References

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References


References


References

Graph notation

We try to use as much as possible graph notation compatible with the textbook of Diestel [138]. Here we recall the most basic definitions.

An undirected graph is a pair \( G = (V, E) \), where \( V \) is some set and \( E \) is a family of 2-element subsets of \( V \). The elements of \( V \) are the vertices of \( G \), and the elements of \( E \) are the edges of \( G \). The vertex set of a graph \( G \) is denoted by \( V(G) \) and the edge set of \( G \) is denoted by \( E(G) \). In this book all the graphs are finite, i.e., the sets \( V(G) \) and \( E(G) \) are finite. We use shorthands \( n = |V(G)| \) and \( m = |E(G)| \) whenever their use is not ambiguous. An edge of an undirected graph with endpoints \( u \) and \( v \) is denoted by \( uv \); the endpoints \( u \) and \( v \) are said to be adjacent, and one is said to be a neighbor of the other. We also say that vertices \( u \) and \( v \) are incident to edge \( uv \).

Unless specified otherwise, all the graphs are simple, which means that no two elements of \( E(G) \) are equal, and that all edges from \( E(G) \) have two different endpoints. If we allow multiple edges with the same endpoints (allowing \( E(G) \) to be a multiset), or edges having both endpoints at the same vertex (allowing elements of \( E(G) \) to be multisets of size 2 over \( E(G) \), called loops), then we arrive at the notion of a multigraph. Whenever we use multigraphs, we state it explicitly, making clear whether multiple edges or loops are allowed.

Similarly, a directed graph is a pair \( G = (V, E) \), where \( V \) is the set of vertices of \( G \), and \( E \) is the set of edges of \( G \), which this time are ordered pairs of two different vertices from \( V \). Edges in directed graphs are often also called arcs. For an edge \( (u, v) \in E(G) \), we say that it is directed from \( u \) to \( v \), or simply that it goes from \( u \) to \( v \). Then \( v \) is an outneighbor of \( u \) and \( u \) is an inneighbor of \( v \). For an arc \( a = (u, v) \), \( u \) is the tail and \( v \) is the head of \( a \). Again, we can allow multiple edges with the same endpoints, as well as edges with the same head as tail (loops). Thus we arrive at the notion of a directed multigraph.
For an undirected graph $G$ and edge $uv \in E(G)$, by contracting edge $uv$ we mean the following operation. We remove $u$ and $v$ from the graph, introduce a new vertex $w_{uv}$, and connect it to all the vertices $u$ or $v$ were adjacent to. Note that the operation, as we defined it here, transforms a simple graph into a simple graph; in other words, if $u$ and $v$ had a common neighbor $x$, then the new vertex $w_{uv}$ will be connected to $x$ only via one edge. We may also view the operation of contraction as an operation on multigraphs: every edge $ux$ and $vx$ for some $x \in V(G)$ gives rise to one new edge $w_{uv}x$, and in particular every copy of the edge $uv$ apart from the contracted one gives rise to a new loop at $w_{uv}$. For us the default type of contraction is the one defined on simple graphs, and whenever we use the multigraph version, we state it explicitly.

Graph $H$ is a subgraph of graph $G$ if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. Similarly, graph $H$ is a supergraph of graph $G$ if $V(H) \supseteq V(G)$ and $E(H) \supseteq E(G)$. For a subset $S \subseteq V(G)$, the subgraph of $G$ induced by $S$ is denoted by $G[S]$; its vertex set is $S$ and its edge set consists of all the edges of $E(G)$ that have both endpoints in $S$. For $S \subseteq V(G)$, we use $G - S$ to denote the graph $G[V\setminus S]$, and for $F \subseteq E(G)$ by $G - F$ we denote the graph $(V(G),E(G)\setminus F)$. We also write $G - v$ instead of $G - \{v\}$. The neighborhood of a vertex $v$ in $G$ is $N_G(v) = \{u \in V : uv \in E(G)\}$ and the closed neighborhood of $v$ is $N_G[v] = N_G(v) \cup \{v\}$. For a vertex set $S \subseteq V$, we define $N_G[S] = \bigcup_{v \in S}N[v]$ and $N_G(S) = N_G[S] \setminus S$. We denote by $d_G(v)$ the degree of a vertex $v$ in graph $G$, which is just the number of edges incident to $v$. We may omit indices if the graph under consideration is clear from the context. A graph $G$ is called $r$-regular if all vertices of $G$ have degree $r$. For directed graphs, the notions of inneighborhood, outneighborhood, indegree, and outdegree are defined analogously.

In an undirected graph $G$, a walk of length $k$ is a nonempty sequence of vertices $v_0, \ldots, v_k$ such that for every $i = 0, \ldots, k - 1$ we have $v_iv_{i+1} \in E(G)$. If $v_0 = v_k$, then the walk is closed. A path is a walk where no two vertices appear twice, and a cycle is a closed walk where no two vertices appear twice, apart from the vertex that appears at the beginning and at the end. We often think of paths and cycles as graphs induced by vertices and edges traversed by them. Then a path is a graph $P$ of the form

\[ V(P) = \{v_0, v_1, \ldots, v_k\} \quad E(P) = \{v_0v_1, v_1v_2, \ldots, v_{k-1}v_k\}. \]

If $P = v_0v_1 \ldots v_k$ is a path, then the graph obtained from $P$ by adding edge $x_kx_0$ is a cycle. Again, the length of a path or cycle is equal to the cardinality of its edge set. We denote by $\text{dist}(u,v)$ the distance between $u$ and $v$ in a graph $G$, which is the shortest length of a path between $u$ and $v$. The notions of walks, paths, and cycles can be naturally generalized to directed graphs by requesting compliance with the edge directions.

The girth of a graph $G$ is the shortest length of a cycle in $G$. A Hamiltonian path (cycle) in a graph $G$ is a path (cycle) passing through all the vertices of
A $G$ graph is bipartite if $V(G)$ can be partitioned into two subsets $(A, B)$ such that every edge of $E(G)$ has one endpoint in $A$ and the second in $B$. Partition $(A, B)$ is also called a bipartition of $G$. Equivalently, $G$ is bipartite if and only if $G$ does not have any cycle of odd length.

An undirected graph $G$ is connected if for every pair $u, v$ of its vertices there is a path between $u$ and $v$. A vertex set $X \subseteq V(G)$ is connected if the subgraph $G[X]$ is connected.

A tree $T$ is a connected undirected graph without cycles. A vertex set $X \subseteq V(T)$ is connected if the subgraph $G[X]$ is connected. A tree $T$ can be rooted at a vertex $r \in V(T)$, which imposes on $V(T)$ natural parent-child and ancestor-descendant relations. A forest $F$ is an undirected graph without cycles; thus all the connected components of $F$ are trees. A spanning tree $T$ of a graph $G$ is a tree such that $V(T) = V(G)$ and $E(T) \subseteq E(G)$. A directed graph $G$ is acyclic, or a DAG, if it does not contain any directed cycles.

A matching $M$ in a graph $G$ is a set of edges that pairwise do not share endpoints. A vertex of $G$ is saturated if it is incident to an edge in the matching. Otherwise the vertex is unsaturated. For a given matching $M$, an alternating path is a path in which the edges alternately belong to the matching and do not belong to the matching. An augmenting path is an alternating path that starts from and ends at an unsaturated vertex. A perfect matching is a matching $M$ that saturates all the vertices of the graph, i.e., every vertex of the graph is an endpoint of an edge in $M$.

