

References

1. Adams, R.A.: Sobolev Spaces. Academic Press, New York (1975)
2. Adimurthi, Tintarev, K.: Hardy inequalities for weighted Dirac operator. *Ann. Mat. Pura Appl.* **189**, 241–251 (2010)
3. Aharonov, Y., Bohm, D.: Significance of electromagnetic potentials in quantum theory. *Phys. Rev.* **115**, 485–491 (1959)
4. Aharonov, Y., Bohm, D.: Further considerations on electromagnetic potentials in quantum theory. *Phys. Rev.* **123**, 1511–1524 (1961)
5. Ahlfors, L.V.: Complex Analysis. An Introduction to the Theory of Analytic Functions of One Complex Variable. International Series in Pure and Applied Mathematics, 3rd edn. McGraw-Hill Book Co., New York (1978)
6. Allegretto, W.: Nonoscillation theory of elliptic equations of order $2n$. *Pac. J. Math.* **64**(1), 1–16 (1976)
7. Ancona, A.: On strong barriers and an inequality of Hardy for domains in \mathbb{R}^2 . *J. Lond. Math. Soc.* **34**, 274–290 (1986)
8. Armitage, D.H., Kuran, Ü.: The convexity of a domain and the superharmonicity of the signed distance function. *Proc. Am. Math. Soc.* **93**(4), 598–600 (1985)
9. Aubin, T.: Problemes isoperimetriques et espaces de Sobolev. *J. Differ. Geom.* **11**, 573–598 (1976)
10. Avkhadiiev, F.G.: Hardy type inequalities in higher dimensions with explicit estimate of constants. *Lobachevskii J. Math.* **21**, 3–31 (2006). <http://ijm.ksu.ru>
11. Avkhadiiev, F.G., Laptev, A.: Hardy inequalities for non-convex domains. In: Around the Research of Vladimir Maz'ya. I, International Mathematical Series (N. Y.), vol. 11, pp. 1–12. Springer, New York (2010)
12. Avkhadiiev, F.G., Wirths, K.: Unified Poincaré and Hardy inequalities with sharp constants for convex domains. *Z. Angew. Math. Mech.* **87**(8–9), 632–642 (2007)
13. Avron, J., Herbst, I., Simon, B.: Schrödinger operators with magnetic fields. I. General Interactions. *Duke Math. J.* **45**(4), 847–883 (1978)
14. Balinsky, A.A.: Hardy type inequalities for Aharonov-Bohm magnetic potentials with multiple singularities, *Math. Res. Lett.* **10**, 169–176 (2003)
15. Balinsky, A., Evans, W.D.: On the zero modes of Pauli operators. *J. Funct. Anal.* **179**, 120–135 (2001)
16. Balinsky, A., Evans, W.D.: Some recent results on Hardy-type inequalities. *Appl. Math. Inf. Sci.* **4**(2), 191–208 (2010)
17. Balinsky, A., Evans, W.D., Hundertmark, D., Lewis, R.T.: On inequalities of Hardy-Sobolev type. *Banach J. Math. Anal.* **2**(2), 94–106 (2008)

