

Appendix A

Definitions and Conventions Used Throughout the Work

This appendix contains the definitions and conventions which are used throughout the work. They are ordered in the way they are used in the text. In some cases, more than one, slightly different, definition is available for the same subject. In this case a choice is made and if needed, it will be argued why this particular definition is chosen.

A.1 The Fourier Transform and Fourier Series

A signal can be represented in the time domain as well as in the frequency domain. A difference is made between non-periodic and periodic waveforms.

A.1.1 Non-periodic Waveforms

To obtain the frequency spectrum of a signal, the Fourier transform is used [34]:

Definition A.1 The Fourier transform of a waveform $w(t)$ is:

$$W(f) = \mathcal{F}[w(t)] = \int_{-\infty}^{\infty} w(t) \cdot e^{-j2\pi f \cdot t} dt \quad (\text{A.1})$$

where $\mathcal{F}[w(t)]$ denotes the Fourier transform of $w(t)$ and f is the frequency parameter with units of Hz. This *defines* the term frequency, which is the parameter f in the Fourier transform.

Note that instead of the frequency, also the angular frequency $\omega = 2 \cdot \pi \cdot f$ can be used in the Fourier transform. Although this sometimes simplifies the mathematics, it is less intuitive when measuring for instance a spectrum with a spectrum analyzer. The Fourier transform results in a *two-sided* spectrum since both positive and negative frequencies are obtained from the definition. Often the Fourier transform of a waveform is split in a magnitude and a phase function:

$$W(f) = |W(f)| \cdot e^{j\theta(f)} \quad (\text{A.2})$$

$$\text{in which: } |W(f)| = \sqrt{\Im[W(f)]^2 + \Re[W(f)]^2} \quad (\text{A.3})$$

$$\text{and: } \theta(f) = \arctan\left(\frac{\Im[W(f)]}{\Re[W(f)]}\right) \quad (\text{A.4})$$

which is called the *polar form* of the Fourier transform. The inverse Fourier transform can be used to, starting from a complex frequency spectrum, obtain the original waveform:

Definition A.2 The inverse Fourier transform of a frequency spectrum $F(f)$ is:

$$w(t) = \mathcal{F}^{-1}[W(f)] = \int_{-\infty}^{\infty} W(f) \cdot e^{j2\pi \cdot f \cdot t} df \quad (\text{A.5})$$

where $\mathcal{F}^{-1}[W(f)]$ denotes the Inverse Fourier transform of $W(f)$.

$w(t)$ and $W(f)$ are called Fourier pairs. The existence of the Fourier transform of a waveform is, however, not always guaranteed since the infinite integral can diverge. Different conditions can be used to check whether the Fourier transform exists for a waveform $w(t)$. These conditions are all *sufficient*, but *not necessary* [34]:

- $w(t)$ is absolutely integrable:

$$\int_{-\infty}^{\infty} |w(t)| dt < \infty \quad (\text{A.6})$$

and, over any time interval of finite width, the function $w(t)$ is a single-valued function with a finite number of maxima and minima, and the number of discontinuities (if any) is finite.

- The normalized energy (2-norm) of the waveform is finite:

$$E = \int_{-\infty}^{\infty} |w(t)|^2 dt < \infty \quad (\text{A.7})$$

The first conditions are called the *Dirichlet conditions*, the second is the finite-energy condition which is somewhat weaker and satisfied for every physical waveform!

A.1.1.1 Properties of the Fourier transform

When doing calculations with waveforms or when calculating the spectrum of a waveform, different properties can be useful. Some of them are briefly summarized in this section, a more complete overview can be found in [34].

- The spectrum of a real waveform is symmetrical:

$$W(-f) = \overline{W(f)} \quad (\text{A.8})$$

in which $\overline{W(f)}$ is the complex conjugate. When using the polar notation, this means:

$$W(-f) = |W(f)| \cdot e^{-j\theta(f)} \quad (\text{A.9})$$

- The frequency f is just a parameter of the Fourier transform that specifies the frequency of interest. The Fourier transform looks for this frequency over all time $-\infty < t < \infty$.
- $W(f)$ can be complex, also for real waveforms $w(t)$.
- Linearity: $\mathcal{F}[a_1 \cdot f(t) + a_2 \cdot g(t)] = a_1 \cdot F(f) + a_2 \cdot G(f)$
- Time delay: $\mathcal{F}[w(t - \tau)] = W(f) \cdot e^{-j\omega \cdot \tau}$
- Complex signal frequency translation: $\mathcal{F}[w(t) \cdot e^{j\omega_c \cdot t}] = W(f - f_c)$
- Differentiation:

$$\mathcal{F} \left[\frac{d^n w(t)}{dt^n} \right] = (j \cdot 2 \cdot \pi \cdot f)^n \cdot W(f) \quad (\text{A.10})$$

- Integration:

$$\mathcal{F} \left[\int_{-\infty}^t w(t) \right] = (j \cdot 2 \cdot \pi \cdot f)^{-1} \cdot W(f) + \frac{1}{2} \cdot W(0) \cdot \delta(f) \quad (\text{A.11})$$

- Convolution:

$$\mathcal{F}[f(t) * g(t)] = F(f) \cdot G(f) \quad (\text{A.12})$$

- Multiplication:

$$\mathcal{F}[f(t) \cdot g(t)] = F(f) * G(f) \quad (\text{A.13})$$

Extra attention is needed for a commonly used property of a signal and its Fourier transform, called *Parseval's theorem*:

Theorem A.1 For two signals $f(t)$ and $g(t)$, with Fourier transforms $F(f)$ and $G(f)$:

$$\int_{-\infty}^{\infty} f(t) \cdot \overline{g(t)} dt = \int_{-\infty}^{\infty} F(f) \cdot \overline{G(f)} df \quad (\text{A.14})$$

where $\overline{g(t)}$ denotes the complex conjugate of $g(t)$. When $f(t) = g(t)$, this reduces to:

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} |F(f)|^2 df \quad (\text{A.15})$$

which is known as Rayleigh's energy theorem.

A.1.2 Periodic Waveforms

It can be shown that a waveform $w(t)$ can be represented by a sum of orthogonal functions $\varphi_n(t)$ in the interval (a, b) using the following theorem [34]:

Theorem A.2 $w(t)$ can be represented over the interval (a, b) by the series

$$w(t) = \sum_n a_n \cdot \varphi_n(t) \quad (\text{A.16})$$

where the orthogonal coefficients are given by

$$a_n = \frac{1}{K_n} \cdot \int_a^b w(t) \cdot \overline{\varphi_n(t)} dt \quad (\text{A.17})$$

$$\text{and: } K_n = \int_a^b \varphi_n(t) \cdot \overline{\varphi_n(t)} dt \quad (\text{A.18})$$

and the range of n is over the integer values that correspond to the subscripts that were used to denote the orthogonal functions in the complete orthogonal set.

A complete set $\{\varphi_n(t)\}$ of orthogonal functions means that any function can be represented with an arbitrarily small error using the functions in $\{\varphi_n(t)\}$. This is, however, difficult to prove. A set of complex exponential functions and a set of harmonic sinusoids can, however, be proven to be complete [36]. These sets are used to define the Fourier series.

Theorem A.3 A physical waveform can be represented over the interval $a < t < a + T_0$ by the complex exponential Fourier series

$$w(t) = \sum_{n=-\infty}^{n=\infty} c_n \cdot e^{j \cdot n \cdot \omega_0 \cdot t} \quad (\text{A.19})$$

where the complex Fourier coefficients are:

$$c_n = \frac{1}{T_0} \cdot \int_a^{a+T_0} w(t) \cdot e^{-j \cdot n \cdot \omega_0 \cdot t} dt \quad (\text{A.20})$$

and where $\omega_0 = 2 \cdot \pi \cdot f_0 = 2 \cdot \pi / T_0$.

For waveforms which are periodic with period T_0 , this Fourier series is a representation of the function over all time because all functions of the orthogonal set are periodic with the same fundamental period as $w(t)$, T_0 . The Fourier series can therefore be seen as a special case of the Fourier transform for periodic functions. The spectrum in this case consists of discrete lines at the harmonic frequencies. Also the properties of the Fourier transform can be used for the Fourier series:

- If $w(t)$ is real, $c_n = \overline{c_{-n}}$
- If $w(t)$ is real and $w(t) = w(-t)$, $\Im[c_n] = 0$
- If $w(t)$ is real and $w(t) = -w(-t)$, $\Re[c_n] = 0$
- Parseval's theorem is:

$$\frac{1}{T_0} \cdot \int_a^{a+T_0} |w(t)|^2 dt = \sum_{n=-\infty}^{n=\infty} |c_n|^2 \quad (\text{A.21})$$

In some applications it can be useful to use the *quadrature* form of the Fourier series:

Theorem A.4 A physical waveform $w(t)$ can be represented over the interval $a < t < a + T_0$ by a sum of sinusoids:

$$w(t) = \sum_{n=0}^{n=\infty} a_n \cdot \cos(n \cdot \omega_0 \cdot t) + \sum_{n=1}^{n=\infty} b_n \cdot \sin(n \cdot \omega_0 \cdot t) \quad (\text{A.22})$$

where the orthogonal functions are $\sin(n \cdot \omega_0 \cdot t)$ and $\cos(n \cdot \omega_0 \cdot t)$, and the coefficients are given by:

$$a_n = \begin{cases} \frac{1}{T_0} \cdot \int_a^{a+T_0} w(t) dt & \text{for } n = 0 \\ \frac{2}{T_0} \cdot \int_a^{a+T_0} w(t) \cdot \cos(n \cdot \omega_0 \cdot t) dt & \text{for } n \geq 1 \end{cases} \quad (\text{A.23})$$

and:

$$b_n = \frac{2}{T_0} \cdot \int_a^{a+T_0} w(t) \cdot \sin(n \cdot \omega_0 \cdot t) dt \quad \text{for } n \geq 1 \quad (\text{A.24})$$

The relation between the coefficients of the complex and the quadrature Fourier series is:

$$a_n = \begin{cases} c_0 & \text{for } n = 0 \\ 2 \cdot \Re(c_n) & \text{for } n \geq 1 \end{cases} \quad (\text{A.25})$$

and,

$$b_n = -2 \cdot \Im(c_n) \quad \text{for } n \geq 1 \quad (\text{A.26})$$

A last form of the Fourier series is the polar form:

Theorem A.5 *A physical waveform $w(t)$ can be represented over the interval $a < t < a + T_0$ by a sum of sinusoids:*

$$w(t) = D_0 + \sum_{n=1}^{n=\infty} D_n \cdot \cos(n \cdot \omega_0 \cdot t + \varphi_n) \quad (\text{A.27})$$

where $w(t)$ is real and:

$$D_n = \begin{cases} a_0 = c_0 & \text{for } n = 0 \\ \sqrt{a_n^2 + b_n^2} = 2 \cdot |c_n| & \text{for } n \geq 1 \end{cases} \quad (\text{A.28})$$

and:

$$\varphi_n = -\arctan\left(\frac{b_n}{a_n}\right) = \angle c_n \quad \text{for } n \geq 1 \quad (\text{A.29})$$

In this work, both the complex and the quadrature form of the Fourier series are used, depending on the situation. It is clear that both forms have their benefits: the ease of mathematical calculations for the complex waveform and the intuitive approach of the quadrature and polar form.

A.2 The Autocorrelation and PSD of a Signal

The autocorrelation and the Power Spectral Density (PSD) of a time signal contain the same information about a signal since the PSD of a random signal is defined as the Fourier transform of its autocorrelation function [41]. Before formulating the formulas of the autocorrelation and the PSD, the definitions of a stationary signal and an ergodic signal are needed:

Definition A.3 A random process $x(t)$ is said to be stationary to the order N if, for any t_1, t_2, \dots, t_N :

$$f_x(x(t_1), x(t_2), \dots, x(t_N)) = f_x(x(t_1 + t_0), x(t_2 + t_0), \dots, x(t_N + t_0)) \quad (\text{A.30})$$

where t_0 is an arbitrary real constant and $f_x(\mathbf{x})$ is an N-dimensional Probability Density Function (PDF). The process is said to be strictly stationary if it is stationary to the order $N \rightarrow \infty$.