An independent set $I$ in a graph $G$ is a subset of the vertex set $V(G)$ such that the vertices of $I$ are pairwise nonadjacent. A clique $C$ in a graph $G$ is a subset of the vertex set $V(G)$ such that the vertices of $C$ are pairwise adjacent. A vertex cover $X$ of a graph $G$ is a subset of the vertex set $V(G)$ such that $X$ covers the edge set $E(G)$, i.e., every edge of $G$ has at least one endpoint in $X$. A dominating set $D$ of a graph $G$ is a subset of the vertex set $V(G)$ such that every vertex of $V(G) \setminus D$ has a neighbor in $D$. A proper coloring of a graph $G$ assigns a color to each vertex of $G$ in such a manner that adjacent vertices receive distinct colors. The chromatic number of $G$ is the minimum $\chi$ such that there is a proper coloring of $G$ using $\chi$ colors.

We now define the notion of a planar graph and an embedding of a graph into the plane. First, instead of embedding into the plane we will equivalently embed our graphs into a sphere: in this manner, we do not distinguish unnecessarily the outer face of the embedding. Formally, an embedding of a graph $G$ into a sphere is a mapping that maps (a) injectively each vertex of $G$ into a point of the sphere, and (b) each edge $uv$ of $G$ into a Jordan curve connecting the images of $u$ and $v$, such that the curves are pairwise disjoint (except for the endpoints) and do not pass through any other image of a vertex. A face is a connected component of the complement of the image of $G$ in the sphere; if $G$ is connected, each face is homeomorphic to an open disc. A planar graph is a graph that admits an embedding into a sphere, and a plane graph is a planar graph together with one fixed embedding.

The classic theorems of Kuratowski and Wagner state that a graph is planar if and only if it does not contain $K_5$ or $K_{3,3}$ as a minor (equivalently, as
a topological minor). In Chapters 6 and 7 we briefly mention graphs embed-
dable into more complicated surfaces than plane or sphere. We refer to the
bibliographic notes at the end of Chapter 6 for pointers to the literature on
this topic.

### SAT notation

Let $\text{Vars} = \{x_1, x_2, \ldots, x_n\}$ be a set of Boolean variables. A variable $x$ or
a negated variable $\neg x$ is called a literal. A propositional formula $\varphi$ is in
conjunctive normal form, or is a CNF formula, if it is of the form:

$$\varphi = C_1 \land C_2 \land \ldots \land C_m.$$  

Here, each $C_i$ is a clause of the form

$$C_i = \ell_1^i \lor \ell_2^i \lor \ldots \lor \ell_{r_i}^i,$$

where $\ell_j^i$ are literals of some variables of $\text{Vars}$. The number of literals $r_i$ in
a clause $C_i$ is called the length of the clause, and is denoted by $|C_i|$. The size of
formula $\varphi$ is defined as $|\varphi| = \sum_{i=1}^{m} |C_i|$. The set of clauses of a CNF formula
is usually denoted by $\text{Cls}$.

For $q \geq 2$, a CNF formula $\varphi$ is in $q$-CNF if every clause from $\varphi$ has at
most $q$ literals. If $\varphi$ is a formula and $X$ a set of variables, then we denote
by $\varphi - X$ the formula obtained from $\varphi$ after removing all the clauses that
contain a literal of a variable from $X$.

For a CNF formula $\varphi$ on variables $\text{Vars}$, a truth assignment is a mapping
$\psi: \text{Vars} \to \{\bot, \top\}$. Here, we denote the false value as $\bot$, and the truth
value as $\top$. This assignment can be naturally extended to literals by taking
$\psi(\neg x) = \neg \psi(x)$ for each $x \in \text{Vars}$. A truth assignment $\psi$ satisfies a clause
$C$ of $\varphi$ if and only if $C$ contains some literal $\ell$ with $\psi(\ell) = \top$; $\psi$ satisfies
formula $\varphi$ if it satisfies all the clauses of $\varphi$. A formula is satisfiable if it is
satisfied by some truth assignment; otherwise it is unsatisfiable.

The notion of a truth assignment can be naturally generalized to partial
assignments that valuate only some subset $X \subseteq \text{Vars}$; i.e., $\psi$ is a mapping
from $X$ to $\{\bot, \top\}$. Here, a clause $C$ is satisfied by $\psi$ if and only if $C$
contains some literal $\ell$ whose variable belongs to $X$, and which moreover satisfies
$\psi(\ell) = \top$. 

Notation
Problem definitions

**Problem definitions**

**$(p,q)$-Cluster**

**Input:** A graph $G$, a vertex $v \in V(G)$ and integers $p$ and $q$.

**Question:** Does there exist a $(p,q)$-cluster containing $v$, that is, a set $C \subseteq V(G)$ such that $v \in C$, $|C| \leq p$, and $d(C) \leq q$?

**$(p,q)$-Partition**

**Input:** A graph $G$ and integers $p$ and $q$.

**Question:** Does there exist a partition of $V(G)$ into $(p,q)$-clusters? Here, a set $C \subseteq V(G)$ is a $(p,q)$-cluster if $|C| \leq p$ and $d(C) \leq q$.

**2-Matroid Intersection**

**Input:** A universe $U$, two matrices representing matroids $M_1, M_2$ over $U$, and an integer $k$.

**Question:** Does there exist a set $S \subseteq U$ of size at least $k$ that is independent both in $M_1$ and $M_2$?

**2-SAT**

**Input:** A CNF formula $\varphi$, where every clause consists of at most two literals.

**Question:** Does there exist a satisfying assignment for $\varphi$?

**2-degenerate Vertex Deletion**

**Input:** A graph $G$ and an integer $k$.

**Question:** Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X$ is 2-degenerate?

**2k × 2k Bipartite Permutation Independent Set**

**Input:** An integer $k$ and a graph $G$ with the vertex set $[2k] \times [2k]$, where every edge is between $I_1 = [k] \times [k]$ and $I_2 = ([2k] \setminus [k]) \times ([2k] \setminus [k])$.

**Question:** Does there exist an independent set $X \subseteq I_1 \cup I_2$ in $G$ that induces a permutation of $[2k]$?

**3-Coloring**

**Input:** A graph $G$.

**Question:** Does there exist a coloring $c : V(G) \to \{1, 2, 3\}$ such that $c(u) \neq c(v)$ for every $uv \in E(G)$?

**3-Hitting Set**

**Input:** A universe $U$, a family $\mathcal{A}$ of sets over $U$, where each set in $\mathcal{A}$ is of size at most 3, and an integer $k$.

**Question:** Does there exist a set $X \subseteq U$ of size at most $k$ that has a nonempty intersection with every element of $\mathcal{A}$?
Problem definitions

3-Matroid Intersection

**Input:**
A universe $U$, three matrices representing matroids $M_1, M_2, M_3$ over $U$, and an integer $k$.

**Question:**
Does there exist a set $S \subseteq U$ of size at least $k$ that is independent in every matroid $M_i$, $i = 1, 2, 3$?

3-SAT

**Input:**
A CNF formula $\varphi$, where every clause consists of at most three literals.

**Question:**
Does there exist a satisfying assignment for $\varphi$?

$\mathcal{G}$ Vertex Deletion

**Input:**
A graph $G$, an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X \in \mathcal{G}$? Here, $\mathcal{G}$ denotes any graph class that is a part of the problem definition.

$\ell$-Matroid Intersection

**Input:**
A universe $U$, $\ell$ matrices representing matroids $M_1, M_2, \ldots, M_\ell$ over $U$, and an integer $k$.

**Question:**
Does there exist a set $S \subseteq U$ of size at least $k$ that is independent in every matroid $M_i$, $1 \leq i \leq \ell$?

