapter 1

Section 1.3

1 $y(t) = 2 - t^2 - 2t^3$. **3** $y(t) = t\cos t$. **5** $y(x) = x^2 - 3x + 2$.

7 $y(x) = (1 - x)\ln x$. **9** $v(t) = 2t$, $s(t) = t^2 - 4$.

11 $v(t) = 6(t + 4)^{1/2} - 13$, $s(t) = 4(t + 4)^{3/2} - 13t - 31$. **13** 26.4.

Section 1.4

1 Nonlinear, homogeneous, fourth-order ordinary DE with variable coefficients.

3 Nonlinear, nonhomogeneous, first-order ordinary DE with constant coefficients.

5 Nonlinear, homogeneous, third-order ordinary DE with constant coefficients.

7 Linear, nonhomogeneous, first-order partial DE with variable coefficients.

Chapter 2

Section 2.1

1 $y(x) = 1/(2x^2 + 1)$. **3** $y(x) = e^{-2e^{3x}}$.

5 $y(x) = e^{4+3x-x^2}$. **7** $y(x) = (1 + x - e^{-x})^{1/2} - 1$.

9 $y(x) = \sin^{-1}(x^2 + 1/2\,)$. **11** $y(x) = -3 + (1 + 2x)^{1/2}$.

13 $y(x) = (2x^3 + x^2 + 3)^{1/2} - 2$. **15** $y(x) = \tan[(2x^2 - 12x + \pi)/4]$.

17 $y^4 + y^3 = 2 - \cos(2x)$. **19** $y(x) = 4y^2 - e^{-2y} = 2x^3 + 2x - 1$.

Section 2.2

1 $y = 2e^{-4t} - 4$. **3** $y(t) = 2e^{-t/3} + 3 - t + t^2/6$.

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5 $y(t) = 4e^{-t} - (2t+1)e^{-3t}$. **7** $y(t) = (2t^2 - 3t)e^{t^2/2}$.

9 $y(t) = 2t^7 - t^6 + t^4$. **11** $y = 2t^{-1/2} - 3t^{3/2}$.

13 $y = [2\cos(2t) + 4(t-2)\sin(2t)]/(t-2)$. **15** $y(t) = (2t-1)e^{2t}/t^3$.

17 $y(t) = (t^3 - 2t^2 + 3)/(t^2 + 2)$.

19 $y(t) = (t^3 - 3t^2 + 3t)(t+1)^2/(t-1)^2$.

Section 2.3

1 $y(x) = (x^3 + x)/2$. **3** $y(x) = x(\ln|x| + C)/3$.

5 $y(x) = -2x - (7x^2 - 3)^{1/2}$. **7** $y(x) = x^2/(2-x)$.

9 $y(x) = (x^2 + 2x)/(2x - 1)$. **11** $y(x) = 2x\tan[\ln(x^2)]$.

13 $y(x) = x(C - 1 + \ln|x|)/(C + \ln|x|)$.

15 $y(x) = -x[1 + (2x+5)^{1/2}]/2$.

17 $y(x) = x[\ln(y^2/|x|) + C]$. **19** $y(x) = x(C + \ln|x|)^{1/3}$.

Section 2.4

1 $y(t) = (2e^{2t} - 1)^{-1/2}$. **3** $y(t) = e^{-2t}/4$. **5** $y(t) = [(t^2+2)e^{t/2}]^{1/4}$.

7 $y(t) = (3 - 2/t^2)^{3/4}$. **9** $y(t) = [(\ln|t| + C)/t]^{2/5}$.

Section 2.5

1 $y(t) = (3t+2)/(2t)$. **3** $y(t) = (t^3 - t - 1)/(t^2 + t)$.

5 $y(t) = 1 + 1/(3e^t - 2t - 2)$. **7** $y(t) = -t - (\cot t)/2$.

9 $y(t) = 1 - t + 1/(e^{t^2/2} - 2)$.

Section 2.6

1 $2x^2y - 3y^2 + 1 = 0$. **3** $x^2 - 5xy - 3y^2 + 2y = 9$.

5 $2x + 3xy - y^2 + 2ye^{x/2} + 3 = 0$. **7** $x^2\sin y - y\sin x = \pi^2/4 - \pi/2$.

9 $x^{-2} - 2y^{-3} + x^3y^{-2} - x^{-1}y^2 = 3$.

11 $\cos(2x)\sin y + e^{2x} - y^{-2} = 2 - 4/\pi^2$.

13 $\mu(y) = y,\quad y(x) = (4/x - 3)^{1/2}$.

15 $\mu(x) = 1/x,\quad x^2 + \ln|x| - y - 2y^2 = C$.

17 $\mu(x) = x^2,\quad y(x) = (x^3/2 - 2)^{1/2}$.

19 $\mu(y) = 1/y^2,\quad x^2 - xy + 2xy^2 - y^{-1} = 5$.

Section 2.7

1 $-1/2 < t < \infty$. **3** $1 < t < 2$. **5** $0 < t < e^2$.

7 $\{(t, y) : y = -t/2,\ -\infty < t < \infty\}$.

9 $\{(t, y) : y = (9 - t^2)^{1/2}$ or $y = -(9 - t^2)^{1/2},\ -\infty < t \le 0$ or $3 \le t < \infty\}$.

11 $\{(t, y) : y \le -t,\ -\infty < t < \infty\}$.

13 $y(t) = 2/(1 - 4t^2),\ -1/2 < t < 1/2;\ y(t) = 2/(3 - 4t^2),\ -\infty < t < -\sqrt{3}/2;$
$y(t) = 1/(17 - 2t^2),\ \sqrt{17/2} < t < \infty$.

15 $y(t) = 2 + (t^2 - 3)^{1/2},\ \sqrt{3} < t < \infty;$
$y(t) = 2 - (t^2 - 3)^{1/2},\ -\infty < t < -\sqrt{3};$
$y(t) = 2 - (t^2 + 1)^{1/2},\ -\infty < t < \infty;\ y(t) = 2 \pm (t^2 - 1)^{1/2},\ 1 < t < \infty$.

17 $y(t) = 4t^2 + 8t + 5,\ -1 < t < \infty;\ y(t) = 4t^2 + 12t + 10,\ -3/2 < t < \infty$.

19 $y(t) = (t + 1)^{2/3},\ -1 < t < \infty;\ y(t) = (t^2 + 2t - 2)^{1/3},\ -1 + \sqrt{3} < t < \infty;$
$y(t) = (t^2 + 2t + 9)^{1/3},\ -\infty < t < \infty;$
$y(t) = (t^2 + 2t - 16)^{1/3},\ -1 - \sqrt{17} < t < -1 + \sqrt{17}$.

Chapter 3

Section 3.1

1 $t = 2(1 + \sqrt{2}\,) \approx 4.83;\ t = 2(3 + 2\sqrt{3}\,) \approx 12.93$.

3 $N_0 = 6000\sqrt{2} \approx 8485$. **5** $t = 3$ years.

Section 3.2

1 $t \approx 22.7$. **3** $t = 1 + (\ln(3/2))/(\ln(18/17)) \approx 8.09;\ T = 80/17 \approx 4.7$.

5 $Q = 2/65 + (8/17)e^{-\pi/4} - (16/65)e^{-3\pi/16} \approx 0.1087$.

7 $p = 1,210.64$/month; \$90,554.60; 8.56 years earlier; \$41,332.30.

Section 3.3

1 $0,\ 150;\ y(t) = 0$ (unstable), $y(t) = 150$ (asymptotically stable);
$y = 150y_0/[y_0 - (y_0 - 150)e^{-300t}]$;
population with logistic growth; $r = 300,\ B = 150$.

3 $0,\ 30;\ y(t) = 0$ (unstable), $y(t) = 30$ (asymptotically stable);
$y = 30y_0/[y_0 - (y_0 - 30)e^{-15t}]$;
population with logistic growth; $r = 15,\ B = 30$.

5 1, 3; $y(t) = 1$ (unstable), $y(t) = 3$ (asymptotically stable);
$y = [3(y_0 - 1) - (y_0 - 3)e^{-4t}]/[y_0 - 1 - (y_0 - 3)e^{-4t}]$;
population with logistic growth and harvesting; $r = 8$, $B = 4$, $\alpha = 6$.

7 0, 40; $y(t) = 0$ (asymptotically stable), $y(t) = 40$ (unstable);
$y = 40y_0/[y_0 - (y_0 - 40)e^{80t}]$; population with a critical threshold; $r = 80$, $T = 40$.

9 0, 20; $y(t) = 0$ (asymptotically stable), $y(t) = 20$ (unstable);
$y = 20y_0/[y_0 - (y_0 - 20)e^{4t}]$; population with a critical threshold; $r = 4$, $T = 20$.

11 1, 5; $y(t) = 1$ (asymptotically stable), $y(t) = 5$ (unstable);
$y = [y_0 - 5 - 5(y_0 - 1)e^{-8t}]/[y_0 - 5 - (y_0 - 1)e^{-8t}]$; chemical reaction;
$c_1 = 1$, $c_2 = 5$, $a = 2$.

13 -2, 4; $y(t) = -2$ (unstable), $y(t) = 4$ (asymptotically stable);
$y = [4(y_0 + 2) + 2(y_0 - 4)e^{-6t}]/[y_0 + 2 - (y_0 - 4)e^{-6t}]$.

15 -3, 2; $y(t) = -3$ (asymptotically stable), $y(t) = 2$ (unstable);
$y = [-3(y_0 - 2) - 2(y_0 + 3)e^{-5t}]/[y_0 - 2 - (y_0 + 3)e^{-5t}]$.

17 1, 2; $y(t) = 1$ (unstable), $y(t) = 2$ (semi-stable).

19 1, 2, 3; $y(t) = 1$ (semi-stable), $y(t) = 2$ (asymptotically stable);
$y(t) = 3$ (unstable).

Chapter 4

Section 4.2

1 $W[f_1, f_2](t) = (4t^2 - 2t - 1)e^{2t}$. **3** $W[f_1, f_2](t) = -t^2$.

5 $W[f_1, f_2](t) = -e^{4t}$. **7** $W[f_1, f_2](t) = a\cos(ac)$.

