

References

1. Abraham, U., Magidor, M.: Cardinal arithmetic. In: Foreman, M., Kanamori, A. (eds.) *Handbook of Set Theory*, vol. 2, pp. 1149–1228. Springer, Berlin (2010)
2. Andretta, A., Neeman, I., Steel, J.: The domestic levels of k^c are iterable. *Isr. J. Math.* **125**, 157–201 (2001)
3. Bartoszynski, T., Judah, H.: *Set Theory. On the Structure of the Real Line*. A K Peters, Wellesley, MA (1995)
4. Becker, H., Kechris, A.: The descriptive set theory of Polish group actions. *London Mathematical Society Lecture Notes Series*, vol. 232 (1996)
5. Blass, A.: Combinatorial cardinal characteristics of the continuum. In: Kanamori, A., Foreman, M. (eds.) *Handbook of Set Theory*, vol. 1, pp. 395–489. Springer, Berlin (2010)
6. Blau, U.: *Die Logik der Unbestimmtheiten und Paradoxien*. Synchron-Verlag (2008)
7. Bilinsky, E., Gitik, M.: A model with a measurable which does not carry a normal measure. *Archive Math. Logic* **51**, 863–876 (2012)
8. Caicedo, A., Ketchersid, R.: A trichotomy theorem in natural models of AD^+ . In: Babinkostova, L., Caicedo, A., Geschke, S., and Scheepers, M. (eds.) *Set Theory and Its Applications, Contemporary Mathematics*, vol. 533, pp. 227–258, American Mathematical Society, Providence, RI (2011)
9. Cummings, J.: Iterated forcing and elementary embeddings. In: Foreman, M., Kanamori, A. (eds.) *Handbook of Set Theory*, vol. 2, pp. 775–883. Springer, Berlin (2010)
10. Devlin, K., Ronald, B.: Jensen. Marginalia to a theorem of Silver. *Lecture Notes in Mathematics* # 499, pp. 115–142. Springer, Berlin (1975)
11. Enderton, H.B.: *A Mathematical Introduction to Logic*, 2nd edn. Harcourt Academic Press, Burlington (2011)
12. Gao, S.: *Invariant Descriptive Set Theory*. CRC Press, Boca Raton (2009)
13. Halbeisen, L.: *Combinatorial Set Theory*. Springer, Berlin (2012)
14. Hjorth, G., Kechris, A.: New dichotomies for Borel equivalence relations. *Bull. Symbolic Logic* **3**, 329–346 (1997)
15. Jech, T.: *Set Theory*, 3rd edn. Springer, Berlin (2002)
16. Jensen, Ronald B.: The fine structure of the constructible hierarchy. *Ann. Math. Logic* **4**, 229–308 (1972)
17. Jensen, R.B., Schimmerling, E., Schindler, R., Steel, J.: Stacking mice. *J. Symb. Logic* **74**, 315–335 (2009)
18. Kanamori, A.: *The Higher Infinite*, 2nd edn. Springer, Berlin (2009)
19. Kanovei, V., Sabok, M., Zapletal, J.: *Canonical Ramsey theory on Polish spaces*. *Cambridge Tracts in Mathematics*. Cambridge University Press, Cambridge (2013)