$\phi$-Maximization

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a set $S$ of at least $k$ vertices of $G$ such that $\phi(G, S)$ is true? Here, $\phi$ denotes any computable Boolean predicate that is a part of the problem definition.

$\phi$-Minimization

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a set $S$ of at most $k$ vertices of $G$ such that $\phi(G, S)$ is true? Here, $\phi$ denotes any computable Boolean predicate that is a part of the problem definition.

$d$-Bounded-Degree Deletion

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that the maximum degree of $G - X$ is at most $d$?

$d$-Clustering

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that the maximum degree of $G - X$ is at most $d$?

$d$-Hitting Set

**Input:**
A universe $U$, a family $\mathcal{A}$ of sets over $U$, where each set in $\mathcal{A}$ is of size at most $d$, and an integer $k$.

**Question:**
Does there exist a set $X \subseteq U$ of size at most $k$ that has a nonempty intersection with every element of $\mathcal{A}$?

$d$-Set Packing

**Input:**
A universe $U$, a family $\mathcal{A}$ of sets over $U$, where each set in $\mathcal{A}$ is of size at most $d$, and an integer $k$.

**Question:**
Does there exist a family $\mathcal{A}' \subseteq \mathcal{A}$ of $k$ pairwise disjoint sets?

$k$-Tree

A synonym for Tree Subgraph Isomorphism.

$k \times k$ Clique

**Input:**
An integer $k$ and a graph $G$ with a vertex set $[k] \times [k]$.

**Question:**
Does there exist a set $X \subseteq V(G)$ that is a clique in $G$ and that contains exactly one element in each row $\{i\} \times [k]$, $1 \leq i \leq k$?
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| **$k \times k$ Permutation Hitting Set with thin sets** |
| **Input:** | An integer $k$ and a family $\mathcal{A}$ of subsets of $[k] \times [k]$, where every element of $\mathcal{A}$ contains at most one element in each row $\{i\} \times [k]$, $1 \leq i \leq k$. |
| **Question:** | Does there exist a set $X \subseteq [k] \times [k]$ that has a nonempty intersection with every element of $\mathcal{A}$ and that contains exactly one element in each row $\{i\} \times [k]$, $1 \leq i \leq k$? |

| **$k \times k$ Permutation Clique** |
| **Input:** | An integer $k$ and a graph $G$ with a vertex set $[k] \times [k]$. |
| **Question:** | Does there exist a set $X \subseteq V(G)$ that is a clique in $G$ and that induces a permutation of $[k]$? |

| **$k \times k$ Permutation Hitting Set** |
| **Input:** | An integer $k$ and a family $\mathcal{A}$ of subsets of $[k] \times [k]$. |
| **Question:** | Does there exist a set $X \subseteq [k] \times [k]$ that has a nonempty intersection with every element of $\mathcal{A}$ and that induces a permutation of $[k]$? |

| **$k \times k$ Permutation Hitting Set with thin sets** |
| **Input:** | An integer $k$ and a family $\mathcal{A}$ of subsets of $[k] \times [k]$, where every element of $\mathcal{A}$ contains at most one element in each row $\{i\} \times [k]$, $1 \leq i \leq k$. |
| **Question:** | Does there exist a set $X \subseteq [k] \times [k]$ that has a nonempty intersection with every element of $\mathcal{A}$ and that induces a permutation of $[k]$? |

| **$q$-Coloring** |
| **Input:** | A graph $G$. |
| **Question:** | Does there exist a coloring $c : V(G) \rightarrow [q]$ such that $c(u) \neq c(v)$ for every $uv \in E(G)$? |

| **$q$-SAT** |
| **Input:** | A CNF formula $\varphi$, where every clause consists of at most $q$ literals. |
| **Question:** | Does there exist a satisfying assignment for $\varphi$? |

| **$q$-Set Cover** |
| **Input:** | A universe $U$, a family $\mathcal{F}$ over $U$, where every element of $\mathcal{F}$ is of size at most $q$, and an integer $k$. |
| **Question:** | Does there exist a subfamily $\mathcal{F}' \subseteq \mathcal{F}$ of size at most $k$ such that $\bigcup \mathcal{F}' = U$? |

| **r-Center** |
| **Input:** | A graph $G$ and an integer $k$. |
| **Question:** | Does there exist a set $X$ of at most $k$ vertices of $G$ such that every vertex of $G$ is within distance at most $r$ from at least one vertex of $X$? |

| **s-Way Cut** |
| **Input:** | A graph $G$ and integers $s$ and $k$. |
| **Question:** | Does there exist a set $X$ of at most $k$ edges of $G$ such that $G - X$ has at least $s$ connected components? |
**Almost 2-SAT**

**Input:**
A CNF formula \( \varphi \), where every clause consists of at most two literals, and an integer \( k \).

**Question:**
Is it possible to make \( \varphi \) satisfiable by deleting at most \( k \) clauses?

**Annotated Bipartite Coloring**

**Input:**
A bipartite graph \( G \), two sets \( B_1, B_2 \subseteq V(G) \), and an integer \( k \).

**Question:**
Does there exist a set \( X \) of at most \( k \) vertices of \( G \) such that \( G - X \) has a proper 2-coloring \( c : V(G) \setminus X \to \{1, 2\} \) (i.e., \( c(u) \neq c(v) \) for every edge \( uv \)) that agrees with the sets \( B_1 \) and \( B_2 \), that is, \( f(v) = i \) whenever \( v \in B_i \setminus X \) and \( i = 1, 2 \)?

**Annotated Satisfiable Almost 2-SAT**

**Input:**
A satisfiable formula \( \varphi \) in CNF form, where every clause of \( \varphi \) consists of at most two literals, two sets of variables \( V^\top \) and \( V^\perp \), and an integer \( k \).

**Question:**
Does there exist a set \( X \) of at most \( k \) variables of \( \varphi \) and a satisfying assignment \( \psi \) of \( \varphi - X \) such that \( \psi(x) = \top \) for every \( x \in V^\top \setminus X \) and \( \psi(x) = \bot \) for every \( x \in V^\perp \setminus X \)? Here, \( \varphi - X \) denotes the formula \( \varphi \) with every clause containing at least one variable of \( X \) deleted.

**Bar Fight Prevention**

**Input:**
A bar, a list of \( n \) potential guests, for every pair of guests a prediction whether they will fight if they are admitted together to the bar, and an integer \( k \).

**Question:**
Can you identify a set of at most \( k \) troublemakers, so that if you admit to the bar everybody except for the troublemakers, no fight breaks out among the admitted guests?

**Bipartite Matching**

**Input:**
A bipartite graph \( G \) and an integer \( k \).

**Question:**
Does there exist an edge set \( S \subseteq E(G) \) of size at least \( k \) such that no two edges in \( S \) share an endpoint?

**CNF-SAT**

**Input:**
A formula \( \varphi \) in conjunctive normal form (CNF).

**Question:**
Does there exist a satisfying assignment for \( \varphi \)?

**Chordal Completion**

**Input:**
A graph \( G \) and an integer \( k \).

**Question:**
Can one add at most \( k \) edges to \( G \) to turn it into a chordal graph?

**Chromatic Number**

**Input:**
A graph \( G \).

**Question:**
What is the minimum integer \( q \), such that there exists a coloring \( c : V(G) \to [q] \) satisfying \( c(u) \neq c(v) \) for every edge \( uv \in E(G) \)?