9 $W[f_1, f_2](x) = a\sinh(ac)$. **11** $f_2(t) = t^2 + t$.

13 $f_2(t) = e^{-2t}$. **15** $f_2(t) = \sin t$.

Subsection 4.3.1

1 $t > 0$. **3** $t < 1$. **5** $-2 < t < 2$. **7** $\pi < t < 2\pi$. **9** $t > \ln 2$.

11 $W[y_1, y_2](2) = 1$. **13** $W[y_1, y_2](1) = -\sqrt{2}$. **15** $W[y_1, y_2](\pi/2) = -1$.

Subsection 4.3.2

1 (i) $y(x) = 2\cos(x/2) - 4\sin(x/2)$;
(ii) $y(x) = 2\cos(x/2) + c\sin(x/2)$, c = const arbitrary; (iii) no solution.

3 (i) No solution; (ii) $y(x) = e^{-x}[\cos(3x) + \sin(3x)]$;
$e^{-x}[\cos(3x) + c\sin(3x)]$, c = const arbitrary.

5 (i) $y(x) = e^{2x}\{\cos(2x) + c[\sin(2x) - \cos(2x)]\}$, $c =$ const arbitrary.
(ii) $y(x) = e^{2x}[\cos(2x] - 3\sin(2x)]$; (iii) no solution.

7 (i) $y(x) = e^{x/2}\{4\cos(x/2) + c[\cos(x/2) - \sin(x/2)]\}$, $c =$ const arbitrary;
(ii) no solution; (iii) $y(x) = e^{x/2}[2\sin(x/2) - \cos(x/2)]$.

9 (i) No solution; (ii) $y(x) = e^{-x}(3\cos x - 2\sin x)$;
(iii) $y(x) = e^{-x}(\cos x + c\sin x)$, $c =$ const arbitrary.

11 (i) $y(x) = e^{-4x}[c\sin(4x) - 3\cos(4x)]$, $c =$ const arbitrary;
(ii) $y(x) = e^{-4x}[\cos(4x) + 2\sin(4x)]$; (iii) no solution.

13 (i) $y(x) = 3\cos(2x/3) - 2\sin(2x/3)$; (ii) no solution;
(iii) $y = c\cos(2x/3) + 3\sin(2x/3)$, $c =$ const arbitrary.

15 (i) $y(x) = e^{-2x}[-(1/2)\cos x + c(\cos x + 2\sin x)]$, $c =$ const arbitrary;
(ii) no solution; (iii) $y(x) = e^{-2x}(-2\cos x + 3\sin x)$.

Subsection 4.4.1

1 $y(t) = 3e^{2t} - e^{-3t}$. **3** $y(t) = 3e^{-t} + e^{t/2}$. **5** $y(t) = -3 + e^{-4t}$.
7 $y(t) = 3e^{-t/3} - e^{t/2}$. **9** $y(t) = 3e^{3t} - 2e^{-t/2}$. **11** $y(t) = 6e^{-t} + e^{-4t}$.
13 $y(t) = 3e^{t} - e^{5t/2}$. **15** $y(t) = e^{-5t}$. **17** $y(x) = e^{-x} - e^{3x}$.
19 $y(x) = 3 - 2e^{2x}$. **21** $y(x) = (\operatorname{csch} 2)[3\sinh(2x) - 2\sinh(2(x-1))]$.
23 $y(x) = (\operatorname{sech} 8)[3\sinh(4x) - \cosh(4(x-2))]$.

Subsection 4.4.2

1 $y(t) = (2t - 3)e^{-3t}$. **3** $y(t) = 5(t + 2)e^{2t/5}$. **5** $y(t) = (2 - t)e^{-2t/3}$.
7 $y(t) = (t + 4)e^{-t/6}$. **9** $y(x) = (2x + 1)e^{x/3}$. **11** $y(t) = te^{-6t}$.
13 $y_2(t) = t^2$. **15** $y_2(t) = t^{-1}\ln t$. **17** $y_2(t) = -(t^2 + 1)$.
19 $y_2(t) = t^2 - 1$.

Subsection 4.4.3

1 $y(t) = e^{-t}[\cos(2t) - 2\sin(2t)]$. **3** $y(t) = e^{3t}[(1/2)\cos t - 3\sin t]$.
5 $y(t) = e^{t/2}[2\cos(2t) + 6\sin(2t)]$. **7** $y(t) = e^{t/2}[6\cos(t/3) + 12\sin(t/3)]$.
9 $y(t) = e^{-t/2}[\cos(3t) - \sin(3t)]$. **11** $y(t) = e^{2t}[3\cos(5t) + 2\sin(5t)]$.

13 (i) $y(x) = e^{x}[\cos(2x) - 2\sin(2x)]$; (ii) no solution;
(iii) $y(x) = e^{x}[\cos(2x) + c\sin(2x)]$, $c =$ const arbitrary.

15 (i) No solution; (ii) $y(x) = e^{2x}[\cos(2x) + 3\sin(2x)]$;
(iii) $y(x) = e^{2x}\{4\cos(2x) + c[\sin(2x) - \cos(2x)]\}$, $c =$ const arbitrary.

17 (i) $y(x) = e^{-3x}[-\cos(3x) + c\sin(3x)]$, $c = \text{const arbitrary}$;

(ii) $y(x) = e^{3x}[-\cos(3x) - 2\sin(3x)]$; (iii) no solution.

19 (i) $y(x) = e^{2x/3}[3\cos x + (1/2)\sin x]$;

(ii) $y(x) = e^{2x/3}\{(15/4)\cos x + c[\sin x - (3/2)\cos x]\}$, $c = \text{const arbitrary}$;

(iii) no solution.

Subsection 4.5.1

1 $y_p(t) = 4t - 2$, $y(t) = 3e^{-2t} - 2e^{-t} + 4t - 2$.

3 $y_p(t) = 3t^2$, $y(t) = (t-2)e^{4t} + 3t^2$.

5 $y_p(t) = t^2 - 3t + 2$, $y(t) = e^t(\cos t + 2\sin t) + t^2 - 3t + 2$.

7 $y_p(t) = 2e^{-2t}$, $y(t) = 2e^{2t} - e^{-4t} + 2e^{-2t}$.

9 $y_p(t) = 2e^{-t}$, $y(t) = (t-3)e^{-2t/3} + 2e^{-t}$.

11 $y_p(t) = 2e^{3t}$, $y(t) = e^{t/2}[2\cos t - (1/2)\sin t] + 2e^{3t}$.

13 $y_p(t) = 2\sin(2t)$, $y(t) = 3e^{-3t} + e^t + 2\sin(2t)$.

15 $y_p(t) = -4\cos(2t) - 3\sin(2t)$, $y(t) = (10t+4)e^{-t} - 4\cos(2t) - 3\sin(2t)$.

17 $y_p(t) = (3/2)\cos(t/3)$, $y(t) = e^{3t}(-\cos t + \sin t) + (3/2)\cos(t/3)$.

19 $y_p(t) = -2te^{2t}$, $y(t) = e^{-t} - 2e^{t/2} - 2te^{2t}$.

21 $y_p(t) = (t-2)e^t$, $y(t) = (t-2)(e^{2t} + e^t)$.

23 $y_p(t) = 2e^{-t}\sin t$, $y(t) = e^t - 4e^{t/4} + 2e^{-t}\sin t$.

25 $y_p(t) = e^{-2t}(\cos t - 2\sin t)$, $y(t) = (1-2t)e^{-t/2} + e^{-2t}(\cos t - 2\sin t)$.

27 $y_p(t) = t\sin(2t)$, $y(t) = -e^{-t} + t\sin(2t)$.

29 $y_p(t) = -(t+1)\cos(2t)$, $y(t) = \cos t - (t+1)\cos(2t)$.

31 $y_p(x) = 2e^{2x}$, $y(x) = e^{-x} + 2e^{2x}$.

33 $y_p(x) = -x$, $y(x) = (\operatorname{sech} 2)[\cosh(2x) + \sinh(2(x-1))] - x$.

35 $y_p(x) = -2\cos(\pi x)$, $y(x) = (3-x)e^{2x} - 2\cos(\pi x)$.

37 $y_p(x) = -\cos(2x) + \sin(2x)$, $y(x) = e^{2x}(-2\cos x - \sin x) - \cos(2x) + \sin(2x)$.

Subsection 4.5.2

1 $y_p(t) = 2te^{3t}$, $y(t) = e^{2t} + 2te^{3t}$.

3 $y_p(t) = -2te^{-t/2}$, $y(t) = (3-2t)e^{-t/2} + e^t$.

5 $y_p(t) = (2t - t^2)e^{4t}$, $y(t) = -3e^{-2t} + (2t - t^2)e^{4t}$.

7 $y_p(t) = (3t - 2t^2)e^{3t/2}$, $y(t) = (4 + 3t - 2t^2)e^{3t/2} - e^{-t}$.

9 $y_p(t) = 2t$, $y(t) = 2e^{-3t} + 2t - 1$.

11 $y_p(t) = 3t$, $y(t) = (1/2)e^{2t/3} + 3t - 2$.

13 $y_p(t) = 3t^2 - t$, $y(t) = 4e^t + 3t^2 - t$.