20. Kechris, A.: Classical Descriptive Set Theory Graduate Texts in Mathematics # 156. Springer, Berlin (1994)
21. Kechris, A., Miller, B.: Topics in orbit equivalence. Lecture Notes in Mathematics 1852. Springer, Berlin (2004).
22. Koellner, P., Woodin, W.H.: Large cardinals from determinacy. In: Foreman, M., Kanamori, A. (eds.) Handbook of Set Theory, vol. 3, pp. 1951–2119. Springer, Berlin (2010)
23. Kunen, K.: Set Theory. An Introduction to Independence Proofs. Elsevier, Amsterdam (1980)
24. Larson, P.: The Stationary Tower. In: Woodin, W.H. (ed.) Notes on a Course. AMS, New York (2004)
25. Larson, P.: AD^+ . Monograph in preparation
26. Martin, D.A., Steel, J.: A proof of projective determinacy. J. Am. Math. Soc. **2**, 71–125 (1989)
27. Mochovakis, Y.: Descriptive Set Theory, 2nd edn. AMS, New York (2009)
28. Mitchell, W.J., Schimmerling, E.: Weak covering without countable closure. Math. Res. Lett. **2**, 595–609 (1995)
29. Mitchell, W.J., Schimmerling, E., Steel, J.: The covering lemma up to a woodin cardinal. Ann. Pure Appl. Logic **84**, 219–255 (1997)
30. Mitchell, W.J., Steel, J.R.: Fine structure and iteration trees. Lecture Notes in Logic, vol. 3 (1994).
31. Neeman, Itay: Inner models in the region of a woodin limit of woodin cardinals. Ann. Pure Appl. Logic **116**, 67–155 (2002)
32. Neeman, I.: The Determinacy of Long Games. de Gruyter, Berlin (2004)
33. Neeman, I.: Determinacy in $L(\mathbb{R})$. In: Foreman, M., Kanamori, A. (eds.) Handbook of Set Theory, vol. 3, pp. 1877–1950. Springer, Berlin (2010)
34. Sargsyan, G.: Descriptive inner model theory. Bull. Symbolic Logic **19**, 1–55 (2013)
35. Schimmerling, E., Zeman, M.: A characterization of \square_κ in core models. J. Math. Logic **4**, 1–72 (2004)
36. Schindler, R., Steel, J.: The core model induction. Monograph in preparation. http://wwwmath.uni-muenster.de/logik/Personen/rds/core_model_induction.pdf
37. Shelah, S.: Proper and Improper Forcing, 2nd edn. Springer, Berlin (1998)
38. Shiryaev, A.: Probability Theory, 2nd edn. Springer, New York (1996)
39. Steel, J.: An outline of inner model theory. In: Kanamori, A., Foreman, M. (eds.) Handbook of Set Theory, vol. 3, pp. 1595–1684. Springer, Berlin (2010)
40. Steel, J.: The core model iterability problem. Lecture Notes in Logic, vol. 8 (1996).
41. Steel, J.: A stationary tower free proof of the derived model theorem. In: Gao, S., Zhang, J. (eds.) Advances in Logic. The North Texas Logic Conference 8–10 Oct 2004, pp. 1–8, AMS, New York (2007).
42. Steel, J.: Derived models associated to mice. In: Chong, C., Feng, Q., Slaman, T.A., Woodin, W.H., Yong, Y. (eds.) Computational Aspects of Infinity, Part I: Tutorials pp. 105–194, Singapore (2008).
43. WindBus, K.: Projektive Determiniertheit. Diplomarbeit, Universität Bonn (1993)
44. Woodin, W.H.: The Axiom of Determinacy, Forcing Axioms, and the Nonstationary Ideal, 2nd edn. de Gruyter, Berlin (2010)
45. Woodin, W.H.: Suitable extender models i. J. Math. Logic **10**, 101–339 (2010)
46. Woodin, W.H.: Suitable extender models ii: beyond ω -huge. J. Math. Logic **11**, 115–436 (2011)
47. Zeman, M.: Inner models and large cardinals. de Gruyter Series in Logic and Its Applications, vol. 5 (2002).

Index

A

- Absolute, 68
- Absolute between V and M , 68
- Absoluteness of well-foundedness, 69
- Accumulation point, 4
- Ackermann's set theory, 20
- AD, 280
- $a \Delta b$, 87
- $\text{AD}_{\mathbb{R}}$, 301
- \aleph_{α} , 34
- Almost disjoint, 7
- Amenable, 73
- $A_M^{n,p}$, 250
- Antichain, 95
 - maximal, 95
- A_M^p , 237
- Approximation property, 122
- AST, 20
- At most countable, 3
- Axiom
 - of choice, 12
 - of extensionality, 9
 - of foundation, 9, 30
 - of infinity, 10
 - of pairing, 10
 - of power set, 10
 - of replacement, 11
 - of separation, 11
 - of union, 10
- Axiom of Determinacy, 280
- Axiom of Real Determinacy, 301