**Clique**

**Input:**
A graph \( G \) and an integer \( k \).

**Question:**
Does there exist a set of \( k \) vertices of \( G \) that is a clique in \( G \)?

**CliqueLog**

**Input:**
A graph \( G \) and an integer \( k \) such that \( k \leq \log_2 |V(G)| \).

**Question:**
Does there exist a set of \( k \) vertices of \( G \) that is a clique in \( G \)?
**Closest String**

**Input:**
A set of $k$ strings $x_1, \ldots, x_k$, each string over an alphabet $\Sigma$ and of length $L$, and an integer $d$.

**Question:**
Does there exist string $y$ of length $L$ over $\Sigma$ such that $d_H(y, x_i) \leq d$ for every $1 \leq i \leq k$? Here, $d_H(x, y)$ is the Hamming distance between strings $x$ and $y$, that is, the number of positions where $x$ and $y$ differ.

**Closest Substring**

**Input:**
A set of $k$ strings $x_1, \ldots, x_k$ over an alphabet $\Sigma$, and integers $L$ and $d$

**Question:**
Does there exist a string $s$ of length $L$ such that, for every $1 \leq i \leq k$, $x_i$ has a substring $x_i'$ of length $L$ with $d_H(s, x_i) \leq d$. Here, $d_H(x, y)$ is the Hamming distance between strings $x$ and $y$, that is, the number of positions where $x$ and $y$ differ.

**Cluster Editing**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a string $s$ of length $L$ such that every connected component of $G - X$ contains at most $k$ vertices? Here, a cluster graph is a graph where every connected component is a clique.

**Cluster Vertex Deletion**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X$ is a cluster graph? Here, a cluster graph is a graph where every connected component is a clique.

**Cochromatic Number**

**Input:**
A graph $G$.

**Question:**
What is the minimum integer $q$, such that there exists a coloring $c : V(G) \to [q]$ satisfying $c(u) \neq c(v)$ for every pair of nonadjacent vertices $u, v$?

**Colored Red-Blue Dominating Set**

**Input:**
A bipartite graph $G$ with bipartition classes $R \uplus B = V(G)$, an integer $\ell$ and a partition of $R$ into $\ell$ sets $R^1, R^2, \ldots, R^\ell$.

**Question:**
Does there exist a set $X \subseteq R$ that contains exactly one element of every set $R^i$, $1 \leq i \leq \ell$ and such that $N_G(X) = B$?

**Colorful Graph Motif**

**Input:**
A graph $G$, an integer $k$ and a coloring $c : V(G) \to [k]$.

**Question:**
Does there exist a connected subgraph of $G$ that contains exactly one vertex of each color?

**Component Order Integrity**

**Input:**
A graph $G$ and two integers $k$ and $\ell$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that every connected component of $G - X$ contains at most $\ell$ vertices?

**Connected Bull Hitting**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G[X]$ is connected and $G - X$ does not contain a bull as an induced subgraph? Here, a bull is a 5-vertex graph $H$ with $V(H) = \{a, b, c, d, e\}$ and $E(H) = \{ab, bc, ac, bd, ce\}$.

**Connected Dominating Set**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G[X]$ is connected and $N_G[X] = V(G)$?

**Connected Feedback Vertex Set**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G[X]$ is connected and $G - X$ is a forest?
**Connected Vertex Cover**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G[X]$ is connected and $G - X$ is edgeless?

**Cycle Packing**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist in $G$ a family of $k$ pairwise vertex-disjoint cycles?

**Digraph Pair Cut**

**Input:**
A directed graph $G$, a designated vertex $s \in V(G)$, a family of pairs of vertices $F \subseteq (V(G))^2$, and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ edges of $G$, such that for each pair $\{u,v\} \in F$, either $u$ or $v$ is not reachable from $s$ in the graph $G - X$?

**Directed Edge Multicut**

**Input:**
A directed graph $G$, a set of pairs $(s_i,t_i)_{i=1}^{\ell}$ of vertices of $G$, and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ edges of $G$ such that for every $1 \leq i \leq \ell$, vertex $t_i$ is not reachable from vertex $s_i$ in the graph $G - X$?

**Directed Edge Multiway Cut**

**Input:**
A directed graph $G$, a set $T \subseteq V(G)$, and an integer $k$.

**Question:**
Does there exist a set $X \subseteq V(G) \setminus T$ of size at most $k$ such that for every two distinct vertices $t_1,t_2 \in T$, vertex $t_2$ is not reachable from vertex $t_1$ in the graph $G - X$?

**Directed Feedback Arc Set**

**Input:**
A directed graph $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ edges of $G$ such that $G - X$ is acyclic?

**Directed Feedback Arc Set Compression**

**Input:**
A directed graph $G$, a set $W \subseteq V(G)$ such that $G - W$ is acyclic, and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ edges of $G$ such that $G - X$ is acyclic?

**Directed Feedback Vertex Set**

**Input:**
A directed graph $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X$ is acyclic?

**Directed Max Leaf**

**Input:**
A directed graph $G$ and an integer $k$.

**Question:**
Does there exist an out-tree in $G$ with at least $k$ leaves? Here, an out-tree is a directed graph whose underlying undirected graph is a tree, and every vertex of the out-tree, except for one, has indegree exactly one; a leaf of an out-tree is a vertex with outdegree zero.

**Directed Steiner Tree**

**Input:**
A directed graph $G$, a designated root vertex $r$, a set $T \subseteq V(G)$, and an integer $\ell$.

**Question:**
Does there exist a subgraph $H$ of $G$ with at most $\ell$ edges, such that every vertex $t \in T$ is reachable from $r$ in $H$?

**Directed Vertex Multiway Cut**

**Input:**
A directed graph $G$, a set $T \subseteq V(G)$, and an integer $k$.

**Question:**
Does there exist a set $X \subseteq V(G) \setminus T$ of size at most $k$ such that for every two distinct vertices $t_1,t_2 \in T$, vertex $t_2$ is not reachable from vertex $t_1$ in the graph $G - X$?
**Disjoint Factors**

**Input:**
A word $w$ over an alphabet $\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_s\}$.

**Question:**
Does there exist pairwise disjoint subwords $u_1, u_2, \ldots, u_s$ of $w$ such that each $u_i$ is of length at least two and begins and ends with $\gamma_i$?

**Disjoint Feedback Vertex Set**

**Input:**
A graph $G$, a set $W \subseteq V(G)$ such that $G - W$ is a forest, and an integer $k$.

**Question:**
Does there exist a set $X \subseteq V(G) \setminus W$ of size at most $k$ such that $G - X$ is a forest?

**Disjoint Feedback Vertex Set in Tournaments**

**Input:**
A tournament $G$, a set $W \subseteq V(G)$ such that $G - W$ is acyclic, and an integer $k$.

**Question:**
Does there exist a set $X \subseteq V(G) \setminus W$ of size at most $k$ such that $G - X$ is acyclic?

**Disjoint Odd Cycle Transversal**

**Input:**
A graph $G$, a set $W \subseteq V(G)$ such that $G - W$ is bipartite, and an integer $k$.

**Question:**
Does there exist a set $X \subseteq V(G) \setminus W$ of size at most $k$ such that $G - X$ is bipartite?

**Disjoint Planar Vertex Deletion**

**Input:**
A graph $G$, a set $W \subseteq V(G)$ such that $G - W$ is planar, and an integer $k$.

**Question:**
Does there exist a set $X \subseteq V(G) \setminus W$ of size at most $k$ such that $G - X$ is planar?

**Disjoint Vertex Cover**

**Input:**
A graph $G$, a set $W \subseteq V(G)$ such that $G - W$ is edgeless, and an integer $k$.

**Question:**
Does there exist a set $X \subseteq V(G) \setminus W$ of size at most $k$ such that $G - X$ is edgeless?

**Distortion**

**Input:**
A graph $G$ and an integer $d$.

**Question:**
Does there exist an embedding $\eta : V(G) \to \mathbb{Z}$ such that for every $u, v \in V(G)$ we have $\text{dist}_G(u, v) \leq |\eta(u) - \eta(v)| \leq d \cdot \text{dist}_G(u, v)$?

