

# Glossary

- @Risk** © Risk Analysis Software using Monte Carlo Simulation software developed by Palisade
- Abandonment option** Option to sell or close down a project (a simple put option).  
Synonym of option to abandon
- Advanced option** Antonym of simple option
- American option** Option that can be exercised at any moment until the maturity date. See Sect. 1.1. Antonym of European option
- Analytical method** Uses mathematical techniques such as calculus or trigonometry to solve mathematical problems in order to find a closed-form solution. See Sect. 1.5.1. Antonym of numerical method
- Arbitrage** Opportunity of taking advantage of a price difference between two or more markets. Antonym of no-arbitrage
- Bellman equation** Necessary condition for optimality to solve mathematical problems using dynamic programming. Synonym of fundamental equation of optimality
- Binomial option pricing formula** A discrete-time model developed by Cox, Ross and Rubinstein for valuing options. See Sect. 3.2.2.
- Binomial tree** A tree with exactly two possibilities at each node that represents how an asset price or underlying project can evolve under the binomial model. Most binomial trees in real options analysis are recombining
- Black-Scholes formula** Closed-form solution developed by Black and Scholes for financial option valuation. See Sect. 3.1.1.
- Bonds** A long-term debt investment of a firm that obligates the issuer to make specified payments to the holder over a specific period.
- Brownian motion** Special type of continuous-time stochastic process to model the behavior of stock prices. See Sect. 3.1.1. Synonym of Wiener process.
- Call option** An option to buy an asset at a certain price by a certain date. See Sect. 1.1. Antonym of put option
- Capital budgeting** The process of planning and managing a firm's long-term investments

- Cash flow** Cash generated by the firm and paid to creditors and shareholders
- Closed-form solution** For models such as Black-Scholes, where there exist equations that can be solved given a set of input assumptions. See Sect. 1.5.1.
- Compound option** An option on an option See Sect. 1.1. Synonym of growth option
- Contingent claims** Approach to option valuation assuming the value of a claim is contingent on an underlying asset. See Sect. 1.5.3.
- Continuous-time stochastic process** Stochastic process where changes can take place at any time. Antonym of discrete-time stochastic process
- Continuation value** Value representing uncertain future cash flows. In an option valuation lattice this is the value to continue the existing operations of a project without exercising the option.
- Contraction option** An option to reduce the scale of a project's operation. Synonym of option to contract and option to scale back
- Crystal Ball ©** Spreadsheet-based application for predictive modeling, forecasting, simulation, and optimization developed by Oracle
- Cumulative distribution** Describes the probability that a random variable will be less than or equal to  $x$  as a function of  $x$ . The cumulative distribution function  $F(x)$  is given by

$$F(x) = P(X \leq x)$$

- Decision tree** A graphical representation of alternative sequential decisions and the possible outcomes of those decisions. See Sects. 3.2.5 and 3.3.3
- Decision tree analysis** Analysis to value a project which takes the flexibility of the project into account. Traditional Decision Tree Analysis, using discounted cash flows, does incorporate flexibility, but fails to adjust the discount rate in order to account for changes in the risk pattern of a project's cash flows. See Sect. 2.3
- Deferral option** American call option found in most projects where one has the right to delay the start of a project. Synonym of option to defer
- Discount rate** The rate used to calculate the present value of future cash flows
- Discrete-time process** Stochastic process where the value of the variable can change only at certain fixed points in time. Antonym of continuous-time stochastic process
- Dividend** A payment made by a firm to its owners, in the form of either cash or stock
- DTA** See Decision tree analysis
- Dynamic programming** Approach to option valuation based on the idea that each decision has both immediate and future consequences. See Sect. 1.5.2.
- European option** Option that can be exercised only at the end of its life, i.e. at maturity date. See Sect. 1.1. Antonym of American option
- Event tree** Is intended to model the uncertainty that drives the value of the underlying risky asset through time. See Sect. 3.2.5 and 3.3.3

**Exercise price** Price at which the holder of an option can buy (in case of a call option) or sell (in case of a put option) the underlying asset. Synonym of strike price

**Expansion option** An option to make further investments and increase the output if conditions are favorable. Synonym of option to expand, option to scale up and growth option

**Free cash flow** Cash flow after taxes, interest and all positive NPV opportunities. Synonym of cash flow from assets

**GBM** See Geometric Brownian motion

**Generalized Wiener process** Continuous-time stochastic process composed of a constant drift and noise by a Wiener process, which is scaled by a constant factor. A generalized Wiener process is given by  $dx = a dt + b dz$ . See Sect. 3.1