18. Balinsky, A., Evans, W.D., Lewis, R.T.: On the number of negative eigenvalues of Schrodinger operators with an Aharonov-Bohm magnetic field. *Proc. R. Soc. Lond.* **457**, 2481–2489 (2001)
19. Balinsky, A., Evans, W.D., Lewis, R.T.: Sobolev, Hardy and CLR inequalities associated with Pauli operators in \mathbb{R}^3 . *J. Phys. A* **34**(5), L19–L23 (2001)
20. Balinsky, A., Evans, W.D., Lewis, R.T.: Hardy’s inequality and curvature. *J. Funct. Anal.* **262**, 648–666 (2012) [Available online 13 October 2011]
21. Balinsky, A., Evans, W.D., Umeda, T.: The Dirac-Hardy and Dirac-Sobolev inequalities in L^1 . *Publ. Res. Inst. Math. Sci.* **47**(3), 791–801 (2011)
22. Balinsky, A., Laptev, A., Sobolev, A.: Generalized Hardy inequality for the magnetic Dirichlet forms. *J. Stat. Phys.* **116**(114), 507–521 (2004)
23. Barbatis, G., Filippas, S., Tertikas, A.: A unified approach to improved L^p Hardy inequalities with best constants. *Trans. Am. Math. Soc.* **356**(6), 2169–2196 (2004)
24. Bargmann, V.: On the number of bound states in a central field of force. *Proc. Nat. Acad. Sci. USA* **38**, 961–966 (1952)
25. Batelaan, H., Tonomura, A.: The Aharonov–Bohm effects: variations on a subtle theme. *Phys. Today* **62**(9), 38–43 (2009)
26. Bateman, H.: *Higher Transcendental Functions*, vol. I. McGraw-Hill, New York (1953)
27. Benguria, R.D., Van Den Bosch, H.: A criterion for the existence of zero modes for the Pauli operator with fastly decaying fields. *J. Math. Phys.* **56**, 052104 (2015)
28. Benguria, R.D., Frank, R.L., Loss, M.: The sharp constant in the Hardy-Sobolev-Maz’ya inequality in the three dimensional upper half-space. *Math. Res. Lett.* **15**(4), 613–622 (2008)
29. Bennett, D.M.: An extension of Rellich’s inequality. *Proc. Am. Math. Soc.* **106**, 987–993 (1989)
30. Brezis, H., Marcus, M.: Hardy’s inequalities revisited. Dedicated to Ennio De Giorgi. *Ann. Scuola Norm. Sup. Pisa Cl. Sci.* **25**(4), 217–237 (1997)
31. Brezis, H., Vázquez, J.V.: Blow-up solutions of some nonlinear elliptic problems. *Revista Matemática de la Universidad Complutense de Madrid* **10**(2), 443–469 (1997)
32. Brown, R.C., Edmunds, D.D., Rákosník, J.: Remarks on inequalities of Poincaré type. *Czech. Math. J.* **45**, 351–377 (1995)
33. Bunt, L.H.N.: *Bijdrage tot de Theorie der convexe Puntverzamelingen*. Thesis, University of Groningen, Amsterdam (1934)
34. Caldiroli, P., Musina, R.: Rellich inequality with weights (English summary). *Calc. Var. PDE.* **45**(1–2), 147–164 (2012)
35. Cannarsa, P., Sinestrari, C.: Semiconcave functions, Hamilton Jacobi equations and optimal control. In: *Progress in Nonlinear Differential equations and their Applications*, vol. 58, Birkhäuser, Boston (2004)
36. Chambers, R.G.: Shift of an electron interference pattern by enclosed magnetic flux. *Phys. Rev. Lett.* **5**(3), (1960)
37. Chisholm, R.S., Everitt, W.N.: On bounded integral operators in the space of square integrable functions. *Proc. R. Soc. Edinb. A* **69**, 199–204 (1971)
38. Conlon, J.P.: A new proof of the Cwikel-Lieb-Rosenbljum bound. *Rocky Mt. J. Math.* 117–122 (1985)
39. Cwikel, M.: Weak type estimates for singular values and the number of bound states of Schrödinger operators. *Ann. Math.* **106**, 93–100 (1977)
40. Davies, E.B.: *One-Parameter Semigroups*. Academic, London (1980)
41. Davies, E.B.: Some norm bounds and quadratic form inequalities for Schrödinger operators (II). *J. Oper. Theory* **12**, 177–196 (1984)
42. Davies, E.B.: *Heat Kernels and Spectral Theory*. Cambridge Tracts in Mathematics, vol. 92. Cambridge University Press, Cambridge (1989)
43. Davies, E.B.: *Spectral Theory and Differential Operators*. Cambridge Studies in Advanced Mathematics, vol. 42. Cambridge University Press, Cambridge (1995)
44. Davies, E.B.: The Hardy constant. *Q. J. Math. Oxf. (2)* **46**, 417–431 (1995)

