

References

1. Alfa, A.S., Neuts, M.F.: Modelling vehicular traffic using the discrete time Markovian arrival process. *Transport. Sci.* **29**(2), 109–117 (1995)
2. Artalejo, J.R., Gomez-Corral, A., He, Q.M.: Markovian arrivals in stochastic modelling: a survey and some new results. *SORT* **34**(2), 101–156 (2010)
3. Asmussen, S.: Phase-type distributions and related point processes: fitting and recent advances. In: Chakravarthy, S.R., Alfa, A.S. (eds.) *Matrix-Analytic Methods in Stochastic Models*. Lecture Notes in Pure and Applied Mathematics, pp. 137–149. Dekker, New York (1997)
4. Asmussen, S.: *Applied Probability and Queues*. Springer, New York (2003)
5. Asmussen, S., Bladt, M.: Point processes with finite-dimensional conditional probabilities. *Stoch. Process. Their Appl.* **82**, 127–142 (1999)
6. Asmussen, S., Nerman, O., Olsson, M.: Fitting phase-type distributions via the EM-algorithm. *Scand. J. Stat.* **23**(4), 419–441 (1996)
7. Atkinson, K.A.: *An Introduction to Numerical Analysis*, 2nd edn. Wiley, New York (1989)
8. Badescu, A.L., Drekić, S., Landriault, D.: Analysis of a threshold divided strategy for a MAP risk model. *Scand. Actuar. J.* **4**, 227–247 (2007)
9. Badescu, A.L., Cheung, E.K., Landriault, D.: Dependent risk models with bivariate phase-type distributions. *J. Appl. Probab.* **46**(1), 113–131 (2009)
10. Balsamo, S., de Nitto Persone, V., Onvural, R.: *Analysis of Queueing Networks with Blocking*. International Series on Operations Research and Management Science. Kluwer Academic Publishers, Boston (2001)
11. Bause, F., Buchholz, P., Kriege, J.: A comparison of Markovian arrival processes and ARMA/ARTA processes for the modelling of correlated input processes. In: *Proceedings of the Winter Simulation Conference* (2009)
12. Bause, F., Buchholz, P., Kriege, J.: ProFiDo: the processes fitting toolkit Dortmund. In: *Proceedings of the 7th International Conference on Quantitative Evaluation of Systems (QEST 2010)*, pp. 87–96. IEEE Computer Society, Williamsburg (2010)
13. Bause, F., Gerloff, P., Kriege, J.: ProFiDo: a toolkit for fitting input models. In: Müller-Clostermann, B., Echtle, K., Rathgeb, E.P. (eds.) *Proceedings of the 15th International GI/ITG Conference on Measurement, Modelling, and Evaluation of Computing Systems and Dependability and Fault Tolerance*. Lecture Notes in Computer Science, vol. 5987, pp. 311–314. Springer, Berlin (2010)
14. Biller, B., Gunes, C.: Introduction to simulation input modeling. In: Johansson, B., Jain, S., Montoya-Torres, J., Hugin, J., Yücesan, E. (eds.) *Proceedings of the Winter Simulation Conference (WSC)*, pp. 49–58 (2010)

15. Bilmes, J.: A gentle tutorial on the EM algorithm and its application to parameter estimation for Gaussian mixture and hidden Markov models. Technical Report TR-97-021, University of Berkeley (1997)
16. Bini, D.A., Latouche, G., Meini, B.: Numerical Methods for Structured Markov Chains. Oxford Science Publications, Oxford (2005)
17. Bladt, M.: A review on phase-type distributions and their use in risk theory. *Astin Bull.* **35**(1), 145–161 (2005)
18. Bobbio, A., Cumani, A.: ML estimation of the parameters of a PH distribution in triangular canonical form. In: Balbo, G., Serazzi, G. (eds.) *Computer Performance Evaluation*, pp. 33–46. Elsevier, Amsterdam (1992)
19. Bobbio, A., Telek, M.: Parameter estimation of phase type distributions. Technical Report R.T.423, Istituto Elettrotecnico Nazionale Galileo Ferraris (1997)