**Geometric Brownian motion** A stochastic process often assumed for asset prices where the logarithm of the underlying variable follows a generalized Wiener process. Ito-process used in finance and economics to model the behavior of stock prices. The GBM is given by

$$dS = \mu S dt + \sigma S dz.$$

Synonym of random walk. See Sect. 3.1

**Geometric series** Infinite additive series with a constant ratio between successive terms. See Sect. 1.2

**Growth option** Option for an early investment that is associated with a greater ability to expand in the future. See Sect. 1.1. Synonym of expansion option, option to scale up and growth option

**Hedging** Taking a position in two or more securities that are negatively correlated (taking opposite trading positions) to reduce risk

**Hedging probabilities** Mathematical constructed probabilities to allow cash flows to be discounted at the risk-free rate. See Sect. 2.4. Synonym of risk-neutral probabilities and risk-adjusted probabilities

**Fundamental equation of optimality** Necessary condition for optimality when solving a mathematical problem by dynamic programming. Synonym of Bellman equation

**In-the-money** An option whose exercise would produce profits. Either (a) a call option where the asset price is greater than the strike price or (b) a put option where the asset price is less than the strike price. Antonym of out-of-the-money

**Interest rate** The price paid for borrowing money

**Itô calculus** Extends the methods of calculus to stochastic processes such as Brownian motion

**Itô process** Continuous-time stochastic process composed of a variable drift and noise by a Wiener process, which is scaled by a variable factor. An Itô process is given by  $dx = a(x, t) dt + b(x, t) dz$ . See Sect. 3.1

**Jump process** Is a type of stochastic process that has discrete movements, called jumps, rather than small continuous movements.

- Law of one price** Two assets that have exactly the same payouts in every state of nature are perfect substitutes and must, therefore, have exactly the same price (or value).
- Log normal distribution** A variable that has a continuous lognormal distribution when the logarithm of the variable has a normal distribution.
- Long position** Position involving the purchase of an asset. Antonym of short position. See Sect. 1.1.1
- MAD** See Marketed Asset Disclaimer
- Marketed Asset Disclaimer** Assumption that the present value of the value of the cash flows of the project (asset) without flexibility (i.e. the traditional NPV) is the best unbiased estimate of the market value of the project were it a traded asset. Used by Copeland and Antikarov
- Markov process** A stochastic process where the behavior of the variable over a short period of time depends solely on the value of the variable at the beginning of the period, not on its past history.
- Markov property** Refers to the memoryless property of a stochastic process, i.e. a Markov process
- Maturity date** The end of the life of a contract
- Margin requirements** Percentage of the value of a security that may be used as a collateral for a loan to finance its purchase
- Mean** Measure of the central tendency of a stochastic variable or a probability distribution
- Monte Carlo simulation** A procedure for randomly sampling changes (simulations) in market variables in order to value a derivative such as a real option. See Sect. 1.5.5
- Multiplicative process** Process in which the value of each state, except for the initial state, equals the value of the previous state multiplied by a constant factor
- Net present value** The present value of discounted future cash flows minus the present value of the cost of the investment
- No-arbitrage** No opportunity of taking advantage of a price difference between two or more markets. Antonym of arbitrage
- Normal distribution** The standard bell-shaped distribution of statistics
- NPV** See Net present value
- Numerical method** Uses techniques like Monte Carlo simulation to solve mathematical problems. A method of valuing an option when no formula is available. See Sect. 1.5.1. Antonym of analytical method
- Objective probabilities** The actual probability that an event will occur
- Option** A right, but not an obligation, to buy or sell underlying assets at a fixed price during a specified time period. See Sect. 1.1
- Option to abandon** Option to sell or close down a project. Synonym of abandonment option
- Option to contract** An option to reduce the scale of a project's operation. Synonym of contraction option and option to scale back