45. Davies, E.B., Hinz, A.M.: Explicit constants for Rellich inequalities in $L_p(\Omega)$. *Math. Z.* **227**(3), 511–523 (1998)
46. Dolbeault, J., Esteban, M.J., Séré, E.: On the eigenvalues of operators with gaps. Application to Dirac operators. *J. Funct. Anal.* **174**(1), 208–226 (2000)
47. Dolbeault, J., Esteban, M.J., Loss, M., Vega, L.: An analytic proof of Hardy-like inequalities related to the Dirac operator. *J. Funct. Anal.* **216**, 1–21 (2004)
48. Edmunds, D.E., Evans, W.D.: *Spectral Theory and Differential Operators*. Oxford University Press, Oxford (1987) [OX2 GDP]
49. Edmunds, D.E., Evans, W.D.: *Hardy Operators, Function Spaces, and Embeddings*. Springer Monographs in Mathematics. Springer, Berlin/Heidelberg/New York (2004)
50. Ehrenberg, W., Siday, R.: The refractive index in electron optics and the principles of dynamics. *Proc. Phys. Soc. B* **62**, 821 (1949)
51. Esteban, M.J., Loss, M.: Self-adjointness of Dirac operators via Hardy-Dirac inequalities. *J. Math. Phys.* **48**, 112107 (2007)
52. Evans, L.C., Gariepy, R.F.: *Measure Theory and Fine Properties of Functions*. Studies in Advanced Mathematics. CRC, Boca Raton/London/New York/Washington, DC (1992)
53. Evans, W.D., Harris, D.J.: Sobolev embeddings for generalized ridged domains, *Proc. Lond. Math. Soc.* **54**, 141–175 (1987)
54. Evans, W.D., Lewis, R.T.: On the Rellich inequality with magnetic potentials. *Math. Z.* **251**, 267–284 (2005)
55. Evans, W.D., Lewis, R.T.: Counting eigenvalues of biharmonic operators with magnetic fields. *Math. Nachrichten* **278**(12–13), 1524–1537 (2005)
56. Evans, W.D., Lewis, R.T.: Hardy and Rellich inequalities with remainders. *J. Math. Inequal.* **1**(4), 473–490 (2007)
57. Falconer, K.: *Fractal Geometry: Mathematical Foundations and Applications*. Wiley, New York (2007)
58. Federer, H.: Curvature measures. *Trans. Am. Math. Soc.* **93**(3), 418–491 (1959)
59. Federer, H., Fleming, W.: Normal and integral currents. *Ann. Math.* **72**, 458–520 (1960)
60. Filippas, S., Maz'ya, V., Tertikas, A.: On a question of Brezis and Marcus. *Calc. Var.* **25**(4), 491–501 (2006)
61. Filippas, S., Maz'ya, V., Tertikas, A.: Critical Hardy-Sobolev inequalities. *J. Math. Pures Appl.* **87**, 37–56 (2007)
62. Frank, R.L., Loss, M.: Hardy-Sobolev-Maz'ya inequalities for arbitrary domains. *J. Math. Pures Appl.* **97**, 39–54 (2012)
63. Frank, R.L., Lieb, E., Seiringer, R.: Hardy-Lieb-Thirring inequalities for fractional Schrödinger operators. *J. Am. Math. Soc.*, **21**, 925–950 (2008)
64. Frank, R.L., Lieb, E.H., Seiringer, R.: Equivalence of Sobolev inequalities and Lieb-Thirring inequalities. In: Exner, P. (ed.) *Proceedings of the XVIth International Congress on Mathematical Physics*, Prague, 2009, pp. 523–535, World Scientific (2010)
65. Fremlin, D.H.: Skeletons and central sets. *Proc. Lond. Math. Soc.* **74**, 701–720 (1997)
66. Frohlich, J., Lieb, E., Loss, M.: Stability of Coulomb systems with magnetic fields I. The one-electron atom. *Commun. Math. Phys.* **104**(2), 251–270 (1986)
67. Gagliardo, E.: Proprietà di alcune classi di funzioni di più variabili. *Ricerche Mat.* **7**, 102–137 (1958)
68. Gilbarg, D., Trudinger, N.S.: *Elliptic Partial Differential Equations of Second Order*. Springer Classics in Mathematics. Springer, Berlin/Heidelberg/New York (2001). Reprint of the 1998 Edition
69. Gkikas, K.T.: Hardy-Sobolev inequalities in unbounded domains and heat kernel estimates. *J. Funct. Anal.* **264**(3), 837–893 (2013)
70. Hadwiger, H.: *Vorlesungen über Inhalt, Oberfläche und Isoperimetrie*. Springer, Berlin/Göttingen/Heidelberg (1957)
71. Hajlasz, P.: Pointwise Hardy inequalities. *Proc. Am. Math. Soc.* **127**(2), 417–423 (1999)
72. Hardy, G.H.: Note on a theorem of Hilbert. *Math. Zeit.* **6**, 314–317 (1920)
73. Hardy, G.H.: An inequality between integrals. *Messenger Math.* **54**, 150–156 (1925)