**Dominating Set**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $N_G[X] = V(G)$?

**Dominating Set on Tournaments**

**Input:**
A tournament $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that for every $v \in V(G) \setminus X$ there exists $u \in X$ with $(u, v) \in E(G)$?

**Dominating Set with Pattern**

**Input:**
An integer $k$, and a graph $H$ on vertex set $[k]$.

**Question:**
Does there exist a tuple $(v_1, v_2, \ldots , v_k)$ of distinct vertices of $G$ such that $N_G[[v_1, v_2, \ldots , v_k]] = V(G)$ and $v_i, v_j \in E(G)$ if and only if $ij \in E(H)$?

**Dual-Coloring**

**Input:**
A graph $G$, an integer $k$, and a graph $H$ on vertex set $[k]$.

**Question:**
Does there exist a coloring $c : V(G) \to [n - k]$ such that $c(u) \neq c(v)$ for every edge $uv$?

**Ed-Hitting Set**

**Input:**
A universe $U$, a family $\mathcal{A}$ of sets over $U$, where each set in $\mathcal{A}$ is of size exactly $d$, and an integer $k$.

**Question:**
Does there exist a set $X \subseteq U$ of size at most $k$ that has a nonempty intersection with every element of $\mathcal{A}$?
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**Exact Odd Set**

**Input:**
A universe $U$, a set $F$ of subsets of $U$, and an integer $k$.

**Question:**
Does there exist a nonempty set $X \subseteq U$ of size exactly $k$ such that $|A \cap X|$ is odd for every $A \in F$?

**Exact Unique Hitting Set**

**Input:**
A universe $U$, a set $A$ of subsets of $U$, and an integer $k$.

**Question:**
Does there exist a set $X \subseteq U$ of size exactly $k$ such that $|A \cap X| = 1$ for every $A \in A$?

**FAST**
An abbreviation for Feedback Arc Set in Tournaments.

**FVST**
An abbreviation for Feedback Vertex Set in Tournaments.

**Face Cover**

**Input:**
A graph $G$ embedded on a plane and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ faces of the embedding of $G$ such that every vertex of $G$ lies on at least one face of $X$?

**Feedback Arc Set in Tournaments**

**Input:**
A tournament $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ edges of $G$ such that $G - X$ is acyclic?

**Feedback Vertex Set**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X$ is a forest?

**Feedback Vertex Set in Tournaments**

**Input:**
A tournament $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X$ is acyclic?

**Feedback Vertex Set in Tournaments Compression**

**Input:**
A tournament $G$, a set $W \subseteq V(G)$ such that $G - W$ is acyclic, and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X$ is acyclic?

**Graph Genus**

**Input:**
A graph $G$ and an integer $g$.

**Question:**
Can $G$ be embedded on a surface of Euler genus $g$?

**Graph Isomorphism**

**Input:**
Two graphs $G$ and $H$.

**Question:**
Are $G$ and $H$ isomorphic?

**Graph Motif**

**Input:**
A graph $G$, an integer $k$, a coloring $c : V(G) \to [k]$, and a multiset $M$ with elements from $[k]$.

**Question:**
Does there exist a set $X \subseteq V(G)$ such that $G[X]$ is connected and the multiset $\{c(u) : u \in X\}$ equals $M$?

**Grid Tiling**

**Input:**
An integer $k$, an integer $n$, and a collection $S$ of $k^2$ nonempty sets $S_{i,j} \subseteq [n] \times [n] (1 \leq i,j \leq k)$.

**Question:**
Can one choose for each $1 \leq i,j \leq k$ a pair $s_{i,j} \in S_{i,j}$ such that
- If $s_{i,j} = (a,b)$ and $s_{i,j+1} = (a',b')$, then $a = a'$.
- If $s_{i,j} = (a,b)$ and $s_{i,j+1} = (a',b')$, then $b = b'$.
Problem definitions

**GRID TILING with \( \leq \)**

**Input:**
An integer \( k \), an integer \( n \), and a collection \( S \) of \( k^2 \) nonempty sets \( S_{i,j} \subseteq [n] \times [n] \) (\( 1 \leq i,j \leq k \)).

**Question:**
Can one choose for each \( 1 \leq i,j \leq k \) a pair \( s_{i,j} \in S_{i,j} \) such that
- If \( s_{i,j} = (a,b) \) and \( s_{i+1,j} = (a',b') \), then \( a \leq a' \).
- If \( s_{i,j} = (a,b) \) and \( s_{i,j+1} = (a',b') \), then \( b \leq b' \).

**HALL SET**

**Input:**
A bipartite graph \( G \) with bipartition classes \( A \cup B = V(G) \) and an integer \( k \).

**Question:**
Does there exist a set \( X \subseteq A \) of size at most \( k \) such that \( |N_G(X)| < |X| \)?

**HALTING**

**Input:**
A description of a deterministic Turing machine \( M \), an input word \( x \).

**Question:**
Does \( M \) halt on \( x \)?

**HAMILTONIAN CYCLE**

**Input:**
A graph \( G \).

**Question:**
Does there exist a simple cycle \( C \) in \( G \) such that \( V(C) = V(G) \)?

**HAMILTONIAN PATH**

**Input:**
A graph \( G \).

**Question:**
Does there exist a simple path \( P \) in \( G \) such that \( V(P) = V(G) \)?

**HITTING SET**

**Input:**
A universe \( U \), a family \( \mathcal{A} \) of sets over \( U \), and an integer \( k \).

**Question:**
Does there exist a set \( X \subseteq U \) of size at most \( k \) that has a nonempty intersection with every element of \( \mathcal{A} \)?

**IMBALANCE**

**Input:**
A graph \( G \) and an integer \( k \).

**Question:**
Does there exist a bijective function \( \pi : V(G) \to \{ 1, 2, \ldots, |V(G)| \} \) (called an ordering) whose imbalance is at most \( k \)?

Here, the imbalance of a vertex \( v \in V(G) \) in an ordering \( \pi \) is defined as

\[
\iota_{\pi}(v) = |\{ u \in N_G(v) : \pi(u) < \pi(v) \}| - |\{ u \in N_G(v) : \pi(u) > \pi(v) \}|,
\]

and the imbalance of an ordering \( \pi \) is defined as

\[
\iota(\pi) = \sum_{v \in V(G)} \iota_{\pi}(v).
\]

**INDUENT DOMINATING SET**

**Input:**
A graph \( G \) and an integer \( k \).

**Question:**
Does there exist a set \( X \) of at most \( k \) vertices of \( G \) such that \( G[X] \) is edgeless and \( N_G[X] = V(G) \)?

**INDEPENDENT FEEDBACK VERTEX SET**

**Input:**
A graph \( G \) and an integer \( k \).

**Question:**
Does there exist a set \( X \) of at most \( k \) vertices of \( G \) such that \( G[X] \) is edgeless and \( G - X \) is a forest?