15 $y_p(t) = 3t^2 - t^3, \quad y(t) = 8e^{-t/4} + 3t^2 - t^3.$

17 $y_p(t) = 3t^2e^{-2t}, \quad y(t) = (3t^2 + t - 2)e^{-2t}.$

19 $y_p(t) = (1/2)t^2e^{t/3}, \quad y(t) = [(1/2)t^2 - t + 3)e^{t/3}.$

21 $y_p(t) = (t^3 - t^2)e^{3t/4}, \quad y(t) = (t^3 - t^2 - 2t)e^{3t/4}.$

23 $y_p(t) = -t\cos(2t), \quad y(t) = (1 - t)\cos(2t) + \sin(2t).$

25 $y_p(t) = -t[\cos(3t) - 3\sin(3t)], \quad y(t) = (2 - t)\cos(3t) + (3t + 1)\sin(3t).$

27 $y_p(t) = (t - t^2)\sin t, \quad y(t) = 2\cos t + (t - t^2)\sin t.$

29 $y_p(t) = (t^2 - 3t)\cos(t/3) - t^2\sin(t/3),$
$y(t) = (t^2 - 3t + 2)\cos(t/3) + (1 - t^2)\sin(t/3).$

31 $y_p(t) = -2te^t\sin t, \quad y(t) = e^t(3\cos t - 2t\sin t).$

33 $y_p(t) = -te^{2t}\cos(3t), \quad y(t) = e^{2t}[(1 - t)\cos(3t) + 3\sin(3t)].$

35 $y_p(t) = (t - t^2)e^{-2t}\cos(2t), \quad y(t) = e^{-2t}[(2 + t - t^2)\cos(2t) - 4\sin(2t)].$

37 $y_p(t) = e^{2t/3}[(t^2 + t)\cos t + (1 - 2t^2)\sin t],$
$y(t) = e^{2t/3}[(t^2 + t + 2)\cos t - 2(1 + t^2)\sin t].$

39 $y_p(t) = \alpha_0 t + \alpha_1 + t(\beta_0 t + \beta_1)e^t + (\gamma_0 t^2 + \gamma_1 t + \gamma_2)e^{-t} + \delta\cos t + \varepsilon\sin t.$

41 $y_p(t) = t(\alpha_0 t + \alpha_1) + (\beta_0 t + \beta_1)\cos(3t) + (\gamma_0 t + \gamma_1)\sin(3t) + t(\delta_0 t^2 + \delta_1 t + \delta_2)e^{2t/3} + (\varepsilon_0 t + \varepsilon_1)e^t.$

43 $y_p(t) = \alpha + (\beta_0 t + \beta_1)e^{-3t} + t^2(\gamma_0 t + \gamma_1)e^{3t} + e^{3t}[\delta\cos(2t) + \varepsilon\sin(2t)].$

45 $y_p(t) = (\alpha_0 t + \alpha_1)\cos t + (\beta_0 t + \beta_1)\sin t + (\gamma_0 t^2 + \gamma_1 t + \gamma_2)e^{3t} + \delta e^t + te^{3t}(\varepsilon\cos t + \zeta\sin t).$

Subsection 4.5.3

1 $y_p(t) = 3t - 1.$ **3** $y_p(t) = 2t + 7.$ **5** $y_p(t) = -t^2.$

7 $y_p(t) = 2e^{-2t}.$ **9** $y_p(t) = e^t.$ **11** $y_p(t) = -2e^{-t}.$ **13** $y_p(t) = 6e^{t/2}.$

15 $y_p(t) = t^2e^{-t}.$ **17** $y_p(t) = te^{3t}.$ **19** $y_p(t) = -(2t + 1/3)e^{-3t}.$

21 $y_p(t) = 2t - 1.$ **23** $y_p(t) = (1/4)\cos(2t) + t\sin(2t).$ **25** $y_p(t) = (1/2)t^{-1}e^t.$

27 $y_p(t) = t^2 - t + 1.$ **29** $y_p(t) = (2t - 1)^2e^{2t}.$

Section 4.6

1 $y(t) = t^2 - 3t^{-2}.$ **3** $y(t) = t^2 - 4t^{-1/3}.$ **5** $y(t) = 2t^{1/2} - 4t^{-1/3}.$

7 $y(t) = 3t^{-2} - t^{1/2}.$ **9** $y(t) = -3t^{-2}.$ **11** $y(t) = t(1 - 2\ln t).$

13 $y(t) = t^{-1}(2 - 3\ln t).$ **15** $y(t) = t^3(2 + \ln t).$

17 $y(t) = t^{-2}[3\cos(\ln t) - \sin(\ln t)].$ **19** $y(t) = t^{1/2}[2\cos(2\ln t) - 5\sin(2\ln t)].$

21 $y_p(t) = t^4/13.$ **23** $y_p(t) = -t^{3/4}/4.$ **25** $y_p(t) = -2t^{-3}.$

Section 4.7

1 $y(t) = (1/2)e^{2t} - 3/2, \quad -\infty < t < \infty.$

3 $y(t) = (1/3)\ln(t/(3-t)), \quad 0 < t < 3.$

5 $y(t) = \ln((1-t)/(2-t)), \quad -\infty < t < 1.$

7 $y(t) = -\tan^{-1} t, \quad -\pi/2 < t < \pi/2.$

9 $y(t) = (1-2t)^{1/2}, \quad -\infty < t < 1/2.$

11 $y(t) = (1/2)(t^2 + 4t + 5), \quad -\infty < t < \infty.$

13 $y(t) = 3e^{2t} - 1/2, \quad -\infty < t < \infty.$

15 $y(t) = 2e^t - 1, \quad -\infty < t < \infty.$

Chapter 5

Subsection 5.1.1

1 $y(t) = 2\cos t - 3\sin t; \quad y(0) = 2, \quad y'(0) = -3; \quad A = \sqrt{13}.$

3 (i) $y(t) = 2\cos(3t)$. (ii) $t = \pi/6,\ \pi/2,\ 5\pi/6.$

5 $\omega = 2; \quad y(t) = \begin{cases} 3\sin(4t), & 0 < t \le 3\pi/8, \\ (3/\sqrt{2}\,)[\cos(2t) - \sin(2t)], & t > 3\pi/8. \end{cases}$

Subsection 5.1.2

1 (i) Critical damping. (ii) $y(t) = (2-t)e^{-3t}$. (iii) $v = -e^{-6}$.

3 (i) $y(t) = \begin{cases} 2\sin t, & 0 < t \le \pi/2, \\ (8/3)e^{\pi/4 - t/2} - (2/3)e^{\pi - 2t}, & t > \pi/2. \end{cases}$ (ii) Overdamping.

5 (i) Overdamping.

(ii) $y'' + 2\sqrt{3}\,y' + 3y = 0, \quad y(\ln(3/2)) = 41/27, \quad y'(\ln(3/2)) = -41\sqrt{3}/27.$

(iii) $y(t) = \begin{cases} -(1/2)e^{-3t} + (5/2)e^{-t}, & 0 < t \le \ln(3/2), \\ (41/27)(3/2)^{\sqrt{3}}e^{-\sqrt{3}\,t}, & t > \ln(3/2). \end{cases}$

Subsection 5.2.1

1 $y(t) = \begin{cases} \cos(t/2), & 0 < t \le \pi, \\ \cos(t/2) - 2\sin(t/2) - 2\cos t, & t > \pi. \end{cases}$

3 $y(t) = \begin{cases} -\cos t - \cos(2t), & 0 < t \le \pi/2, \\ -\cos t + \sin t, & t > \pi/2. \end{cases}$

5 $y(t) = \begin{cases} (2+t)\sin(3t), & 0 < t \le \pi/6, \\ -(1/3)\cos(3t) + (2+\pi/6)\sin(3t), & t > \pi/6. \end{cases}$

Subsection 5.2.2

1 (i) Overdamping.

(ii) $y'' + 4y' + 3y = 0, \quad y(\pi) = 3e^{-3\pi} - 5e^{-\pi} - 2, \quad y'(\pi) = -9e^{-3\pi} + 5e^{-\pi} + 2.$

(iii) $y(t) = \begin{cases} 3e^{-3t} - 5e^{-t} + 2\cos t + 4\sin t, & 0 < t \le \pi, \\ 3e^{-3t} - (2e^{\pi} + 5)e^{-t}, & t > \pi. \end{cases}$

3 (i) Overdamping.

(ii) $y'' + 4y' + 4y = 25\cos t, \quad y(\pi) = 5e^{-4\pi} - 16e^{-\pi} + 8,$

$y'(\pi) = (5/\pi - 10)e^{-4\pi} + 32e^{-\pi} - 26.$

(iii) $y(t) = \begin{cases} 5e^{-4t} - 16e^{-t} + 12\cos(t/2) + 8\sin(t/2), & 0 < t \le \pi, \\ [11e^{2\pi} - 16e^{\pi} + (5/\pi)e^{-2\pi}t]e^{-2t} + 3\cos t + 4\sin t, & t > \pi. \end{cases}$

Section 5.3

1 (i) $Q(t) = e^{-2t} + 7\cos t + 6\sin t; \quad I(t) = -2e^{-2t} + 6\cos t - 7\sin t.$

(ii) Forced response $\approx \sqrt{85}\,\cos(t - 0.7); \quad t \approx 0.7.$

3 $I(t) = \begin{cases} \sin t + 4\cos(t/2), & 0 < t \le \pi, \\ (3/25)(35\pi - 16 - 35t)e^{\pi - t} + 12\cos(t/2 + 16\sin(t/2), & t > \pi. \end{cases}$

Chapter 6

Subsection 6.2.1

1 $-15.$ **3** $-8.$ **5** $-20.$ **7** $a = -1.$ **9** $a = 3, -9/2.$

Subsection 6.2.2

1 $x_1 = -1, \quad x_2 = 2, \quad x_3 = 1.$

3 $x_1 = -4 - 4a, \quad x_2 = 6 + 6a, \quad x_3 = a.$

5 No solution. **7** $x_1 = -3, \quad x_2 = -1, \quad x_3 = 2.$

9 $x_1 = -1, \quad x_2 = 2, \quad x_3 = -2, \quad x_4 = 1.$

11 $x_1 = -2a, \quad x_2 = -1 + 3a, \quad x_3 = 2 - 9a, \quad x_4 = 5a.$

13 No solution. **15** $x_1 = -1, \quad x_2 = 2, \quad x_3 = 4, \quad x_4 = 1.$

Subsection 6.2.3

1 $W[f_1, f_2, f_3](x) = 0$; linearly dependent.

3 $W[f_1, f_2, f_3](t) = (6t - 5)e^{2t}$; linearly independent.

5 $W[f_1, f_2, f_3](t) = 2$; $a = 1$, $b = 2$, $c = -1$.

7 $W[f_1, f_2, f_3](x) = e^{2x}$; $a = 3$, $b = -2$, $c = 1$.

9 $W[f_1, f_2, f_3](x) = 2\cos x$; $a = 3$, $b = -1$, $c = 2$.

