Appendix A

A.1 Degeneracy of UCTP Conflict Graphs

In this section, we document the degeneracy of conflict graphs for various University Course Timetabling Problem (UCTP) benchmark sets that do not have availability requirements. This is supplementary data for Section 2.3.2. The tables A.1 and A.2 show degeneracy values of UCTP instance conflict graphs which were computed by repeatedly removing vertices of minimal degree (see Sect. 2.3.1). Entries in bold face indicate that the clash-free timetables are connected with respect to the Kempe-exchange operation due to Thm. 2.10.

A.2 A Meta-heuristics Toolbox

Since course timetabling problems are typically computationally hard, practitioners and researches typically resort to meta-heuristics as solution approaches. In this section, we briefly review several basic meta-heuristic techniques for solving search and optimization problems. This section serves as a supplement to our discussion of different solution approaches to timetabling problems in the previous chapters.

Many search and optimization problems are hard, that is, no algorithms are known that find optimal solutions in polynomial time. Meta-heuristics are a popular class of approaches that attempt to find *good* solutions within reasonable time. They are often the last resort to problems (e.g., course timetabling problems), which cannot be dealt with satisfactorily by other approaches. On the downside, not much has been proved concerning useful properties of the solutions generated by meta-heuristics. Their performance is typically determined experimentally. A large number of meta-heuristics exists that are inspired by how nature solves optimization/adaptation tasks, for example Evolutionary Algorithm [BFM97] and particle swarm optimization (PSO) [Ken10], and ant colony optimization (ACO) [DS04].

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Table A.1 Degeneracy values of the conflict graphs

Instance	deg(G)	Instance	deg(G)	Instance	deg(G)
small_1	54	med_1	59	big_1	60
small_2	**41**	med_2	67	big_2	68
small_3	98	med_3	67	big_3	64
small_4	69	med_4	69	big_4	80
small_5	84	med_5	87	big_5	75
small_6	**24**	med_6	101	big_6	93
small_7	68	med_7	120	big_7	111
small_8	84	med_8	98	big_8	82
small_9	124	med_9	121	big_9	77
small_10	136	med_10	64	big_10	77
small_11	**34**	med_11	97	big_11	76
small_12	**22**	med_12	78	big_12	76
small_13	146	med_13	105	big_13	84
small_14	100	med_14	92	big_14	74
small_15	79	med_15	101	big_15	127
small_16	118	med_16	145	big_16	115
small_17	120	med_17	126	big_17	184
small_18	60	med_18	188	big_18	131
small_19	141	med_19	173	big_19	159
small_20	**28**	med_20	153	big_20	144

The instances are the CB-CTT, PE-CTT and Erlangen benchmarking instances from [DGS]. For each instance there are 45 timeslots. The degeneracy values are marked in bold face if the connectedness of clash-free timetables is established by Theorem 2.10

Table A.2 Degeneracy values of the conflict graphs of the metaheuristic network instances

Instance	deg(G)	Instance	deg(G)	Instance	deg(G)
easy01	**15**	medium01	49	hard01	68
easy02	**19**	medium02	53	hard02	67
easy03	**13**	medium03	52		
easy04	**12**	medium04	51		
easy05	**20**	medium05	47		

For each instance there are 45 timeslots. Degeneracy values are marked in bold face if the connectedness of clash-free timetables is established by Theorem 2.10

Most of the nature-inspired approaches operate on a population of candidate solutions to a problem instance. Another class of metaheuristics is based on the Local Search (LS) paradigm, which aims for an iterative improvement of a single candidate solution [PS98, p. 454]. Examples of LS variants are Simulated Annealing (SA) [KGV83], Tabu Search (TS) [GL97], and Iterated Local Search (ILS) [LMS03]. Without loss of generality, we will talk about minimization problems (Tables A.1 and A.2).

A.2.1 Local Search

In order to find an optimal solution to a problem instance $\mathcal{I}$, an LS algorithm traverses the search space of a problem instance $\mathcal{I}$ using some neighborhood N on the solutions:

$$\mathrm{N} : \mathcal{S}(\mathcal{I}) \rightarrow \mathcal{P}(\mathcal{S}(\mathcal{I})) \ .$$

Algorithm 6 shows the basic LS procedure as given in [PS98, p. 454]. The function `step` selects a neighbor from the neighborhood N:

$$\texttt{step}(t) = \begin{cases} \text{some } s \in \mathrm{N}(t) \text{ s.t. } f(s) < f(t) & \text{if it exists} \\ \text{“no”} & \text{otherwise} \end{cases} ,$$

where f is the objective function. Stochastic Local Search (SLS) algorithms introduce randomness during the initialization and/or the neighborhood selection [HS04, pp. 30–31]. Furthermore, neighbors may be selected even if they are not strict improvements over the current solution in order to escape local optima. This class of algorithms includes for example SA, TS, and ILS.