The first-order stationarity of a random process can thus easily be checked by determining whether its first-order PDF is a function of time.

Definition A.4 A random process is said to be ergodic if all time averages of any sample function are equal to the corresponding ensemble averages (expectations).

Since ergodicity is not the most straightforward concept to understand, an example is given to illustrate this:

Example A.1 In electrical engineering, two commonly used averages are the **dc** and **rms** values. These values are both defined (and measured) as time averages. However, when the process is ergodic, also ensemble averages can be used.

- The dc value of a random process $x(t)$ is by definition equal to:

$$x_{dc} \triangleq \lim_{T \rightarrow \infty} \frac{1}{T} \cdot \int_{-T/2}^{T/2} [x(t)]dt \tag{A.31}$$

which is, in the case of an ergodic process, equivalent to:

$$E[x(t)] = \int_{-\infty}^{\infty} [x \cdot f_x(x)]dx = m_x \tag{A.32}$$

where $E[x(t)]$ or m_x is the expected or mean value of $x(t)$.

- The rms value of a random process $x(t)$ is by definition equal to:

$$x_{rms} \triangleq \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \cdot \int_{-T/2}^{T/2} [x(t)^2]dt} \tag{A.33}$$

which is, in the case of an ergodic process, equivalent to:

$$\sqrt{E[x(t)^2]} = \sqrt{\sigma_x^2 + m_x^2} \quad (\text{A.34})$$

where $E[x(t)^2]$ is the expected or mean value of $x(t)^2$ and σ_x^2 is the variance of $x(t)$.

A.2.1 The Autocorrelation

The autocorrelation of a signal is defined as the correlation between the signal and a time-shifted version of itself.

Definition A.5 The autocorrelation of a quadratically infinitely integrable signal $f(t)$ is most often defined as the continuous cross-correlation of the signal with itself:

$$R_f(\tau) = \int_{-\infty}^{\infty} f(t) \cdot \overline{f(t - \tau)} dt \quad (\text{A.35})$$

$$= \int_{-\infty}^{\infty} f(t + \tau/2) \cdot \overline{f(t - \tau/2)} dt \quad (\text{A.36})$$

$$= \int_{-\infty}^{\infty} f(t + \tau) \cdot \overline{f(t)} dt \quad (\text{A.37})$$

where $\overline{f(t)}$ is the complex conjugate of $f(t)$. For real signals this complex conjugate can be omitted.

For signals which last forever and are therefore often not quadratically infinitely integrable, an alternative definition of the autocorrelation is used:

Definition A.6

$$R_f(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \cdot \int_{-T/2}^{T/2} \overline{f(t)} \cdot f(\tau - t) dt \quad (\text{A.38})$$

when $f(t)$ is ergodic or stationary in the wide sense.

In this thesis, since most of the discussed signals are oscillator outputs which can last forever, definition (A.6) is used.

A.2.2 The Power Spectral Density

The power spectral density of a signal is a representation of the power in a signal in the frequency domain. It can be defined as the Fourier transform of the autocorrelation function of the signal (the *Wiener-Khinchine* theorem) [34]:

Definition A.7 The Power Spectral Density of a signal $f(t)$ with autocorrelation function $R_f(\tau)$ is calculated as the Fourier transform of $R_f(\tau)$:

$$S_f(\omega) = \int_{-\infty}^{\infty} R_f(\tau) \cdot e^{-j\omega\tau} d\tau \quad (\text{A.39})$$

It follows that the autocorrelation of a signal $f(t)$ with PSD $S_f(\omega)$ can be calculated as:

$$R_f(\omega) = \frac{1}{2 \cdot \pi} \cdot \int_{-\infty}^{\infty} R_f(\tau) \cdot e^{j\omega\tau} d\tau \quad (\text{A.40})$$

The power of a signal in a certain frequency band $[\omega_1, \omega_2]$ can then be calculated by integrating over positive as well as negative frequencies:

$$P_{\omega_1, \omega_2} = \int_{\omega_1}^{\omega_2} (S_f(\omega) + S_f(-\omega)) d\omega \quad (\text{A.41})$$

For real signals, the PSD is symmetrical around zero. Alternatively, the power spectral density can be defined as follows:

Definition A.8 The average power in a signal $f(t)$ is defined as:

$$\mathcal{P}\{f(t)\} = \lim_{T \rightarrow \infty} \frac{1}{T} \cdot \int_{-T/2}^{T/2} |f(t)|^2 dt \quad (\text{A.42})$$

Let:

$$f_T(t) = f(t) \cdot \Pi\left(\frac{t}{T}\right) \quad (\text{A.43})$$

and using Parseval's theorem, this can be written as:

$$\mathcal{P}\{f(t)\} = \lim_{T \rightarrow \infty} \frac{1}{T} \cdot \int_{-\infty}^{\infty} |F_T(f)|^2 df \quad (\text{A.44})$$

$$= \int_{-\infty}^{\infty} S_f(f) df \quad (\text{A.45})$$

where

$$S_f(f) = \lim_{T \rightarrow \infty} \frac{1}{T} \cdot |F_T(f)|^2 \quad (\text{A.46})$$

is called the *Power Spectral Density*.

It must be noted that both definitions lead to exactly the same result. The first calculation method is often called the *indirect* calculation method; the second, *direct*, calculation method has no need for calculating the autocorrelation first. The unit of the PSD is *watt per hertz*, W/Hz. For practical reasons mostly the single-sided PSD is used, as will be indicated throughout the work.

A.3 Generalized and Special Functions

In this section some of the commonly used functions are defined in order to avoid any confusion with other definitions which are sometimes used. The definitions used are the same as in [34].

A.3.1 The Dirac Delta Function

Definition A.9 The Dirac delta function $\delta(x)$ is defined by:

$$\int_{-\infty}^{\infty} w(x) \cdot \delta(x) = w(0) \quad (\text{A.47})$$

where $w(x)$ is a continuous function at $x = 0$.

Depending on the application x can be time or frequency or some other variable. An alternative definition of $\delta(x)$ is:

Definition A.10 The Dirac delta function $\delta(x)$ is defined by:

$$\int_{-\infty}^{\infty} \delta(x) = 1 \quad (\text{A.48})$$

and:

$$\delta(x) = \begin{cases} \infty & \text{for } x = 0 \\ 0 & \text{for } x \neq 0 \end{cases} \quad (\text{A.49})$$

Since the Dirac function is not a real function (it contains singularities), it is called a singular function. In some situations, it can be useful to use the equivalent integral of the δ function:

$$\delta(x) = \int_{-\infty}^{\infty} e^{\pm j \cdot 2 \cdot \omega \cdot x \cdot y} dy \quad (\text{A.50})$$

This can be proven by noting that the Fourier transform of the delta function is equal to 1. By calculating the inverse Fourier transform, the equivalent integral is obtained.

A.3.2 The Step Function

The step or Heaviside function is often used when calculating Laplace transforms.

Definition A.11 The unit step function $u(t)$ is

$$u(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases} \quad (\text{A.51})$$

The relation between the unit step function and the Dirac function is:

$$\int_{-\infty}^t \delta(x) dx = u(t) \quad (\text{A.52})$$

$$\frac{du(t)}{t} = \delta(t) \quad (\text{A.53})$$

The Fourier transform of the unit step function is equal to:

$$\mathcal{F}[u(t)] = \frac{1}{2} \cdot \delta(f) + \frac{1}{j \cdot 2 \cdot \pi \cdot f} \quad (\text{A.54})$$

A.3.3 A Rectangular Pulse

A rectangular pulse is often used to consider a function only over a certain interval.

Definition A.12 Let $\Pi[\cdot]$ denote a single rectangular pulse with duration T , then:

$$\Pi \left[\frac{t}{T} \right] \triangleq \begin{cases} 1, & |t| \leq \frac{T}{2} \\ 0, & |t| > \frac{T}{2} \end{cases} \quad (\text{A.55})$$

The Fourier transform of this function is equal to the *sinc* function:

$$\text{sinc}(x) = \frac{\sin(x)}{x} \quad (\text{A.56})$$

which makes:

$$\mathcal{F} \left[\Pi \left(\frac{t}{T} \right) \right] = T \cdot \text{sinc}(\pi \cdot T \cdot f) \quad (\text{A.57})$$

Note that the definition of a Dirac impulse sometimes uses this rectangular pulse:

$$\delta(t) = \lim_{T \rightarrow 0} \frac{1}{T} \cdot \Pi \left[\frac{t}{T} \right] \quad (\text{A.58})$$

It is easy to confirm that the properties mentioned above indeed hold for this definition.

A.3.4 A Triangular Pulse

The triangular function is defined as follows.

Definition A.13 Let $\Lambda[\cdot]$ denote the triangular function:

$$\Lambda\left[\frac{t}{T}\right] \triangleq \begin{cases} 1 - \frac{|t|}{T}, & |t| \leq T \\ 0, & |t| > T \end{cases} \quad (\text{A.59})$$

The Fourier transform of this function is equal to:

$$\mathcal{F}\left[\Lambda\left(\frac{t}{T}\right)\right] = T \cdot \text{sinc}(\pi \cdot T \cdot f)^2 \quad (\text{A.60})$$

Indeed, a triangular pulse can be constructed by the convolution of two rectangular pulses. Also this function is sometimes used to define the Dirac impulse:

$$\delta(t) = \lim_{T \rightarrow 0} \frac{1}{T} \cdot \Lambda\left[\frac{t}{T}\right] \quad (\text{A.61})$$

A.3.5 The Derivative Triangular Pulse

A last function is the derivative of the triangular pulse. It is defined as:

Definition A.14 Let $\mathcal{E}[\cdot]$ denote the derivative triangular function:

$$\mathcal{E}\left[\frac{t}{T}\right] \triangleq \begin{cases} \frac{1}{T}, & T \leq t \leq 0 \\ \frac{-1}{T}, & 0 < t \leq T \\ 0, & |t| > T \end{cases} \quad (\text{A.62})$$

The Fourier transform of this function is equal to:

$$\mathcal{F}\left[\mathcal{E}\left(\frac{t}{T}\right)\right] = 2 \cdot T \cdot \frac{\sin(\pi \cdot T \cdot f)^2}{-j \cdot \pi \cdot T \cdot f} \quad (\text{A.63})$$

Appendix B

Influence of a Nonlinear Amplifier

This appendix explains some parts of the discussion in Sect. 4.2 on the influence of a nonlinear amplifier on the frequency of a harmonic oscillator. Especially the lengthy mathematical derivations are omitted in the text and derived here.