20. Bobbio, A., Horváth, A., Scarpa, M., Telek, M.: Acyclic discrete phase type distributions: properties and a parameter estimation algorithm. *Perform. Eval.* **54**(1), 1–32 (2003)
21. Bobbio, A., Horváth, A., Telek, M.: Matching three moments with minimal acyclic phase type distributions. *Stoch. Model.* **21**(2–3), 303–326 (2005)
22. Bodrog, L., Heindl, A., Horváth, G., Telek, M., Horváth, A.: A Markovian canonical form of second-order matrix-exponential processes. *Eur. J. Oper. Res.* **160**(1), 51–68 (2008)
23. Bodrog, L., Heindl, A., Horváth, G., Telek, M., Horváth, A.: Current results and open questions on PH and MAP characterization. In: Bini, D., Meini, B., Ramaswami, V., Remiche, M., Taylor, P. (eds.) *Numerical Methods for Structured Markov Chains*, No. 07461 in Dagstuhl Seminar Proceedings (2008)
24. Bodrog, L., Buchholz, P., Kriege, J., Telek, M.: Canonical form based MAP(2) fitting. In: *Proceedings of the 7th International Conference on the Quantitative Evaluation of Systems (QEST)*, pp. 107–116. IEEE Computer Society, Williamsburg (2010)
25. Breuer, L.: An EM algorithm for batch Markovian arrival processes and its comparison to a simpler estimation procedure. *Ann. OR* **112**(1–4), 123–138 (2002)
26. Breuer, L., Kume, A.: An EM algorithm for Markovian arrival processes observed at discrete times. In: Müller-Clostermann, B., Echtle, K., Rathgeb, E. (eds.) *Measurement, Modelling, and Evaluation of Computing Systems and Dependability and Fault Tolerance. Lecture Notes in Computer Science*, vol. 5987, pp. 242–258. Springer, Berlin (2010)
27. Brickner, C., Indrawan, D., Williams, D., Chakravarthy, S.R.: Simulation of a stochastic model for a service system. In: Johansson, B., Jain, S., Montoya-Torres, J., Hugan, J., Yücesan, E. (eds.) *Proceedings of the Winter Simulation Conference (WSC)*, pp. 1636–1647 (2010)
28. Buchholz, P.: A class of hierarchical queueing networks and their analysis. *Queueing Syst.* **15**(1), 59–80 (1994)
29. Buchholz, P.: Exact and ordinary lumpability in finite Markov chains. *J. Appl. Probab.* **31**, 59–75 (1994)
30. Buchholz, P.: Structured analysis approaches for large Markov chains. *Appl. Numer. Math.* **31**(4), 375–404 (1999)
31. Buchholz, P.: An EM-algorithm for MAP fitting from real traffic data. In: Kemper, P., Sanders, W.H. (eds.) *Computer Performance Evaluation/TOOLS. Lecture Notes in Computer Science*, vol. 2794, pp. 218–236. Springer, New York (2003)
32. Buchholz, P., Kriege, J.: A heuristic approach for fitting MAPs to moments and joint moments. In: *Proceedings of the 6th International Conference on the Quantitative Evaluation of Systems (QEST)*, pp. 53–62. IEEE Computer Society, Budapest (2009)
33. Buchholz, P., Kriege, J.: Equivalence transformations for acyclic phase type distributions. Technical Report 827, Department of Computer Science, TU Dortmund. http://www.cs.uni-dortmund.de/nps/de/Forschung/Publikationen/Graue_Reihe1/Ver_ffentlichungen_2009/827.pdf (2009)
34. Buchholz, P., Kriege, J.: Markov modeling of availability and unavailability data. In: *Proceedings of the 10th European Dependable Computing Conference (EDCC)*, IEEE Computer Society, Newcastle upon Tyne (2014)

35. Buchholz, P., Panchenko, A.: An EM algorithm for fitting of real traffic traces to PH-distribution. In: Proceedings of the International Conference on Parallel Computing in Electrical Engineering, PARELEC, pp. 283–288. IEEE Computer Society, Dresden (2004)