Section 6.3

1 $y(t) = c_1e^{-2t} + c_2 + c_3e^{4t}$. **3** $y(t) = (c_1 + c_2t)e^{-t} + c_3e^t$.

5 $y(t) = (c_1 + c_2t)e^{t/2} + e^{-t}[c_3\cos(2t) + c_4\sin(2t)]$.

7 $y(t) = e^{2t}[c_1\cos(3t) + c_2\sin(3t)] + (c_3t + c_4)e^{-2t} + c_5$.

9 $y(t) = 2 - e^{-2t}$. **11** $y(t) = (2 - t)e^t + 2e^{-3t}$.

13 $y(t) = -2e^{2t} + e^{-t/2}(4\cos t + \sin t)$.

15 $y(t) = e^{-t} + e^{3t}(2\cos t - \sin t)$. **17** $y(t) = 2e^{-t} - 3e^t - e^{-2t} + 8e^{t/2}$.

19 $y(t) = -6 + e^{2t} - 2e^t\cos t$.

Subsection 6.4.1

1 $y_p(t) = \alpha + \beta e^{2t} + t(\gamma_1 + \gamma_2t)e^t + t^2(\delta_1 + \delta_2t)e^{4t}$.

3 $y_p(t) = \alpha t + t^2(\beta_1 + \beta_2t + \beta_3t^2)e^{-3t} + \gamma_1\cos t + \gamma_2\sin t + t[\delta_1\cos(2t) + \delta_2\sin(2t)]$.

5 $y_p(t) = (\alpha_1 + \alpha_2t)e^t + t^2(\beta_1 + \beta_2t + \beta_3t^2)e^{t/3} + e^t(\gamma_1\cos t + \gamma_2\sin t) + t[(\delta_1 + \delta_2t)\cos(4t) + (\varepsilon_1 + \varepsilon_2t)\sin(4t)]$.

7 $y_p(t) = t^3(\alpha_1 + \alpha_2t) + t^2(\beta_1 + \beta_2t + \beta_3t^2)e^{2t} + (\gamma_1 + \gamma_2t)e^t + e^{-t}(\delta_1\cos t + \delta_2\sin t)$.

9 $y(t) = 1 + e^{-2t} - 2e^{4t} + 3e^t$. **11** $y(t) = (3 - t)e^{-t} + e^{4t} - t^2e^{-t}$.

13 $y(t) = -e^{-2t} + e^t(\cos t + 2\sin t) + t - 3$.

15 $y(t) = (2t - 1)e^t - t^2 - t - 2$. **17** $y(t) = e^t - e^{-2t} + (2t + 2)\cos t$.

19 $y(t) = 2e^{-t} - 3e^{2t} + (2t^2 + t - 1)e^t$.

Subsection 6.4.2

1 $y_p(t) = 2t$. **3** $y_p(t) = (t - 7/12)e^t$. **5** $y_p(t) = (t^2 - 1/3)e^{-t}$.

7 $y_p(t) = t[\cos(2t) + \sin(2t)]$. **9** $y_p(t) = t^3e^{-t}$.

Chapter 7

Subsection 7.2.1

1 $\begin{pmatrix} 3 & -2 \\ -8 & 8 \end{pmatrix}$, $\begin{pmatrix} 8 & -3 \\ -5 & -2 \end{pmatrix}$, $\begin{pmatrix} 0 & -4 \\ -1 & 8 \end{pmatrix}$, $\begin{pmatrix} -2 & 1 \\ -16 & 10 \end{pmatrix}$,

$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$, $\frac{1}{4}\begin{pmatrix} -4 & 0 \\ -2 & 1 \end{pmatrix}$.

3 $\begin{pmatrix} 3 & -5 \\ 9 & 4 \end{pmatrix}$, $\begin{pmatrix} 8 & -18 \\ -4 & -1 \end{pmatrix}$, $\begin{pmatrix} -22 & -2 \\ 3 & 8 \end{pmatrix}$, $\begin{pmatrix} 4 & 7 \\ 14 & -18 \end{pmatrix}$,

$\frac{1}{10}\begin{pmatrix} 1 & 4 \\ -2 & 2 \end{pmatrix}$, $\frac{1}{17}\begin{pmatrix} -2 & 3 \\ 5 & 1 \end{pmatrix}$.

5 $\begin{pmatrix} 1 & -7 \\ 8 & 0 \end{pmatrix}$, $\begin{pmatrix} 12 & -14 \\ -9 & 7 \end{pmatrix}$, $\begin{pmatrix} -4 & -6 \\ 6 & -1 \end{pmatrix}$, $\begin{pmatrix} -1 & 12 \\ -3 & -4 \end{pmatrix}$,

$\frac{1}{5}\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$, $\frac{1}{8}\begin{pmatrix} 1 & -3 \\ -1 & -1 \end{pmatrix}$.

7 $\begin{pmatrix} 7 & -2 & -2 \\ -3 & 4 & 1 \\ -1 & 8 & 6 \end{pmatrix}$, $\begin{pmatrix} 7 & 4 & -3 \\ -8 & -1 & 12 \\ 2 & 12 & -5 \end{pmatrix}$, $\begin{pmatrix} 4 & -6 & -4 \\ -3 & 6 & 5 \\ 3 & 8 & -8 \end{pmatrix}$, $\begin{pmatrix} 7 & -2 & -5 \\ -1 & -10 & 0 \\ -3 & 16 & 5 \end{pmatrix}$,

$\frac{1}{13}\begin{pmatrix} 7 & 4 & -1 \\ -2 & -3 & 4 \\ 8 & 12 & -3 \end{pmatrix}$, $\frac{1}{10}\begin{pmatrix} 8 & 8 & 6 \\ -1 & 4 & 3 \\ 2 & 2 & 4 \end{pmatrix}$.

9 $\begin{pmatrix} 2 & 0 & 3 \\ 0 & 3 & -7 \\ 12 & -1 & -14 \end{pmatrix}$, $\begin{pmatrix} 10 & -7 & 1 \\ 0 & 1 & -7 \\ 4 & 9 & -21 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 & 3 \\ -12 & 10 & -1 \\ -36 & 30 & 3 \end{pmatrix}$, $\begin{pmatrix} 0 & 5 & -15 \\ -4 & 0 & 4 \\ 8 & -7 & 13 \end{pmatrix}$,

A is singular, $\frac{1}{6}\begin{pmatrix} 3 & 3 & 3 \\ 4 & 4 & 2 \\ 4 & -2 & 2 \end{pmatrix}$.

Subsection 7.2.2

1 Linearly independent. **3** Linearly dependent.

5 Linearly independent if $a \neq -3$. Linearly dependent if $a = -3$.

7 Linearly dependent only at $t = 0, -1, 31/34$. Linearly independent on the set of real numbers.

9 Linearly dependent for every real value of t. Linearly dependent on the set of real numbers.

Subsection 7.2.3

1 $r = -3, -1;\quad \mathbf{v}^{(1)} = (2, 1)^{\mathrm{T}},\quad \mathbf{v}^{(2)} = (3, 1)^{\mathrm{T}}.$

3 $r = i, -i;\quad \mathbf{v}^{(1)} = (1 + i, 1)^{\mathrm{T}},\quad \mathbf{v}^{(2)} = (1 - i, 1)^{\mathrm{T}}.$

5 $r = 0, 2, -1;\quad \mathbf{v}^{(1)} = (-1, 0, 1)^{\mathrm{T}},\quad \mathbf{v}^{(2)} = (1, 1, 0)^{\mathrm{T}},\quad \mathbf{v}^{(3)} = (0, 1, 2)^{\mathrm{T}}.$

7 $r = 1, 1;\quad \mathbf{v} = (1, 2)^{\mathrm{T}}.$

9 $r = 0, 1, 1;\quad \mathbf{v}^{(1)} = (1, 2, 1)^{\mathrm{T}},\quad \mathbf{v}^{(2)} = (-1, 1, 0)^{\mathrm{T}},\quad \mathbf{v}^{(3)} = (0, 2, 1)^{\mathrm{T}}.$

11 $r = 1, -1, -1;\quad \mathbf{v}^{(1)} = (2, -1, 0)^{\mathrm{T}},\quad \mathbf{v}^{(2)} = (1, 0, 1)^{\mathrm{T}}.$

13 $r = 1, 1, 1;\quad \mathbf{v}^{(1)} = (1, 1, 0)^{\mathrm{T}},\quad \mathbf{v}^{(2)} = (-2, 0, 1)^{\mathrm{T}}.$

15 $r = -1, -1, -1;\quad \mathbf{v} = (0, 1, 2)^{\mathrm{T}}.$

Section 7.3

1 $\mathbf{x}(t) = \begin{pmatrix} 4e^{2t} - 3e^{3t} \\ 2e^{2t} - 3e^{3t} \end{pmatrix}.$ **3** $\mathbf{x}(t) = \begin{pmatrix} (2 - 8t)e^{t} \\ (1 + 4t)e^{t} \end{pmatrix}.$

5 $\mathbf{x}(t) = \begin{pmatrix} 2\cos t - 5\sin t \\ -\cos t - 12\sin t \end{pmatrix}.$ **7** $\mathbf{x}(t) = \begin{pmatrix} 4e^{-t} - 3e^{-4t} + 2t + 1 \\ 2e^{-t} - 3e^{-4t} + t \end{pmatrix}.$

9 $\mathbf{x}(t) = \begin{pmatrix} (3t + 7)e^{-2t} - \sin t \\ -(2t + 5)e^{-2t} + 2\cos t \end{pmatrix}.$

Subsection 7.4.1

1 $\mathbf{x}(t) = \begin{pmatrix} 2e^{-4t} + 8e^{-t} \\ -2e^{-4t} + 4e^{-t} \end{pmatrix}.$ **3** $\mathbf{x}(t) = \begin{pmatrix} 9e^{-2t} + e^{5t} \\ -3e^{-2t} + 2e^{5t} \end{pmatrix}.$

5 $\mathbf{x}(t) = \begin{pmatrix} 3e^{2t} + 2e^{6t} \\ 2e^{2t} - 4e^{6t} \end{pmatrix}.$ **7** $\mathbf{x}(t) = \begin{pmatrix} -3e^{-6t} + 8e^{t} \\ 12e^{-6t} - 4e^{t} \end{pmatrix}.$

9 $\mathbf{x}(t) = \begin{pmatrix} 2e^{t/2} + 2e^{4t} \\ -e^{t/2} + 6e^{4t} \end{pmatrix}.$ **11** $\mathbf{x}(t) = \begin{pmatrix} -2e^{-11t} - 6e^{-t} \\ 6e^{-11t} - 2e^{-t} \end{pmatrix}.$

13 $\mathbf{x}(t) = \begin{pmatrix} 2e^{t} - e^{2t} \\ 2e^{t} \\ -e^{2t} \end{pmatrix}.$ **15** $\mathbf{x}(t) = \begin{pmatrix} 3e^{-2t} - 2e^{t} + e^{2t} \\ -e^{t} + e^{2t} \\ 3e^{-2t} - e^{t} + e^{2t} \end{pmatrix}.$

17 $\mathbf{x}(t) = \begin{pmatrix} -2 - e^{t} \\ 2 + e^{-t} \\ -e^{-t} + 2e^{t} \end{pmatrix}.$

Subsection 7.4.2

1 $\mathbf{x}(t) = \begin{pmatrix} 2\cos(2t) - \sin(2t) \\ 3\cos(2t) - 4\sin(2t) \end{pmatrix}$. **3** $\mathbf{x}(t) = \begin{pmatrix} -4\cos(4t) - 3\sin(4t) \\ -2\cos(4t) + 2\sin(4t) \end{pmatrix}$.