B.1 Derivation of Eq. (4.37)

In Sect. 4.2 it is shown that when the applied current waveform is known, the resulting frequency shift for a certain feedback network is given by (4.27):

$$\Im [Z_1] + \sum_2^{\infty} \Im [k \cdot Z_k] \cdot \delta_k^2 = 0 \tag{B.1}$$

where $\delta_k = I_k/I_1$ is the ratio of the harmonics and the fundamental component of the applied current. Z_k is the impedance of the tuned feedback network at the frequency $k \cdot \omega_0$ where ω_0 is the fundamental angular frequency of the oscillator. Assume a parallel RLC network with transfer function (2.66):

$$H_P(s) = \frac{\frac{s}{C}}{s^2 + \frac{1}{C \cdot R_P} \cdot s + \frac{1}{L \cdot C}} = \frac{\omega_n \cdot \sqrt{\frac{L}{C}} \cdot s}{s^2 + \frac{\omega_n}{Q} \cdot s + \omega_n^2} \tag{B.2}$$

When substituting $s = j \cdot k \cdot \omega_0$, the imaginary part of the resulting impedance is equal to:

$$\Im [Z_k] = \frac{-k \cdot \omega_0 \cdot Q^2 \cdot (k^2 \cdot \omega_0^2 - \omega_n^2)}{C \cdot (k^4 \cdot \omega_0^4 \cdot Q^2 - 2 \cdot k^2 \cdot Q^2 \cdot \omega_0^2 \cdot \omega_n^2 + \omega_n^4 \cdot Q^2 + k^2 \cdot \omega_0^2 \cdot \omega_n^2)} \tag{B.3}$$

When $k = 1$, this equation simplifies to:

$$\Im[Z_1] = \frac{-\omega_0 \cdot Q^2 \cdot (\omega_0 + \omega_n) \cdot (\omega_0 - \omega_n)}{C \cdot (\omega_0^4 \cdot Q^2 - 2 \cdot Q^2 \omega_0^2 \cdot \omega_n^2 + \omega_n^4 \cdot Q^2 + \omega_0^2 \cdot \omega_n^2)} \quad (\text{B.4})$$

$$= \frac{-\omega_0 \cdot Q^2 \cdot (\omega_0 + \omega_n) \cdot \Delta\omega_0}{C \cdot (\omega_0^4 \cdot Q^2 - 2 \cdot Q^2 \omega_0^2 \cdot \omega_n^2 + \omega_n^4 \cdot Q^2 + \omega_0^2 \cdot \omega_n^2)} \quad (\text{B.5})$$

where $\Delta\omega_0 = \omega_0 - \omega_n$. Apart from the factor $\Delta\omega_0$, which is close to zero, it can be assumed that $\omega_0 \approx \omega_n$:

$$\Im[Z_1] \approx \frac{2 \cdot Q^2 \cdot \Delta\omega_0}{C \cdot \omega_n^2} \quad (\text{B.6})$$

When $k \neq 1$, it can also be assumed that $\omega_0 \approx \omega_n$:

$$\Im[Z_k] = \frac{-k \cdot \omega_n^3 \cdot Q^2 \cdot (k^2 - 1)}{C \cdot (k^4 \cdot \omega_n^4 \cdot Q^2 - 2 \cdot k^2 \cdot Q^2 \cdot \omega_n^4 + \omega_n^4 \cdot Q^2 + k^2 \cdot \omega_n^4)} \quad (\text{B.7})$$

$$= \frac{-k \cdot (k^2 - 1)}{C \cdot \omega_n \cdot (\frac{k^2}{Q^2} + k^4 - 2 \cdot k^2 + 1)} \quad (\text{B.8})$$

$$= \frac{-k \cdot (k^2 - 1)}{C \cdot \omega_n \cdot (\frac{k^2}{Q^2} + (k^2 - 1)^2)} \quad (\text{B.9})$$

When $Q \gg 1$ this simplifies to:

$$\Im[Z_k] = \frac{-k \cdot (k^2 - 1)}{C \cdot \omega_n \cdot (k^2 - 1)^2} = \frac{-k}{C \cdot \omega_n \cdot (k^2 - 1)} \quad (\text{B.10})$$

The estimated frequency deviation, using (4.27), results in Eq. (4.37):

$$\frac{\Delta\omega_0}{\omega_n} = -\frac{1}{2 \cdot Q^2} \cdot \sum_{k=2}^{\infty} \frac{k^2}{k^2 - 1} \cdot \delta_k^2 \quad (\text{B.11})$$

B.2 Waveform in a Nonlinear Harmonic Oscillator

In this section the waveform of a harmonic oscillator with a softly nonlinear amplifier is calculated. This, however, is not an easy task. Instead of the linear oscillator in Fig. B.1a, an oscillator with a linear feedback network (which is mostly the case) and with a nonlinear amplifier is studied, Fig. B.1b. The representation of this oscillator, using s-functions, is not correct since the amplifier is no longer a linear building block.

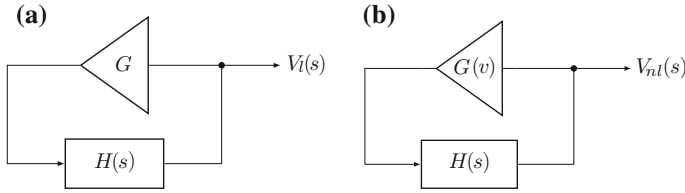


Fig. B.1 Typical block diagram of a harmonic (linear) oscillator (a). An oscillator with a linear feedback network and a nonlinear amplifier (b)

Similar to the calculations in Sect. 4.2.2, the calculations start with the Fourier series of the nonlinear, but periodic, output waveform:

$$v(t) = \sum_{k=-\infty}^{+\infty} c_k \cdot e^{j \cdot k \cdot \omega_0 \cdot t} \tag{B.12}$$

where, for real waveforms, $c_{-k} = \overline{c_k}$ and ω_0 is the fundamental frequency. This expression will be used to calculate the output current of the transconductance amplifier, which is the input current of the feedback network.

B.2.1 The Feedback Network

This waveform results from the injection of a current into the feedback network $H(s)$. Since the feedback network is assumed to be completely linear, the injected current can easily be calculated by dividing each frequency component c_k of the output voltage by the complex impedance $H(s)$ of the feedback network:

$$V(s) = I(s) \cdot H(s) \tag{B.13}$$

$$\Rightarrow i(t) = \sum_{k=-\infty}^{+\infty} \frac{c_k}{H(j \cdot k \cdot \omega_0)} \cdot e^{j \cdot k \cdot \omega_0 \cdot t} \tag{B.14}$$

For a parallel RLC feedback network, the transfer function can be written as:

$$H(s) = \frac{\frac{s}{C}}{s^2 + \frac{\omega_n}{Q} \cdot s + \omega_n^2} \tag{B.15}$$

where ω_n is the natural frequency of the feedback network and Q represents the quality factor. Since, for the steady-state behavior of the network:

$$1/H(j \cdot k \cdot \omega_0) = \left[j \cdot k \cdot \omega_0 \cdot C + \frac{\omega_n \cdot C}{Q} + \frac{C \cdot \omega_n^2}{j \cdot k \cdot \omega_0} \right] \quad (\text{B.16})$$

$$= \frac{\omega_n \cdot C}{Q} \cdot \left[1 + j \cdot \left(\frac{k \cdot \omega_0 \cdot Q}{\omega_n} - \frac{Q \cdot \omega_n}{k \cdot \omega_0} \right) \right] \quad (\text{B.17})$$

when the frequency deviation due to the nonlinearity of the amplifier is small compared to the natural frequency ω_n , this results in:

$$\frac{1}{H(j \cdot k \cdot \omega_0)} \approx \frac{\omega_n \cdot C}{Q} \cdot \left[1 + j \cdot k \cdot Q \left(1 - \frac{1}{k^2} \right) \right] \quad (\text{B.18})$$

$$= \frac{\omega_n \cdot C}{Q} \cdot \sqrt{1 + k^2 \cdot Q^2 \cdot \left(1 - \frac{1}{k^2} \right)^2} \cdot e^{j \arctan(kQ(1-1/k^2))} \quad (\text{B.19})$$

The complex impedance (conductance) experienced by each harmonic can be calculated with this formula.

B.2.2 The Nonlinear Amplifier

Another relationship between the voltage over the feedback network and the injected current is the amplifier characteristic. Although the Barkhausen criterion (Theorem 2.2) is only valid for linear systems, similar conclusions can be drawn for nonlinear systems. Since it is assumed that the oscillator's output signal is periodic and has a stable amplitude, the current-voltage relationship of both the feedback network and the amplifier must be the same! This equality results in, similar to what the Barkhausen criterion does for linear systems, the oscillator equation:

$$G(v(t)) = i(t) = \sum_{k=-\infty}^{+\infty} \frac{c_k}{H(j \cdot k \cdot \omega_0)} \cdot e^{j \cdot k \cdot \omega_0 \cdot t} \quad (\text{B.20})$$

Solving this equation, if possible, results in the steady-state waveform of the oscillator. Note that startup behavior and transition effects are not taken into account in this equation. An amplifier with the following transfer characteristic is assumed:

$$G(v) = A_1 \cdot v + A_3 \cdot v^3 \quad (\text{B.21})$$

where A_1 and A_3 are real coefficients determining the gain and nonlinearity of the amplifier and v is the input voltage. In order to obtain a stable output waveform, limited by the nonlinearity in the amplifier, A_1 must be positive and A_3 negative. Furthermore, to guarantee the startup of the oscillator, the gain for small input signals A_1 needs to compensate for the attenuation in the tank around the oscillation frequency ω_0 . This amplifier characteristic, which is perfectly symmetric, is typical for differential circuits. It results in only odd harmonics in the output waveform.

B.2.2.1 Signals with an Infinite Bandwidth

In a first attempt to calculate the resulting waveform, one can try to insert the Fourier series of the voltage output signal into the amplifier characteristic. This, however, results in an infinite sum of cross products which is rather difficult to handle:

$$G(v(t)) = A_1 \cdot \sum_{k=-\infty}^{+\infty} c_k \cdot e^{j \cdot k \cdot \omega_0 \cdot t} + A_3 \cdot \sum_{k=-\infty}^{+\infty} c_k^3 \cdot e^{j \cdot 3 \cdot k \cdot \omega_0 \cdot t} \quad (\text{B.22})$$

$$+ 3 \cdot A_3 \cdot \sum_{k=-\infty}^{+\infty} \left\{ c_k \cdot e^{j \cdot k \cdot \omega_0 \cdot t} \cdot \sum_{l=-\infty}^{+\infty, l \neq k} c_l^2 \cdot e^{j \cdot 2 \cdot k \cdot \omega_0 \cdot t} \right\} \quad (\text{B.23})$$

This means that an infinite set of equations is needed to be solved to obtain all Fourier coefficients c_k . An alternative method is calculating the coefficients numerically in an iterative way. As will be shown, however, limiting the bandwidth of the output signal results in a solution with an acceptable accuracy.

B.2.2.2 Bandwidth-Limited Signals

A smarter approach is to assume that the output signal is band limited. A periodic voltage waveform, containing only the first, third and fifth harmonic, is assumed:

$$v(t) = \sum_{k=-5, -3, \dots, 5} c_k \cdot e^{j \cdot k \cdot \omega_0 \cdot t} \quad (\text{B.24})$$

