Part I
Tessellations

Exercises

Question 1. (20 points). Why are there only five Platonic solids? Did we miss some, or is there a reason there is not a sixth one?
Question 2. (20 points). No two edges of the Delaunay triangulation of a finite set of points in the plane cross each other. Give a convincing argument why this is the case.
Question 3. (20 points). All faces of an embedding of a maximal planar graph with three or more vertices in the plane are bounded by exactly tree edges. Give a convincing argument that this is the case.

HINT you might find Kuratowski's Theorem on planar graphs useful, which says, among other things, that the complete graph of 5 vertices is not planar.
Question 4. (20 points). Let S be a finite set of sites in $\mathbb{R}^2$, each with a real weight. Show that every region in the Apollonius diagram of S is connected.
Question 5. (20 points). Let $a = (a_1, a_2), b = (b_1, b_2), x = (x_1, x_2)$ be three points in $\mathbb{R}^2$. Prove that *a, b, x* form a left-turn iff $\det \begin{bmatrix} 1 & \mathrm{a}_1 & \mathrm{a}_2 \\ 1 & \mathrm{b}_1 & \mathrm{b}_2 \\ 1 & \mathrm{x}_1 & \mathrm{x}_2 \end{bmatrix} > 0$.
Question 6. ($20 = 10 + 10$ points). Consider the Delaunay triangulation of the BCC lattice in $\mathbb{R}^3$. Recall that it consists of congruent copies of only one tetrahedron, which has two long edges (of length 2) and four short edges (of length $\sqrt{3}$).

(a) Draw the four tetrahedra in the star of a long edge and highlight the link of the long edge.
(b) Draw the six tetrahedra in the star of a short edge and highlight the link of the short edge.