74. Hardy, G.H.: *A Mathematician's Apology*. Cambridge University Press, Cambridge (1992). With a foreword by C.P. Snow. Reprint of the 1967 edition
75. Hardy, G.H., Littlewood, J.E., Pólya, G.: *Inequalities*, 2nd edn. Cambridge University Press, Cambridge (1952)
76. Helffer, B., Mohamed, A.: Caractérisation du spectre essentiel de l'opérateur de Schrödinger avec un champ magnétique. *Ann. Inst. Fourier* **38**(2), 95–112 (1988)
77. Herbst, I.: Spectral theory of the operator $(p^2 + m^2)^{1/2} - ze^2/r$. *Commun. Math. Phys.* **53**(3), 285–294 (1977)
78. Hoffmann-Ostenhof, M., Hoffmann-Ostenhof, T., Laptev, A.: A geometrical version of Hardy's inequality. *J. Funct. Anal.* **189**, 539–548 (2002)
79. Hörmander, L.: *The Analysis of Linear Partial Differential Operators I*. Springer, Berlin/Heidelberg (1983)
80. Hörmander, L.: *Notions of Convexity*. Birkhäuser, Boston/Basel/Berlin (1994)
81. Itoh, J.-I., Tanaka, M.: The Lipschitz continuity of the distance function to the cut locus. *Trans. Am. Math. Soc.* **353**(1), 21–40 (2001)
82. Kalf, H.: A characterization of the Friedrichs extension of Sturm-Liouville operators. *J. Lond. Math. Soc.* **17**, 511–521 (1978)
83. Kato, T.: *Perturbation Theory for Linear Operators*, 2nd edn. Springer, Berlin/Heidelberg/New York (1976)
84. Kato, T.: Schrödinger operators with singular potentials. *Isr. J. Math.* **13**(1–2), 135–148 (1972)
85. Kinnunen, J., Martio, O.: Hardy's inequality for Sobolev functions. *Math. Res. Lett.* **4**, 489–500 (1997)
86. Kinnunen, J., Korte, R.: *Characterizations of Hardy's Inequality. Around the research of Vladimir Maz'ya I*, pp. 239–254. Springer, New York (2010)
87. Koskela, P., Zhong, X.: Hardy's inequality and the boundary size. *Proc. Am. Math. Soc.* **131**(4), 1151–1158 (2002)
88. Krantz, S.: *Complex Analysis: The Geometric Viewpoint*. Carus Mathematical Monographs, vol. 23. Mathematical Association of America, Washington, DC (1990)
89. Kregar, A.: *Aharonov-Bohm Effect*, University of Ljubljana, Department of Physics, March 2011
90. Kreyszig, E.: *Differential Geometry*. Dover, New York (1991)
91. Kufner, A., Maligranda, L., Persson, L.-E.: *The Hardy Inequality – About Its History and Some Related Results*. Vydavatelský servis, Pilsen (2007)
92. Kovalenko, V.F., Perelmutter, M.A., Semenov, Ya.A.: Schrödinger operators with $L_w^{1/2}(R^l)$ potentials. *J. Math. Phys.* **22**(5), 1033–1044 (1981)
93. Kuratowski, K.: *Topology*, vol. II. Academic, New York (1968)
94. Landau, E.: A note on a theorem concerning series of positive terms. *J. Lond. Math. Soc.* **1**, 38–39 (1926)
95. Landau, L.D., Lifshitz, E.M.: *Quantum Mechanics (Nonrelativistic Theory)*. Pergamon, Oxford (1977)
96. Laptev, A.: Spectral inequalities for partial differential equations and their applications. *AMS/IP Stud. Adv. Math.* **51**, 629–643 (2012)
97. Laptev, A., Netrusov, Yu.: On the negative eigenvalues of a class of Schrödinger operators. *Differential Operators and Spectral Theory*. *Am. Math. Soc. Transl.* **2** **189**, 173–186 (1999)
98. Laptev, A., Sobolev, A.V.: Hardy inequalities for simply connected planar domains. *Am. Math. Soc. Transl. Ser. 2* **225**, 133–140 (2008)
99. Laptev, A., Weidl, T.: Hardy inequalities for magnetic Dirichlet forms. *Oper. Theory: Adv. Appl.* **108**, 299–305 (1999)
100. Lehrbäck, J.: Pointwise Hardy inequalities and uniformly fat sets. *Proc. Am. Math. Soc.* **136**(6), 2193–2200 (2008)
101. Lehrbäck, J.: Weighted Hardy inequalities and the size of the boundary. *Manuscripts Math.* **127**, 249–273 (2008)

102. Lehrbäck, J., Tuominen, H.: A note on the dimensions of Assouad and Aikawa. *J. Math. Soc. Jpn.* **65**,2, 343–356 (2013)
103. Levin, D., Solomyak, M.: The Rozenbljum-Lieb-Cwikel inequality for Markov generators. *J. Anal. Math.* **71**, 173–193 (1997)
104. Lewis, J.L.: Uniformly fat sets. *Trans. Am. Math. Soc.* **308**, 177–196 (1988)
105. Lewis, R.T.: Singular elliptic operators of second order with purely discrete spectra. *Trans. Am. Math. Soc.* **271**, 653–666 (1982)
106. Lewis, R.T.: Spectral properties of some degenerate elliptic differential operators. In: *Operator Theory: Advances and Applications*, vol. 219, pp. 139–156. Springer, Basel (2012)
107. Lewis, R.T., Li, J., Li, Y.: A geometric characterization of a sharp Hardy inequality. *J. Funct. Anal.* **262**(7), 3159–3185 (2012)
108. Li, Y., Nirenberg, L.: The distance to the boundary, Finsler geometry, and the singular set of viscosity solutions of some Hamilton-Jacobi equations. *Commun. Pure Appl. Math.* **18**(1), 85–146 (2005)
109. Li, P., Yau, S.-T.: On the Schrödinger equation and the eigenvalue problem. *Commun. Math. Phys.* **88**, 309–318 (1983)
110. Lieb, E.H.: Bounds on the number of eigenvalues of Laplace and Schrödinger operators. *Bull. Am. Math. Soc.* **82**, 751–753 (1976)
111. Lieb, E.H., Loss, M.: *Analysis*. Graduate Studies in Mathematics, vol. 14, 2nd edn. American Mathematical Society, Providence (2001)
112. Lieb, E.H., Seiringer, R.: *The Stability of Matter in Quantum Mechanics*. Cambridge University Press, New York (2010)
113. Luukkainen, J.: Assouad dimension: Antifractal metrization, porous sets and homogeneous measures. *J. Korean Math. Soc.* **1**, 23–76 (1998)
114. MacLaurin, C.: A second letter to Martin Folkes, Esq.; concerning the roots of equations, with the demonstration of other rules in algebra. *Philos. Trans.* **36**, 59–96 (1729)
115. Mantegazza, C., Mennucci, A.C.: Hamilton-Jacobi equations and distance functions on Riemannian manifolds. *Appl. Math. Optim.* **47**, 1–25 (2003)
116. Marcus, M., Mizel, V., Pinchover, Y.: On the best constant for Hardy’s inequality in \mathbb{R}^n . *Trans. Am. Math. Soc.* **350**(8), 3237–3255 (1998)
117. Matskewich, T., Sobolevskii, P.: The best possible constant in generalized Hardy’s inequality for convex domain in \mathbb{R}^n . *Nonlinear Anal. Theory Methods Appl.* **28**(9), 1601–1610 (1997)
118. Maz’ya, V.G.: Classes of domains and embedding theorems for function spaces. *Dokl. Akad. Nauk. SSSR* **133**, 527–530 (1960). English transl.: *Sov. Math. Dokl.* **1**, 882–885
119. Maz’ya, V.G.: *Sobolev Spaces*. Springer, Berlin (1985)
120. Motzkin, T.S.: Sur quelques propriétés caractéristiques des ensembles convexes. *Atti Real. Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. Serie VI* **21**, 562–567 (1935)
121. Muckenhoupt, B.: Hardy’s inequality with weights. *Stud. Math.* **44**, 31–38 (1972)
122. Nenciu, G., Nenciu, I.: On confining potentials and essential self-adjointness for Schrödinger operators on bounded domains in \mathbb{R}^n . *Ann. Henri Poincaré* **10**, 377–394 (2009)
123. Newton, I.: *Sive de compositione et resolutione arithmetica liber. Arithmetica universalis* (1707)
124. Nirenberg, L.: On elliptic partial differential equations. *Ann. Sc. Norm. Pisa* **13**, 1–48 (1959)
125. Okazawa, N.: L^p -theory of Schrödinger operators with strongly singular potentials. *Jpn J. Math.* **22**(2), 200–239 (1996)
126. Opic, B., Kufner, A.: *Hardy-type Inequalities*. Pitman Research Notes in Mathematics, Series, vol. 219. Longman Science & Technology, Harlow (1990)
127. Psaradakis, G.: L^1 Hardy inequalities with weights. *J. Geom. Anal.* **23**(4), 1703–1728 (2013)
128. Rellich, F.: Halbeschränkte Differentialoperatoren höherer Ordnung. In: Gerretsen, J.C.H., de Groot, J. (eds.) *Proceedings of the International Congress of Mathematicians 1954*, vol. III, pp.243–250. Noordhoff, Groningen (1956)
129. Rellich, F., Berkowitz, J.: *Perturbation Theory of Eigenvalue Problems*. Gordon and Breach, New York/London/Paris (1969)