$$\begin{aligned} G(v(t)) = & A_3 \cdot \bar{c}_5^3 \cdot e^{-15 \cdot j \cdot \omega_0 \cdot t} + 3 \cdot A_3 \cdot \bar{c}_3 \cdot \bar{c}_5^2 \cdot e^{-13 \cdot j \cdot \omega_0 \cdot t} \\ & + 3 \cdot A_3 \cdot (\bar{c}_3^2 \cdot \bar{c}_5 + \bar{c}_1 \cdot \bar{c}_5^2) \cdot e^{-11 \cdot j \cdot \omega_0 \cdot t} \\ & + A_3 \cdot (\bar{c}_3^3 + 3 \cdot \bar{c}_1 \cdot \bar{c}_5^2) \cdot e^{-9 \cdot j \cdot \omega_0 \cdot t} \\ & + 3 \cdot A_3 \cdot (\bar{c}_1^2 \cdot \bar{c}_5 + \bar{c}_1 \cdot \bar{c}_3^2 + \bar{c}_3 \cdot \bar{c}_5^2) \cdot e^{-7 \cdot j \cdot \omega_0 \cdot t} \\ & + [A_1 \cdot \bar{c}_5 + 3 \cdot A_3 \cdot (\bar{c}_1^2 \cdot \bar{c}_3 + \bar{c}_1 \cdot \bar{c}_3^2 + \bar{c}_5 \cdot \bar{c}_5^2)] \cdot e^{-5 \cdot j \cdot \omega_0 \cdot t} \\ & + [A_1 \cdot \bar{c}_3 + A_3 \cdot \bar{c}_1^3 + 3 \cdot A_3 \cdot (\bar{c}_1^2 \cdot \bar{c}_5 + \bar{c}_3 \cdot \bar{c}_3^2)] \cdot e^{-3 \cdot j \cdot \omega_0 \cdot t} \\ & + [A_1 \cdot \bar{c}_1 + 3 \cdot A_3 \cdot (\bar{c}_1 \cdot \bar{c}_1^2 + \bar{c}_3^2 \cdot \bar{c}_5)] \cdot e^{-1 \cdot j \cdot \omega_0 \cdot t} \\ & + [A_1 \cdot \bar{c}_1 + 3 \cdot A_3 \cdot (\bar{c}_1 \cdot \bar{c}_1^2 + \bar{c}_3^2 \cdot \bar{c}_5)] \cdot e^{1 \cdot j \cdot \omega_0 \cdot t} \\ & + [A_1 \cdot \bar{c}_3 + A_3 \cdot \bar{c}_1^3 + 3 \cdot A_3 \cdot (\bar{c}_1^2 \cdot \bar{c}_5 + \bar{c}_3 \cdot \bar{c}_3^2)] \cdot e^{3 \cdot j \cdot \omega_0 \cdot t} \\ & + [A_1 \cdot \bar{c}_5 + 3 \cdot A_3 \cdot (\bar{c}_1^2 \cdot \bar{c}_3 + \bar{c}_1 \cdot \bar{c}_3^2 + \bar{c}_5 \cdot \bar{c}_5^2)] \cdot e^{5 \cdot j \cdot \omega_0 \cdot t} \\ & + 3 \cdot A_3 \cdot (\bar{c}_1^2 \cdot \bar{c}_5 + \bar{c}_1 \cdot \bar{c}_3^2 + \bar{c}_3 \cdot \bar{c}_5^2) \cdot e^{7 \cdot j \cdot \omega_0 \cdot t} \\ & + A_3 \cdot (\bar{c}_3^3 + 3 \cdot \bar{c}_1 \cdot \bar{c}_5^2) \cdot e^{9 \cdot j \cdot \omega_0 \cdot t} \\ & + 3 \cdot A_3 \cdot (\bar{c}_3^2 \cdot \bar{c}_5 + \bar{c}_1 \cdot \bar{c}_5^2) \cdot e^{11 \cdot j \cdot \omega_0 \cdot t} \\ & + 3 \cdot A_3 \cdot \bar{c}_3 \cdot \bar{c}_5^2 \cdot e^{13 \cdot j \cdot \omega_0 \cdot t} + A_3 \cdot \bar{c}_5^3 \cdot e^{15 \cdot j \cdot \omega_0 \cdot t} \end{aligned} \quad (\text{B.25})$$

where \bar{x} is the complex conjugate of x . Since the signals in the oscillator are real signals, $c_{-k} = \bar{c}_k$. It appears from this expression that in the real system *all* odd

harmonics will be present due to the up-conversion in the nonlinear network. However, except for the harmonics from -5 to 5 , higher harmonics are neglected in this derivation. Since the current input of the feedback network is assumed to be equal to the output of the amplifier, using (B.19), this results in 3 independent complex equations. Due to the fact the signals are real, the complex conjugate equations can be left out:

$$c_1 \cdot \frac{\omega_0 \cdot C}{Q} = [A_1 \cdot c_1 + 3 \cdot A_3 \cdot (\overline{c_1} \cdot c_1^2 + \overline{c_5} \cdot c_3^2)] \quad (\text{B.26})$$

$$\begin{aligned} c_3 \cdot \frac{\omega_0 \cdot C}{Q} \cdot \sqrt{1 + 9 \cdot Q^2 \cdot \left(\frac{8}{9}\right)^2} \cdot e^{j \cdot \arctan\left(3 \cdot Q \cdot \frac{8}{9}\right)} \\ = [A_1 \cdot c_3 + A_3 \cdot c_1^3 + 3 \cdot A_3 \cdot (c_5 \cdot \overline{c_1}^2 + \overline{c_3} \cdot c_3^2)] \end{aligned} \quad (\text{B.27})$$

$$\begin{aligned} c_5 \cdot \frac{\omega_0 \cdot C}{Q} \cdot \sqrt{1 + 25 \cdot Q^2 \cdot \left(\frac{24}{25}\right)^2} \cdot e^{j \cdot \arctan\left(5 \cdot Q \cdot \frac{24}{25}\right)} \\ = [A_1 \cdot c_5 + 3 \cdot A_3 \cdot (c_3 \cdot c_1^2 + \overline{c_1} \cdot c_3^2 + \overline{c_5} \cdot c_3^2)] \end{aligned} \quad (\text{B.28})$$

This set of equations can be solved for a certain feedback network and amplifier in order to obtain the output waveform. The bandwidth limitation of the signal, however, requires a careful approach, certainly for a high gain in the amplifier and/or networks with a low Q factor. A high Q results in a sharper peak in the transfer function; hence, this results in a suppression of the harmonics.

B.2.3 Application to a Known Network

Let us illustrate this now with an example. Suppose an oscillator with a feedback network similar to Fig. 2.9a (a parallel RLC-tank). The resonant angular frequency is normalized and therefore equal to 1 rad/s. The Q -factor is assumed to be equal to 10. The transfer function then looks as follows:

$$H(s) = \frac{s/C}{s^2 + \frac{1}{10} \cdot s + 1} \quad (\text{B.29})$$

At the resonant frequency, this transfer function is a real function with a value of (B.19):

$$H(j \cdot \omega_n) = \frac{Q}{\omega_n \cdot C} = \frac{10}{C} \quad (\text{B.30})$$

which means that the higher the value of the capacitor in the network (for a certain resonant frequency and Q -factor), the lower the impedance at resonance. Note that, using $\omega_n = 1/\sqrt{L \cdot C}$ and $Q = R_P \cdot \sqrt{C/L}$ (2.64), this equation equals R_P . For the sake of keeping the problem simple and understandable, the value of the capacitor is chosen equal to 10, which makes the impedance of the network 1 at the resonant

frequency. This means that the gain of a linear amplifier must also be equal to 1, according to Barkhausen. Therefore, the following characteristic is assumed:

$$G(v) = 1.1 \cdot v - 0.1 \cdot v^3 \quad (\text{B.31})$$

This means, for an input voltage of ± 1 V, that the gain indeed decreases to 1. As a result, an output amplitude slightly higher than 1 V is expected. Solving the oscillator Eqs. (B.26–B.28), results in the following solution:

$$c_1 = 2.0301 \times 10^{-1} + j \cdot 5.4048 \times 10^{-1} \quad (\text{B.32})$$

$$c_3 = 4.5033 \times 10^{-6} - j \cdot 7.6881 \times 10^{-6} \quad (\text{B.33})$$

$$c_5 = -1.8225 \times 10^{-8} + j \cdot 3.5200 \times 10^{-9} \quad (\text{B.34})$$

It is clearly visible that the impact of the harmonics in the resulting signal is low compared to the fundamental waveform. This, however, changes when the (small-signal) gain, A_1 , becomes higher. The resulting coefficients for different values of the gain are shown in Table B.1. Because of the ‘soft’ distortion in the amplifier, the influence on the frequency is rather low. The case of an amplifier with strong distortion is discussed in Sect. 4.2.2. The waveform of the output voltage over the tank and the output current of the amplifier are shown in Fig. B.2. In this figure, the impact of the nonlinearity is clearly visible in the applied current. The resonant feedback network, however, hides most of these nonlinearities in the output voltage signal.

Note that this method to calculate the harmonics in the output waveform starts from the assumption that the center frequency is fixed. By doing this, one degree of freedom is taken away in the set of equations. When the frequency is left as a variable, another variable (such as the initial phase shift of the fundamental frequency) has to be chosen freely. Furthermore, due to this assumption, this method only works for circuits with a limited amount of harmonics. Increasing the number of harmonics in the output waveform increases the accuracy but also causes an explosion of terms in the output waveform.

B.3 Proof of Eq. (4.42)

The sum of the infinite series in Sect. 4.2.2 can be written as:

$$\sum_{k \in \{3,5,\dots\}}^{\infty} \frac{1}{k^2 - 1} = \sum_{k=1}^{\infty} \frac{1}{(2 \cdot k + 1)^2 - 1} \quad (\text{B.35})$$

$$= \frac{1}{4} \cdot \sum_{k=1}^{\infty} \frac{1}{k \cdot (k + 1)} \quad (\text{B.36})$$

The sum can be calculated using the following formula:

Table B.1 Coefficients of the different harmonics of the output waveform for the example oscillator

	$Q = 10$	$Q = 1$
$i(v) = 1.1 \cdot v - 0.1 \cdot v^3$		
c_1	$2.030 \times 10^{-1} + j \cdot 5.405 \times 10^{-1}$	$4.638 \times 10^{-1} + j \cdot 3.438 \times 10^{-1}$
c_3	$4.503e - 6 - j \cdot 7.688e - 6$	$-2.156 \times 10^{-5} - j \cdot 1.816 \times 10^{-5}$
c_5	$-1.823e - 8 + j \cdot 3.520e - 9$	$1.611 \times 10^{-7} - j \cdot 9.622 \times 10^{-8}$
$\Delta\omega_0/\omega_n$	-3.572×10^{-10}	-3.577×10^{-9}
$i(v) = 1.3 \cdot v - 0.1 \cdot v^3$		
c_1	$5.676 \times 10^{-1} - j \cdot 8.233 \times 10^{-1}$	$4.314 \times 10^{-1} - j \cdot 9.022 \times 10^{-1}$
c_3	$1.265 \times 10^{-5} - j \cdot 4.454 \times 10^{-5}$	$1.816 \times 10^{-5} - j \cdot 1.454 \times 10^{-4}$
c_5	$-2.692 \times 10^{-8} - j \cdot 2.881 \times 10^{-7}$	$-1.231 \times 10^{-6} - j \cdot 2.658 \times 10^{-6}$
$\Delta\omega_0/\omega_n$	-3.215×10^{-9}	-3.223×10^{-8}
$i(v) = 1.5 \cdot v - 0.1 \cdot v^3$		
c_1	$4.601 \times 10^{-1} + j \cdot 1.206 \times 10^0$	$-6.181 \times 10^{-1} + j \cdot 1.133 \times 10^0$
c_3	$4.901 \times 10^{-5} - j \cdot 8.673 \times 10^{-5}$	$-8.824 \times 10^{-5} + j \cdot 3.026 \times 10^{-4}$
c_5	$-1.012 \times 10^{-6} + j \cdot 2.314 \times 10^{-7}$	$1.896 \times 10^{-6} + j \cdot 1.035 \times 10^{-5}$
$\Delta\omega_0/\omega_n$	-8.934×10^{-9}	-8.971×10^{-8}
$i(v) = 2.0 \cdot v - 0.1 \cdot v^3$		
c_1	$-6.213 \times 10^{-1} + j \cdot 1.717 \times 10^0$	$7.372 \times 10^{-1} - j \cdot 1.670 \times 10^0$
c_3	$1.332 \times 10^{-4} + j \cdot 2.483 \times 10^{-4}$	$2.870 \times 10^{-5} - j \cdot 8.913 \times 10^{-4}$
c_5	$5.772 \times 10^{-6} + j \cdot 1.059 \times 10^{-6}$	$-3.532 \times 10^{-5} - j \cdot 4.815 \times 10^{-5}$
$\Delta\omega_0/\omega_n$	-3.577×10^{-8}	-3.622×10^{-7}

Also the influence of the harmonics on the center frequency is shown in the table

Theorem B.1

$$\sum_{k=1}^l \frac{1}{k \cdot (k + 1)} = \frac{l}{l + 1} \tag{B.37}$$

Proof This proof uses induction: first it is shown that the explicit formula holds for $l = 1$; afterwards it is shown that, if the formula holds for l , it will also hold for $l + 1$.