B

- Baire Category Theorem, 6
- Baire space, 127
- Banach-Mazur game, 287

Base, 288

- Bernstein property, 155
- \sqsupset_{α} , 63
- BG, 18
- BGC, 19
- Bijjective, 14

C

- Cantor normal form, 30
- Cantor space, 127
- Cantor's Discontinuum, 8
- Cantor–Bendixson
 - Theorem of, 5
- Cantor–Bendixson rank, 144
- Cantor–Schröder–Bernstein
 - Theorem of, 2
- Cardinal, 33
 - α -Erdős, 230
 - α -strong, 58
 - λ -supercompact, 59
 - (strongly) Mahlo, 46
 - (strongly) inaccessible, 46
 - ineffable, 50
 - limit, 33
 - measurable, 53
 - regular, 36
 - Reinhardt, 52
 - remarkable, 232
 - singular, 36
 - strong, 58
 - strong limit, 46
 - strong up to δ with respect to A , 227
 - subcompact, 61
 - subtle, 90
 - successor, 33
 - supercompact, 59

threadable, 277
 weakly compact, 50
 weakly inaccessible, 46
 weakly Mahlo, 46
 with the tree property, 50
 Woodin, 227
 Cardinal successor, 33
 Cardinality, 33
 Cartesian product, 13
 CH, 36
 Closed, 40
 δ -, 40
 in $[\theta]^{<\kappa}$, 45
 Closure, 128
 Club
 δ -in α , 40
 in $[\theta]^{<\kappa}$, 45
 in α , 40
 Club filter, 41
 Cofinal, 36, 239
 Cofinality, 36
 Cohen forcing, 94
 at κ , 109
 Collection principle, 30
 Compatible, 93
 Comprehension
 Σ_0 -, 73
 axiom for φ , 19
 schema, 19
 Condensation
 full, 81
 local, 81
 Condensation lemma, 80
 Condensation point, 5
 Cone
 S -, 288
 Turing, 288
 Constructible universe, 74
 Continuum Hypothesis, 3, 36
 Continuum Problem, 3
 Countable, 3
 Countable chain condition, 105
 Covering game, 286
 Critical point, 50
 $\text{crit}(\pi)$, 50

D
 Decides
 $p - \varphi(\tau_1, \dots, \tau_k)$, 98
 Δ -Lemma, 106
 Δ_1^T , 68
 Δ -system, 106

Dense, 93
 below p , 93
 Dependent choice, 31
 Determinacy
 OD_S -, 291
 \diamond , 82
 \diamond_κ , 82
 $\diamond_\kappa(R)$, 82
 \diamond_κ^+ , 90
 $\diamond_\kappa^+(R)$, 90
 \diamond_κ^* , 83
 $\diamond_\kappa^*(R)$, 83
 \diamond_κ^* , 83
 $\diamond_{\lambda, \kappa}^*$, 275
 Distance, 127
 Domain, 13
 Downward absolute, 68

E

Elementary embedding, 50
 non-trivial, 50
 Embedding normal form, 303
 κ -closed, 303
 additivity, 307
 Extender, 210
 λ -closed, 217
 λ -complete, 219
 continuum-complete, 220
 countably complete, 220
 derived from π , E_π , 211
 long, 210
 short, 210
 strength of an, 216
 Extension of embeddings lemma
 downward, 240
 general downward, 253
 general upward, 253
 upward, 243
 Extensional, 28

F

Filter, 40, 93
 $< \mu$ -closed, 40
 Fréchet, 41
 normal, 41
 nowhere dense, 180
 Ramsey, 180
 rapid, 165
 selective, 180
 uniform, 53
 weakly normal, 59

Finite, 3
 Forces, 97
 Forcing
 Mathias, 181
 Namba, 276
 notion of, 93
 Prikry, 188
 supercompact tree Prikry, 233