130. Rosenberger, R.: A new characterization of the Friedrichs extension of semi-bounded Sturm-Liouville operators. *J. Lond. Math. Soc. (2)* **31**, 501–510 (1985)
131. Rosenbljum, G.V.: The distribution of the discrete spectrum for singular differential operators. *Soviet Math. Dokl.* **13**, 245–249 (1972)
132. Schmidt, K.M.: A short proof for Bargmann-type inequalities. *R. Soc. Lond. Proc. Ser. A Math. Phys. Eng. Sci.* **458**(2027), 2829–2832 (2002)
133. Schmincke, U.-W.: Essential selfadjointness of a Schrödinger operator with strongly singular potential. *Math. Z.* **124**, 47–50 (1972)
134. Seiringer, R.: Inequalities for Schrödinger operators and applications to the stability of matter problem. Lectures given in Tucson, Arizona, 16–20 March 2009
135. Shen, Zh.: Eigenvalue asymptotics and exponential decay of eigenfunctions of Schrödinger operators with magnetic fields, *Trans. Am. Math. Soc.* **348**, 4465–4488 (1996)
136. Simon, B.: Essential self-adjointness of Schrödinger operators with singular potentials. *Rat. Anal. Mech.* **52**(1), 44–48 (1973)
137. Sobolev, S.L.: On a theorem of functional analysis. *Mat. Sb. Am. Math. Soc. Transl. II Ser.* **34**, 39–68 (1938); **46**, 471–497 (1963)
138. Solomyak, M.Z.: A remark on the Hardy inequalities. *Integr. Equ. Oper. Theory* **19**, 120–124 (1994)
139. Stein, E.M.: *Singular Integrals and Differentiability properties of Functions*. Princeton University Press, Princeton (1970)
140. Talenti, G.: Best constant in Sobolev inequality. *Ann. Mat. Pura Appl.* **110**, 353–372 (1976)
141. Thaller, B.: *The Dirac Equation*. Springer, Berlin (1992)
142. Thomas, J.C.: *Some Problems Associated with Sum and Integral Inequalities*. Ph.D. thesis, Cardiff University, Wales (2007)
143. Thorpe, J.: *Elementary Topics in Differential Geometry*. Undergraduate Texts in Mathematics, Springer, New York (1994)
144. Tidblom, J.: A geometrical version of Hardy’s inequality for $W_0^{1,p}(\Omega)$. *Proc. Am. Math. Soc.* **132**(8), 2265–2271 (2004)
145. Tidblom, J.: *Improved L^p Hardy Inequalities*. Ph.D. thesis, Stockholm University (2005)
146. Ward, A.: *On Essential Self-Adjointness, Confining Potential and the L_p Hardy inequality*. Ph.D. thesis, Massey University, Albany (2014)
147. Weder, R.A.: Spectral properties of one-body relativistic spin-zero Hamiltonians. *Ann. Inst. H. Poincaré Sect. A (NS)* **20**, 211–220 (1974)
148. Weder, R.A.: Spectral analysis of pseudodifferential operators. *J. Funct. Anal.* **20**(4), 319–337 (1975)
149. Wen, G.-C.: *Conformal Mappings and Boundary-Value Problems*. Translations of Mathematical Monographs, vol. 166. American Mathematical Society, Providence (1992)
150. Whittaker, E.T., Watson, G.N.: *A Course of Modern Analysis*, 4th edn. The University Press, Cambridge (1940)
151. Yafaev, D.: Sharp constants in the Hardy-Rellich inequalities. *J. Funct. Anal.* **168**, 121–144 (1999)