36. Buchholz, P., Telek, M.: Stochastic Petri nets with matrix exponentially distributed firing times. *Perform. Eval.* **67**(12), 1373–1385 (2010)
37. Buchholz, P., Telek, M.: Rational arrival processes associated to labelled Markov processes. *J. Appl. Probab.* **49**(1), 40–59 (2012)
38. Buchholz, P., Telek, M.: On minimal representations of rational arrival processes. *Ann. Oper. Res.* **202**(1), 35–58 (2013)
39. Buchholz, P., Kemper, P., Kriege, J.: Multi-class Markovian arrival processes and their parameter fitting. *Perform. Eval.* **67**(11), 1092–1106 (2010)
40. Buchholz, P., Felko, I., Kriege, J.: Transformation of acyclic phase type distributions for correlation fitting. In: Proceedings of the Analytical and Stochastic Modeling Techniques and Applications (ASMTA). Lecture Notes in Computer Science, pp. 96–111. Springer, Berlin (2013)
41. Casale, G., Zhang, E.Z., Smirni, E.: KPC-toolbox: simple yet effective trace fitting using Markovian arrival processes. In: Proceedings of the 5th International Conference on the Quantitative Evaluation of Systems (QEST), pp. 83–92. IEEE Computer Society, St. Malo (2008)
42. Casale, G., Zhang, E.Z., Smirni, E.: Trace data characterization and fitting for Markov modeling. *Perform. Eval.* **67**(2), 61–79 (2010)
43. Ching, W.K.: Iterative Methods for Queuing and Manufacturing Systems. Monographs in Mathematics. Springer, London (2001)
44. Collection of availability traces. <http://www.cs.illinois.edu/~pbg/availability/>
45. Cox, D.R.: A use of complex probabilities in the theory of stochastic processes. *Math. Proc. Camb. Phil. Soc.* **51**, 313–319 (1955)
46. Crowder, M.J., Kimber, A.C., Smith, R.L., Sweeting, T.J.: Statistical Analysis of Reliability Data. CRC Press, Boca Raton (1994)
47. Cumani, A.: On the canonical representation of homogeneous Markov processes modeling failure-time distributions. *Micrelectron. Reliab.* **22**(3), 583–602 (1982)
48. Dayar, T.: On moments of discrete phase-type distributions. In: Bravetti, M., Kloul, L., Zavattaro, G. (eds.) Proceedings of the EPEW/WS-FM. Lecture Notes in Computer Science, vol. 3670, pp. 51–63. Springer, New York (2005)
49. Dayar, T.: Analyzing Markov Chains Using Kronecker Products. Briefs in Mathematics. Springer, New York (2012)
50. Dayar, T., Meriç, A.: Kronecker representation and decompositional analysis of closed queueing networks with phase-type service distributions and arbitrary buffer sizes. *Ann. OR* **164**(1), 193–210 (2008)
51. Dempster, A., Laird, N., Rubin, D.: Maximum likelihood from incomplete data via the EM algorithm. *J. R. Stat. Soc. Ser. B* **39**(1), 1–38 (1977)
52. El-Rayes, A., Kwiatkowska, M., Norman, G.: Solving infinite stochastic process algebra models through matrix-geometric methods. In: Hillston, J., Silva, M. (eds.) Proceedings of the 7th Process Algebras and Performance Modelling Workshop, pp. 41–62 (1999)
53. Erlang, A.K.: Solution of some problems in the theory of probabilities of significance in automatic telephone exchanges. *Elektroteknikeren* **13**, 5–13 (1917)
54. Fackrell, M.: Characterization of matrix-exponential distributions. Ph.D. thesis, School of Applied Mathematics, The University of Adelaide (2003)
55. Fackrell, M.: Modelling healthcare systems with phase-type distributions. *Health Care Manag. Sci.* **12**, 11–26 (2009)
56. Failure trace archive. <http://fta.scem.uws.edu.au/>
57. Fang, Y.: Hyper-Erlang distribution model and its application in wireless mobile networks. *Wirel. Netw.* **7**(3), 211–219 (2001)