- $l = 1$

It can easily be seen that:

$$\sum_{k=1}^{l=1} \frac{1}{k \cdot (k + 1)} = 1/2 = \frac{l}{l + 1} \tag{B.38}$$

- $l + 1$

When the formula holds for l , the sum of the series from 1 to $l + 1$ can be written as:

$$\sum_{k=1}^{l+1} \frac{1}{k \cdot (k + 1)} = \sum_{k=1}^l \frac{1}{k \cdot (k + 1)} + \frac{1}{(l + 1) \cdot (l + 2)} \tag{B.39}$$

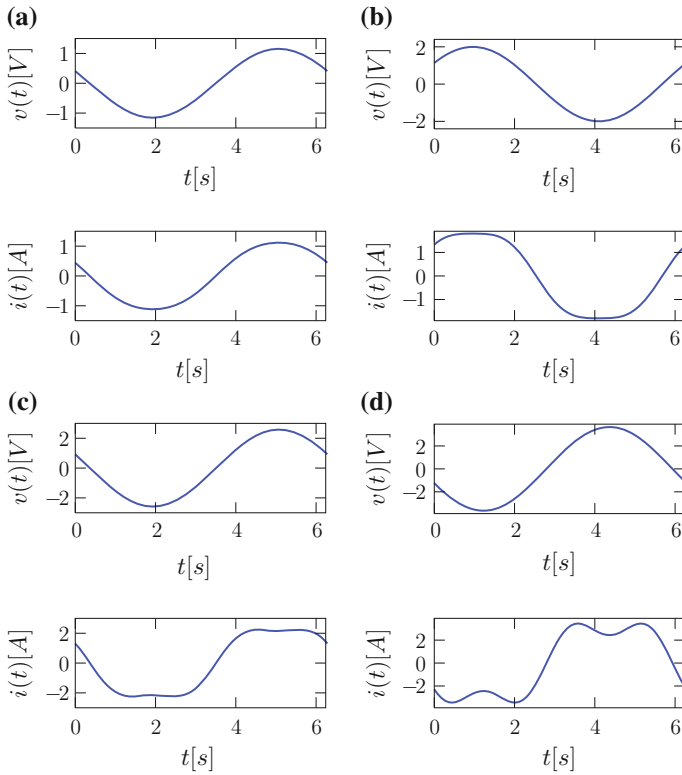


Fig. B.2 Output voltage waveform and injected current for different transconductance amplifiers with soft distortion. The nonlinearity is necessary to control the amplitude but causes harmonics in the output waveform. The harmonics, on their turn, cause a small frequency drop. **a** $Q = 10$, $i(v) = 1.5 \cdot v - 0.1 \cdot v^3$. **b** $Q = 10$, $i(v) = 2.0 \cdot v - 0.1 \cdot v^3$

$$= \frac{l}{l+1} + \frac{1}{(l+1) \cdot (l+2)} \tag{B.40}$$

$$= \frac{l^2 + 2 \cdot l + 1}{(l+1) \cdot (l+2)} \tag{B.41}$$

$$= \frac{l+1}{l+2} \tag{B.42}$$

which proves Theorem B.1. ■

It can easily be seen that:

$$\lim_{l \rightarrow +\infty} \sum_{k=1}^l \frac{1}{k \cdot (k+1)} = \lim_{l \rightarrow +\infty} \frac{l}{l+1} = 1 \tag{B.43}$$

which is used to obtain the result of (4.42).

Appendix C

Measurement Issues for Jitter and Phase Noise

Measuring jitter and phase noise in an oscillator output signal is a difficult task. It is mainly the colored noise which complicates the measurements because of the ultra-slow frequency variations, which causes frequency drift. A possible consequence of this, when measuring the cycle-to-cycle jitter, is that the subsequent measurements do not converge! Due to the frequency drift, the measured period ‘drifts away’ from the average period depending on the moment at which the measurement is performed. In [92, 180] different methods are discussed to overcome these problems.

C.1 White Noise Jitter Divergence

Different measures are available to express the instabilities in an oscillator output signal. Most of them are based on the *variance* of variations in the frequency, phase or phase-time. The phase deviation $\phi(t)$ of an oscillator modulated with white noise is given by a *random walk* about the ideal phase $\omega_0 \cdot t$ (the resulting PSD is proportional to $1/f^2$). When considering the variance σ_ϕ of the phase fluctuations, this causes a continuous increase with the observation time (this is similar to the absolute jitter, Definition 3.1; the uncertainty of the next clock edge will increase over time). This can be understood by the knowledge that the VCO is an ideal integrator, of which the output (the phase) can grow forever. It is this very same integration that shapes the PSD of the phase fluctuations $S_\phi(f)$, which makes the PSD go to infinity. This integration operation is the same as the so-called superposition integral in Hajimiri’s theory, Eq. (3.20).

To bound this error, the VCO can be put in a phase-locked loop (PLL), which limits the phase deviation compared to a reference clock. In the time domain, this can be interpreted as a periodical *reset* of the phase deviation. In the frequency domain, this means that the low-frequency content of the output phase deviation is filtered. The transfer function of a PLL acts as a high-pass filter on the oscillator output. In this configuration, the jitter can easily be measured, because it does not drift away from the ideal (reference) phase. Furthermore this technique allows to measure

the frequency deviation of the oscillator by means of the VCO input signal, which is proportional to the frequency deviation between the oscillator and the reference input. The carrier frequency is not present in this control signal, which allows to exploit the full dynamic range of the measurement setup (if a strong carrier signal is present, the phase noise or frequency is dominated by the carrier signal). The main drawback of this *closed-loop, clock-referenced* technique is the fact the output is filtered by the PLL transfer function, which needs to be corrected afterwards [92, 180, 181].

Another method to avoid the divergence of frequency stability measures in the time domain is limiting the time interval over which they are measured. So-called *open-loop, self-referenced* techniques [180] start a measurement at a certain clock edge and measure the jitter of the following clock edges compared to the first clock edge. When starting a new measurement, ($t = 0$) the phase error is reset. In this way, the phase error can only drift away during the measurement interval. When the jitter of the subsequent clock cycles is uncorrelated, the following standard deviation as a function of the measurement interval t can be expected (3.59):

$$\sigma_{abs,OL}(t) = \sqrt{c} \cdot \sqrt{t} \quad (\text{C.1})$$

which is equivalent to Eq. (3.59) and valid when only white noise is injected in the oscillator ($1/f^2$ in the oscillator spectrum). This means that the cycle-to-cycle jitter is an open-loop, self-referenced jitter measurement for which the measurement time is equal to the oscillator period, $t = 1/f_{osc}$. c is an important Figure of Merit of an oscillator, which is related to the shape of the Lorentzian noise spectrum (3.50). Although the cycle-to-cycle jitter is a straightforward measurement to quantify the phase noise, a high time resolution or a specialized measurement setup is needed to do these measurements [92]. An interesting question is how both measurements, *closed-loop* and *open-loop*, can be related.

The measured jitter of a closed-loop measurement strongly depends on the bandwidth of the PLL. In [180] it is illustrated that, for a PLL of which the transfer function can be approximated by the low-pass characteristic:

$$H(s) = \frac{2 \cdot \pi \cdot f_L}{s + 2 \cdot \pi \cdot f_L} \quad (\text{C.2})$$

where f_L is the loop bandwidth of the PLL, the measured jitter stabilizes when the measurement time is high compared to $1/(2 \cdot \pi \cdot f_L)$. This can be understood as follows: when performing a self-referenced measurement starting at $t = 0$, the jitter increases with the square root of the measurement interval. At the moment the measurement time approaches

$$t_L = \frac{1}{2 \cdot \pi \cdot f_L}, \quad (\text{C.3})$$

the PLL is able to compensate for higher phase deviations and the jitter becomes constant as a function of time. When the absolute jitter of a clock edge, compared

to the reference clock, is called σ_x , than the jitter between two clock edges, which are separated a period $\Delta t \gg t_L$ from each other, is equal to $\sqrt{2} \cdot \sigma_x$, assuming that this jitter is uncorrelated. In [180, 282] this characteristic, which is a function of the PLL bandwidth f_L , is analyzed. It is shown that:

$$\sigma_x = \sqrt{c} \cdot \sqrt{\frac{1}{4 \cdot \pi \cdot f_L}} \quad (\text{C.4})$$

Indeed: the resulting jitter is a factor $\sqrt{2}$ different from the jitter at the -3 dB ($= 1/\sqrt{2}$) bandwidth f_L of the PLL. These conclusions are important when performing the *closed-loop, clock-referenced* measurement discussed earlier to compensate for the PLL behavior. The jitter can be reduced in two ways: by improving the oscillator jitter (c) and by increasing the PLL bandwidth (f_L). This clearly shows the relationship between a closed-loop and an open-loop measurement.

C.1.1 Frequency Fluctuations as a Noise Measure

The instantaneous relative frequency fluctuations are represented by (2.41):

$$y(t) = \frac{\frac{d\phi(t)}{dt}}{\omega_0} \quad (\text{C.5})$$

As a matter of fact, the frequency cannot be measured instantaneously (opposite to phase measurements, in the absence of amplitude noise), which makes it necessary to measure the frequency drift over a certain time span τ :

$$\langle y \rangle_{t_0, \tau} = \frac{1}{\tau} \cdot \int_{t_0 - \tau/2}^{t_0 + \tau/2} y(t) dt. \quad (\text{C.6})$$

Using (2.41), this equation can be written as:

$$\langle y \rangle_{t_0, \tau} = \frac{\phi(t_0 + \tau/2) - \phi(t_0 - \tau/2)}{\omega_0 \cdot \tau} = \frac{1}{\tau} \cdot [x(t_0 + \tau/2) - x(t_0 - \tau/2)] \quad (\text{C.7})$$

This means that the averaged instantaneous frequency fluctuations over a certain time interval can be calculated as the normalized phase-time $x(t)$ error over the very same time interval. Since $y(t)$, $\phi(t)$ and therefore also $x(t)$ are random processes, this averaged error is a random variable with a mean value and a standard deviation. The mean value is equal to zero, which makes the variance equal to the mean-square value:

$$\sigma_{\langle y \rangle, \tau}^2 = E[\langle y \rangle_{t_0, \tau}^2] \quad (\text{C.8})$$

where $E[\cdot]$ stands for the expected value calculated over an infinite set of samples. It can therefore be stated that $\sigma_{\langle y \rangle, \tau}^2$ is the true variance of $\langle y \rangle_{t_0, \tau}$. In a practical situation, however, it is not possible to create an infinite set of samples, which makes the variance itself a random variable over the different (limited) sample sets. It is considered to be an estimator for the true variance (based on a limited set). A good, unbiased, estimator is an estimator of which the average value converges to the true variance. An unbiased estimator for the variance is given by:

$$s_{\langle y \rangle, \tau}^2 = \frac{1}{N-1} \cdot \sum_{i=1}^N \left(\langle y \rangle_{t_i, \tau} - \frac{1}{N} \cdot \sum_{j=1}^N \langle y \rangle_{t_j, \tau} \right)^2 \quad (\text{C.9})$$

where N is the number of samples of $\langle y \rangle$ in the subset and $N \geq 2$ [217]. This estimator holds as long as the samples are uncorrelated. Note also the difference in notation between the true variance of an infinite set σ_x^2 and the estimated variance of a subset s_x^2 . The relationship between this variance and the cycle-to-cycle jitter is:

$$\sigma_{\langle y \rangle, T_0} = \frac{\sigma_c}{T_0} \quad (\text{C.10})$$

C.1.2 Relation Between the Variance and the PSD

The link between the PSD and the variance of the fractional frequency fluctuations is obtained by considering the fact that the time interval τ over which the frequency fluctuations are averaged, can be treated as a filter in the time domain. In this way, the samples of $\langle y \rangle_{t_0, \tau}$ are considered to be a moving average operation:

$$\langle y \rangle_{t_0, \tau} = \frac{1}{\tau} \cdot \int_{t_0 - \tau/2}^{t_0 + \tau/2} y(t) dt \quad (\text{C.11})$$

$$= \frac{1}{\tau} \cdot \int_{-\infty}^{\infty} y(t) \cdot \Pi\left(\frac{t}{\tau}\right) dt \quad (\text{C.12})$$

$$= \frac{1}{\tau} \cdot \int_{-\infty}^{\infty} y(t) \cdot \Pi\left(\frac{t_0 - t}{\tau}\right) dt \quad (\text{C.13})$$

which is a convolution integral. The variance is then obtained by integrating over the PSD of the resulting waveform:

$$\sigma_{(y),\tau}^2 = \int_{-\infty}^{\infty} S_y(f) \cdot \left(\frac{\sin(\pi \cdot f \cdot \tau)}{\pi \cdot f \cdot \tau} \right)^2 df \quad (\text{C.14})$$

$$= \int_{-\infty}^{\infty} \frac{f^2}{f_0^2} \cdot S_{\phi}(f) \cdot \left(\frac{\sin(\pi \cdot f \cdot \tau)}{\pi \cdot f \cdot \tau} \right)^2 df \quad (\text{C.15})$$

where the factor $\left(\frac{\sin(\pi \cdot f \cdot \tau)}{\pi \cdot f \cdot \tau} \right)^2$ is called the *transfer function of the time-domain measure of frequency stability* [92, 217]. Using (C.10) it can be seen that this equation is equivalent to (3.67). In case of a $1/f^2$ phase noise profile, this integral will converge.