**INDEPENDENT SET**

**Input:**
A graph \( G \) and an integer \( k \).

**Question:**
Does there exist a set \( X \) of at most \( k \) vertices of \( G \) such that \( G[X] \) is edgeless?

**INDUCED MATCHING**

**Input:**
A graph \( G \) and an integer \( k \).

**Question:**
Does there exist a set \( X \) of exactly \( 2k \) vertices of \( G \) such that \( G[X] \) is a matching consisting of \( k \) edges?
**Problem definitions**

**Integer Linear Programming**

**Input:**
Integers $m$ and $p$, a matrix $A \in \mathbb{Z}^{m \times p}$, and vectors $b \in \mathbb{Z}^m$, $c \in \mathbb{Z}^p$.

**Question:**
Find a vector $x \in \mathbb{Z}^p$ that satisfies $Ax \leq b$ and that minimizes the value of the scalar product $c \cdot x$.

**Integer Linear Programming Feasibility**

**Input:**
Integers $m$ and $p$, a matrix $A \in \mathbb{Z}^{m \times p}$, and a vector $b \in \mathbb{Z}^m$.

**Question:**
Does there exist a vector $x \in \mathbb{Z}^p$ such that $Ax \leq b$?

**Linear Programming**

**Input:**
Integers $m$ and $p$, a matrix $A \in \mathbb{R}^{m \times p}$, and vectors $b \in \mathbb{R}^m$, $c \in \mathbb{R}^p$.

**Question:**
Find a vector $x \in \mathbb{R}^p$ that satisfies $Ax \leq b$ and that minimizes the value of the scalar product $c \cdot x$.

**Linkless Embedding**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist an embedding of $G$ into $\mathbb{R}^3$ such that any pairwise linked family of cycles in $G$ has size at most $k$? Here, we say that two vertex-disjoint cycles of $G$ are linked if they cannot be separated by a continuous deformation (i.e., they look like two consecutive links of a chain).

**List Coloring**

**Input:**
A graph $G$ and a set $L(v)$ for every $v \in V(G)$.

**Question:**
Can one choose a color $c(v) \in L(v)$ for every $v \in V(G)$ such that $c(u) \neq c(v)$ for every $uv \in E(G)$?

**Long Induced Path**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist an induced subgraph of $G$ that is isomorphic to a path on $k$ vertices?

**Longest Common Subsequence**

**Input:**
Two strings $a$ and $b$.

**Question:**
What is the length of the longest common subsequence of $a$ and $b$? That is, we ask for the largest possible integer $n$ for which there exist indices $1 \leq i_1 < i_2 < \ldots < i_n \leq |a|$ and $1 \leq j_1 < j_2 < \ldots < j_n \leq |b|$ such that $a[i_r] = b[j_r]$ for every $1 \leq r \leq n$.

**Longest Cycle**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a cycle in $G$ of length at least $k$?

**Longest Path**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a path in $G$ consisting of $k$ vertices?

**Matroid Ed-Set Packing**

**Input:**
An integer $k$ and a matrix $M$ representing a matroid $\mathcal{M}$ of rank $d \cdot k$ over a universe $U$, and a family $\mathcal{A}$ of sets over $U$, where each set in $\mathcal{A}$ is of size exactly $d$.

**Question:**
Does there exist a family $\mathcal{A}' \subseteq \mathcal{A}$ of $k$ pairwise disjoint sets, such that $\bigcup_{A \in \mathcal{A}'} A$ is independent in $\mathcal{M}$?

**Matroid Parity**

**Input:**
A matrix $A$ representing a matroid $\mathcal{M}$ over a universe $U$ of size $2n$, a partition of $U$ into $n$ pairs $P_1, P_2, \ldots, P_n$, and an integer $k$.

**Question:**
Does there exist an independent set $X$ in $\mathcal{M}$ of size $2k$ that is a union of $k$ pairs $P_i$?
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**Min-Ones-2-SAT**

**Input:**
A CNF formula $\varphi$, where every clause consists of at most two literals, and an integer $k$.

**Question:**
Does there exist an assignment $\psi$ that satisfies $\varphi$ and sets at most $k$ variables to true?

**Min-Ones-$r$-SAT**

**Input:**
A CNF formula $\varphi$, where every clause consists of at most $r$ literals, and an integer $k$.

**Question:**
Does there exist an assignment $\psi$ that satisfies $\varphi$ and sets at most $k$ variables to true?

**Minimum Bisection**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a partition $A \uplus B$ of $V(G)$ such that $|A| - |B| \leq 1$ and at most $k$ edges of $G$ have one endpoint in $A$ and the second endpoint in $B$?

**Minimum Maximal Matching**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist an inclusion-wise maximal matching in $G$ that consists of at most $k$ edges?

**Multicolored Biclique**

**Input:**
A bipartite graph $G$ with bipartition classes $A \uplus B = V(G)$, an integer $k$, a partition of $A$ into $k$ sets $A_1, A_2, \ldots, A_k$, and a partition of $B$ into $k$ sets $B_1, B_2, \ldots, B_k$.

**Question:**
Does there exist a set $X \subseteq A \cup B$ that contains exactly one element of every set $A_i$ and $B_i$, $1 \leq i \leq \ell$ and that induces a complete bipartite graph $K_{k,k}$ in $G$?

**Multicolored Clique**

**Input:**
A graph $G$, an integer $k$, and a partition of $V(G)$ into $k$ sets $V_1, V_2, \ldots, V_k$.

**Question:**
Does there exist a set $X \subseteq V(G)$ that contains exactly one element of every set $V_i$ and that is a clique in $G$?

**Multicolored Independent Set**

**Input:**
A graph $G$, an integer $k$, and a partition of $V(G)$ into $k$ sets $V_1, V_2, \ldots, V_k$.

**Question:**
Does there exist a set $X \subseteq V(G)$ that contains exactly one element of every set $V_i$ and that is an independent set in $G$?

**Multicut**

A synonym for Vertex Multicut.

**NAE-SAT**

**Input:**
A CNF formula $\varphi$.

**Question:**
Does there exist an assignment $\psi$ such that for every clause $C$ in $\varphi$, at least one literal of $C$ is evaluated to true and at least one literal is evaluated to false by $\psi$?

**Odd Cycle Transversal**

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X$ is bipartite?

**Odd Set**

**Input:**
A universe $U$, a set $F$ of subsets of $U$, and an integer $k$.

**Question:**
Does there exist a nonempty set $X \subseteq U$ of size at most $k$ such that $|A \cap X|$ is odd for every $A \in F$?