58. Feldmann, A., Whitt, W.: Fitting mixtures of exponentials to long-tail distributions to analyze network performance models. *Perform. Eval.* **31**, 245–279 (1998)

59. Fischer, W., Meier-Hellstern, K.S.: The Markov-modulated Poisson process (MMPP) cookbook. *Perform. Eval.* **18**(2), 149–171 (1993)
60. Fox, B.L., Glynn, P.W.: Computing Poisson probabilities. *Commun. ACM.* **31**(4), 440–445 (1988)
61. Garg, L., Masala, G., McClean, S.I., Micocci, M., Cannas, G.: Using phase type distributions for modelling HIV disease progression. In: *Proceedings of the 25th International Symposium on Computer-Based Medical Systems (CBMS)*, pp. 1–4. IEEE, Computer Society (2012)
62. Gerhardt, I., Nelson, B.L.: On capturing dependence in point processes: matching moments and other techniques. Technical Report, Northwestern University (2009)
63. Goseva-Popstojanova, K., Trivedi, K.S.: Effects of failure correlation on software in operation. In: *Proceedings of the 2000 Pacific Rim International Symposium on Dependable Computing (PRDC)*, pp. 69–76. IEEE Computer Society, Los Angeles (2000)
64. Haddad, S., Moreaux, P., Chiola, G.: Efficient handling of phase-type distributions in generalized stochastic Petri nets. In: Azéma, P., Balbo, G. (eds.) *Proceedings of the 18th International Conference on ICATPN. Lecture Notes in Computer Science*, vol. 1248, pp. 175–194. Springer, Berlin (1997)
65. He, Q.M., Neuts, M.: Markov arrival processes with marked transitions. *Stoch. Process. Their Appl.* **74**, 37–52 (1998)
66. He, Q.M., Zhang, H.: A note on unicyclic representations of phase type distributions. *Stoch. Model.* **21**, 465–483 (2005)
67. He, Q.M., Zhang, H.: On matrix exponential distributions. *Adv. Appl. Probab.* **39**(1), 271–292 (2007)
68. Heckmüller, S., Wolfinger, B.E.: Using load transformations for the specification of arrival processes in simulation and analysis. *Simulation* **85**(8), 485–496 (2009)
69. Heindl, A.: Inverse characterization of hyperexponential MAP(2)s. In: *Proceedings of the Analytical and Stochastic Modelling Techniques and Applications (ASMTA)*, pp. 183–189 (2004)
70. Heindl, A., Telek, M.: Output models of MAP/PH/1/(K) queues for an efficient network decomposition. *Perform. Eval.* **49**(1/4), 321–339 (2002)
71. Hornig, R., Varga, A.: An Overview of the OMNeT++ Simulation Environment. In: *Proceedings of 1st International Conference on Simulation Tools and Techniques for Communications, Networks and Systems (SIMUTools)* (2008)
72. Heindl, A., Mitchell, K., van de Liefvoort, A.: Correlation bounds for second-order MAPs with application to queueing network decomposition. *Perform. Eval.* **63**(6), 553–577 (2006)
73. Heindl, A., Horváth, G., Gross, K.: Explicit inverse characterizations of acyclic MAPs of second order. In: Horváth, A., Telek, M. (eds.) *Proceedings of the 3rd European Performance Engineering Workshop: EPEW. Lecture Notes in Computer Science*, vol. 4054, pp. 108–122. Springer, Berlin (2006)
74. Horváth, A., Telek, M.: Approximating heavy tailed behavior with phase type distributions. In: *Proceedings of the 3rd International Conference on Matrix-Analytic Methods in Stochastic Models*. Leuven, Belgium (2000)
75. Horváth, A., Telek, M.: PhFit: a general purpose phase type fitting tool. In: *Proceedings of the Performance Tools 2002. Lecture Notes in Computer Science*, vol. 2324, pp. 82–91. Springer, Berlin (2002)
