Notation List

$\mathrm{Im}(\varphi)$	The image of φ
$\mathrm{Ker}(\varphi)$	The kernel of φ
$\mathrm{Coker}(\varphi)$	The cokernel of φ
$\mathrm{Proj}\,\mathbf{R}$	The isomorphism class of finitely-generated projective $\mathbf{R}$-modules
$\mathrm{K}_0(\mathbf{R})$	The Groethendieck group of $\mathbf{R}$
$\mathrm{Syz}(a_1,\ldots,a_n)$	The syzygy module of $(a_1,\ldots,a_n)$
$\mathrm{GL}_s(\mathbf{R})$	The group of invertible matrices of size $s \times s$ with entries in $\mathbf{R}$
$\mathrm{SL}_s(\mathbf{R})$	The subgroup of $\mathrm{GL}_s(\mathbf{R})$ formed by matrices of determinant 1
$\mathrm{M}_{n,m}(\mathbf{R})$	The set of matrices of size $n \times m$ with entries in $\mathbf{R}$
$\mathrm{M}_n(\mathbf{R})$	The set of matrices of size $n \times n$ with entries in $\mathbf{R}$
$\mathrm{M}(\mathbf{R})$	The set $\cup_{n\geq 1}\mathrm{M}_n(\mathbf{R})$
$\mathrm{Idem}(\mathbf{R})$	The set of idempotent matrices in $\mathrm{M}(\mathbf{R})$
$\mathrm{GL}(\mathbf{R})$	The group $\cup_{n\geq 1}\mathrm{GL}_n(\mathbf{R})$
$\mathrm{E}_s(\mathbf{R})$	The subgroup of $\mathrm{SL}_s(\mathbf{R})$ generated by elementary matrices
$\mathscr{D}_k(G)$	The determinantal ideal of order k of G
$\mathscr{F}_n(T)$	The nth Fitting ideal of T
$\mathbf{R}^\times$	The group of units of $\mathbf{R}$
$\mathrm{I}_{r,m}$	The standard projection matrix $\begin{pmatrix} \mathrm{I}_r & 0_{r,m-r} \\ 0_{m-r,r} & 0_{m-r,m-r} \end{pmatrix}$
$\mathrm{Um}_n(\mathbf{R})$	The set of unimodular rows (or vectors) of length n with entries in $\mathbf{R}$
$\mathrm{Rad}(\mathbf{R})$	The set of all $x \in \mathbf{R}$ such that $1 + x\mathbf{R} \subseteq \mathbf{R}^\times$
$a^{\mathbb{N}}$	The monoid $\{a^n;\, n \in \mathbb{N}\}$
$\mathscr{M}(a)$	The monoid $a^{\mathbb{N}}$
$S^{-1}\mathbf{R}$ or $\mathbf{R}_S$	The localization of $\mathbf{R}$ at S

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I. Yengui, *Constructive Commutative Algebra*, Lecture Notes in Mathematics 2138,
DOI 10.1007/978-3-319-19494-3