**Partial Dominating Set**

**Input:**
A graph $G$ and integers $k$ and $r$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $|N_G[X]| \geq r$?
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<tr>
<td>A synonym for <strong>Multicolored Clique</strong>.</td>
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<tr>
<td><strong>Perfect $d$-Set Matching</strong></td>
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<tr>
<td><strong>Input:</strong> A universe $U$, a family $A$ of sets over $U$, where each set in $A$ is of size exactly $d$, and an integer $k$.</td>
</tr>
<tr>
<td><strong>Question:</strong> Does there exist a family $A' \subseteq A$ of $k$ pairwise disjoint sets such that $\bigcup A' = U$?</td>
</tr>
<tr>
<td><strong>Perfect Code</strong></td>
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<tr>
<td><strong>Input:</strong> A graph $G$ and an integer $k$.</td>
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<tr>
<td><strong>Question:</strong> Does there exist a set $X$ of at most $k$ vertices of $G$ such that for every $u \in V(G)$ there exists exactly one vertex $v \in X$ for which $u \in N_G[v]$?</td>
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<tr>
<td><strong>Permutation Composition</strong></td>
</tr>
<tr>
<td><strong>Input:</strong> A family $P$ of permutations of a universe $U$, additional permutation $\pi$ of $U$, and an integer $k$.</td>
</tr>
<tr>
<td><strong>Question:</strong> Does there exist a sequence $\pi_1, \pi_2, \ldots, \pi_k \in P$ such that $\pi = \pi_1 \circ \pi_2 \circ \ldots \circ \pi_k$?</td>
</tr>
<tr>
<td><strong>Planar 3-Coloring</strong></td>
</tr>
<tr>
<td><strong>Input:</strong> A planar graph $G$.</td>
</tr>
<tr>
<td><strong>Question:</strong> Does there exist a coloring $c : V(G) \to {1, 2, 3}$ such that $c(u) \neq c(v)$ for every $uv \in E(G)$?</td>
</tr>
<tr>
<td><strong>Planar 3-SAT</strong></td>
</tr>
<tr>
<td><strong>Input:</strong> A CNF formula $\varphi$, such that every clause of $\varphi$ consists of at most three literals and the incidence graph of $\varphi$ is planar. Here, an incidence graph of a formula $\varphi$ is a bipartite graph with vertex sets consisting of all variables and clauses of $\varphi$ where a variable is adjacent to all clauses it appears in.</td>
</tr>
<tr>
<td><strong>Question:</strong> Does there exist a satisfying assignment for $\varphi$?</td>
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<td><strong>Planar Vertex Deletion</strong></td>
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<tr>
<td><strong>Input:</strong> A graph $G$ and an integer $k$.</td>
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<tr>
<td><strong>Question:</strong> Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X$ is planar?</td>
</tr>
<tr>
<td><strong>Planar Deletion Compression</strong></td>
</tr>
<tr>
<td><strong>Input:</strong> A graph $G$, a set $W \subseteq V(G)$ such that $G - W$ is planar, and an integer $k$.</td>
</tr>
<tr>
<td><strong>Question:</strong> Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X$ is planar?</td>
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<tr>
<td><strong>Planar Diameter Improvement</strong></td>
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<tr>
<td><strong>Input:</strong> A planar graph $G$ and an integer $d$.</td>
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<td><strong>Question:</strong> Does there exist a supergraph of $G$ that is planar and has diameter at most $d$?</td>
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<td><strong>Planar Feedback Vertex Set</strong></td>
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<tr>
<td><strong>Input:</strong> A planar graph $G$ and an integer $k$.</td>
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<tr>
<td><strong>Question:</strong> Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X$ is a forest?</td>
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<td><strong>Planar Hamiltonian Cycle</strong></td>
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<tr>
<td><strong>Input:</strong> A planar graph $G$.</td>
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<tr>
<td><strong>Question:</strong> Does there exist a simple cycle $C$ in $G$ such that $V(C) = V(G)$?</td>
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Planar Longest Cycle

**Input:**
A planar graph $G$ and an integer $k$.

**Question:**
Does there exist a cycle in $G$ of length at least $k$?

Planar Longest Path

**Input:**
A planar graph $G$ and an integer $k$.

**Question:**
Does there exist a path in $G$ consisting of $k$ vertices?

Planar Vertex Cover

**Input:**
A planar graph $G$, an integer $k$.

**Question:**
Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X$ is edgeless?

Point Line Cover

**Input:**
A set $P$ of points in the plane and an integer $k$.

**Question:**
Does there exist a family $L$ of at most $k$ lines on the plane such that every point in $P$ lies on some line from $L$?

Polynomial Identity Testing

**Input:**
Two polynomials $f$ and $g$ over a field $F$, given as arithmetic circuits with addition, subtraction and multiplication gates.

**Question:**
Is it true that $f(x) = g(x)$ for every $x \in F$?

Pseudo Achromatic Number

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a partition of $V(G)$ into $k$ sets $V_1, V_2, \ldots, V_k$ such that for every $1 \leq i < j \leq k$ there exists at least one edge of $G$ with one endpoint in $V_i$ and the second endpoint in $V_j$?

Ramsey

**Input:**
A graph $G$ and an integer $k$.

**Question:**
Does there exist a set $X$ of exactly $k$ vertices of $G$ such that $G[X]$ is a clique or $G[X]$ is edgeless?

Red-Blue Dominating Set

**Input:**
A bipartite graph $G$ with bipartition classes $R \uplus B = V(G)$ and an integer $k$.

**Question:**
Does there exist a set $X \subseteq R$ of size at most $k$ such that $N_G(X) = B$?

Satellite Problem

**Input:**
A graph $G$, integers $p$ and $q$, a vertex $v \in V(G)$, and a partition $V_0, V_1, \ldots, V_r$ of $V(G)$ such that $v \in V_0$ and there is no edge between $V_i$ and $V_j$ for any $1 \leq i < j \leq r$.

**Question:**
Does there exist a $(p,q)$-cluster $C$ satisfying $V_0 \subseteq C$ such that for every $1 \leq i \leq r$, either $C \cap V_i = \emptyset$ or $V_i \subseteq C$? Here, a set $C \subseteq V(G)$ is a $(p,q)$-cluster if $|C| \leq p$ and $d(C) \leq q$.

Scattered Set

**Input:**
A graph $G$ and integers $r$ and $k$.

**Question:**
Does there exist a set $X$ of at least $k$ vertices of $G$ such that $\text{dist}_{G}(u,v) > r$ for every distinct $u,v \in X$?

Set Cover

**Input:**
A universe $U$, a family $\mathcal{F}$ over $U$, and an integer $k$.

**Question:**
Does there exist a subfamily $\mathcal{F}' \subseteq \mathcal{F}$ of size at most $k$ such that $\bigcup \mathcal{F}' = U$?