76. Horváth, A., Telek, M.: Markovian modeling of real data traffic: Heuristic phase type and MAP fitting of heavy tailed and fractal like samples. In: Calzarossa, M.C., Tucci, S. (eds.) *Proceedings of the Performance 2002. Lecture Notes in Computer Science*, vol. 2459, pp. 405–434. Springer, Berlin (2002)
77. Horváth, A., Telek, M.: Matching more than three moments with acyclic phase type distributions. *Stoch. Model.* **23**, 167–194 (2007)
78. Horváth, G., Telek, M.: On the canonical representation of phase type distributions. *Perform. Eval.* **66**, 396–409 (2009)
79. Horváth, A., Horváth, G., Telek, M.: A traffic based decomposition of two-class queueing network with priority service. *Comput. Netw.* **53**(8), 1235–1248 (2009)

80. Horváth, A., Rácz, S., Telek, M.: Moments characterization of order 3 matrix exponential distributions. In: Al-Begain, K., Fiems, D., Horváth, G. (eds.) *Proceedings of the Analytical and Stochastic Modeling Techniques and Applications (ASMTA)*. Lecture Notes in Computer Science, vol. 5513, pp. 174–188. Springer, Berlin (2009)
81. Horváth, G., Reinecke, P., Telek, M., Wolter, K.: Efficient generation of PH-distributed random variates. In: Al-Begain, K., Fiems, D., Vincent, J.M. (eds.) *Proceedings of the Analytical and Stochastic Modeling Techniques and Applications (ASMTA)*. Lecture Notes in Computer Science, vol. 7314, pp. 271–285. Springer, Berlin (2012)
82. Horváth, G., Telek, M., Buchholz, P.: A MAP fitting approach with independent approximation of the inter-arrival time distribution and the lag-correlation. In: *Proceedings of the 2nd International Conference on the Quantitative Evaluation of Systems (QEST)*, pp. 124–133. IEEE CS Press, Torino (2005)
83. Ide, I.: Superposition of interrupted Poisson processes and its application to packetized voice multiplexers. In: *Proceedings of the International Teletraffic Congress (ITC12)* (1988)
84. The internet traffic archive. <http://ita.ee.lbl.gov/>
85. Iosup, A., Jan, M., Sonmez, O., Epema, D.H.: On the dynamic resource availability in grids. In: *Proceedings of the 8th IEEE/ACM International Conference on Grid Computing* (2007)
86. Javadi, B., Kondo, D., Iosup, A., Epema, D.H.J.: The failure trace archive: enabling the comparison of failure measurements and models of distributed systems. *J. Parallel Distrib. Comput.* **73**(8), 1208–1223 (2013)
87. Johnson, M.: Selecting parameters of phase distributions: combining nonlinear programming, heuristics, and Erlang distributions. *INFORMS J. Comput.* **5**(1), 69–83 (1993)
88. Johnson, M., Taaffe, M.: Matching moments to phase distributions: mixtures of Erlang distributions of common order. *Stoch. Model.* **4**(5), 711–743 (1989)
89. Johnson, M., Taaffe, M.: Matching moments to phase distributions: nonlinear programming approaches. *Stoch. Model.* **2**(6), 259–281 (1990)
90. Jordan, M.I., Jacobs, R.A.: Hierarchical mixtures of experts and the EM algorithm. *Neural Comput.* **6**(2), 181–214 (1994)
91. Kawanishi, K.: On the counting process for a class of Markovian arrival processes with an application to a queueing system. *Queueing Syst.* **49**, 93–122 (2005)
92. Kelton, W.D., Sadowski, R.P., Sadowski, D.A.: *Simulation with Arena*, 4th edn. McGraw-Hill, New York (2007)
93. Kemeny, J.G., Snell, J.L.: *Finite Markov Chains*, repr edn. University Series in Undergraduate Mathematics. VanNostrand, New York (1969)
94. Khayari, R.E.A., Sadre, R., Haverkort, B.: Fitting world-wide web request traces with the EM-algorithm. *Perform. Eval.* **52**, 175–191 (2003)