$\mathrm{Spec}(\mathbf{R})$	The set of prime ideals of $\mathbf{R}$
$\mathscr{M}(U)$	The monoid generated by U
$\mathscr{I}_{\mathbf{R}}(I)$ or $\mathscr{I}(I)$	The ideal generated by I
$\mathscr{S}(I;U)$	The monoid $\mathscr{M}(U)+\mathscr{I}(I)$
$\mathscr{M}(u_1,\dots,u_\ell)$	The monoid $\mathscr{M}(\{u_1,\dots,u_\ell\})$
$\mathscr{I}(a_1,\dots,a_k)$	The ideal $\mathscr{I}(\{a_1,\dots,a_k\})$
$\mathscr{S}(a_1,\dots,a_k;u_1,\dots,u_\ell)$	The monoid $\mathscr{M}(u_1,\dots,u_\ell)+\mathscr{I}(a_1,\dots,a_k)$
$\mathbf{R}_a$	The localization of the ring $\mathbf{R}$ at the monoid $a^{\mathbb{N}}$
M_a	The localization of the module M at the monoid $a^{\mathbb{N}}$
$\mathrm{Res}(f,g)$	The resultant of f and g
$\mathrm{Res}_X(f,g)$	The resultant of f and g with respect to X
$\gcd(f,g)$	The greatest common divisor of f and g
$\mathbf{R}\langle X\rangle$	The localization of $\mathbf{R}[X]$ at the monoid of monic polynomials
$\mathbf{R}(X)$	The localization of $\mathbf{R}[X]$ at the monoid of primitive polynomials
$\mathbf{R}_{\mathfrak{p}}$	The localization of $\mathbf{R}$ at the monoid $\mathbf{R}\setminus\mathfrak{p}$, where $\mathfrak{p}$ is a prime ideal
$\mathbf{R}_{\mathrm{red}}$	The reduced ring associated to the ring $\mathbf{R}$
$\mathrm{LC}(f)$	The leading coefficient of the polynomial f
$\mathrm{LM}(f)$	The leading monomial of the polynomial f
$\mathrm{LT}(f)$	The leading term of the polynomial f
$\mathrm{LT}(I)$	The ideal $\langle \mathrm{LT}(f) : f\in I\rangle$
$\mathrm{mdeg}(f)$	The multidegree of the polynomial f
$\mathrm{tdeg}(f)$	The total degree of the polynomial f
$E_{i,j}(a)$	The matrix with 1s on the diagonal, a on position (i,j) and 0s elsewhere
$\bar{S}$	The saturation of the monoid S
$\mathrm{D}_{\mathbf{R}}(\mathfrak{a})$	The radical $\sqrt{\mathfrak{a}}$ of the ideal $\mathfrak{a}$
$\mathrm{D}_{\mathbf{R}}(x_1,\dots,x_n)$	The radical ideal $\mathrm{D}_{\mathbf{R}}(\langle x_1,\dots,x_n\rangle)$
$\mathrm{Zar}\,\mathbf{R}$	The Zariski lattice of the ring $\mathbf{R}$, i.e., the set $\{\mathrm{D}_{\mathbf{R}}(x_1,\dots,x_n)\mid n\in\mathbb{N}\ \&\ x_1,\dots,x_n\in\mathbf{R}\}$
$I:J$	The conductor of J in I, i.e., $\{x\in\mathbf{R}\mid xJ\subseteq I\}$, where I,J are ideals of the ring $\mathbf{R}$
$I:a$	The conductor $I:\langle a\rangle$ of $\langle a\rangle$ in I
$\mathrm{Ann}(x)$	The annihilator of x, i.e., $\langle 0\rangle:\langle x\rangle$
$\mathrm{K}_{\mathbf{R}}(x)$	The Krull boundary ideal of x, i.e., $\langle x\rangle+(\mathrm{D}_{\mathbf{R}}(0):x)$
$\mathbf{R}^{\{x\}}$	The upper Krull boundary of x in $\mathbf{R}$, i.e., $\mathbf{R}/\mathrm{K}_{\mathbf{R}}(x)$
$S_{\{x\}}$	The Krull boundary monoid of x., i.e., $x^{\mathbb{N}}(1+x\mathbf{R})$
$\mathbf{R}_{\{x\}}$	The lower Krull boundary of x in $\mathbf{R}$, i.e., $\mathbf{R}_{S_{\{x\}}}$
$\mathrm{Kdim}\mathbf{R}$ or $\dim\mathbf{R}$	The Krull dimension of $\mathbf{R}$