Forcing conditions, 93

Formula

Π_n^- , 67

Σ_0^- , 67

Σ_n^- , 67

F^+ , 42

Fullness, 101, 123

Function, 13

rudimentary (in E), 70

simple, 72, 296

uniformizing, 139

G

Gödel pairing function, 35

GCH, 36

Generalized Continuum Hypothesis, 36

Generic

\mathbb{P} -over M , 94

\mathcal{D} -, 93

Generic extension, 96

Good embedding, 243

H

Hausdorff formula, 38

HC, 38

Height of a tree T , 46

Hereditarily in, 86

Hereditarily ordinal definable from ω , 86

Hereditarily smaller than κ , 38

HF, 38

H_κ , 38

h_M , 193

$h_M^{n+1,p}$, 252

H_M

HOD, 86

HOD $_z$, 86

Homomorphism, 113

dense, 113

I

III-founded, 26

Inaccessible to the reals

ω_1 is -, 141

Incompatible, 93

Induction principle, 23, 26

Inductive, 10, 23

Ineffable, 50

Injective, 14

Inner model, 51

Interpretation

G -of τ , 96

Invariant

S -, 288

Turing-, 288

Isomorphic, 15

Isomorphism, 15

Iterability

of a ppm \mathcal{M} , 197

Iterable by U and its images, 184

Iteration

linear – of V of length γ given by U , 183

of a ppm \mathcal{M} of length α , 197

putative – of a ppm \mathcal{M} of length α , 196

putative linear – of V of length γ given by U , 183

J

J_α , 74

$J_\alpha[E]$, 73

J -structure, 79

J -structure

acceptable, 235

K

κ -chain condition, 105

κ -Knaster, 105

KP, 90

Kripke–Platek set theory, 90

Kurepa's Hypothesis, 90

L

L , 74

$\lambda^{<\kappa}$, 37

$L[E]$, 73

Lebesgue measurable, 148

Length (of a well-ordering), 29

$< \kappa$ -closed, 109

$< \kappa$ -distributive, 109

$<_{\mathcal{M}}$, 65

Level of s in T , 46

$\text{lv}_T(s)$, 46

Levy collapse, 111

Limit point, 40

Łoś Theorem, 54, 195, 213

M

Measure, 53
 Mitchell order, 65, 233
 $M^{n,p}$, 250
 Monotone enumeration, 29
 Mostowski collapse, 28
 Mouse
 x -, 198
 M^p , 237
 Mutual generics, 119

N

Names
 \mathbb{P} -, 96
 n -completion, 254
 n -embedding, 259
 Nice name, 108
 Norm on $A \subset {}^\omega\omega$, 145
 Null set, 7, 147
 Number
 natural, 23
 ordinal, 24

O

OD, 86
 OD_z , 86
 ω -complete, 197
 ω_1^z , 91
 Open, 95
 $x \oplus y$, 133
 Order
 atomless, 95
 linear, 14
 partial, 14, 93
 separative, 115
 Order type, 29
 Order-preserving, 15
 Ordered pair, 13
 Ordinal, 24
 z -admissible, 91
 limit, 25
 successor, 25
 Ordinal definable from $-$, 86
 Otp, 29
 Outer measure, 147

P

P-point, 180
 Parameters
 good, 237, 251
 very good, 238, 251

PD, 307

Perfect subset property, 142
 Pigeonhole Principle, 33
 π_U^M , 57
 P_M , 237
 P_M^n , 251
 Polish space, 127
 Positive sets, 42
 Power set, 2
 $\mathcal{P}(X)$, 2, 10
 Premouse
 x -, 194
 Prikry sequence, 189
 Product, 118
 Projection Lemma, 118
 Projection of T , 128
 Projective Determinacy, 307
 Projectum
 Σ_1 -, 236
 n th, 250

Q

Q-point, 180
 Q -formula, 238

R

Range, 13
 Rank, 28, 30
 Rank initial segment of V , 28
 $\text{rk}_R(x)$, 28
 $\|x\|_R$, 28
 Real
 Cohen over M , 152
 dominating, 181
 generic over M , 153
 random over M , 152
 unbounded, 181
 Recursion theorem, 26
 Reduct, 237
 n^{rth} , 250
 Reduction property, 141
 Reflection Principle, 89
 Regressive, 42
 Relation, 13
 set-like, 26
 Reshaped subset of ω_1 , 163
 Restriction, 14
 $\rho_n(M)$, 250
 $\rho_1(M)$, 236
 Rigid, 202
 R_M , 238
 R_M^n , 251
 $r\Sigma_{n+1}$ elementary, 254