95. Kleinrock, L.: *Queueing Systems*, vol. 1. Wiley, New York (1975)
96. Kleinrock, L.: *Queueing Systems*, vol. 2. Wiley, New York (1976)
97. Klemm, A., Lindemann, C., Lohmann, M.: Modeling IP traffic using the batch Markovian arrival process. *Perform. Eval.* **54**(2), 149–173 (2003)
98. Kriege, J., Buchholz, P.: Correlated phase-type distributed random numbers as input models for simulations. *Perform. Eval.* **68**(11), 1247–1260 (2011)
99. Kriege, J., Buchholz, P.: Simulating stochastic processes with OMNeT++. In: Liu, J., Quaglia, F., Eidenbenz, S., Gilmore, S. (eds.) *Proceedings of the 4th International ICST Conference on Simulation Tools and Techniques (SimuTools'11)*, pp. 367–374. ICST/ACM, Brussels (2011)
100. Krijnen, W.P.: Convergence of the sequence of parameters generated by alternating least squares algorithms. *Comput. Stat. Data Anal.* **51**, 481–489 (2006)
101. Kuczura, A.: The interrupted Poisson process as an overflow process. *The Bell Syst. Tech. J.* **52**(3), 437–448 (1973)
102. Latouche, G.: A phase-type semi-Markov point process. *SIAM J. Algebr. Discrete Meth.* **3**, 77–90 (1982)
103. Latouche, G., Ramaswami, V.: *Introduction to Matrix Analytic Methods in Stochastic Modeling*. ASA-SIAM Series on Statistics and Applied Probability. Society for Industrial and Applied Mathematics, Philadelphia (1987)

104. Latouche, G., Ramaswami, V.: *Introduction to Matrix Analytic Methods in Stochastic Modeling*. Society for Industrial and Applied Mathematics, Philadelphia (1999)
105. Law, A.M., Kelton, W.D.: *Simulation Modeling and Analysis*, 3rd edn. McGraw-Hill, Boston (2000). ISBN 0-07-059292-6
106. Law, A.M., McComas, M.G.: ExpertFit distribution-fitting software: how the ExpertFit distribution-fitting software can make your simulation models more valid. In: Chick, S.E., Sanchez, P.J., Ferrin, D.M., Morrice, D.J. (eds.) *Proceedings of the Winter Simulation Conference*, pp. 169–174. ACM, Berlin (2003)
107. Lawson, C.L., Hanson, B.J.: *Solving Least Squares Problems*. Prentice-Hall, Englewood Cliffs (1974)
108. Lazowska, E.D., Zahorjan, J., Graham, G.S., Sevcik, K.C.: *Quantitative system performance-computer system analysis using queueing network models*. Prentice Hall, Upper Saddle River (1984)
109. Leland, W.E., Taqqu, M.S., Willinger, W., Wilson, D.V.: On the self-similar nature of ethernet traffic (extended version). *IEEE/ACM Trans. Netw.* **2**(1), 1–15 (1994)
110. van de Liefvoort, A.: The moment problem for continuous distributions. Technical Report WP-CM-1990-02, University of Missouri, Kansas City (1990)
111. Lipsky, L.: *Queueing Theory: A Linear Algebraic Approach*. Springer, New York (2008)
112. Loan, C.F.: The ubiquitous Kronecker product. *J. Comput. Appl. Math.* **123**(1–2), 85–100 (2000)
113. Lucantoni, D.M.: New results on the single server queue with a batch Markovian arrival process. *Stoch. Model.* **7**(1), 1–46 (1991)
114. Lucantoni, D.M.: The BMAP/G/1 queue: a tutorial. In: Donatiello, L., Nelson, R.D. (eds.) *Performance/SIGMETRICS Tutorials. Lecture Notes in Computer Science*, vol. 729, pp. 330–358. Springer, Berlin (1993)
115. Lucantoni, D.M., Meier-Hellstern, K.S., Neuts, M.F.: A single-server queue with server vacations and a class of non-renewal arrival processes. *Adv. Appl. Probab.* **22**(3), 676–705 (1990)
116. Maier, R.S., O’Cinneide, C.A.: A closure characterisation of phase-type distributions. *J. Appl. Probab.* **29**(1), 92–103 (1992)