The previous discussion assumes white, uncorrelated noise. In Sect. 3.6 it has been shown that in the case of correlated or colored noise sources, the linear increase over time for an open-loop measurement does not hold. Only measuring the jitter over a certain time interval to obtain the cycle-to-cycle jitter, is therefore not sufficient to do a jitter characterization. In the frequency domain, this is comparable to characterizing the noise based on only one frequency value of the PSD. The influence of the frequency noise shaping ($1/f^\alpha$) is closely related to the evolution of the absolute jitter over time (3.65).

C.2 Colored Noise Jitter

When colored noise is injected in an oscillator, this results in an $1/f^\alpha$ region in the phase spectrum. α is in this case greater or equal to 3. In this case, the absolute jitter does no longer increase proportionally to the square root of the observation time, but also a term proportional to the observation time appears, (3.65). As explained in Sect. 3.6.3, the noise injected during subsequent time intervals is therefore correlated, which causes the phase to drift faster away than in the case of uncorrelated noise sources. As stated in [104], it can be assumed in this case that the standard deviation adds instead of the variance.

C.2.1 Frequency Fluctuations as a Colored Noise Measure

As previously shown, the fractional frequency fluctuations are an interesting measure to measure the frequency stability in the time domain. Moreover, due to its relation to the cycle-to-cycle jitter, it gives an intuitive insight in the behavior and the effects of the jitter on for instance the performance of a clocked circuit. Problems arise, however, when the integral (C.15) is calculated for a colored noise spectrum. Keeping in mind that the transfer function of the time-domain measure of frequency stability is equal to 1 around $f = 0$, it is clear that this integral will diverge, see (2.47) with

$\alpha \leq -1$. Typically, this is solved by limiting the minimum frequency to which this integral is calculated around the carrier. Often this is explained as a limit which comes forth from the limited observation time. This, however, is not satisfactory since the calculated variance strongly depends on this value and therefore does not converge when a larger observation time is used [41].

A solution can be to use the PLL measurement method described above: the frequencies within the bandwidth of the PLL, f_L , are suppressed as a result of the PLL feedback mechanism. In this way, as a result of the continuous tuning of the oscillator to the nominal frequency of the frequency reference, the variance of the frequency fluctuations (and the jitter) converges for an increasing number of samples $\langle y \rangle_{t_0, \tau}$ when the total observation time is larger than the inverse of the loop bandwidth [92].

C.2.2 The Allan Variance

A different way to overcome these convergence problems, is the use of an alternative measure instead of the variance. Instead of using the complete set of average frequency fluctuations $\langle y \rangle_{t_0, \tau}$, one can combine the subsequent averages instead. The Allan variance, which is also an IEEE-recommended measure for frequency stability, is defined as [5]:

$$\sigma_{A, \langle y \rangle, \tau}^2 = E \left[\sum_{i=1}^2 \left(\langle y \rangle_{t_i, \tau} - \frac{1}{2} \cdot \sum_{j=1}^2 \langle y \rangle_{t_j, \tau} \right)^2 \right] \quad (\text{C.16})$$

$$= \frac{1}{2} \cdot E[(\langle y \rangle_{t_2, \tau} - \langle y \rangle_{t_1, \tau})^2] \quad (\text{C.17})$$

where $\langle y \rangle_{t_i, \tau}$ is defined as:

$$\langle y \rangle_{t_i, \tau} = \frac{1}{\tau} \cdot \int_{t_i - \tau/2}^{t_i + \tau/2} y(t) dt \quad (\text{C.18})$$

and $t_{i+1} - t_i = \tau$, which means that there is no *dead time* between the subsequent samples. Similar to (C.7), also this equation can be expressed in terms of the phase-time $x(t_k)$ (2.40):

$$\sigma_{A, \langle y \rangle, \tau}^2 = \frac{1}{2 \cdot \tau^2} \cdot E \left[[x(t_{k+2}) - 2 \cdot x(t_{k+1}) + x(t_k)]^2 \right] \quad (\text{C.19})$$

which shows the tight connection between the Allan variance and the alternative definition of the cycle-to-cycle jitter, (3.62). Although the difference between the Allan variance and the classic variance is subtle, it has a huge impact on the correlated noise. The correlation between the subsequent samples is, due to the definition of the

Allan variance, canceled. The linear trend in the jitter is compensated by combining several samples. To demonstrate the convergence in case of a colored ($1/f$) noise source, the transfer function of this frequency measure has to be determined. The Allan variance is the variance of the random variable:

$$\frac{1}{\sqrt{2}} \cdot (\langle y \rangle_{t_{k+1}, \tau} - \langle y \rangle_{t_k, \tau}) = \frac{1}{\sqrt{2} \cdot \tau} \cdot \int_{-\infty}^{\infty} y(t) \cdot h_A \left(\frac{t_{k+1} - t}{\tau} \right) dt \quad (\text{C.20})$$

where $h_A(t_{k+1} - t)$ is equal to the *derivative triangular function* (see definition A.14):

$$h_A(t) = \Xi(t/\tau) \triangleq \begin{cases} \frac{1}{\tau}, & \tau \leq t \leq 0 \\ \frac{-1}{\tau}, & 0 < t \leq \tau \\ 0, & |t| > \tau \end{cases} \quad (\text{C.21})$$

The resulting PSD, after performing the filtering operation, can then be integrated to obtain the resulting Allan variance of the oscillator signal:

$$\sigma_{A, (y), \tau}^2 = \int_{-\infty}^{\infty} S_y(f) \cdot 2 \cdot \left(\frac{\sin(\pi \cdot f \cdot \tau)}{\pi \cdot f \cdot \tau} \right)^2 df \quad (\text{C.22})$$

$$= \int_{-\infty}^{\infty} \frac{f^2}{f_0^2} \cdot S_\phi(f) \cdot 2 \cdot \left(\frac{\sin(\pi \cdot f \cdot \tau)}{\pi \cdot f \cdot \tau} \right)^2 df \quad (\text{C.23})$$

Because of the low-frequency filtering operation of the Allan variance, the integral is convergent also when $1/f$ noise is injected in the oscillator. However, an upper limit for the noise integration is still needed for white and flicker noise. Typically this bound is taken at the cutoff frequency f_h which is existing in each system. In a nonlinear oscillator, it is reasonable to integrate until 1.5 times the oscillation frequency; every higher frequency is also represented in this bandwidth. In Table C.1 an overview is given of the resulting Allan variance for different slopes of the frequency (phase) spectrum, as a function of the observation time τ . It is clearly seen that only for *flicker frequency fluctuations* the Allan variance is independent of the observation time.

An interesting conclusion after using the Allan variance is the observation that by combining measurements of subsequent periods, the low-frequency content can be filtered away. However, from Table C.1 it appears that the Allan variance is only able to compensate for the first-order colored noise contributions. In the following section, a methodology is demonstrated to also compensate for the higher-order contributions in order to quantify them in the time domain.

Table C.1 The Allan variance for different slopes of the (single-sided) PSD

$S_y(f)$	$S_\phi(f)$	$\sigma_{A,(y),\tau}^2$	Description
$h_{-2} \cdot f^{-2}$	$f_0^2 \cdot h_{-2} \cdot f^{-4}$	$\frac{2 \cdot \pi^2 \cdot h_{-2} \cdot \tau}{3}$	Random walk frequency fluctuations
$h_{-1} \cdot f^{-1}$	$f_0^2 \cdot h_{-1} \cdot f^{-3}$	$2 \cdot \ln(2) \cdot h_{-1}$	Flicker frequency fluctuations
$h_0 \cdot f^0$	$f_0^2 \cdot h_0 \cdot f^{-2}$	$\frac{h_0}{2 \cdot \tau}$	White frequency fluctuations
$h_1 \cdot f^1$	$f_0^2 \cdot h_1 \cdot f^{-1}$	$\frac{h_1}{4 \cdot \pi^2 \cdot \tau^2} \cdot [1.038 + 3 \cdot \ln(2 \cdot \pi \cdot f_h \cdot \tau)]$	Flicker phase fluctuations
$h_2 \cdot f^2$	$f_0^2 \cdot h_2 \cdot f^0$	$\frac{3 \cdot h_2 \cdot f_h}{4 \cdot \pi^2 \cdot \tau^2}$	White phase fluctuations

It is clearly seen that only for $1/f$ noise injected in the oscillator, a constant value is obtained [5, 217]

C.2.3 The Use of Structure Functions

The use of structure functions in the area of oscillator phase noise analysis was first introduced by Lindsey and Chie [164]. It will be shown below that this theory succeeds in capturing the previously discussed time-domain noise measures (variances) in a unifying methodology. Structure functions are a mathematical tool which can be used to avoid the singularities which appear at low frequencies in the case of colored noise sources. The underlying idea is that the structure functions are able to de-trend the measurement data, similar to what the Allan variance does for first-order colored noise sources.

C.2.3.1 Increments of a Function

The increment of a random process $x(t)$ is defined as follows:

Definition C.1 Let $\Delta^N x(t, \tau)$ denote the N -th increment of $x(t)$ with time step τ . The first-order increment of a random process $x(t)$ is defined as:

$$\Delta^1 x(t, \tau) = x(t + \tau) - x(t) \tag{C.24}$$

The N -th-order increment of a function can recursively be defined as:

$$\Delta^N x(t, \tau) = \Delta^{N-1}[\Delta^1 x(t, \tau)] = \Delta^1[\Delta^{N-1} x(t, \tau)] \tag{C.25}$$

or can be written explicitly as:

$$\Delta^N x(t, \tau) = \sum_{k=0}^N (-1)^k \cdot \binom{N}{k} \cdot x[t + (N - k) \cdot \tau] \tag{C.26}$$

where $\binom{N}{k}$ denotes the combinations of k samples out of N .

Note that this definition results in the same formulas as used to approximate the N -th-order derivative of $x(t)$ using only forward differences. This observation helps to understand the working principle of this method. When calculating the N -th-order increment, $N + 1$, τ -spaced samples of the function are needed. Moreover, when M , τ -spaced samples are available, $M - N$ N -th-order increments can be calculated, which results in a high data utilization for large numbers of samples. Because the adjacent samples are combined, the increments are able to detect or cancel trends or correlations between the different data points. Generally speaking, the *variance* of the result of a certain increment is larger for uncorrelated samples than for correlated data samples. This is the same working principle of the Allan variance for adjacent oscillation periods! Note that the N -th increment of a polynomial of an order smaller than N , is equal to zero. This corresponds to the fact that these increments are an estimator of the N -th derivative.