5 $\mathbf{x}(t) = \begin{pmatrix} e^t(-\cos t + 2\sin t) \\ 5e^t \sin t \end{pmatrix}$.

7 $\mathbf{x}(t) = \begin{pmatrix} e^{-t}[4\cos(t/2) + (1/2)\sin(t/2)] \\ e^{-t}[-2\cos(t/2) + 3\sin(t/2)] \end{pmatrix}$.

9 $\mathbf{x}(t) = \begin{pmatrix} e^{-3t}[-(3/2)\cos t - (1/2)\sin t] \\ e^{-3t}[\cos t - (1/2)\sin t] \end{pmatrix}$. **11** $\mathbf{x}(t) = \begin{pmatrix} 4 - \cos t - \sin t \\ 2 - 2\sin t \\ -\sin t \end{pmatrix}$.

13 $\mathbf{x}(t) = \begin{pmatrix} -e^t \sin t \\ 2e^{-t} + e^t \cos t \\ -e^t \cos t \end{pmatrix}$. **15** $\mathbf{x}(t) = \begin{pmatrix} -2e^t + 2e^{-t}\sin(2t) \\ -2e^t + e^{-t}\sin(2t) \\ e^{-t}\cos(2t) \end{pmatrix}$.

Subsection 7.4.3

1 $\mathbf{x}(t) = \begin{pmatrix} 4te^{-t} \\ (1-2t)e^{-t} \end{pmatrix}$. **3** $\mathbf{x}(t) = \begin{pmatrix} -(t+3)e^{t/3} \\ (t+4)e^{t/3} \end{pmatrix}$.

5 $\mathbf{x}(t) = \begin{pmatrix} -9te^{-2t} \\ (1-3t)e^{-2t} \end{pmatrix}$. **7** $\mathbf{x}(t) = \begin{pmatrix} 1 + e^t \\ 1 - e^t \\ 1 + 2e^t \end{pmatrix}$.

9 $\mathbf{x}(t) = \begin{pmatrix} 2e^{-t} + 2e^{2t} \\ e^{-t} - 2e^{2t} \\ e^{-t} + 2e^{2t} \end{pmatrix}$. **11** $\mathbf{x}(t) = \begin{pmatrix} 4 + (1-t)e^t \\ (t-2)e^t \\ 2 - e^t \end{pmatrix}$.

13 $\mathbf{x}(t) = \begin{pmatrix} t - 1 \\ 3 - 2t \\ 1 - t \end{pmatrix} e^{-t}$. **15** $\mathbf{x}(t) = \begin{pmatrix} 1 - 4t - 2t^2 \\ 1 - 2t \\ -1 - t - t^2 \end{pmatrix} e^t$.

Section 7.5

1 $e^{At} = \begin{pmatrix} -e^{-t} + 2e^t & -2e^{-t} + 2e^t \\ e^{-t} - e^t & 2e^{-t} - e^t \end{pmatrix}$, $\mathbf{x}(t) = \begin{pmatrix} -4e^{-t} + 2e^t \\ 4e^{-t} - e^t \end{pmatrix}$.

3 $e^{At} = \begin{pmatrix} \cos t & -2\sin t \\ (1/2)\sin t & \cos t \end{pmatrix} e^t, \quad \mathbf{x}(t) = \begin{pmatrix} -2\cos t - 2\sin t \\ \cos t - \sin t \end{pmatrix} e^t.$

5 $e^{At} = \begin{pmatrix} 1-t & t \\ -t & t+1 \end{pmatrix} e^{-2t}, \quad \mathbf{x}(t) = \begin{pmatrix} t-1 \\ t \end{pmatrix} e^{-2t}.$

7 $e^{At} = \begin{pmatrix} 3-2e^{2t} & -6+6e^{2t} & 3-3e^{2t} \\ 1-e^{-t} & -2+3e^{-t} & 1-e^{-t} \\ -2e^{-t}+2e^{2t} & 6e^{-t}-6e^{2t} & -2e^{-t}+3e^{2t} \end{pmatrix}, \quad \mathbf{x}(t) = \begin{pmatrix} -3+3e^{2t} \\ -1+2e^{-t} \\ 4e^{-t}-3e^{2t} \end{pmatrix}.$

9 $\mathbf{x}(t) = \begin{pmatrix} 2e^{-t} - \cos t + 2\sin t \\ -e^{-t} + \cos t - 2\sin t \\ 2\cos t + \sin t \end{pmatrix}.$ **11** $\mathbf{x}(t) = \begin{pmatrix} 3e^{-2t} - e^{2t} \\ -3e^{-2t} + 2e^{2t} \end{pmatrix}.$

13 $\mathbf{x}(t) = \begin{pmatrix} 2e^{t/2} + e^{2t} \\ -e^{t/2} - e^{2t} \end{pmatrix}.$ **15** $\mathbf{x}(t) = \begin{pmatrix} -\sin(t/2) \\ \cos(t/2) + \sin(t/2) \end{pmatrix}.$

Section 7.6

1 $\mathbf{x}(t) = \begin{pmatrix} e^t - 1 \\ -e^t + t + 2 \end{pmatrix}.$ **3** $\mathbf{x}(t) = \begin{pmatrix} 2e^{-t} - 2e^{4t} - 1 + e^{2t} \\ 2e^{-t} - e^{4t} + e^{2t} \end{pmatrix}.$

5 $\mathbf{x}(t) = \begin{pmatrix} -3e^{-2t} + (t+1)e^{3t} \\ -3e^{-2t} - 2(t+1)e^{3t} \end{pmatrix}.$ **7** $\mathbf{x}(t) = \begin{pmatrix} -2e^{-2t} + e^{-t} + 4e^t \\ 2e^{-2t} \\ -2e^{-2t} - 1 + 2e^t \end{pmatrix}.$

9 $\mathbf{x}(t) = \begin{pmatrix} e^{-t} + t + 2 \\ 2e^{-t} + t - 1 \end{pmatrix}.$ **11** $\mathbf{x}(t) = \begin{pmatrix} 6e^{3t} + e^{2t} + 2e^{-t} \\ 3e^{3t} + e^{2t} - e^{-t} \end{pmatrix}.$

13 $\mathbf{x}(t) = \begin{pmatrix} (2t-1)e^t - e^{t/2} \\ (3-t)e^t - 2e^{t/2} \end{pmatrix}.$ **15** $\mathbf{x}(t) = \begin{pmatrix} -2e^{-t} + e^{2t} \\ 2e^t + 2e^{2t} \\ e^{-t} - 2e^t + 2e^{2t} \end{pmatrix}.$

17 $\mathbf{x}(t) = \begin{pmatrix} 2\sin(t/2) + 2e^t \\ -2\cos(t/2) - 4\sin(t/2) + 2 - 8e^t \end{pmatrix}.$

19 $\mathbf{x}(t) = \begin{pmatrix} (2t-1)e^{-2t} + e^t + 1 \\ (6t-1)e^{-2t} + 6e^t + 1 \end{pmatrix}.$ **21** $\mathbf{x}(t) = \begin{pmatrix} -\sin t + 4 + 2t \\ -\sin t + 2 + t \\ -\cos t + 2 \end{pmatrix}.$

23 $\mathbf{x}(t) = \begin{pmatrix} 4\cos(2t) - 2\sin(2t) + 3 \\ 2\cos(2t) + 2 \end{pmatrix}.$ **25** $\mathbf{x}(t) = \begin{pmatrix} (4t+2)e^{-t} + 1 \\ -(2t+2)e^{-t} - 2 \end{pmatrix}.$

Chapter 8

Section 8.1

1 $F(s) = \dfrac{2 - 3e^{-2s}}{s}$. **3** $F(s) = \dfrac{1 - e^{-s}}{s^2} - \dfrac{2e^{-s}}{s}$. **5** $F(s) = \dfrac{e^{6-3s}}{s-2}$.

7 $F(s) = \dfrac{3}{s^2} - \dfrac{2}{s} + \dfrac{4}{s^2+16}$. **9** $F(s) = \dfrac{3}{s-2} - \dfrac{2s}{s^2+9}$.

11 $f(t) = e^{2t} - 1$. **13** $f(t) = 2e^t - 3e^{-2t}$. **15** $f(t) = 2 - e^t + 3e^{-t}$.

17 $f(t) = 1 - 2t + e^{-t}$. **19** $f(t) = 2 - \cos t$. **21** $f(t) = 2 + 3\cos t - \sin t$.

23 $f(t) = e^t - 2\cos t$. **25** $f(t) = 1 - e^{-4t}$. **27** $f(t) = e^{2t} - e^{-3t}$.

29 $f(t) = 1 - \cos(2t)$.