117. McLachlan, G.J., Krishnan, T.: *The EM Algorithm and Extensions*. Wiley, Hoboken (1997)
118. Mészáros, A., Telek, M.: A two-phase MAP fitting method with APH interarrival time distribution. In: *Proceedings of the Winter Simulation Conference*. ACM, Berlin (2012)
119. Meyer, C.D.: *Matrix Analysis and Applied Linear Algebra*. Society for Industrial and Applied Mathematics, Philadelphia (2004)
120. Minin, V.N., Suchard, M.A.: Counting labeled transitions in continuous-time Markov models of evolution. *J. Math. Biol.* **56**, 391–412 (2008)
121. Mocanu, S., Commault, C.: Sparse representations of phase-type distributions. *Stoch. Model.* **15**, 759–778 (1999)
122. Montoro-Cazorla, D., Pérez-Ocón, R.: A maintenance model with failures and inspection following Markovian arrival processes and two repair modes. *Eur. J. Oper. Res.* **186**(2), 694–707 (2008)
123. Narayana, S., Neuts, M.: The first two moments matrices of the counts for the Markovian arrival process. *Stoch. Model.* **8**, 694–707 (1992)
124. Neuts, M.F.: A versatile Markovian point process. *J. Appl. Probab.* **16**, 764–779 (1979)
125. Neuts, M.F.: *Matrix-geometric solutions in stochastic models*. Johns Hopkins University Press, Baltimore (1981)
126. Neuts, M.F., Meier, K.S.: On the use of phase type distributions in reliability modelling of systems with two components. *OR Spectr.* **2**(4), 227–234 (1981)
127. Nielsen, B.F.: Lecture notes on phase-type distributions for 02407 stochastic processes. <http://www2.imm.dtu.dk/courses/02407/> (2012)
128. Nightingale, E.B., Douceur, J.R., Orgovan, V.: Cycles, cells and platters: an empirical analysis of hardware failures on a million consumer PCs. In: Kirsch, C.M., Heiser, G. (eds.) *Proceedings of the EuroSys*, pp. 343–356. ACM, Salzburg (2011)

129. O’Cinneide, C.A.: On non-uniqueness of representations of phase-type distributions. *Stoch. Model.* **5**, 247–259 (1989)
130. O’Cinneide, C.A.: Characterization of phase-type distributions. *Stoch. Model.* **6**, 1–57 (1990)
131. O’Cinneide, C.A.: Phase type distributions and invariant polytopes. *Adv. Appl. Prob.* **23**, 515–535 (1991)
132. O’Cinneide, C.A.: Phase-type distributions: open problems and a few properties. *Stoch. Model.* **15**(4), 731–757 (1999)
133. Okamura, H., Dohi, T., Trivedi, K.S.: Markovian arrival process parameter estimation with group data. *IEEE/ACM Trans. Netw.* **17**(4), 1326–1339 (2009)
134. Okamura, H., Dohi, T., Trivedi, K.S.: A refined EM algorithm for PH distributions. *Perform. Eval.* **68**(10), 938–954 (2011)
135. Okamura, H., Dohi, T., Trivedi, K.S.: Improvement of expectation-maximization algorithm for phase-type distributions with grouped and truncated data. *Appl. Stoch. Model. Bus. Ind.* **29**(2), 141–156 (2012)
136. Olsson, M.: The EMpht-programme. Technical Report, Chalmers University of Technology (1998)
137. Osogami, T., Harchol-Balter, M.: A closed-form solution for mapping general distributions to minimal PH distributions. In: Kemper, P., Sanders, W.H. (eds.) *Computer Performance Evaluation. Modelling Techniques and Tools. Lecture Notes in Computer Science*, vol. 2794, pp. 200–217. Springer, Berlin (2003)
138. Osogami, T., Harchol-Balter, M.: Necessary and sufficient conditions for representing general distributions by Coxians. In: Kemper, P., Sanders, W.H. (eds.) *Computer Performance Evaluation. Modelling Techniques and Tools. Lecture Notes in Computer Science*, vol. 2794, pp. 182–199. Springer, Berlin (2003)