Algorithm 6: LocalSearch

input : $\mathcal{I}$: problem instance
output: A solution to $\mathcal{I}$

$s \longleftarrow$ initial solution
while `step`$(s) \neq$ *“no”* **do**
  $s \longleftarrow$ `step`(s)
return s

Simulated Annealing

Simulated Annealing (SA) is a SLS optimization algorithm inspired by the minimization of free energy during the annealing process in metallurgy [KGV83]. An overview of different variants of the algorithm and theoretical results is given in [HJJ03a]. The selection of the initial solution is specific to the problem to be solved. For example, the initialization step for solving the MMF-CB-CTT problem may consist of solving the underlying search problem, see [MW12*]. In each iteration, the SA algorithm selects a neighbor s of the current solution t at random from $\mathrm{N}(t)$. The neighbor s replaces t (“s is accepted”), if it is better. If it is worse then it is accepted with a certain probability that tends to decrease over time. More specifically, the probability of accepting a worsening depends on (i) the current temperature ϑ and (ii) the *energy difference* $\Delta E(t,s)$ between the current solution t and the neighbor s. The accepted solution is determined as follows:

$$\texttt{accept}(t,s) = \begin{cases} s & \text{if } f(s) < f(t) \text{ or } \mathrm{u}(0,1) \leq \exp(-\Delta E(t,s)/\vartheta) \\ t & \text{otherwise} \end{cases} ,$$

where $\mathrm{u}(0,1)$ draws a number from the interval $[0,1]$ uniformly at random. The energy difference is typically set to $\Delta E(t,s) = f(t) - f(s)$. In summary, several things happen during each iteration: First, a neighbor s is picked from $\mathrm{N}(t)$ at random. Then, t is updated to $\texttt{accept}(s,t)$. Finally, the temperature ϑ is decreased according to the cooling schedule.

A wealth of cooling schedules is available in the literature, see for example [HJJ03b]. A rather generic cooling schedule is *geometric cooling*. It has been applied for example for solving timetabling problems in [TD98, Kos05, MW12*]. Given an initial temperature ϑ_{max}, the cooling schedule decreases the temperature exponentially:

$$\vartheta(i) = \beta^i \cdot \vartheta_{max} \quad ,$$

where i is the iteration.

Tabu Search

Tabu Search (TS) is a SLS variant which utilizes a memory structure, the *tabu list*, in order to guide the search process. We follow the general description of TS given in [Glo89]. The neighborhood exploration is based on so-called *moves*, which transform a solution into a neighbor solution. The tabu list contains *forbidden* and thus renders a part of the neighborhood "tabu". In each iteration, a set S of solutions is sampled from the neighborhood, excluding neighbors resulting from tabu moves. Then the current solution is updated to one of the solutions in S, for example one which minimizes the objective function over S. Then the tabu list is updated, for example, such that the tabu list contains the inverse moves of the last k moves performed, for some $k \in \mathbb{N}$. The *aspiration criterion* may override the tabu status of a neighbor. For instance, if a tabu neighbor improves on the best solution found so far it may still be considered.

A.2.2 Evolutionary Algorithms

Evolutionary Algorithms (EAs) is a class of algorithm inspired by the evolution of species. For an overview of theory and applications of evolutionary algorithms see [BFM97, MF04]. An EA typically maintains a population of candidate solutions

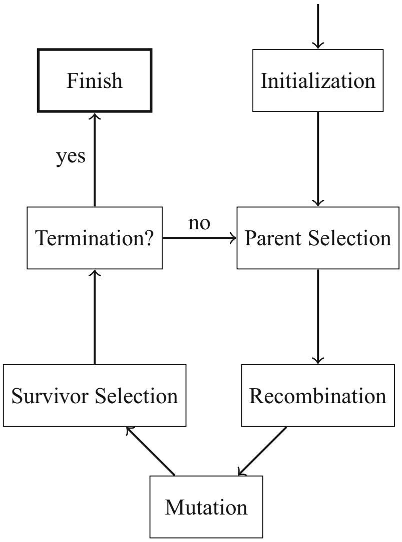

Fig. A.1 The evolutionary cycle, adapted from [Wei02, p. 43]

to a problem instance. The search space is explored by "evolving" the population over time, that is, new candidate solutions are created by mechanisms such *mutation* or *crossover*. Selection mechanisms keep good candidates with respect to the objective function and remove others from the population.

Figure A.1 shows a very general and high-level view of evolutionary algorithms. The initialization step creates at random a population of candidate solutions. Then a selection of promising solutions for recombination is performed. The recombination creates new solutions ("offsprings") by combining features of two or more parent solutions. The offsprings are mutated, that is, modified at random. Then the survivor selection determines from the parents and the offsprings a set of solutions that forms the population in the next iteration. This cycle is repeated until the termination criterion is satisfied. Typically, the algorithm terminates when sufficiently good solutions have been found or a maximum number of iterations has been performed.

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