- Option to defer** Call option found where one has the right to delay the start of a project. Synonym of deferral option
- Option to expand** An option to make further investments and increase the output if conditions are favorable. Synonym of expansion option, option to scale up and growth option
- Option to scale back** An option to reduce the scale of a project's operation. Synonym of option to contract and contraction option
- Option to scale up** An option to make further investments and increase the output if conditions are favorable. Synonym of option to expand, expansion option and growth option
- Option to switch** Are portfolios of American call and put options that allow their owner to switch at a fixed cost (or costs) between two modes of operation. Synonym of switching option
- Out-of-the-money** Describes an option whose exercise would not be profitable. Either (a) a call option where the asset price is less than the strike price or (b) a put option where the asset price is greater than the strike price. Antonym of in-the-money
- Partial differential equation** Equation containing partial derivatives of multi-variate functions
- Present value** The current value of future cash flows discounted at the appropriate discount rate.
- Probability distribution** Function that assigns a probability to each measurable subset of the possible outcomes of a stochastic variable
- Put option** Right, but not the obligation, to sell an asset for a certain price by a certain date. See Sect. 1.1. Antonym of call option
- Random walk** Theory that stock price changes from day to day are at random; the changes are independent of each other and have the same probability distribution. An example of this is the Ito-process that is used in finance and economics to model the behavior of stock prices. Synonym of geometric Brownian motion. See Sect. 3.1
- Real option** Option involving real (as opposed to financial) assets. Real assets include land, plant and machinery.
- Recombining binomial tree** Binomial tree with the property that multiple paths within the tree end up in the same node, i.e. two or more paths of a tree merge or recombine. A way to restrict the number of nodes in the tree and thereby the number of computations
- Recursive equation** Equation that should be solved multiple times. The outcome of the equation is dependent on the previous outcome
- Replicating portfolio** Collection of assets with exactly the same payouts as another collection of assets. Assumes that there are no arbitrage opportunities and that there exist a number of traded assets in the market that can be obtained to replicate the existing asset's payout profile.
- Replicating portfolio method** Method based on contingent claims. See Sect. 2.4

- Risk-adjusted probabilities** Mathematical constructed probabilities to allow cash flows to be discounted at the risk-free rate. See Sect. 2.4. Synonym of risk-neutral probabilities and hedging probabilities
- Risk-neutral probabilities** Mathematical constructed probabilities to allow cash flows to be discounted at the risk-free rate. See Sect. 2.4. Synonym of hedging probabilities and risk-adjusted probabilities
- Risk-neutral probability method** Method based on contingent claims. See Sect. 2.4
- Riskless rate of return** The rate of interest that can be earned without assuming any risks.
- Samuelson's proof** Proof of the theorem that the rate of return on any security will be a random walk regardless of the pattern of cash flows that it is expected to generate in the future as long as investors have complete information about those cash flows.
- Security** A tradable asset of every kind
- Set of equations** Mathematical, internally consistent description of a real life problem
- Short position** A position assumed when traders sell shares they do not own. Antonym of long position. See Sect. 1.1.1
- Short selling** Selling in the market shares that have been borrowed from another investor.
- Simple option** Option that is not dependent on other options. See Sect. 1.1. Antonym of advanced option
- Standard deviation** Measure of the dispersion about the mean of a stochastic variable or a probability distribution. Square root of variance
- Standardized normal distribution** Normal distribution with mean 0 and standard deviation 1
- Stochastic process** Collection of random values describing the probabilistic behaviour of a stochastic variable.
- Stochastic variable** Variable that follows a stochastic process, i.e. a variable whose future value is uncertain.
- Stock price** Price of a stock
- Strike price** The price at which the asset may be bought or sold in an option contract. Synonym of exercise price
- Switching option** Are portfolios of American call and put options that allow their owner to switch at a fixed cost (or costs) between two modes of operation. Synonym of option to switch
- Twin security** Project with exactly the same payouts as another project. A priced security whose cash payouts in every state of nature over the life of the project are perfectly correlated with those of the project.
- Underlying asset** Asset on which a contingent claim is dependent
- Underlying value** Value of the underlying asset

**Value at Risk** Refers to certain loss that will not be exceeded at some specified confidence level. Single number summarizing the total risk in a portfolio of financial assets, stating that the analyst is  $X$  percent certain there will not be a loss of more than  $V$  dollars in the next  $N$  days

**Variance** Measure of the dispersion about the mean of a stochastic variable or a probability distribution. Square of standard deviation

**Volatility** A measure of the uncertainty of the return realized on an asset. Dispersion of a stochastic variable or the measure of the dispersion of a stochastic variable

**WACC** See Weighted average cost of capital

**Weak form of market efficiency** Theory that market is efficient with respect to historical price information. In other words past information related to prices is fully reflected in the current market prices and hence it cannot be used to earn excess return.

**Weighted average cost of capital** Weighted average of the marginal costs of capital after taxes.

**Wiener process** A stochastic process where the change in a variable during each short period of time of length  $\delta t$  has a normal distribution with a mean equal to zero and a variance equal to  $\delta t$ . Continuous-time stochastic process used in finance and economics to model volatility. See Sect. 3.1.1. Synonym of Brownian motion.

**Zero-sum game** A situation in which one person's gain is equivalent to another's loss, so that the net change in wealth or benefit is zero.

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