C.2.3.2 Structure Functions

Based on these increments, structure functions can be defined:

Definition C.2 The N -th-order structure function $D_x^N(t_s, \tau)$ is defined as the autocorrelation function of the N -th increment:

$$D_x^N(t_s, \tau) \triangleq \left\langle (\Delta^N x(t, \tau)) \cdot (\Delta^N x(t + t_s, \tau)) \right\rangle \quad (\text{C.27})$$

$$= \lim_{T \rightarrow \infty} \int_{-T/2}^{T/2} (\Delta^N x(t, \tau)) \cdot (\Delta^N x(t + t_s, \tau)) dt \quad (\text{C.28})$$

When the time shift t_s is equal to zero, this can shortly be written as:

$$D_x^N(\tau) = \left\langle (\Delta^N x(t, \tau))^2 \right\rangle \quad (\text{C.29})$$

In this work, the term *structure function* refers to the structure function with $t_s = 0$, which is in fact the variance of the N -th-order increment of $x(t)$. In [164] it is shown that these functions can easily be related to the output PSD of the oscillator [92].

$$D_x^N(\tau) = 2^{2 \cdot N} \cdot \int_{-\infty}^{\infty} S_x(f) \cdot \sin^{2 \cdot N}(\pi \cdot f \cdot \tau) df \quad (\text{C.30})$$

$$D_x^N(\tau) = 2^{2 \cdot N - 2} \cdot \int_{-\infty}^{\infty} S_y(f) \cdot \frac{\sin^{2 \cdot N}(\pi \cdot f \cdot \tau)}{(\pi \cdot f)^2} df \quad (\text{C.31})$$

$$\frac{D_x^N(\tau)}{\tau^2} = 2^{2 \cdot N - 2} \cdot \int_{-\infty}^{\infty} S_y(f) \cdot \frac{\sin^{2 \cdot N}(\pi \cdot f \cdot \tau)}{(\pi \cdot f \cdot \tau)^2} df \quad (\text{C.32})$$

where the last equation is just the structure function normalized to the observation interval τ . These equations can be obtained by using the *gated integration* approach as previously demonstrated for the variance and the Allan variance. If f is close to zero, the transfer function of the timing measure can be approximated by:

$$|H_{D,N}(f)| = \left| \frac{\sin^N(\pi \cdot f \cdot \tau)}{(\pi \cdot f \cdot \tau)} \right| \approx (\pi \cdot f \cdot \tau)^{N-1} \tag{C.33}$$

This means that the singularities in the spectrum can be compensated by using the correct structure function. Since:

$$S_y(f) \cdot \frac{\sin^{2-N}(\pi \cdot f \cdot \tau)}{(\pi \cdot f \cdot \tau)^2} \sim f^\alpha \cdot f^{2N-2} \tag{C.34}$$

the integration of the spectrum is convergent as long as $\alpha > -(2 \cdot N - 1)$ or $N > \frac{1-\alpha}{2}$. This indicates that it is always possible to take a high enough order of increment N to obtain a bounded structure function. The effect of taking a higher-order structure function is demonstrated in Table C.2. Interesting to note is the fact that the structure functions have a close relationship with the previously defined time-domain jitter measures:

$$\sigma_{(y),\tau}^2 = \frac{1}{\tau^2} \cdot D_x^1(\tau) \tag{C.35}$$

$$\sigma_{A,(y),\tau}^2 = \frac{1}{2 \cdot \tau^2} \cdot D_x^2(\tau) \tag{C.36}$$

C.2.3.3 Structure Functions to Measure Polynomial Drift

Colored or correlated noise sources result in a polynomial drift in for instance the phase-time error. It is interesting to compare the results of the random noise process

Table C.2 Values of the different structure functions, corresponding to the different areas in the power-law noise spectrum

$S_\phi(f)$	$D_x^1(\tau)$	$D_x^2(\tau)$	$D_x^3(\tau)$
$f_0^2 \cdot h_{-2} \cdot f^{-4}$	Non-convergent	$\frac{4}{3} \cdot \pi^2 \cdot h_{-2} \cdot \tau^3$	$2 \cdot \pi^2 \cdot h_{-2} \cdot \tau^3$
$f_0^2 \cdot h_{-1} \cdot f^{-3}$	Non-convergent	$4 \cdot \ln(2) \cdot h_{-1} \cdot \tau^2$	$6.75 \cdot h_{-1} \cdot \tau^2$
$f_0^2 \cdot h_0 \cdot f^{-2}$	$\frac{1}{2} \cdot h_0 \cdot \tau$	$h_0 \cdot \tau$	$3 \cdot h_0 \cdot \tau$
$f_0^2 \cdot h_1 \cdot f^{-1}$	$\frac{1}{2 \cdot \pi^2} \cdot h_1 \cdot [\ln(\pi \cdot \tau \cdot f_h) + 1.27]$	$\frac{3}{2 \cdot \pi^2} \cdot h_1 \cdot [\ln(\pi \cdot \tau \cdot f_h) + 1.04]$	$\frac{6}{\pi^2} \cdot h_1 \cdot [\ln(\pi \cdot \tau \cdot f_h) + 0.96]$
$f_0^2 \cdot h_2 \cdot f^0$	$\frac{1}{2 \cdot \pi^2} \cdot h_2 \cdot f_h$	$\frac{3}{2 \cdot \pi^2} \cdot h_2 \cdot f_h$	$\frac{5}{\pi^2} \cdot h_2 \cdot f_h$

A single-sided (positive) PSD is assumed, which requires integration from 0 to infinity. It can clearly be seen that, although it has an impact on the convergence, the dependence on τ does not change as a function of the order of the structure function [92, 164]

as it was previously analyzed to the outcome of structure functions of a deterministic function. Assume the following deterministic phase time error:

$$x(t) = \sum_{k=0}^M \frac{A_k}{k!} \cdot t^k \tag{C.37}$$

where M denotes the maximum polynomial drift order. Since the N -th increment of this function corresponds to the N -th derivative (or at least its numerical approximation), the outcome of the increment is equal to zero. This means that the drift on the time-domain measurement of the phase error can completely be suppressed by choosing a high order of the structure function. If $N = M$ the outcome will be independent of t and be equal to:

$$\Delta^M x(t) = \tau^N \cdot A_N = \sqrt{D_x^M(\tau)} \tag{C.38}$$

which only depends on the observation time τ . The τ -dependency of this deterministic increment looks very similar to the previously obtained values of the stochastic process in Table C.2. Can a distinction be made between these types of processes? Table C.3 compares the τ -dependency of the structure functions for different orders of polynomial drift. This shows that the dependency to τ of the structure functions is always different for deterministic and stochastic processes. This allows to determine unambiguously the highest polynomial phase drift order. By subtracting the resulting orders, it is possible to determine the lower-order terms and even the complete polynomial. In this way, noise measurements can completely be detrended and analyzed in detail.

In [164] an algorithm to determine the highest-order of frequency drift and for the presence of (higher order) Flicker-noise:

- [Initialize] Set order $i = 1$

Table C.3 τ -dependency of structure functions of deterministic and stochastic processes

Deterministic		Stochastic	
M	$\frac{\sqrt{D_x^M(\tau)}}{\tau}$	α_m	$\frac{\sqrt{D_x^M(\tau)}}{\tau}$
2	$A_2 \cdot \tau$	-2	$\sim \tau^{1/2}$
3	$A_3 \cdot \tau^2$	-4	$\sim \tau^{3/2}$
4	$A_4 \cdot \tau^3$	-6	$\sim \tau^{5/2}$

In the left column, the maximum order of the polynomial is given. The second column shows the outcome of the structure function of order $N = M$. The third column shows the lowest α , α_m for which the stochastic process $S_y(f) = h_\alpha \cdot f^\alpha$ has a bounded structure function of order M . The last column shows the outcome of the normalized structure function for the particular α_m power-law in $S_y(f)$ [92, 164]

- [Test for stationarity] Calculate $\hat{D}_\phi^{(i)}(\tau)$ and check whether it is stationary (time-independent), using several samples. If not, increase i by 1 and try again. If yes, set order of frequency drift to $i - 1$ and go further.
- [Test for boundedness] If $\hat{D}_\phi^{(i)}(\tau)$ is not bounded; increase i by 1 and try again. If yes, set $M = i$ and go further.
- [End] Calculate $S_\phi(\omega)$ from $\hat{D}_\phi^{(i)}(l\tau)$, $l = 1, \dots, l_{max}$.

The last step, however, appears to be a difficult task; in [164] two approaches are demonstrated. One can also use $\hat{D}_\phi^{(i)}(l\tau)$, $l = 1, \dots, l_{max}$ to do a curve-fitting using the terms in Table C.2.

Appendix D

Comparison to the State of the Art

This appendix contains four tables containing a comparison of oscillator implementations from literature. The reported specifications of all references are spread over 4 tables, Tables D.1, D.2, D.3 and D.4. The specifications of the implementations which have been discussed in Part II of this work are found at the bottom of every table. Values and references printed in *italic*, are based on simulations only.

Some of the phase noise FoM values are calculated using (3.63) when only the jitter is reported. Note that this results in an overestimate of the noise spectrum, since this also includes jitter coming from the output circuitry (which is not accumulated in the oscillator).

Table D.1 Comparison to the state of the art

References	Tech. (μm)	$f_{\text{tank}}(f_{\text{out}})$ (MHz)	Topology	Voltage (V)	Power (mW)
[178], JSSC	0.35	1,536 (96)	LC harm.	5 (3.3)	31.35
	0.35	1,536 (12)	LC harm.	5 (3.3)	31.35
[177], ISCAS	0.25	900 (25)	LC harm.	3.3	59.4
	0.25	900 (25)	LC harm.	3.3	59.4
[175], FCS	0.13	3,000 (6–133)	LC harm.	3.3	6.6
[281], ISSCC	0.35	9,800	LC harm.	2.2	11.88
	0.35	9,800	LC harm.	2.2	11.88
[252], MWCL	0.18	4,610	LC harm.	1.5	3
	0.18	5,000	LC harm.	1.5	3
[117], MWCL	0.18	5,600	LC harm.	1.2	2.4
[165], EL	0.18	5,470	LC harm.	1.2	5.04
	0.18	5,470	LC harm.	1.2	5.04
[166], EL	0.18	5,500	LC harm.	1.2	3
[77], WAS	0.25	1,800	LC harm.	1.1	0.17
	0.25	1,800	LC harm.	1.1	0.17

(continued)

Table D.1 (continued)

References	Tech. (μm)	$f_{\text{tank}}(f_{\text{out}})$ (MHz)	Topology	Voltage (V)	Power (mW)
[150], MWCL	0.18	2,300	LC harm.	1.5	0.97
	0.18	2,300	LC harm.	1.5	0.97
[148], MWCL	0.18	2,630	LC harm.	0.45	0.43
	0.18	2,645	LC harm.	0.45	0.43
[119], ICECS	0.25	2,400	LC harm.	1.5	0.08
	0.25	2,400	LC harm.	1.5	0.08
[155], JSSC	0.13	5,121	LC Ring	0.5	4.01
	0.13	5,341	LC harm.	0.5	1.01
[234], FCS	0.18	1,700 (1–133)	LC harm.	3.3	23.1 (25 MHz)
[3], FCS	0.18	–(1–133)	LC harm.	3.3	23.43 (25 MHz)
[238], ISSSE	Bip.	0.01–10	LR	–	–
[250, 251], JSSC	0.18	14	RC rel.	1.8	45 μ
	0.18	14	RC rel.	1.8	45 μ
[29], ISSCC	0.13	3.2	IV-C rel.	1.5	38.4 μ
[94], JSSC	0.8	1.5	IV-C ring	5	1.8
[90], ISSCC	65 n	12	IV-C rel.	1.2	90 μ
[192], SBCCI	0.5	12.8	IV-C rel.	3	0.4
[275], SBCCI	0.5	11.6/21.4	IV-C rel.	3	0.4
[195], ISCAS	0.13	2	IV-C rel.	1.8	3 μ
[162], ESSCIRC	0.18	31.25 k	IV-C rel.	1.8	360 n
[254], ICECS	0.18	6.66 k	IV-C rel.	1.5	940 n
	0.18	6.66 k	IV-C rel.	1.5	940 n
[197], ISSCC	65 n	9–30 k	IV-C rel.	1	120 n
	65 n	9–30 k	IV-C rel.	1	120 n
[244], JSSC	0.25	7	Ring	2.4	1.5
[149], VLSI	0.18	10	Ring	1.2	80 μ
[257], ESSCIRC	0.35	2–100	Ring	1.8	0.18 (30 MHz)
	0.35	2–100	Ring	1.8	0.18 (30 MHz)
[225], JSSC	65 n	0.1	MV-C rel.	1.2	41 μ
[227], JSSC	65 n	0.15 (20 Hz)	MV-C rel.	1.2	51 μ
	65 n	0.15 (20 Hz)	MV-C rel.	1.2	51 μ
[71], TCAS-I	0.35	3.3 k	MV-C rel.	1	11 n
[126], JSSC	0.7	1.6	ETF-FLL	5	7.8
T-Wien bridge	65 n	6	RC harm.	1.2	66 μ
	65 n	6	RC harm.	1.2	66 μ
V-Wien bridge	0.13	24	RC harm.	0.9	33 μ
Pulsed oscillator	0.13	48–1,540	Pulsed LC	1.1	46 μ
Inj.-locked 1	0.13	950–1,150	RC harm.	1.0	127 μ
Inj.-locked 2	40 n	23–36	RC harm.	1.0	72 μ