Section 8.2

1 $F(s) = \dfrac{1}{s^2}\, e^{-4s}$. **3** $F(s) = \dfrac{1}{s+1}\, e^{-3s}$.

5 $F(s) = \dfrac{3s+2}{s^2}\, e^{-2s}$. **7** $F(s) = \dfrac{1}{s-2}\, e^{2-s}$.

9 $F(s) = \dfrac{2s-7}{(s-4)^2}$. **11** $F(s) = -\dfrac{2s}{(s+1)^3}$.

13 $F(s) = \dfrac{2}{s+1} - \dfrac{6}{s^2+2s+5}$. **15** $F(s) = \dfrac{s-7}{s^2-2s+5}$.

17 $f(t) = (t-3)H(t-1)$. **19** $f(t) = -\sin(\pi(t-1))H(t-1)$.

21 $f(t) = tH(t-1) + e^tH(t-2)$. **23** $f(t) = (2t-1)e^{-2t}$.

25 $f(t) = (t^2+t)e^{-t}$. **27** $f(t) = e^{-t/2}[3\cos(2t) + 2\sin(2t)]$.

29 $f(t) = (t-1)e^{2(t-1)}H(t-1)$.

Subsection 8.3.1

1 $y(t) = e^{-t} - 2e^{-3t} - 1$. **3** $y(t) = (t-2)e^{-t} + 4$.

5 $y(t) = 3\cos t - \sin t + 1$. **7** $y(t) = t - e^{2t} - 2e^{3t}$.

9 $y(t) = te^{3t} + 2 - t$. **11** $y(t) = e^{2t}(2\cos t - 3\sin t) + 2t - 1$.

13 $y(t) = e^t - 2e^{-5t} - 2e^{2t}$. **15** $y(t) = (2t-3)e^{2t/3} + 2e^{-t}$.

17 $y(t) = 2e^{-t}[\sin(t/2) - 2\cos(t/2)] - e^{2t}$.

19 $y(t) = 3 - e^{2t} + 2\cos(2t) + \sin(2t)$.

21 $y(t) = -2e^{-3t} - \cos(2t) + \sin(2t)$. **23** $y(t) = e^{2t} - e^{-5t} + e^{-t} - 3$.

25 $y(t) = 3t + 1 + e^{-t}$. **27** $y(t) = e^{-2t} - 2e^{2t} - 2t + 3$.

29 $y(t) = 2 + (t-1)e^t + e^{2t}$. **31** $y(t) = e^t - 2e^{-2t} + 1$.

Subsection 8.3.2

1 $y(y) = (e^{t-1} - t)H(t-1)$.

3 $y(t) = 2e^{t/2} - t - 2 + [t + 1 - 2e^{(t-1)/2}]H(t-1)$.

5 $y(t) = [e^{-2(t-1/2)} + 2e^{t-1/2} - 3]H(t - 1/2)$.

7 $y(t) = [1 + (t-3)e^{t-2}]H(t-2)$.

9 $y(t) = \{1 - e^{2(t-1)}[\cos(t-1) - 2\sin(t-1)]\}H(t-1)$.

11 $y(t) = [3e^{2(t-1)} - 8e^{t-1} + 2t + 3]H(t-1)$.

13 $y(t) = 2e^{2t} - 2e^{-t/2} + [e^{2(t-3)} + 4e^{(t-3)/2} - 5]H(t-3)$.

15 $y(t) = (3t+2)e^{-t} + [t - 2 + (10 - 3t)e^{4-t}]H(t-4)$.

17 $y(t) = [t - 4 + 3e^{(1-t)/3}]H(t-1) + [5 - t - 3e^{(2-t)/3}]H(t-2)$.

19 $$y(t) = [3t - 3\cos(3(t-1)) - \sin(3(t-1))]H(t-1) + [\cos(3(t-2)) - 1]H(t-2).$$

Subsection 8.3.3

1 $y(t) = [e^{2(t-2)} - 1]H(t-2)$. **3** $y(t) = e^t - t - 1 + (1 - e^{t-1})H(t-1)$.

5 $y(t) = -\cos(2t) + (1/2)\sin(2(t-3))H(t-3)$.

7 $y(t) = \sinh t - t - \sinh(t-1)H(t-1)$.

9 $y(t) = 2\cos(3t) + 3\sin(3t) + \sin(3(t-2))H(t-2)$.

11 $y(t) = [e^{1-t} - e^{2(1-t)}]H(t-1)$.

13 $y(t) = 3 - (t+1)e^{-t} - (t - 1/2)e^{1/2-t}H(t-1/2)$.

15 $y(t) = [e^{\pi-t} - e^{3(t-\pi)}]H(t-\pi)$.

17 $y(t) = 1 + 2(1 - e^{1-t})H(t-1) - (1 - e^{2-t})H(t-2)$.

19 $$y(t) = \cosh(2t) - (1/2)\sinh(2t) + (1/2)\sinh(2t-1)H(t-1/2) + (1/2)\sinh(2(t-1))H(t-1).$$

Subsection 8.3.4

1 $y(t) = -t^2$. **3** $y(t) = 2t^2 + 3$. **5** $y(t) = 3t - 1$. **7** $y(t) = 5 - 2t$.

9 $y(t) = t^2 - t$. **11** $y(t) = t^2 - 3t - 1$. **13** $y(t) = 3 - t$. **15** $y(t) = 2t - 3$.

Section 8.4

1 $\mathbf{x}(t) = \begin{pmatrix} 2e^{-t} - e^{3t} \\ 4e^{-t} - e^{3t} \end{pmatrix}$. **3** $\mathbf{x}(t) = \begin{pmatrix} 2\cos(2t) - \sin(2t) \\ 3\cos(2t) - 4\sin(2t) \end{pmatrix}$.

5 $\mathbf{x}(t) = \begin{pmatrix} 1-6t \\ 3-9t \end{pmatrix} e^{-t}$. **7** $\mathbf{x}(t) = \begin{pmatrix} 2e^t - 3e^{-2t} + 2 \\ e^t - e^{-2t} + 1 \end{pmatrix}$.

9 $\mathbf{x}(t) = \begin{pmatrix} (6t+13)e^{-t} + 3 \\ -(3t+5)e^{-t} - 1 \end{pmatrix}$. **11** $\mathbf{x}(t) = \begin{pmatrix} e^{3t}(\cos t - 2\sin t) + 1 \\ -e^{3t}(2\cos t + \sin t) + 1 \end{pmatrix}$.

13 $\mathbf{x}(t) = \begin{pmatrix} 2e^t - 3e^{t/2} + 2t + 2 \\ -e^t + 3e^{t/2} - 3t - 4 \end{pmatrix}$. **15** $\mathbf{x}(t) = \begin{pmatrix} -e^{t/2}(2\cos t + \sin t) + 4t + 2 \\ e^{t/2}(\cos t - 2\sin t) + 2t + 2 \end{pmatrix}$.

17 $\mathbf{x}(t) = \begin{pmatrix} (1-t)e^t + e^{-t} \\ -te^t - 3e^{-t} \end{pmatrix}$. **19** $\mathbf{x}(t) = \begin{pmatrix} 2\cos(2t) - 2\sin(2t) + 2 \\ -2\sin(2t) + 1 \end{pmatrix} e^t$.

21 $\mathbf{x}(t) = \begin{pmatrix} e^t - e^{-2t} \\ 2 + e^t \\ 4 + e^{-2t} \end{pmatrix}$. **23** $\mathbf{x}(t) = \begin{pmatrix} (t+4)e^{-t} + 1 \\ e^{-t} + 1 \\ (t+2)e^{-t} \end{pmatrix}$.

25 $\mathbf{x}(t) = \begin{pmatrix} e^t(\sin t - \cos t) - 3 \\ e^t(\cos t - \sin t) + 2 \\ -2e^t(\cos t + \sin t) - 3 \end{pmatrix}$.

Chapter 9

Section 9.2

1 $y(t) = c_1\big(1 + \frac{1}{2}t^2 - \frac{1}{6}t^3 + \frac{1}{12}t^4 - \frac{1}{40}t^5 + \cdots\big)$
$+ c_2\big(t - \frac{1}{2}t^2 + \frac{1}{3}t^3 - \frac{1}{8}t^4 + \frac{1}{24}t^5 - \cdots\big); \quad -\infty < t < \infty.$

3 $y(t) = c_1\big(1 - \frac{3}{2}t^2 + 2t^3 - \frac{13}{8}t^4 + t^5 - \cdots\big)$
$+ c_2\big(t - 2t^2 + \frac{13}{6}t^3 - \frac{5}{3}t^4 + \frac{121}{120}t^5 - \cdots\big); \quad -\infty < t < \infty.$

5 $y(t) = c_1\big(1 - \frac{1}{4}t^2 - \frac{1}{24}t^3 + \frac{1}{192}t^4 + \frac{1}{640}t^5 + \cdots\big)$
$+ c_2\big(t + \frac{1}{4}t^2 - \frac{1}{24}t^3 - \frac{1}{64}t^4 - \frac{1}{1920}t^5 - \cdots\big); \quad -\infty < t < \infty.$

7 $y(t) = c_1\big(1 - t^2 + \frac{1}{3}t^3 - \frac{1}{12}t^4 + \frac{1}{30}t^5 - \cdots\big)$
$+ c_2\big(t - \frac{1}{2}t^2\big); \quad -\infty < t < \infty.$

9 $y(t) = c_1\big(1 - \frac{1}{2}t^2 - \frac{1}{3}t^3 + \frac{1}{8}t^4 + \frac{7}{60}t^5 + \cdots\big)$
$+ c_2\big(t - \frac{1}{3}t^3 - \frac{1}{6}t^4 + \frac{1}{15}t^5 + \frac{1}{20}t^6 + \cdots\big); \quad -\infty < t < \infty.$

11 $y(t) = c_1\big(1 - \frac{1}{3}t^3 + \frac{1}{45}t^6 - \frac{1}{105}t^7 + \frac{1}{210}t^8 - \cdots\big)$
$+ c_2\big(t + t^2 + \frac{1}{3}t^3 - \frac{1}{6}t^4 - \frac{1}{15}t^5 + \cdots\big); \quad -1 < t < 1.$