rud_E closed, 72
 Rudimentary relation, 71
 Russell's Antinomy, 12

S

$S_\alpha[E]$, 75
 Scale on $A \subset {}^\omega\omega$, 145
 SCH, 39
 Sequence, 29
 Set
 admissible, 91
 determined, 280
 Set of reals
 F_σ , 128
 G_δ , 128
 $\Delta_n^1(x)$, 137
 $\Pi_n^1(x)$, 137
 $\Sigma_1^1(x)$, 137
 $\Sigma_{n+1}^1(x)$, 137
 α -Souslin, 131
 Π^1 , 132
 $\tilde{\Pi}^1$, 132
 $\tilde{\Sigma}_{n+1}^1$, 132
 Σ^1 , 132
 $\tilde{\Sigma}^1$, 132
 $\tilde{\Sigma}_{n+1}^1$, 132
 κ -universally Baire, 149
 $\tilde{\Delta}^1$, 135
 $\tilde{\Delta}_n^1$
 analytic, 132
 basic open, 127
 Borel, 128
 closed, 3, 127
 coanalytic, 132
 complete coanalytic, 134
 δ -homogeneously Souslin, 323
 δ -weakly homogeneously Souslin, 323
 dense, 4
 determined, 280
 homogeneously Souslin, 323
 meager, 6, 148
 nowhere dense, 6, 148
 of first category, 6
 of second category, 6
 open, 3, 127
 perfect, 4
 projective, 132
 small, 174
 Solovay over M , 153
 universal $\tilde{\Sigma}^1$, 133
 universally Baire, 149
 weakly homogeneously Souslin, 324
 Shift map, 186, 198

Shoenfield tree, 131
 σ -algebra, 128
 Silver indiscernibles for $L[x]$, 209
 Singular Cardinal Hypothesis, 39
 Skolem function
 Σ_1^- , 193
 Solid, 258
 1-, 258
 n -, 258
 Solidity witness, 256
 Solovay game, 289
 Solovay sequence, 301
 length of, 302
 Sound, 253
 n -, 253
 \square_κ , 269
 $\square_\kappa(\mathbb{R})$, 269
 Standard code, 237
 n^{rmlth} , 250
 Standard parameter, 255
 n th, 254
 Standard reduct
 n th, 254
 Standard witness, 257
 Stationary, 41
 Stationary set
 reflecting, 278
 Steel forcing, 295
 Stem
 of a Prikry condition, 188
 Strategy, 279
 Subset, 9
 proper, 9
 Support, 105, 110
 Surjective, 14
 Symmetric difference, 87

T

TC($\{x\}$), 27
 $\|R\|$, 28
 Transitive, 23
 Transitive closure, 27
 Transitive collapse, 28
 Tree, 46
 κ -, 47
 κ -Aronszajn, 47
 κ -Kurepa, 47
 κ -Souslin, 47
 on $\omega \times \alpha$, 128
 on X , 127
 perfect, 127
 Turing reducible, 288
 $2^{<\kappa}$, 37

Uult($V; U$), 57

Ultrafilter, 40

Ultrapower

 Σ_0^- , 194, 216 $r\Sigma_{n+1}^-$, 262

Ultrapower embedding, 57

Ultrapower map

 Σ_0^- , 195, 216 $r\Sigma_{n+1}^-$, 262Ultrapower of M by U , 57

Unbounded

in $[\theta]^{<\kappa}$, 45in γ , 40

Uncountable, 3

Upward absolute, 68

V V_α , 28

Ville's Lemma, 91

WWeakly $r\Sigma_{n+1}$ elementary, 254

Well-founded, 26

Well-founded part, 26

Well-ordering, 15

Wfp(B), 26

WindBus tree, 304

Winning strategy, 280

X $x^\#$, 202**Z**

Z, 12

ZC, 12

 $0^\#$, 202

ZF, 12

ZFC, 12

ZFC⁻, 12ZFC^{-∞}, 12

Zorn's Lemma, 16