Table D.2 Comparison to the state of the art: noise

References	Jit. (ps)	PN (Δf) (dB) (MHz)	FoM_{PN} (dB)	Tuning (Range) (%/V) (V)	$FoM_{PN,tuned}$ (dB)
[178]	6.78	–	128.6 ^a	–	–
	8.96	–	135.2 ^a	–	–
[177]	3.927	–114(0.1)	136.4 ^a	–	–
	3.927	–143(1)	153.2	–	–
[175]	2	–73.97 (12 k)	152.0 ^a	–	–
[281]	–	–93 (0.1)	182.1	1.37 (2)	183.5
	–	–118 (1)	187.1	1.37 (2)	188.5
[252]	–	–120.99 (1)	189.5	3.38 (2.4)	194.8
	–	–120.42 (1)	189.6	3.38 (2.4)	194.9
[117]	–	–119.13 (1)	190.3	8.93 (1.2)	199.8
[165]	–	–102 (0.1)	190.2	–	–
	–	–122.4 (1)	189.7	–	–
[166]	–	–121.3 (1)	191.1	–	–
[77]	–	–126.2 (1)	199.0	–	–
	–	–144.4 (8)	199.1	–	–
[150]	–	–111 (1)	179.2	8.99 (1.5)	188.7
	–	–133 (1, locked)	200.4	–	–
[148]	–	–105.9 (0.4)	185.9	7.30 (1.05)	194.6
	–	–106.4 (0.4)	184.8	7.30 (1.05)	193.4
[119]	–	–73.62 (0.6)	156.6	5.19 (1.5)	163.8
	–	–82.44 (1)	161.0	5.19 (1.5)	168.2
[155]	–	–121.6 (0.6)	194.2	–	–
	–	–116.1 (0.6)	195.1	–	–
[234]	2.8 (100 MHz)	–82 (0.01)	148.4	–	–
[3]	2 (125 MHz)	–82 (0.01)	150.2	–	–
[238]	–	–	–	–	–
[250, 251]	–	– (4, 100 k)	146.0	–	–
	–	– (10 k)	148.0	–	–
[29]	455	–	135.9 ^a	–	–
[94]	65	–102 (10 k)	150.7	–	–
[90]	–	–82.125 (10 k)	162.0	–	–
[192]	<0.1 %	–	–	–	–
[275]	<0.1 %	–	–	–	–
[195]	–	–	–	–	–
[162]	–	–	–	–	–
[254]	–	–50 (10 Hz)	136.7	–	–
	–	–89 (1 kHz)	137.4	–	–
[197]	–	–	–	–	–
	–	–	–	–	–
[244]	–	–	–	–	–

(continued)

Table D.2 (continued)

References	Jit. (ps)	PN (Δf) (dB) (MHz)	FoM_{PN} (dB)	Tuning (Range) (%/V) (V)	$FoM_{PN,tuned}$ (dB)
[149]	–	–	–	–	–
[257]	–	–32 (1 k)	129.0	–	–
	–	–96 (1)	133.0	–	–
[225]	52 n	–	109.6 ^a	–	–
[227]	–	–	–	–	–
	–	–	–	–	–
[71]	–	–	–	–	–
[126]	320	–	119.2 ^a	–	–
T-Wien bridge	127 ^a	–73.7 (10 k)	141.1	–	–
	127 ^a	–94.6 (100 k)	142.0	–	–
V-Wien bridge	–	–	–	–	–
Pulsed oscillator	49.6	–	142.7 ^a	–	–
Inj.-locked 1	–	–	–	–	–
Inj.-locked 2	–	–	–	–	–

^a Calculation based on the reported jitter/phase noise value, using (3.63)

Table D.3 Comparison to the state of the art: voltage dependency

References	V-range (V)	Rel. V-range (%)	Sensitivity (ppm/V)	Sensitivity (ppm/%)	Remarks
[178]	4.5–5.5	20.0	38	1.85	Bandgap reg.
	4.5–5.5	20.0	38	1.85	Bandgap reg.
[177]	2.97–3.63	20.0	91	3.00	Open loop, bandgap
	2.97–3.63	20.0	91	3.00	Open loop, bandgap
[175]	3.0–3.6	18.2	3.3 ^a	0.11 ^a	Bandgap reg.
[281]	–	–	–	–	–
	–	–	–	–	–
[252]	–	–	–	–	–
	–	–	–	–	–
[117]	–	–	–	–	–
[165]	–	–	–	–	–
	–	–	–	–	–
[166]	–	–	–	–	–
[77]	–	–	–	–	–
	–	–	–	–	–
[150]	–	–	–	–	–
	–	–	–	–	–

(continued)

Table D.3 (continued)

References	V-range (V)	Rel. V-range (%)	Sensitivity (ppm/V)	Sensitivity (ppm/%)	Remarks
[148]	–	–	–	–	–
	–	–	–	–	–
[119]	–	–	–	–	–
	–	–	–	–	–
[155]	–	–	–	–	–
	–	–	–	–	–
[234]	3–3.6	18.2	100	3.3	T-Null
[3]	3–3.6	18.2	167	5.5	T-Null
[238]	–	–	–	–	–
[250, 251]	1.7–1.9	11.1	16,000	288	V-avg. feedback
	1.7–1.9	11.1	16,000	288	V-avg. feedback
[29]	1.4–1.6	13.3	40,000	600	Offset canceling
[94]	–	–	–	–	–
[90]	–	–	–	–	–
[192]	2.5–5.5	75.0	5,500	222	Bandgap reg.
[275]	3.0–5.5	58.8	6,400	272	Bandgap reg.
[195]	1.8–2.5	32.6	2.8e5	6140	Self-biasing
[162]	–	–	50,000	900	Self-biasing
[254]	0.8–1.8	76.9	9,800	127	PN-resistor
	0.8–1.8	76.9	9,800	127	PN-resistor
[197]	1.5–3.3	75.0	10,000	240	Bandgap ref.
	1.5–3.3	75.0	10,000	240	Bandgap ref.
[244]	2.4–2.75	13.6	17,700	456	Bandgap reg.
[149]	1.2–3.0	85.7	556	11.7	Bandgap reg.
[257]	1.8–3.0	50.0	40,000	960	Ext. voltage,
	1.8–3.0	50.0	40,000	960	PN-resistor
[225]	1.12–1.4	22.2	7,140	90	Ext. PTAT
[227]	–	–	–	–	–
	–	–	–	–	–
[71]	1.0–2.5	85.7	35,000	613	MOS PTAT
[126]	–	–	–	–	–
T-Wien bridge	1.08–1.32	20	25,000	300	PN-resistor
	1.08–1.32	20	25,000	300	PN-resistor
V-Wien bridge	0.4–1.4	111	104	0.94	Dual. reg.
Pulsed oscillator	0.6–1.6	91	74	0.81	(External) Self-biasing
Inj.-locked 1	0.7–1.6	78	0	0	Inj. locked
Inj.-locked 2	0.7–1.5	73	0	0	AM-Inj. locked

^a Estimated value from total reported frequency deviation

Table D.4 Comparison to the state of the art: temperature dependency

References	T-Range (°C)	Sensitivity (ppm/°C)	Trimming/ Calibration	Area (core) (mm ²)	Abs. accuracy (ppm)
[178]	-10-85	12.3 (8.1)	Digital	(0.22)	±100
	-10-85	12.3 (8.1)	Digital	(0.22)	±100
[177]	-5-75	5.25 (3.8)	Digital	-	±152 ^a
	-5-75	5.25 (3.8)	Digital	-	±152 ^a
[175]	0-70	2.3	Digital	0.81	±277 ^a
[281]	-	-	-	0.40 (0.14)	-
	-	-	-	0.40 (0.14)	-
[252]	-	-	-	0.41	-
	-	-	-	0.41	-
[117]	-	-	-	0.60	-
[165]	-	-	-	-	-
	-	-	-	-	-
[166]	-	-	-	0.30	-
[77]	-	-	-	-	-
	-	-	-	-	-
[150]	-	-	-	0.79	-
	-	-	-	0.79	-
[148]	-	-	-	2.00	-
	-	-	-	2.00	-
[119]	-	-	-	0.59	-
	-	-	-	0.59	-
[155]	-	-	-	0.73	-
	-	-	-	0.73	-
[234]	-40-85	3.5 (1.6)	Dig.ext.wobble	-	±40
[3]	0-70	1 (0.29)	Dig.ext.wobble	-	±50
[238]	-	-	-	-	-
[250, 251]	-40-125	150 (91)	No	(0.04)	±4,000
	-40-125	150 (91)	No	(0.04)	±4,000
[29]	20-60	125	Digital, 1-pt.	(0.073)	±1,500
[94]	-	-	-	(1.17)	-
[90]	-	-	-	(0.03)	-
[192]	-40-125	625 (424)	Digital, 1-pt.	(0.18)	-
[275]	-40-125	303	Digital, 1-pt.	(0.19)	-
[195]	-35-85	417	1-Point	2.9 (0.015)	-

(continued)

Table D.4 (continued)

References	T-Range (°C)	Sensitivity (ppm/°C)	Trimming/ Calibration	Area (core) (mm ²)	Abs. accuracy (ppm)
[162]	−45–80	4,000	1-Point	(0.016)	±155
[254]	−40–120	56.9	No	(0.09)	±8,000
	−40–120	56.9	No	(0.09)	±8,000
[197]	0–90	22.2	No	(0.032)	–
	−40–90	38.5	No	(0.032)	–
[244]	−40–125	315 (60–102)	No	1.6	±22,000
[149]	−20–120	66.7	No	(0.22)	–
[257]	−20–100	90	No	(0.08)	±27,000
	−20–100	90	No	(0.08)	±27,000
[225]	−40–85	320	1-Point	(0.11)	±37,00
[227]	−55–125	55.6	2-Point	(0.2)	±60,000 ^b
	−55–125	300	1-Point	(0.2)	±60,000 ^b
[71]	−20–80	500	No	(0.1)	±2e5
[126]	−55–125	11.2	1-Point	6.75	±1,000
T-Wien bridge	0–120	86.1 (75)	No	(0.03)	±8,800
	0–120	33	No	(0.03)	±8,800
V-Wien bridge	–	1e4	No	(0.03)	±4,600
Pulsed oscillator	−40–100	48–1,540	No	2.63	±7,600
Inj.-locked 1	−20–100	950–1,150	No	0.18 (0.0022)	–
Inj.-locked 2	−20–100	23–36	No	0.165 (0.0017)	–

^a Including all external variations^b Uncompensated, before trimming

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