13 $y(t) = c_1\big(1 + \frac{1}{4}t^2 + \frac{1}{32}t^4 + \frac{1}{160}t^5 + \frac{1}{240}t^6 + \cdots\big)$
$+ c_2\big(t - \frac{1}{4}t^2 + \frac{1}{6}t^3 - \frac{1}{96}t^4 + \frac{7}{480}t^5 + \cdots\big); \quad -2 < t < 2.$

15 $y(t) = c_1\left(1 + \frac{1}{2}t^2 - \frac{1}{24}t^4 - \frac{1}{30}t^5 + \frac{7}{720}t^6 + \cdots\right)$
$+ c_2\left(t + \frac{1}{2}t^2 + \frac{1}{3}t^3 - \frac{1}{24}t^4 - \frac{7}{60}t^5 + \cdots\right); \quad -\infty < t < \infty.$

17 $y(t) = c_1\left(1 + \frac{1}{6}t^3 + \frac{5}{24}t^4 + \frac{1}{6}t^5 + \frac{29}{180}t^6 + \cdots\right)$
$+ c_2\left(t + \frac{1}{2}t^2 + \frac{1}{3}t^3 + \frac{5}{12}t^4 + \frac{11}{24}t^5 + \cdots\right); \quad -1 < t < 1.$

19 $y(t) = c_1\left(1 + t^2 - \frac{1}{3}t^3 + \frac{1}{4}t^4 - \frac{7}{30}t^5 + \cdots\right)$
$+ c_2\left(t + \frac{1}{2}t^3 - \frac{1}{3}t^4 + \frac{1}{8}t^5 - \frac{1}{15}t^6 + \cdots\right); \quad -\infty < t < \infty.$

Section 9.3

1 $t = -2; \quad s = t + 2; \quad sx'' + (s-2)x' - 3x = 0.$

3 $t_1 = 3; \quad s = t - 3; \quad (s^2 + 6s)x'' + (2s+7)x' + (s+3)x = 0;$
$t_2 = -3; \quad s = t + 3; \quad (s^2 - 6s)x'' + (2s-5)x' + (s-3)x = 0.$

5 $t = 0$: regular singular point.

7 $t = 1/2$: irregular singular point.

9 $t = 0$: irregular singular point; $t = -2$: regular singular point.

11 $t = 1$: irregular singular point; $t = -1$: regular singular point.

13 $t = 0$: regular singular point.

Subsection 9.4.1

1 $r_1 = 1, \ r_2 = -\frac{1}{2}; \quad y_1(t) = t\left(1 - t + \frac{1}{2}t^2 - \frac{1}{6}t^3 + \frac{1}{24}t^4 - \cdots\right);$
$y_2(t) = t^{-1/2}\left(1 + 2t - 4t^2 + \frac{8}{3}t^3 - \frac{16}{15}t^4 + \cdots\right); \quad -\infty < t < \infty.$

3 $r_1 = \frac{1}{2}, \ r_2 = 0; \quad y_1(t) = t^{1/2}\left(1 + \frac{2}{3}t + \frac{1}{30}t^2 - \frac{1}{35}t^3 - \frac{19}{7560}t^4 + \cdots\right);$
$y_2(t) = 1 + 2t + \frac{1}{2}t^2 - \frac{1}{15}t^3 - \frac{19}{840}t^4 + \cdots; \quad -\infty < t < \infty.$

5 $r_1 = \frac{1}{3}, \ r_2 = -1; \quad y_1(t) = t^{1/3}\left(1 + \frac{2}{21}t - \frac{1}{630}t^2 + \frac{2}{36855}t^3 - \frac{1}{505440}t^4 + \cdots\right);$
$y_2(t) = t^{-1} - 2 - \frac{1}{2}t; \quad -\infty < t < \infty.$

7 $r_1 = 0, \ r_2 = -\frac{1}{2}; \quad y_1(t) = 1 - \frac{1}{10}t^2 + \frac{1}{105}t^3 + \frac{1}{504}t^4 - \frac{1}{3150}t^5 + \cdots;$
$y_2(t) = t^{-1/2}\left(1 + \frac{1}{2}t - \frac{1}{24}t^2 - \frac{1}{80}t^3 + \frac{23}{2688}t^4 + \cdots\right); \quad -\infty < t < \infty.$

9 $r_1 = \frac{3}{2}, \ r_2 = 1; \quad y_1(t) = t^{3/2}\left(1 - \frac{1}{3}t - \frac{1}{6}t^2 + \frac{5}{126}t^3 + \frac{37}{4536}t^4 + \cdots\right);$
$y_2(t) = t\left(1 - t - \frac{1}{6}t^2 + \frac{13}{90}t^3 + \frac{17}{2520}t^4 + \cdots\right); \quad -\infty < t < \infty.$

11 $r_1 = 0, \ r_2 = -\frac{3}{2}; \quad y_1(t) = 1 - t + \frac{1}{2}t^2 - \frac{7}{54}t^3 + \frac{29}{2376}t^4 + \cdots;$
$y_2(t) = t^{-3/2}\left(1 - \frac{13}{4}t + \frac{127}{32}t^2 - \frac{3013}{1152}t^3 + \frac{87167}{92160}t^4 - \cdots\right); \quad -2 < t < 2.$

13 $r_1 = 2,\ r_2 = \frac{1}{2};\ \ y_1(t) = t^2\Big(1 - \frac{1}{5}\,t + \frac{1}{70}\,t^2 - \frac{1}{1890}\,t^3 + \frac{1}{83160}\,t^4 - \cdots\Big);$

$y_2(t) = t^{1/2}\Big(1 + t - \frac{1}{2}\,t^2 + \frac{1}{18}\,t^3 - \frac{1}{360}\,t^4 + \cdots\Big);\quad -\infty < t < \infty.$

15 $r_1 = \frac{1}{2},\ r_2 = 0;\ \ y_1(t) = t^{1/2}\Big(1 + \frac{1}{3}\,t - \frac{1}{60}\,t^2 - \frac{13}{180}\,t^3 - \frac{131}{12960}\,t^4 + \cdots\Big);$

$y_2(t) = 1 + t + \frac{1}{6}\,t^2 - \frac{11}{90}\,t^3 - \frac{131}{2520}\,t^4 + \cdots;\quad -\infty < t < \infty.$

Subsection 9.4.2

1 $r_1 = r_2 = 0;\ \ y_1(t) = 1 - \frac{1}{2}\,t^2 + \frac{1}{9}\,t^3 + \frac{1}{24}\,t^4 - \frac{7}{450}\,t^5 + \cdots;$

$y_2(t) = y_1(t)\ln t + \Big(-t + \frac{3}{4}\,t^2 + \frac{1}{27}\,t^3 - \frac{37}{288}\,t^4 + \frac{299}{13500}\,t^5 + \cdots\Big);\quad -\infty < t < \infty.$

3 $r_1 = r_2 = 1;\ \ y_1(t) = t\Big(1 + t - \frac{1}{9}\,t^3 - \frac{1}{144}\,t^4 + \frac{1}{240}\,t^5 + \cdots\Big);$

$y_2(t) = y_1(t)\ln t + t\Big(-2t - \frac{1}{2}\,t^2 + \frac{13}{54}\,t^3 + \frac{43}{864}\,t^4 - \frac{67}{7200}\,t^5 + \cdots\Big);\quad -\infty < t < \infty.$

5 $r_1 = r_2 = 0;\ \ y_1(t) = 1 + t + \frac{1}{4}\,t^2 + \frac{5}{36}\,t^3 + \frac{23}{576}\,t^4 + \cdots;$

$y_2(t) = y_1(t)\ln t + \Big(-2t - \frac{1}{2}\,t^2 - \frac{7}{27}\,t^3 - \frac{287}{3456}\,t^4 - \frac{15631}{432000}\,t^5 + \cdots\Big);\quad -\infty < t < \infty.$

7 $r_1 = r_2 = -1;\ \ y_1(t) = t^{-1}\Big(1 + t - \frac{1}{2}\,t^2 + \frac{5}{18}\,t^3 - \frac{5}{36}\,t^4 + \cdots\Big);$

$y_2(t) = y_1(t)\ln t + t^{-1}\Big(-5t + \frac{9}{4}\,t^2 - \frac{137}{108}\,t^3 + \frac{563}{864}\,t^4 - \frac{6361}{21600}\,t^5 + \cdots\Big);\quad -\infty < t < \infty.$

9 $r_1 = r_2 = 0;\ \ y_1(t) = 1 - 2t + \frac{3}{4}\,t^2 + \frac{3}{64}\,t^4 + \frac{3}{160}\,t^5 + \cdots;$

$y_2(t) = y_1(t)\ln t + \Big(3t - \frac{5}{2}\,t^2 + \frac{1}{36}\,t^3 - \frac{145}{1152}\,t^4 - \frac{17}{400}\,t^5 + \cdots\Big);\quad -1 < t < 1.$

Subsection 9.4.3

1 $r_1 = \frac{1}{2},\ r_2 = -\frac{1}{2};\ \ y_1(t) = t^{1/2}\Big(1 - \frac{1}{4}\,t + \frac{1}{24}\,t^2 - \frac{1}{192}\,t^3 + \frac{1}{1920}\,t^4 - \cdots\Big);$

$y_2(t) = t^{-1/2};\quad -\infty < t < \infty.$

3 $r_1 = 0,\ r_2 = -2;\ \ y_1(t) = 1 + \frac{4}{3}\,t + \frac{2}{3}\,t^2 + \frac{1}{15}\,t^3 + \frac{1}{30}\,t^4 + \cdots;$

$y_2(t) = t^{-2}\Big(1 + 8t + \frac{1}{3}\,t^3 + \frac{7}{6}\,t^4 + \frac{1}{72}\,t^6 + \cdots\Big);\quad -1/2 < t < 1/2.$

5 $r_1 = 0,\ r_2 = -2;\ \ y_1(t) = 1 - \frac{1}{6}\,t + \frac{1}{48}\,t^2 - \frac{17}{480}\,t^3 + \frac{37}{5760}\,t^4 + \cdots;$