Set Packing

**Input:**
A universe $U$, a family $\mathcal{A}$ of sets over $U$, and an integer $k$.

**Question:**
Does there exist a family $\mathcal{A}' \subseteq \mathcal{A}$ of $k$ pairwise disjoint sets?

Set Splitting

**Input:**
A universe $U$ and a family $\mathcal{F}$ of sets over $U$.

**Question:**
Does there exist a set $X \subseteq U$ such that $A \cap X \neq \emptyset$ and $A \setminus X \neq \emptyset$ for every $A \in \mathcal{F}$?
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<td>A description of a nondeterministic Turing machine $M$, a string $x$, and an integer $k$.</td>
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<tr>
<td><strong>Question:</strong></td>
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<tr>
<td>Does there exist a computation path of $M$ that accepts $x$ on at most $k$ steps?</td>
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<tr>
<td><strong>Skew Edge Multicut</strong></td>
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<tr>
<td><strong>Input:</strong></td>
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<tr>
<td>A directed graph $G$, a set of pairs $(s_i, t_i)_{i=1}^{\ell}$ of vertices of $G$, and an integer $k$.</td>
</tr>
<tr>
<td><strong>Question:</strong></td>
</tr>
<tr>
<td>Does there exist a set $X$ of at most $k$ edges of $G$ such that for every $1 \leq i \leq j \leq \ell$, vertex $t_j$ is not reachable from vertex $s_i$ in the graph $G - X$?</td>
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<td><strong>Special Disjoint FVS</strong></td>
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<td><strong>Input:</strong></td>
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<tr>
<td>A graph $G$, a set $W \subseteq V(G)$ such that $G - W$ is edgeless and every vertex of $V(G) \setminus W$ is of degree at most three in $G$, and an integer $k$.</td>
</tr>
<tr>
<td><strong>Question:</strong></td>
</tr>
<tr>
<td>Does there exist a set $X \subseteq V(G) \setminus W$ of size at most $k$ such that $G - X$ is a forest?</td>
</tr>
<tr>
<td><strong>Split Edge Deletion</strong></td>
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<tr>
<td><strong>Input:</strong></td>
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<tr>
<td>A graph $G$ and an integer $k$.</td>
</tr>
<tr>
<td><strong>Question:</strong></td>
</tr>
<tr>
<td>Does there exist a set $X$ of at most $k$ edges of $G$ such that $G - X$ is a split graph? Here, a split graph is a graph whose vertex set can be partitioned into two parts, one inducing a clique and one inducing an independent set.</td>
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<tr>
<td><strong>Split Vertex Deletion</strong></td>
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<td><strong>Input:</strong></td>
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<td>A graph $G$ and an integer $k$.</td>
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<td><strong>Question:</strong></td>
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<tr>
<td>Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X$ is a split graph? Here, a split graph is a graph whose vertex set can be partitioned into two parts, one being a clique and one being an independent set.</td>
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<tr>
<td><strong>Steiner Tree</strong></td>
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<tr>
<td><strong>Input:</strong></td>
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<tr>
<td>A graph $G$, a set $K \subseteq V(G)$, and an integer $k$.</td>
</tr>
<tr>
<td><strong>Question:</strong></td>
</tr>
<tr>
<td>Does there exist a connected subgraph of $G$ that contains at most $k$ edges and contains all vertices of $K$?</td>
</tr>
<tr>
<td><strong>Strongly Connected Steiner Subgraph</strong></td>
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<tr>
<td><strong>Input:</strong></td>
</tr>
<tr>
<td>A directed graph $G$, a set $K \subseteq V(G)$, and an integer $k$.</td>
</tr>
<tr>
<td><strong>Question:</strong></td>
</tr>
<tr>
<td>Does there exist a strongly connected subgraph of $G$ that contains at most $k$ edges and contains all vertices of $K$?</td>
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<td><strong>Subgraph Isomorphism</strong></td>
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<td><strong>Input:</strong></td>
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<tr>
<td>Two graphs $G$ and $H$.</td>
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<tr>
<td><strong>Question:</strong></td>
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<tr>
<td>Does there exist a subgraph of $G$ that is isomorphic to $H$?</td>
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<tr>
<td><strong>Subset Sum</strong></td>
</tr>
<tr>
<td><strong>Input:</strong></td>
</tr>
<tr>
<td>A set $S$ of integers and integers $k$ and $m$.</td>
</tr>
<tr>
<td><strong>Question:</strong></td>
</tr>
<tr>
<td>Does there exist a subset $X \subseteq S$ of size at most $k$ whose elements sum up to $m$?</td>
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<tr>
<td><strong>TSP</strong></td>
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<tr>
<td><strong>Input:</strong></td>
</tr>
<tr>
<td>A graph $G$ with edge weights $w : E(G) \to \mathbb{R}_{&gt;0}$.</td>
</tr>
<tr>
<td><strong>Question:</strong></td>
</tr>
<tr>
<td>Find a closed walk of minimum possible total weight that visits all vertices of $G$.</td>
</tr>
<tr>
<td><strong>Total Dominating Set</strong></td>
</tr>
<tr>
<td><strong>Input:</strong></td>
</tr>
<tr>
<td>A graph $G$ and an integer $k$.</td>
</tr>
<tr>
<td><strong>Question:</strong></td>
</tr>
<tr>
<td>Does there exist a set $X$ of at most $k$ vertices of $G$ such that for every $u \in V(G)$ there exists $v \in X$ with $uv \in E(G)$?</td>
</tr>
<tr>
<td>Problem Definition</td>
</tr>
<tr>
<td>-------------------</td>
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<tr>
<td><strong>Tree Spanner</strong></td>
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<tr>
<td><strong>Tree Subgraph Isomorphism</strong></td>
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<td><strong>Treewidth</strong></td>
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<td><strong>Treewidth-$\eta$ Modulator</strong></td>
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<td><strong>Triangle Packing</strong></td>
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<td><strong>Unique Hitting Set</strong></td>
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<td><strong>Unit Square Independent Set</strong></td>
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<td><strong>Variable Deletion Almost 2-SAT</strong></td>
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<td><strong>Vertex $k$-Way Cut</strong></td>
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<td><strong>Vertex Coloring</strong></td>
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<td><strong>Vertex Cover</strong></td>
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<td><strong>Vertex Cover Above Matching</strong></td>
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### Problem definitions

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<tr>
<th><strong>Vertex Cover Above LP</strong></th>
<th><strong>Weighted Independent Set</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> A graph $G$ and an integer $k$.</td>
<td><strong>Input:</strong> A graph $G$ with vertex weights $w : V(G) \rightarrow \mathbb{R}_{\geq 0}$.</td>
</tr>
<tr>
<td><strong>Question:</strong> Does there exist a set $X$ of at most $k$ vertices of $G$ such that $G - X$ is edgeless? Note that this is the same problem as <em>Vertex Cover</em>, but the name <em>Vertex Cover Above LP</em> is usually used in the context of above guarantee parameterization with an optimum solution to a linear programming relaxation as a lower bound.</td>
<td><strong>Question:</strong> Find an independent set in $G$ of the maximum possible total weight.</td>
</tr>
</tbody>
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<table>
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<tr>
<th><strong>Vertex Disjoint Paths</strong></th>
<th><strong>Weighted Longest Path</strong></th>
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</thead>
<tbody>
<tr>
<td><strong>Input:</strong> A graph $G$ and $k$ pairs of vertices $(s_i, t_i)_{i=1}^k$.</td>
<td><strong>Input:</strong> A graph $G$ with vertex weights $w : V(G) \rightarrow \mathbb{N}$ and an integer $k$.</td>
</tr>
<tr>
<td><strong>Question:</strong> Do there exist $k$ pairwise vertex-disjoint paths $P_1, P_2, \ldots, P_k$ such that $P_i$ starts in $s_i$ and ends in $t_i$?</td>
<td><strong>Question:</strong> Find a simple path in $G$ on $k$ vertices of the minimum possible total weight.</td>
</tr>
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<table>
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<tr>
<th><strong>Vertex Multicut</strong></th>
<th><strong>Weighted Circuit Satisfiability</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> A graph $G$, a set of pairs $(s_i, t_i)_{i=1}^\ell$ of vertices of $G$, and an integer $k$.</td>
<td><strong>Input:</strong> A Boolean circuit $C$ and an integer $k$.</td>
</tr>
<tr>
<td><strong>Question:</strong> Does there exist a set $X$ of at most $k$ vertices of $G$, not containing any vertex $s_i$ or $t_i$, such that for every $1 \leq i \leq \ell$, vertices $s_i$ and $t_i$ lie in different connected components of $G - X$?</td>
<td><strong>Question:</strong> Does there exist an assignment to the input gates of $C$ that satisfies $C$ and that sets exactly $k$ input gates to true?</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th><strong>Vertex Multiway Cut</strong></th>
<th><strong>Weighted Independent Set</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> A graph $G$, a set $T \subseteq V(G)$, and an integer $k$.</td>
<td><strong>Input:</strong> A graph $G$ with vertex weights $w : V(G) \rightarrow \mathbb{R}_{\geq 0}$.</td>
</tr>
<tr>
<td><strong>Question:</strong> Does there exist a set $X \subseteq V(G) \setminus T$ of size at most $k$ such that every element of $T$ lies in a different connected component of $G - X$?</td>
<td><strong>Question:</strong> Find an independent set in $G$ of the maximum possible total weight.</td>
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<tr>
<th><strong>Weighted Longest Path</strong></th>
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<tbody>
<tr>
<td><strong>Input:</strong> A graph $G$ with vertex weights $w : V(G) \rightarrow \mathbb{N}$ and an integer $k$.</td>
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