$I_{\mathbf{R}}(a,b) = \cup_{n\in\mathbb{N}}(a^n b^{n+1}\mathbf{R} + a^{n+1}\mathbf{R} : a^n b^n \mathbf{R})$	
$H \triangleleft G$	H is a normal subgroup of G
$H \ntriangleleft G$	H is not a normal subgroup of G
$\mathrm{diag}(u_1,\ldots,u_n)$	The matrix in $\mathrm{M}_n(\mathbf{R})$ with u_i on position (i,i) for $1 \le i \le n$, and 0s elsewhere
$e_{i,j}$	The matrix with 1 on position (i,j) and 0s elsewhere
$u \sim_G u'$	There exists $A \in G$ such that $uA = u'$
$\mathrm{GL}_n(\mathbf{A},\, J)$	The normal subgroup of $\mathrm{GL}_n(\mathbf{A})$ consisting of matrices M which are $\equiv \mathrm{I}_n \bmod \mathrm{M}_n(J)$
$\mathrm{E}_n(\mathbf{A},\, J)$	The normal subgroup of $\mathrm{E}_n(\mathbf{A})$ generated by the elementary matrices $\{E_{i,j}(a);\ a \in J,\ 1 \le i \ne j \le n\}$
$\mathrm{SL}_n(\mathbf{A},\, J)$	The group $\mathrm{SL}_n(\mathbf{A}) \cap \mathrm{GL}_n(\mathbf{A},\, J)$
$\mathbb{M}_n^m$	The set of monomials in $\mathbf{R}[X_1,\ldots,X_n]^m$
$\mathbb{M}_n$	The set of monomials in $\mathbf{R}[X_1,\ldots,X_n]$
$Y^{\uparrow}$	The set $\{Z \in E \mid Z \ge Y\}$
$\mathscr{M}_E^+(Y_1,\ldots,Y_m)$	The final subset of E of finite type $\cup_{i=1}^m Y_i^{\uparrow}$
$\mathscr{F}(E)$	The set of final subsets of finite type of E, including the empty subset considered as generated by the empty family
$\mathscr{M}_d$	The set $\mathscr{F}(\mathbb{N}^d) \setminus \{\emptyset\}$
$\bar{f}^F$	A remainder of f on division by F
$\mathrm{LC}_n(I)$	The ideal generated by the leading coefficients of the elements of I of degree n
$\mathrm{LC}_\infty(I)$	The ideal $\cup_{n\in\mathbb{N}}\mathrm{LC}_n(I)$
$S(f,g)$	The S-polynomial of f and g
$S(f,f)$	The auto-S-polynomial of f
$\mathbb{Z}_{p\mathbb{Z}}$	The localization $\{\frac{a}{b} \in \mathbb{Q} \mid a \in \mathbb{Z}$ and $b \in \mathbb{Z} \setminus p\mathbb{Z}\}$ of $\mathbb{Z}$
$\mathbf{R}\langle X_1,\ldots,X_n\rangle$	The ring $(\mathbf{R}\langle X_1,\ldots,X_{n-1}\rangle)\langle X_n\rangle$
$\mathrm{Ann}(a^\infty)$	The ideal $\cup_{n\in\mathbb{N}}\mathrm{Ann}(a^n)$
$\mathbb{F}_2$	The field with two elements
$\mathbf{R}_{a_1.a_2.\ldots.a_n}$	The ring $\mathscr{M}(a_1,\ldots,a_n)^{-1}\mathbf{R} = \mathbf{R}[\frac{1}{a_1\cdots a_n}]$
$\mathrm{Sat}(S)$	The saturation of S
$(I : a^\infty)$	The ideal $\{x \in \mathbf{A} \mid \exists n \in \mathbb{N} \mid xa^n \in I\}$
$\mathrm{index}(u)$	The position of the last primitive component of u
$\mathrm{PrimMon}(u)$	The last monomial of u which has an invertible coefficient
$\mathrm{PrimCoeff}(u)$	The coefficient of $\mathrm{PrimMon}(u)$
$\mathrm{Prim}(u)$	The primitive version of u
$\mathscr{I}(u)$	The height of u, i.e., the couple $(\mathrm{index}(u), \mathrm{mdeg}(\mathrm{PrimMon}(u)))$

Echel(S)	The list S put in an echelon form
PrimRed$(s;S)$	The reduction u of s modulo S so that $[S,u]$ becomes in an echelon form
$\mathrm{HS}_I(t)$	The Hilbert series of I
$\delta(S)$	The (saturation) defect of the list S
$\delta_S(t)$	The (saturation) defect series of the list S
tdeg(f)	The total degree of f

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I. Yengui, *Constructive Commutative Algebra*, Lecture Notes in Mathematics 2138,
DOI 10.1007/978-3-319-19494-3

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