Author Index

- Adams, A., 13
Adimurthi, 13, 47
Aharonov, Y., 166
Ahlfors, L.V., 191
Allegretto, W., 12, 215, 225
Ancona, A., viii, 81, 92, 95
Armitage, D.H., 67, 69
Aubin, T., 19
Avkhadiev, F.G., 63, 82, 104, 109, 112, 115,
126, 128, 132, 134, 217
Avron, J., 171
- Balinsky, A., viii, 26, 40, 49, 89, 104, 116, 118,
121, 181, 185, 192, 203, 210
Bargmann, G., 181
Batelaan, H., 166
Bateman, H., 233
Benguria, R., 142, 212
Bennett, D.M., 213, 221
Berkowitz, J., 213, 229
Bohm, D., 166
Bosch, H. van den, 212
Brezis, H., 82, 99, 136, 141
Brown, R.C., 74
Bunt, L.H.N., 49, 51, 54
- Caldirolì, P., 215
Cannarsa, P., 61
Chambers, R.G., 166
Chisholm, R.S., 8
Conlon, J.P., 34
Cwikel, M., viii, 27, 34
- Davies, E.B., viii, 11, 81, 85, 86, 95, 96, 99,
213, 218, 219, 221
Dolbeault, J., 45, 47
- Edmunds, D.E., 6, 13, 14, 16–18, 20, 22, 26,
28, 30, 33, 50, 57, 74, 80, 94, 138,
179, 206, 231, 241, 244
Ehrenberg, W., 166
Esteban, M.J., 45, 47
Evans, L.C., 59
Evans, W.D., viii, 6, 13, 14, 16–18, 20, 22,
26, 28, 30, 33, 40, 49–51, 56,
57, 74, 80, 89, 94, 100, 102, 104,
116, 118, 121, 138, 179, 181,
206, 210, 224, 228, 231, 232, 241,
244
Everitt, W.N., 8
- Falconer, K., 79, 80
Federer, H., 18, 49, 51
Filippas, S., ix, 104, 106, 141, 150, 151
Fleming, W., 18
Frank, R.L., ix, 140–142, 144, 147, 162, 213,
216
Fremlin, D.H., 50, 53, 57
Frohlich, J., 205
- Gagliardo, E., 18, 147
Garièpy, R.F., 59
Gilbarg, D., 49, 72, 123
Gkikas, K.T., ix, 157, 158

- Hadwiger, H., 112
 Hajłasz, P., 79
 Hardy, G.H., vii, 41
 Harris, D.J., 49–51, 56, 57
 Helffer, B., 204
 Herbst, I., 39, 141, 171
 Hinz, A.M., ix, 11, 213, 218, 221
 Hoffmann-Ostenhof, M., 100, 108
 Hoffmann-Ostenhof, T., 100, 108
 Hörmander, L., 54
 Hundertmark, D., 104
- Itoh, J.-I., 57, 117
- Kalf, H., 232
 Kato, T., 29, 168, 231, 241, 243
 Kinnunen, J., 78, 80
 Korte, R., 78
 Koskela, P., 80
 Kovalenko, V.F., 218
 Krantz, S., 131
 Kregar, A., 166
 Kreyszig, E., 69
 Kufner, A., vii, 1, 8, 243
 Kuran, Ü, 67, 69
 Kuratowski, K., 53
- Landau, E., 1
 Laptev, A., 81, 97–100, 108, 126, 128, 171, 174, 181, 188, 192, 203, 224, 246
 Lehrbäck, J., 79, 80
 Levin, D., 35, 162
 Lewis, J.L., 77, 78
 Lewis, R.T., viii, 11, 49, 58, 69, 71, 81, 86, 96, 100, 102, 104, 112, 118, 121, 125, 126, 155, 181, 210, 224, 228, 232
 Li, J., viii, 49, 58, 69, 71, 81, 86, 96, 104, 121, 125, 126, 155
 Li, P., 34, 35, 162
 Li, Y., viii, 49, 53, 56–58, 69, 71, 81, 86, 96, 104, 117, 121, 125, 126, 155
 Lieb, E.H., viii, 20, 27, 34, 35, 39, 43, 140, 162, 170, 205
 Littlewood, J.E., vii, 41
 Loss, M., ix, 20, 35, 39, 43, 45, 47, 141, 142, 144, 147, 162, 170, 205
 Luukainen, J., 80
- Marcus, M., 80–82, 87, 88, 99, 126
 Martio, O., 80
 Matskewich, T., 81
 Maz'ya, V.G., ix, 18, 78, 104, 106, 135, 141, 150, 151
 Mennucci, A.C., 57, 117
 Mizel, V., 80, 81, 87, 88, 126
 Mohamed, A., 204
 Motzkin, T.S., 49, 51, 54
 Muckenhoupt, B., 8
 Musina, R., 215
- Nenciu, G., 82
 Nenciu, I., 82
 Netrusov, Yu., 181, 246
 Newton, I., 70
 Nirenberg, L., viii, 18, 53, 56, 57, 117, 147
- Okazawa, N., 213, 218
 Opic, B., 8, 243
- Pólya, G., vii, 41
 Perelmuter, M.A., 218
 Persson, L.-E., vii, 1
 Pinchover, Y., 80, 81, 87, 88, 126
 Psaradakis, G., 62, 63
- Rákosník, J., 74
 Rellich, F., 213, 224, 229, 234
 Rosenberger, R., 232
 Rosenbljum, G.V., viii, 27, 34
- Schmidt, K.M., 181
 Schminke, U.-W., 213
 Seiringer, R., 25, 34, 140, 162
 Semenov, Ya.A., 218
 Séré, E., 45, 47
 Shen, Zh., 204
 Siday, R., 166
 Simon, B., 82, 171
 Sinestrari, C., 61
 Sobolev, A.V., 81, 97–99, 192, 203
 Sobolev, S.L., 18
 Sobolevskii, P., 81
 Solomyak, M.Z., 13, 35, 162
 Stein, E.M., 74
- Talenti, G., 19
 Tanaka, M., 57, 117
 Tertikas, A., ix, 104, 106, 141, 150, 151
- MacLaurin, C., 70
 Maligranda, L., vii, 1
 Mantegazza, C., 57, 117