$y_2(t) = t^{-2}\Big(1 - \frac{1}{2}\,t - \frac{1}{6}\,t^3 + \frac{5}{96}\,t^4 - \frac{1}{192}\,t^5 + \cdots\Big);\quad -\infty < t < \infty.$

7 $r_1 = 4,\ r_2 = -1;\ \ y_1(t) = t^4\Big(1 + \frac{1}{6}\,t + \frac{1}{42}\,t^2 + \frac{1}{336}\,t^3 + \frac{1}{3024}\,t^4 + \cdots\Big);$

$y_2(t) = t^{-1}\Big(1 + t + \frac{1}{2}\,t^2 + \frac{1}{6}\,t^3 + \frac{1}{24}\,t^4\Big);\quad -\infty < t < \infty.$

9 $r_1 = 1,\ r_2 = -3;\ \ y_1(t) = t\Big(1 - \frac{2}{5}\,t + \frac{2}{15}\,t^2 - \frac{4}{105}\,t^3 + \frac{1}{105}\,t^4 - \cdots\Big);$

$y_2(t) = t^{-3}\Big(1 - 2t + 2t^2 - \frac{4}{3}\,t^3\Big);\quad -\infty < t < \infty.$

11 $r_1 = 0,\ r_2 = -1;\ \ y_1(t) = 1 - \frac{1}{2}\,t + \frac{1}{12}\,t^3 + \frac{1}{120}\,t^4 + \frac{1}{240}\,t^5 + \cdots;$

$y_2(t) = t^{-1}\Big(1 + \frac{1}{6}\,t^3 + \frac{1}{24}\,t^4 + \frac{1}{60}\,t^5 + \frac{1}{72}\,t^6 + \cdots\Big);\quad -1 < t < 1.$

13 $r_1 = -1,\ r_2 = -2;\ \ y_1(t) = t^{-1}\Big(1 + t - \frac{1}{2}\,t^2 - \frac{1}{3}\,t^3 + \frac{7}{60}\,t^4 + \cdots\Big);$

$y_2(t) = 3y_1(t)\ln t + t^{-2}\Big(1 - 8t^2 - \frac{7}{12}\,t^3 + \frac{27}{8}\,t^4 - \frac{383}{2400}\,t^5 + \cdots\Big);\quad -\infty < t < \infty.$

15 $r_1 = \frac{1}{2},\ r_2 = -\frac{3}{2};\ \ y_1(t) = t^{1/2}\Big(1 - \frac{1}{12}\,t + \frac{1}{128}\,t^2 - \frac{1}{1536}\,t^3 + \frac{7}{147456}\,t^4 - \cdots\Big);$

$y_2(t) = -\frac{3}{32}\,y_1(t)\ln t + t^{-3/2}\Big(1 - \frac{3}{4}\,t + \frac{1}{192}\,t^3 - \frac{7}{16384}\,t^4 + \frac{9}{327680}\,t^5 - \cdots\Big);$

$-\infty < t < \infty.$

17 $r_1 = 3,\ r_2 = 1;\ \ y_1(t) = t^3\Big(1 - 2t + 2t^2 - \frac{4}{3}\,t^3 + \frac{2}{3}\,t^4 + \cdots\Big);$

$y_2(t) = -4y_1(t)\ln t + t\Big(1 + 2t - 8t^3 + 12t^4 - \frac{88}{9}\,t^5 + \cdots\Big);\quad -\infty < t < \infty.$

19 $r_1 = 2,\ r_2 = -1;\ \ y_1(t) = t^2\Big(1 - \frac{5}{4}\,t + \frac{3}{4}\,t^2 - \frac{7}{24}\,t^3 + \frac{1}{12}\,t^4 + \cdots\Big);$

$y_2(t) = -2y_1(t)\ln t + t^{-1}\Big(1 + t + \frac{3}{2}\,t^2 - \frac{21}{8}\,t^4 + \frac{19}{8}\,t^5 + \cdots\Big);\quad -\infty < t < \infty.$

21 $r_1 = 3,\ r_2 = -1;\ \ y_1(t) = t^3\Big(1 + \frac{4}{5}\,t + \frac{1}{6}\,t^2 - \frac{1}{35}\,t^3 - \frac{1}{60}\,t^4 + \cdots\Big);$

$y_2(t) = t^{-1}\Big(1 + \frac{1}{2}\,t^2 - \frac{1}{3}\,t^3 + \frac{77}{150}\,t^5 + \frac{17}{72}\,t^6 + \cdots\Big);\quad -\infty < t < \infty.$

23 $r_1 = 0,\ r_2 = -1;\ \ y_1(t) = 1 - \frac{1}{2}\,t - \frac{1}{2}\,t^2 + \frac{1}{8}\,t^3 + \frac{11}{160}\,t^4 + \cdots;$

$y_2(t) = -3y_1(t)\ln t + t^{-1}\Big(1 - 6t^2 - \frac{9}{8}\,t^3 + \frac{53}{32}\,t^4 + \frac{331}{1600}\,t^5 + \cdots\Big);\quad -2 < t < 2.$

Chapter 10

Section 10.1

1 $y_{10} = -5.2253,\quad E_{10} = 23.9305;\quad y_{20} = -5.6719,\quad E_{20} = 17.4281;$

$y_{100} = -6.5327,\quad E_{100} = 4.8971;\quad y(3.2) = -6.8691.$

3 $y_{10} = 1.8609,\quad E_{10} = 25.0151;\quad y_{20} = 2.1427,\quad E_{20} = 13.6607;$

$y_{100} = 2.4085,\quad E_{100} = 2.9499;\quad y(1) = 2.4817.$

5 $y_{10} = 3.7108,\quad E_{10} = 2.9905;\quad y_{20} = 3.6535,\quad E_{20} = 1.3992;$

$y_{100} = 3.6127,\quad E_{100} = 0.2675;\quad y(1) = 3.6031.$

7 $y_{10} = -2.2310;\quad h_{20} = -2.4185;\quad y_{100} = -2.5887.$

9 $y_{10} = -1.9898;\quad y_{20} = -2.2508;\quad y_{100} = -2.5760.$

Section 10.2

1 $y_{10} = -6.9103,\quad E_{10} = 0.6001;\quad y_{20} = -6.8888,\quad E_{20} = 0.2870;$

$y_{100} = -6.8704,\quad E_{100} = 0.0185;\quad y(3.2) = -6.8691.$

3 $y_{10} = 2.4427,\quad E_{10} = 1.5707;\quad y_{20} = 2.4712,\quad E_{20} = 0.4214;$

$y_{100} = 2.4812,\quad E_{100} = 0.0178;\quad y(1) = 2.4817.$

5 $y_{10} = 3.5862$, $E_{10} = 0.4667$; $y_{20} = 3.5994$, $E_{20} = 0.1024$;
$y_{100} = 3.6029$, $E_{100} = 0.0037$; $y(1) = 3.6031$.

7 $y_{10} = -2.6097$; $y_{20} = -2.6278$; $y_{100} = -2.6340$.

9 $y_{10} = -2.5749$; $y_{20} = -2.6537$; $y_{100} = -2.6877$.

Section 10.3

1 $y_{10} = -5.5505$, $E_{10} = 19.1965$; $y_{20} = -6.5187$, $E_{20} = 5.1009$;
$y_{100} = -6.8546$, $E_{100} = 0.2118$; $y(3.2) = -6.8691$.

3 $y_{10} = 2.4565$, $E_{10} = 1.0138$; $y_{20} = 2.4750$, $E_{20} = 0.2676$;
$y_{100} = 2.4814$, $E_{100} = 0.0112$; $y(1) = 2.4817$.

5 $y_{10} = 3.5725$, $E_{10} = 0.8485$; $y_)20 = 3.5963$, $E_{20} = 0.1869$;
$y_{100} = 3.6028$, $E_{100} = 0.0069$; $y(1) = 3.6031$.

7 $y_{10} = -2.6288$; $y_{20} = -2.6328$; $y_{100} = -2.6342$.

9 $y_{10} = -2.6295$; $y_{20} = -2.6722$; $y_{100} = -2.6886$.

Section 10.4

1 $y_{10} = -6.8663$, $E_{10} = 0.0414$; $y_{20} = -6.8689$, $E_{20} = 0.0026$;
$y_{100} = -6.8691$, $E_{100} = 0.0000$; $y(3.2) = -6.8691$.

3 $y_{10} = 2.4816$, $E_{10} = 0.0014$; $y_{20} = 2.4817$, $E_{20} = 0.0001$;
$y_{100} = 2.4817$, $E_{100} = 0.0000$; $y(1) = 2.4817$.

5 $y_{10} = 3.6028$, $E_{10} = 0.0056$; $y_{20} = 3.6031$, $E_{20} = 0.0003$;
$y_{100} = 3.6031$, $E_{100} = 0.0000$; $y(1) = 3.6031$.

7 $y_{10} = -2.6343$; $y_{20} = -2.6343$; $y_{100} = -2.6343$.

9 $y_{10} = -2.6894$; $y_{20} = -2.6894$; $y_{100} = -2.6894$.

11 Euler: $y_{100} = 0.9007$; Euler mid-point: $y_{100} = 0.9041$;
Improved Euler: $y_{100} = 0.9041$; Runge–Kutta: $y_{100} = 0.9041$.

13 Euler: $y_{100} = 0.9981$; Euler mid-point: $y_{100} = 0.9998$;
Improved Euler: $y_{100} = 0.9998$; Runge–Kutta: $y_{100} = 1.0000$.

15 Euler: $y_{100} = 0.9664$; Euler mid-point: $y_{100} = 0.9782$;
Improved Euler: $y_{100} = 0.9782$; Runge–Kutta: $y_{100} = 0.9782$.

17 Euler: $y_{100} = 0.3605$; Euler mid-point: $y_{100} = 0.3619$;
Improved Euler: $y_{100} = 0.3619$; Runge–Kutta: $y_{100} = 0.3619$.

19 Euler: $y_{100} = 0.7838$; Euler mid-point: $y_{100} = 0.7847$;
Improved Euler: $y_{100} = 0.7847$; Runge–Kutta: $y_{100} = 0.7847$.

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