139. Panchenko, A., Thümmler, A.: Efficient phase-type fitting with aggregated traffic traces. *Perform. Eval.* **64**(7–8), 629–645 (2007)
140. Paxson, V., Floyd, S.: Wide area traffic: the failure of Poisson modeling. *IEEE/ACM Trans. Netw.* **3**(3), 226–244 (1995)
141. Rahnamay-Naeini, M., Pezoa, J.E., Azar, G., Ghani, N., Hayat, M.M.: Modeling stochastic correlated failures and their effects on network reliability. In: *Proceedings of 20th International Conference on Computer Communications and Networks (ICCCN)*, pp. 1–6 (2011)
142. Realistic vehicular traces. <http://www.lst.inf.ethz.ch/research/ad-hoc/car-traces/>
143. Reinecke, P., Horváth, G.: Phase-type distributions for realistic modelling in discrete-event simulation. In: *Proceedings of the 5th International ICST Conference on Simulation Tools and Techniques. SIMUTOOLS ’12*, pp. 283–290. ICST, Brussels (2012)
144. Reinecke, P., Krauß, T., Wolter, K.: Cluster-based fitting of phase-type distributions to empirical data. *Comput. Math. Appl.* **64**(12), 3840–3851 (2012)
145. Riska, A., Smirni, E.: ETAQA solutions for infinite Markov processes with repetitive structure. *INFORMS J. Comput.* **19**(2), 215–228 (2007)
146. Riska, A., Diev, V., Smirni, E.: An EM-based technique for approximating long-tailed data sets with PH distributions. *Perform. Eval.* **55**, 147–164 (2004)
147. Ruiz-Castro, J.E., Fernández-Villodre, G., Pérez-Ocón, R.: Discrete repairable systems with external and internal failures under phase-type distributions. *IEEE Trans. Reliab.* **58**(1), 41–52 (2009)
148. Sauer, C.H., Chandy, K.M.: *Computer Systems Performance Modeling*. Prentice Hall, Englewood Cliffs (1981)
149. Schmickler, L.: MEDA: mixed Erlang distributions as phase-type representations of empirical distribution functions. *Stoch. Model.* **8**(1), 131–156 (1992)
150. Schwarz, G.: Estimating the dimension of a model. *Ann. Stat.* **6**(2), 461–464 (1978)
151. Stewart, W.J.: *Introduction to the Numerical Solution of Markov Chains*. Princeton University Press, Princeton (1994)
152. Stewart, W.J.: *Probability, Markov Chains, Queues, and Simulation*. Princeton University Press, Princeton (2009)

153. Takahashi, Y.: Asymptotic exponentiality of the tail of the waiting-time distribution in a PH/PH/c queue. *Adv. Appl. Probab.* **13**(3), 619–630 (1981)
154. Telek, M., Heindl, A.: Matching moments for acyclic discrete and continuous phase-type distributions of second order. *Int. J. Simulat. Syst. Sci. Tech.* **3**(3–4), 47–57 (2002). [Special Issue on: Analytical and Stochastic Modelling Techniques]
155. Telek, M., Horváth, G.: A minimal representation of Markov arrival processes and a moments matching method. *Perform. Eval.* **64**(9–12), 1153–1168 (2007)
156. Thümmler, A., Buchholz, P., Telek, M.: A novel approach for phase-type fitting with the EM algorithm. *IEEE Trans. Dep. Sec. Comput.* **3**(3), 245–258 (2006)
157. Trivedi, K.S.: *Probability and Statistics with Reliability, Queuing and Computer Science Applications*, 2nd edn. Wiley, Chichester (2002)
158. Van Houdt, B., Lenin, R.B., Blondia, C.: Delay distribution of (im)patient customers in a discrete time D-MAP/PH/1 queue with age-dependent service times. *Queueing Syst.* **45**(1), 59–73 (2003)
159. Vehicular mobility trace of the city of Cologne, Germany. <http://kolntrace.project.citi-lab.fr/>
160. Wu, C.F.J.: On the convergence properties of the EM algorithm. *Ann. Stat.* **11**(1), 95–103 (1983)

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