Thaller, B., 173, 211
Thomas, J.C., 228, 239
Thorpe, J., 68
Tidblom, J., 82, 83, 86, 96, 106, 108
Tintarev, K., 13, 47
Tonomura, A., 166
Trudinger, N.S., 49, 72, 123
Tuominen, H., 80

Umeda, T., 26

Vázquez, J.L., 136, 141
Vega, L., 45

Ward, A., 77, 82
Watson, G.N., 40
Weidl, T., 171, 174, 188, 224
Wen, G.-C., 128, 186
Weyl, H., 231
Whittaker, E.T., 40
Wirths, K., 82, 104, 109, 115, 217

Yafaev, D., 141
Yau, S-T., 34, 35, 162

Zhong, X., 80

Subject Index

- Adjoint, 28
Aharonov-Bohm effect, 166
Aikawa dimension, 79
Associated Legendre polynomials, 40
- Bargmann estimate, 181, 247
Beurling-Deny conditions, 36
Biharmonic operators $\Delta_A^2 - V$, 236
Birman-Schwinger, 205
- Central set, viii, 50
Chebyshev sets, 51
Circulation, 173
Closed differential form, 183
CLR inequality, viii, 27
Compact embedding, 18
Compact relative to form, 30
Condition (C), 150
Condition (R), 151
Convolution, 23
Critical point, 189
Critical value, 189
Cut locus, viii, 56
Cut point, 56
- Diamagnetic inequality, 168
Differential forms, 167
Dirichlet Laplacian, 24
Distribution function, 21
Doubly connected domains, 128
- Effective field, 192
Elementary symmetric functions, 69
- Elementary symmetric means, 70
Embedding, 18
Equimeasurable, 21
Essential self-adjointness, 231
Extension property, 20
Exterior cone condition, 99
Exterior derivative, 167
Exterior domains, 157
- Flux, 173
Fourier transform, 23
Fractional HSM inequalities, 140
Friedrichs extension, 29, 231
Friedrichs inequality, 15
- Gauge equivalent, 172
Gauge invariance, 172
Gauge transformation, 172
Good set, 53
- Hardy's inequality, viii, 2
Hardy-Sobolev-Maz'ya inequality, ix
Hausdorff dimension, 79
Heisenberg's inequality, 36
Hilbert's inequality, 41
Hölder's inequality, 2
HSM inequalities, 135
- Inequality for punctured planes, 189
Inner Minkowski dimension, 73
Inradius, 82
Interior cone condition, 78

- Interior diameter, 104
 Isoperimetric inequality, 18
- Kato's distributional inequality, 168
 Kato's inequality, viii, 39
 Koebe's $1/4$ theorem, 81, 96
- Landau Hamiltonian, 171
 Landau levels, 171
 Lax-Milgram theorem, 94
 Limit circle, 231
 Limit point, 231
 Lipschitz domain, 78
- Magnetic Dirichlet form, 184
 Magnetic field strength, 192
 Magnetic gradient, 167
 Magnetic potential, 167
 Magnetic Schrödinger operator, 204
 Markov generator, 35
 Massless Dirac operator, 205
 Maximal function, 79
 Maxwell's equations, 165
 Mean convex, 68
 Mean curvature, 68
 Mean distance function, 81, 83
 Measure of non-compactness, 17
 Minimal surface, 68
 Minkowski dimension, 73
 Morse theory, 189
 Motzkin's theorem, 54
 Muckenhoupt condition, 8
 Multiple singularities, 183
 Multiply connected domain, 189
- Near set, 50
 Neumann Laplacian, 24
 Non-increasing rearrangement, 21
 Non-principal solution, 232
 Normalized distance function, 58
 Nullity, 209
- 1-sheeted hyperboloid, 120
 Parseval's formula, 24
 Pauli operator, 204
 p -capacity, 78
 Plancherel's theorem, 23
 Poincaré gauge, 173
 Poincaré inequality, 17
 Pointwise q -Hardy inequality, 79
- Principal curvatures, 63
 Principal directions, 63
 Principal solution, 232
- Quasi-relativistic operator, 44
- Radon measure, 59
 Rearrangements, 21
 Relativistic Hardy-type inequalities, 38
 The Rellich inequality in $L^p(\mathbb{R}^n)$, $n \geq 2$, 218
 Rellich-Kondrachov theorem, 17
 Rellich's inequality, 213
 Rellich-Sobolev inequalities, 216
 Ridge, viii, 50
 Ridge point, 50
 Riemann mapping theorem, 96
 Riesz representation theorem, 59
 Ring torus, 69, 119
- Schwarz space, 23
 Skeleton, viii, 51
 Sobolev conjugate, 18
 Sobolev embedding theorem, 17
 Sobolev's inequality, viii, 18
 Sobolev spaces, 13
 Strong barrier, 92
 Subharmonic, 67
 Superharmonic, 67
 Symmetric non-increasing rearrangement, 22
- θ -cone condition, 85
 Titchmarsh-Weyl solutions, 233
 Transversal gauge, 173
 Truncation rules, 20
- Uncertainty principle, 36
 Uniform p -fatness, 78
- Weak Hardy inequality, 27
 Weakly mean convex, 68
 Weakly mean convex domains, viii, 150
 Weyl-Dirac operator, 45, 205
 Weyl's theorem, 33
 Whitney covering, 73
- Young's inequality, 89
- Zeeman term, 204

Notation

$Y := L^1(\mathbb{R}^+; L^\infty(\mathbb{S}^1); rdr) =$	\hat{f} , 23
$L^1(\mathbb{R}^+; rdr) \otimes L^\infty(\mathbb{S}^1),$	\mathbb{D}_A , 205
$\sqrt{-\Delta + m^2}$, 44	$\mathbb{E}f$, 23
$-\Delta_A$, 167	\mathbb{H}_B^1 , 207
A^\star , 22	$\mathbb{M}f$, 79
$C^{k,\gamma}(\bar{\Omega})$, 19	\mathbb{P} , 206
$C^k(\bar{\Omega})$, 19	\mathbb{P}_A , 204
$C_p(K, \Omega)$, 78	\mathbb{S}_A , 204
$Crit_F$, 190	$\mathcal{B}(W^{k,p}(\Omega), W^{k,p}(\mathbb{R}^n))$, 20
$D_0^{1,p}(\Omega)$, 15	$\mathcal{R}(\Omega)$, 50
$D_{int}(\Omega)$, 104	$\mathcal{R}_C(\Omega)$, 50
E^* , 28	\mathcal{S} , 208
$G(\Omega)$, 53	$\mathcal{S}(\mathbb{R}^n)$, 23
$H(\mathbf{y})$, 68	$\mathcal{S}(\Omega)$, 51
H_A^1 , 207	\mathcal{S}^* , 209
$H^{k,p}(\Omega)$, 14	$\dim_A(E)$, 79
$H_{0,A}^1(\Omega)$, 167	$\dim_{M,\Omega}(\partial\Omega)$, 73
$H_{0,A}^{1,p}(\Omega)$, 168	$\dim_{\mathcal{H}}(E)$, 79
H_A , 170	nul , 209
$L_{loc}^1(\Omega)$, 14	∇_A , 167
$L^{q,\infty}(\mathbb{R}^n)$, 25	$\boldsymbol{\sigma} \cdot \mathbf{B}$, 204
$M_k(\lambda)$, 70	$\sigma_k(\lambda)$, 70
$N(\mathbf{x})$, 50	$\sqrt{-\Delta}$, 44
P_I^k , 40	$\tilde{\kappa}$, 65
$W^{k,p}(\Omega)$, 13	Δ_A , 170
$W_0^{k,p}(\Omega)$, 13	$\Psi(\mathbf{A})$, 173
$X = L^\infty(\mathbb{R}^+; L^2(\mathbb{S}^1); rdr) \equiv$	$\Sigma(\Omega)$, 53
$L^\infty(\mathbb{R}^+; rdr) \otimes L^2(\mathbb{S}^1),$	$d\mathbf{A}$, 167
176	$f * g$, 23
$\ u\ _{k,p,\Omega}$, 14	f^* , 21
$\ u\ _{p,\Omega}$, 14	f^\star , 22
\bar{s} , 56	h_A , 170
δ_M , 83	$m(\mathbf{y})$, 56
$\delta_{M,p}$, 83	$ord_{p_j} F$, 190